Partially Hyperbolic Geodesic Flows

Ygor de Jesus
ygoorartthur@gmail.com
Universidade Estadual de Campinas - Brazil

In this talk we are going to present a construction of Partially Hyperbolic Geodesic flows via conformal metric deformations and a criterion for robust transitivity inside the class of the geodesic flows. Partially hyperbolicity is a topic that has been widely studied during the last 30 years and appears as a generalization of Anosov Systems. It is a classical result of dynamical systems that the geodesic flow is a natural example of an Anosov system when the Riemannian metric has negative sectional curvatures. Many of the techniques that are known nowadays to study Anosov systems were developed during the study of geodesic flows in negative curved manifolds such as the classic Hopf's Argument used to prove ergodicity. The presence of a center bundle implies many difficulties in generalizing techniques used to study uniform hyperbolic cases. For instance, Hopf's Argument can not be applied in general once strong stable and unstable foliations are not transversal for non-Anosov partially hyperbolic systems. For geodesic flows such difficulties can be even harder to overcome once many techniques rely on local perturbation of the system, which is essentially not possible in this setting once a small perturbation on the Riemannian metric can implies big changes in the dynamics of the geodesic flow. Our construction shows the existence of partial hyperbolicity in a new setting and also the existence of systems that are not Anosov but still can present similar interesting behaviors with the presence of a non-trivial center bundle. We are able to give positive answers to questions attributed to Enrique Pujals and Keith Burns on the existence of ergodic partially

hyperbolic geodesic flows that are not Anosov and transitive geodesic flows on Riemannian manifolds with conjugate points.