UNIVERSIDADE ESTADUAL DE CAMPINAS

Faculdade de Ciências Aplicadas

GABRIEL DE SOUZA AMARO

ESTUDOS DE FLEXIBILIDADE EM PROBLEMAS DE PLANEJAMENTO DA PRODUÇÃO

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Dissertação apresentada a Faculdade de Ciências Aplicadas da Universidade Estadual de Campinas como parte dos requisitos exigidos para a obtenção do título de Mestre em Engenharia de Produção e Manufatura na área de Pesquisa Operacional e Gestão de Processos.

Orientador: Diego Jacinto Fiorotto

Coorientador: Washington Alves de Oliveira

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“Os que se encantam com a prática sem a ciência são como os timoneiros que entram no navio sem timão nem bússola, nunca tendo certeza do seu destino”.

(Leonardo da Vinci)

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[1] As opiniões, hipóteses e conclusões ou recomendações expressas neste material são de responsabilidade do autor e não necessariamente refletem a visão da FAPESP.

Resumo

Esse trabalho apresenta um estudo a respeito da flexibilidade de processo em sistemas produtivos. A flexibilidade de processo é a habilidade que um sistema produtivo tem para responder às variações nas circunstâncias, por exemplo, um aumento inesperado na demanda, ou uma redução na capacidade produtiva. Quanto mais flexível o sistema produtivo for, melhor ele poderá responder à essas mudanças. Com os resultados obtidos desse estudo, dois artigos (em revisão) foram submetidos para revistas internacionais e um terceiro está em elaboração. No primeiro trabalho intitulado "*Impact analysis of flexibility on the integrated lot sizing problem and supplier selection*" estudou-se a flexibilidade no problema integrado de dimensionamento de lotes e seleção de fornecedores. Como resultado, observamos que uma quantidade reduzida de fornecedores que entregam poucos produtos foi suficiente para se obter os mesmos benefícios da flexibilidade total. O segundo artigo intitulado "*Evaluating process flexibility in lot sizing problems: an approach based on multicriteria decision making*" utilizou os resultados já conhecidos na literatura para analisar as vantagens da flexibilidade de um ponto de vista multicritério. Os resultados indicaram que embora a flexibilidade total seja vantajosa em termos do custo de produção, ao considerar outros critérios operacionais, essa configuração apareceu nas últimas posições do ranking das alternativas, enquanto a configuração conhecida como cadeia longa em posições intermediárias. Também observou-se que, para a abordagem multicritério, é interessante investir em flexibilidade desde as capacidades mais apertadas. Um terceiro trabalho foi realizado com o objetivo de definir a flexibilidade utilizando uma abordagem que permitiu generalizar, definir e analisar configurações de flexibilidade para sistemas não balanceados (quantidade de itens diferente da quantidade de máquinas). Foi proposto algumas métricas para estudar os benefícios da flexibilidade, os principais resultados mostraram que algumas configurações com uma quantidade de ligações bastante reduzida não utilizaram todas as ligações permitidas. Além disso, mesmo com a melhor alocação dos itens às máquinas, aumentar a quantidade de flexibilidade permitiu melhorar o desempenho das configurações. Os artigos submetidos e em elaboração se encontram no anexo do texto.

Palavras-chaves: problema de dimensionamento de lotes; flexibilidade de processo; programação inteira

Abstract

This work presents a study on process flexibility in production systems. Process flexibility is the ability that a production system has to respond to variations on circumstances, for example, an increase in demand or a reduction in the production capacity. The more flexible a production system is, the better it can respond to the changes. With the research results, two papers (under review) were submitted to international papers and a third paper is in progress. In the first work entitled "Impact analysis of flexibility on the integrated lot sizing problem and supplier selection" we studied the flexibility on the integrated lot sizing and supplier selection problem. The results showed that a reduced amount of selected suppliers producing/delivering a small number of products is enough to obtain the total flexibility benefits. The second paper entitled "Evaluating process flexibility in lot sizing problems: an approach based on multicriteria decision making" used previous results from the literature to analyze the advantages of process flexibility through multicriteria perspectives. The findings showed that although total flexibility is more suitable concerning the total cost, it appeared in the last positions of the alternatives from a based-ranking multicriteria approach. Moreover, the well-known long-chain configuration ranked in the intermediate positions, and it was noticed that it is interesting to invest in process flexibility for all capacity levels. A third work was done to define and analyze flexibility configurations to non-balanced systems (when the number of items is different from the number of machines). We proposed some metrics to study the benefits of flexibility, the main findings showed some configurations with a very reduced amount of links did not use all allowed links. Moreover, even the best designation of items to machines, to increase the amount of flexibility allowed increasing the performance of the configuration. Note that the manuscripts appear attached.

Keywords: lot sizing problem; process flexibility; integer programing

Conteúdo

CAPÍTULO 1

Introdução

Com a globalização e cada vez mais as fronteiras comerciais entre os países se estreitando, é irremediável o aumento da competitividade no mercado. As organizações sofrem cada dia mais pressão por produtos que sejam de qualidade e que superem as expectativas dos clientes. Inevitavelmente, a busca por produtos que atendam esses requisitos atinge o custo dos produtos. Uma vez que o preço é definido pela dinâmica do mercado, não há outra alternativa além de reduzir custos para que seja possível manter a margem de lucro no patamar almejado e se manter competitivo frente os concorrentes.

Uma das principais atividades na indústria consiste na produção de itens. Para diminuir os custos de produção, as grandes indústrias utilizam ferramentas e métodos matemáticos em busca de um planejamento de produção otimizado. Dentre os custos de produção, há o custo de preparação de máquinas que ocorre cada vez que uma máquina é preparada para produzir certo item. Esse custo de preparação envolve o tempo que a máquina fica ociosa, eventuais limpezas que são necessárias, trocas de ferramenta ou molde, entre outros. Outro custo de produção relevante está relacionado a estocagem. O espaço físico disponível para o armazenamento dos produtos finais é limitado e a manutenção desse espaço representa um custo, seja dos trabalhadores da área, da iluminação e segurança do local e principalmente, o custo de oportunidade do capital imobilizado. O problema de dimensionamento de lotes consiste, justamente, em decidir em qual momento e em qual quantidade alocar uma decisão de produção de itens de modo a minimizar o custo, principalmente, o custo de preparação de máquinas e o custo de estocagem. Observe que ao alocar uma ordem de produção para atender toda a demanda ao longo do horizonte de planejamento, será necessário estocar todos esses itens, o que representa um alto custo de estocagem, mas um baixo custo de preparação de máquinas. Em contrapartida, ao alocar diversas ordens de produção para atender somente a demanda imediata, o custo de estocagem será baixo, mas o custo de preparação de máquinas será alto. Assim, é preciso encontrar o planejamento de produção que equilibre estes custos. Vale notar que esse trabalho considera o problema de dimensionamento de lotes com máquinas paralelas em que é possível atender a demanda com atraso em um contexto determinístico.

A *flexibilidade* pode ser definida como uma estratégia que as organizações adotam para responder as variações recorrentes nas circunstâncias. A flexibilidade assegura que as operações de manufatura são eficientes em custos mesmo com mudanças no mercado (GUPTA; GOYAL, 1989). Existem diversos tipos de flexibilidade. Por exemplo, a flexibilidade de preparação de máquinas permite que a preparação inicie no final de um período e termine no começo do próximo. Outra possibilidade é a flexibilidade de operações, nesse caso, a ordem das etapas que são necessárias para a fabricação de um produto pode ser alterada (SETHI; SETHI, 1990; GUPTA; SOMERS, 1992).

A *flexibilidade de máquinas*, ou *flexibilidade de processo* (que é estudada nesse trabalho) pode ser definida como a facilidade com a qual uma máquina pode ser adaptada para fabricar diferentes itens, de forma que quanto mais itens diferentes essa máquina puder fabricar, mais flexível ela é (SETHI; SETHI, 1990). Ao longo desse texto, é discutida a flexibilidade pela ótica do contexto industrial, principalmente sobre máquinas e itens, mas é possível analisar o benefício da flexibilidade de processo no caso de trabalhadores multifuncionais (e até mesmo fora do contexto industrial), por exemplo, uma enfermeira que é treinada para atuar no departamento de oncologia é menos flexível do que uma enfermeira treinada para a pediatria e cirurgia geral. Assim, é possível perceber que a flexibilidade de processo relaciona máquinas e itens, mas também, colaboradores e funções. Daqui em diante, a flexibilidade ocorre sobre as máquinas, salvo em contexto especificado. A relação entre as máquinas e os itens é marcada por uma *configuração de flexibilidade* e pode ser representada por um grafo bipartido em que os vértices de uma partição representam as máquinas e os vértices da outra partição, os itens (JORDAN; GRAVES, 1995). As arestas conectam os vértices entre as partições. Observa-se que em geral, na prática, a quantidade de itens é muito maior do que a quantidade de máquinas, quando isso acontece o sistema é denominado *não balanceado*. Um caso mais simples, porém, amplamente estudado na literatura, é quando a quantidade de itens é igual a quantidade de máquinas, esses sistemas são denominados *balanceados*.

A flexibilidade de máquinas está intimamente ligada com o problema de dimensionamento de lotes. Na formulação clássica do problema, cada máquina pode fabricar todos os produtos (FIOROTTO; ARAUJO; JANS, 2015), o que define a configuração de *flexibilidade total*. Essa situação é raramente encontrada na prática, já que seria extremamente custoso e muitas vezes inviável uma máquina que possa fabricar todos os produtos. Uma situação mais realista é a configuração de *flexibilidade limitada*, em que cada máquina pode produzir um conjunto reduzido de itens.

A Figura 1.1 apresenta algumas configurações de flexibilidade para sistemas balanceados. A Figura 1.1(a) representa a configuração dedicada de flexibilidade, em que cada máquina/planta pode fabricar um produto e cada produto pode ser fabricado em somente uma máquina. Em seguida, a Figura 1.1(b) apresenta uma configuração com $2n$ ligações, em que *n* é o número de máquinas/itens. Essa configuração de flexibilidade é conhecida como *cluster*. Observe que cada máquina pode produzir dois produtos e cada produto pode ser produzido em duas máquinas. A Figura 1.1(c) apresenta a regra da cadeia proposta por Jordan e Graves (1995), note que essa configuração também apresenta $2n$ ligações. Finalmente, a Figura 1.1(d) apresenta a configuração de flexibilidade total com n^2 ligações.

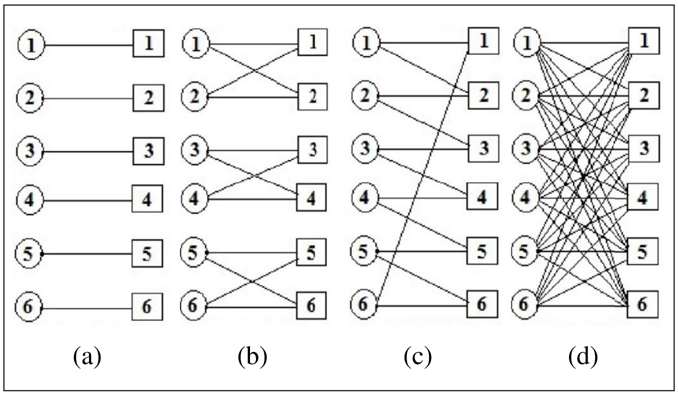
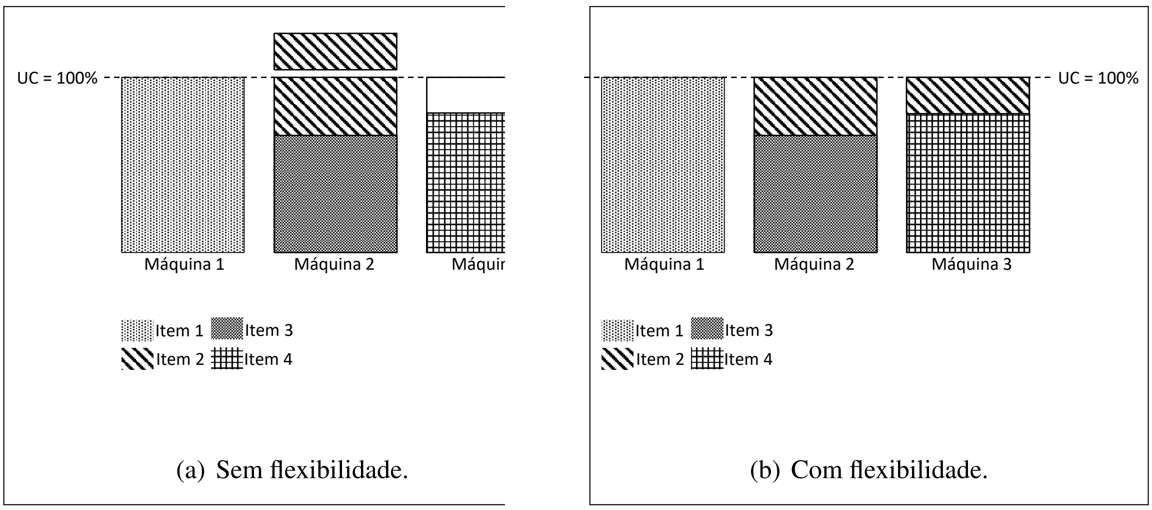


Figura 1.1: Configurações de flexibilidade. Extraído de Fiorotto, Jans e Araujo (2018).

Para entender a importância da flexibilidade em contexto determinístico é interessante analisar um caso ilustrativo. Ao observar Figura 1.2(a), é possível notar um certo sistema produtivo com 3 máquinas e 4 itens. A máquina 1 só é capaz de produzir o item 1, enquanto que a máquina 2 pode produzir os itens 2 e 3. Finalmente, a máquina 3 pode produzir somente o item 4. Somente a máquina 3 ainda tem capacidade disponível (cerca de 20%), e a demanda do item 2 não foi totalmente atendida. Nesse caso, a empresa não consegue atender toda a demanda mesmo ainda tendo capacidade disponível. Se for possível adicionar flexibilidade à máquina 3 para que ela possa fabricar o item 2, será possível atender toda a demanda, como pode ser obs



(a) Sem flexibilidade.

(b) Com flexibilidade.

Figura 1.2: Importância da flexibilidade em contexto determinístico.

O exemplo acima se enquadra em um contexto, de uma decisão tática, em que a demanda é determinística. Fica claro, portanto, que o benefício da flexibilidade é aparente nos contextos determinísticos, quando em um cenário com menos flexibilidade não é possível atender toda a demanda. Além disso, é possível obter benefícios da flexibilidade em um contexto em que a demanda é incerta. Assim, a flexibilidade atenua os impactos da variação da demanda através do uso eficiente da capacidade das máquinas.

A decisão de flexibilidade também pode ser considerada estratégica, por exemplo, no processo de projeto de fábrica é preciso decidir quais produtos serão fabricados na planta e também quais máquinas serão adquiridas, essas máquinas podem ser mais flexíveis e logo, mais caras do que máquinas menos flexíveis. Nesse momento é preciso avaliar a conjectura e a direção do mercado para justificar os investimentos e buscar a rentabilidade a longo prazo.

Esse trabalho estudou a flexibilidade de máquinas no problema de dimensionamento de lotes com máquinas paralelas, diversos itens, com possibilidade de atraso no atendimento da demanda, em contexto determinístico. Inicialmente, o estudo contemplou o problema integrado de dimensionamento de lotes e seleção de fornecedores, em que a planta decide quais produtos comprar de cada fornecedor, sendo que diversas configurações são possíveis, isso é, é possível encontrar um fornecedor que entregue um único produto, um fornecedor que entregue uma quantidade reduzida de produtos, e até um fornecedor que entregue todos os produtos. O desempenho de cada uma dessas configurações de flexibilidade foi analisado afim de se obter resultados à respeito do benefício de cada uma delas. Em se tratando do benefício das configurações de flexibilidade, diversos trabalhos na literatura consideram exclusivamente a função objetivo do problema. O passo seguinte desse trabalho foi analisar o desempenho das configurações de flexibilidade à luz de uma abordagem multicritério, que fornece um ordenamento (ranking) das configurações estudadas. Esse tipo de análise permite considerar aspectos não explorados anteriormente na literatura (como, por exemplo, a utilização de capacidade das máquinas) e que têm importância na tomada de decisão à respeito de qual e como a flexibilidade deve ser implementada. Da mesma maneira, a flexibilidade de máquinas no problema de dimensionamento de lotes não balanceado não foi amplamente estudada, mesmo sendo um caso encontrado recorrentemente na prática. Isso acontece porque as configurações de flexibilidade para o caso não balanceado são mais difíceis de serem caracterizadas. O passo seguinte foi propor uma abordagem que permitiu a construção das configurações nos sistemas não balanceados, e ainda, conseguiu representar as configurações dos sistemas balanceados como um caso particular do não balanceado.

Os resultados desse trabalho se dividem nas classes dos problemas estudados. Quanto à flexibilidade no problema integrado de dimensionamento de lotes e seleção de fornecedores, i) é possível reduzir a quantidade de fornecedores selecionados e obter economias decorrentes do processo de simplificação da gestão de fornecedores; ii) a sazonalidade da demanda tem impacto direto na quantidade de fornecedores selecionados; iii) alguns poucos fornecedores entregando alguns poucos produtos é semelhante, em termos de custo total, de que cada fornecedor possa entregar todos os produtos. Já a análise multicritério do desempenho das configurações de flexibilidade (não considerando o problema integrado de seleção de fornecedores) mostra que i) embora a flexibilidade total seja a melhor em termos da função objetivo do problema, ela aparece sempre nas últimas posições do ranking das configurações de flexibilidade; ii) já as configurações como a regra da cadeia apareceu apenas em posições intermediárias; iii) finalmente, através da análise multicritério é possível verificar que já é vantajoso investir em flexibilidade desde as capacidades mais apertadas ao contrário do que é observado na análise mono-critério (função objetivo). Ao considerar o problema de dimensionamento de lotes não balanceado, os principais resultados são i) a proposta de uma maneira de construção das configurações de flexibilidade; ii) a proposta de novos indicadores de análise de desempenho. Quanto às descobertas: iii) poucas configurações de flexibilidade utilizam todas as ligações permitidas; iv) a decisão da alocação dos itens às máquinas tem impacto substancial nos indicadores; v) a flexibilidade tem impacto positivo nos indicadores, em relação à um cenário sem flexibilidade, mesmo quando a decisão da configuração é realizada pelo modelo.

Essa dissertação está organizada da seguinte maneira. O próximo capítulo apresenta uma revisão da literatura dos principais estudos de flexibilidade nos problemas de planejamento da produção. O Capítulo 3 discute os principais resultados obtidos durante o mestrado. O Capítulo 4 apresenta as conclusões do trabalho bem como as propostas futuras. Finalmente, os artigos desenvolvidos durante o mestrado se encontram nos Anexos.

CAPÍTULO 2

Contextualização da literatura

Este capítulo contextualiza a literatura de flexibilidade em problemas de planejamento da produção. A Seção 2.1 trata de casos de planejamento da produção e também de trabalhadores multifuncionais. Devido a escassez de trabalhos de flexibilidade considerando o contexto determinístico, a contextualização da literatura apresenta diversos artigos em contexto estocástico. Já a Seção 2.2 trata da flexibilidade em problemas de planejamento da produção quando o sistema é não balanceado. Vale notar que esses trabalhos também consideram o contexto estocástico.

2.1 Flexibilidade em problemas de planejamento

O trabalho seminal de Jordan e Graves (1995) foi o primeiro a estudar os benefícios da flexibilidade. Inspirado em um caso real da indústria automobilística norte-americana, os autores analisaram como distribuir a flexibilidade e propuseram uma configuração que ficou conhecida como regra da cadeia. Essa configuração de flexibilidade é especial pois é possível traçar um caminho entre qualquer item e máquina o que gera alta conectividade e permite atenuar as variações da demanda através da redistribuição da produção dos itens entre às máquinas.

Decisões de flexibilidade não envolvem apenas uma planta e seus produtos, ou mesmo uma máquina e sua habilidade de fabricação. É possível pensar sobre flexibilidade de processo em outros contextos como na cadeia de suprimentos, ou no treinamento de trabalhadores. Quanto a cadeia de suprimentos, as decisões de flexibilidade de processo envolvem a distribuição dos itens a jusante (*upstream*) e a montante (*downstream*), e quanto ao treinamento de trabalhadores, se eles serão mais ou menos multifuncionais. Observe que trabalhadores que são mais multifuncionais representam um custo maior, por exemplo, ao pensar em uma linha de produção, para que um trabalhador possa operar uma máquina além daquela que ele foi treinado é preciso um novo treinamento (o que representa um custo), além disso, o nível de responsabilidades de um colaborador pode impactar diretamente no seu salário, dessa maneira, trabalhadores mais multifuncionais são mais caros, portanto, é interessante encontrar um nível de treinamento que seja satisfatório para as tarefas que serão desempenhadas e que permita ainda responder às variações nas circunstâncias da organização.

Como mencionado anteriormente, diversos autores exploraram a flexibilidade na cadeia de suprimentos. Garavelli (2003) consideraram uma cadeia de suprimentos com dois níveis e mostraram que é interessante a flexibilidade tanto do lado dos fornecedores para as plantas, quanto das plantas para os clientes. Cochran e Marquez (2005) apresentaram um modelo de cobertura que é capaz de obter o investimento ótimo em maquinário sendo que cada máquina tem diferentes flexibilidades. Hopp, Iravani e Xu (2010) estudaram os benefícios da flexibilidade em uma cadeia de suprimentos multiníveis, os autores mostraram que quando as entradas nos fornecedores são incertas, a flexibilidade é benéfica e quando a demanda dos clientes é incerta, os fabricantes se beneficiam da flexibilidade.

A respeito dos estudos de flexibilidade para o treinamento de trabalhadores multifuncionais, Tekin, Hopp e Van Oyen (2004) mostraram que a escolha da estratégia de treinamento dos trabalhadores depende de diversos fatores externos, mas que treinamento das habilidades seguindo uma configuração da regra da cadeia geraram o maior rendimento. Iravani, Van Oyen e Sims (2005) apresentaram uma nova maneira de representar as configurações de flexibilidade, os autores propuseram dois indicadores que são a média da matriz que representa a configuração de flexibilidade e os autovalores dessa matriz. Fügener, Pahr e Brunner (2018) propuseram uma nova configuração de flexibilidade que engloba aspectos da flexibilidade total e da regra da cadeia. Essa configuração gerou o maior benefício dentre as configurações estudadas.

Além dos estudos envolvendo situações práticas da flexibilidade, diversos autores desenvolveram modelos analíticos para analisar o benefício da flexibilidade. Chou *et al.* (2010) mostraram que a regra da cadeia funciona bem para diversos formatos de distribuição de demanda e obtiveram um conjunto de condições que quando satisfeitas, garantem que a performance da regra da cadeia é bastante próxima da performance da flexibilidade total. Simchi-Levi e Wei (2012) analisaram o comportamento do benefício marginal quando a regra da cadeia é construída. Os autores demonstraram que quando é possível traçar um caminho entre os itens/máquinas, o benefício marginal é máximo. Désir *et al.* (2016) mostraram que para alguns formatos de distribuição de demanda, a regra da cadeia não apresenta o melhor desempenho e ainda, é possível obter configurações menos conectadas que apresentam uma performance ligeiramente superior a regra da cadeia.

Fiorotto, Jans e Araujo (2018) analisaram os benefícios da flexibilidade (considerando um cenário determinístico) no problema de dimensionamento de lotes. Em diferentes análises, os autores compararam diversas configurações de flexibilidade. Os resultados indicaram que a regra da cadeia obtém um desempenho próximo da flexibilidade total, enquanto que a configuração em *cluster*, com a mesma quantidade de ligações não apresenta um bom desempenho. Foram desenvolvidos ainda dois novos modelos, ambos lineares e determinísticos. O primeiro com o intuito de obter a melhor alocação de itens às máquinas na regra da cadeia e o segundo a melhor configuração sujeito a um orçamento limitado. Novamente, os benefícios da flexibilidade total foram alcançados. Em especial, com apenas $1.5n$ ligações (em que n é o número de itens), já se obtém praticamente o mesmo desempenho, em termos de custo total, que a flexibilidade total.

2.2 Flexibilidade de máquinas em problemas não balanceados

Essa seção trata da flexibilidade nos problemas de planejamento da produção em que a quantidade de itens é diferente da quantidade de máquinas. Note que, em geral, na prática, a quantidade de itens é muito maior do que a quantidade de máquinas daí a importância de se estudar a flexibilidade nesses casos. A literatura ainda é escassa a respeito da flexibilidade em sistemas não balanceados, principalmente devido a dificuldade de se construir e caracterizar as configurações de flexibilidade.

Tanrisever, Morrice e Morton (2012) apresentaram um modelo de gestão de capacidade e propuseram duas configurações de flexibilidade para sistemas não balanceados. A primeira é uma adaptação da regra da cadeia de Jordan e Graves (1995) e a segunda, uma regra da cadeia parcial. Os resultados computacionais mostraram que o desempenho da regra da cadeia adaptada foi o mesmo da configuração de flexibilidade total, em termos da quantidade atrasada. Em contrapartida, ao considerar o custo de cada ligação existente, a regra da cadeia parcial teve um desempenho ligeiramente melhor.

Deng e Shen (2013) propuseram uma regra da cadeia adaptada para sistemas não balanceados, em que, os itens e as máquinas são arranjados de maneira alternada em uma circunferência e então conectados. Os autores mostraram que essa maneira de se construir a regra da cadeia produziu uma configuração com desempenho superior a configuração usando as orientações propostas por Jordan e Graves (1995).

Feng, Wang e Shen (2017) realizaram um estudo de flexibilidade de máquinas em um problema de planejamento da produção não balanceado. Os autores propuseram um modelo em dois estágios, em que no primeiro é definido a designação dos itens às plantas e no segundo, o planejamento da produção. Os autores consideraram que as plantas tem eficiências diferentes para produzir certos produtos. Os resultados mostraram que quando a eficiência da planta diminui, a quantidade de ligações necessárias no sistema aumenta, já quando a eficiência da planta aumenta, mais produtos são designados para ela.

Vale ressaltar que, até o momento, não foram encontrados estudos de flexibilidade de máquinas no problema de dimensionamento de lotes não balanceado em contexto determinístico. Algumas particularidades dos sistemas não balanceados ainda precisam ser definidas o que representa um desafio a ser superado.

CAPÍTULO 3

Discussão dos resultados

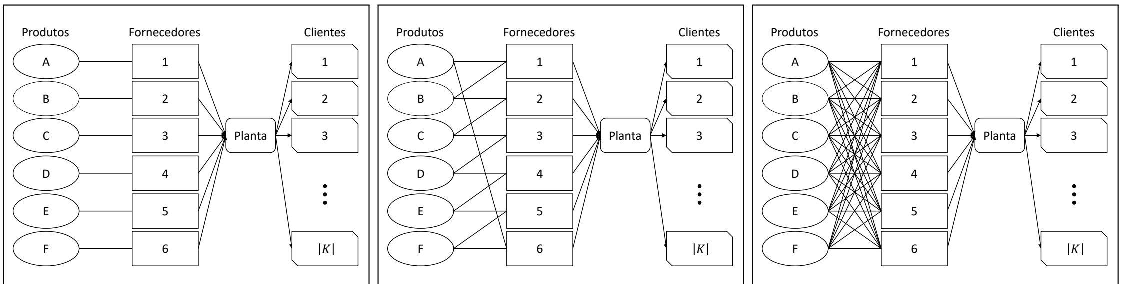
Nesse capítulo é apresentado os desenvolvimentos realizados durante o mestrado e discute-se os principais resultados obtidos. A Seção 3.1 se refere aos resultados do artigo intitulado "*Impact analysis of flexibility on the integrated lot sizing problem and supplier selection*". A Seção 3.2 se refere aos resultados do artigo intitulado "*Evaluating process flexibility in lot sizing problems: an approach based on multicriteria decision making*". Finalmente, a Seção 3.3 apresenta o desenvolvimento e os resultados do estudo de flexibilidade de máquinas no problema de dimensionamento de lotes não balanceado. Ainda não há na literatura um trabalho que realize esse estudo em um contexto determinístico. O artigo referente a este trabalho está em andamento.

3.1 O problema integrado de dimensionamento de lotes e seleção de fornecedores

Nessa seção é apresentado o estudo de flexibilidade de aquisição de produtos, esse tipo de flexibilidade aparece no problema integrado de dimensionamento de lotes com seleção de fornecedores. Observa-se que esse é o primeiro estudo de flexibilidade considerando ambos os problemas de maneira integrada.

Já é conhecido na literatura que o problema de dimensionamento de lotes pode ser integrado com o problema de seleção de fornecedores. O custo de matéria prima é parte significativa do custo total do produto, assim, é extremamente importante lidar com fornecedores confiáveis e buscar uma relação estável e duradoura. Embora, o problema integrado de dimensionamento de lotes e seleção de fornecedores seja conhecido, até o momento, não houve uma investigação a respeito da flexibilidade nesse problema. A Figura 3.1 ilustra a ideia do problema estudado. Uma planta tem a sua disposição diferentes fornecedores, com suas capacidades e seus níveis de flexibilidade. A Figura 3.1(a) apresenta uma possibilidade em que cada fornecedor escolhido pode entregar somente um produto. Em seguida, a Figura 3.1(b) apresenta uma escolha em que os fornecedores são mais flexíveis, isso é, cada fornecedor pode entregar mais produtos do que a configuração anterior. Finalmente, a Figura 3.1(c) apresenta a situação em que cada fornecedor é totalmente flexível para entregar todos os produtos necessários. Vale notar que em cada caso, os fornecedores são diferentes ou então, o contrato com cada fornecedor é alterado para entregar certo conjunto de produtos, desde que disponíveis.

Portanto, uma *configuração de flexibilidade de aquisição de produtos* pode ser definida como um grafo bipartido $G = (U, V, E)$ em que U e V denotam o conjunto dos vértices e E o conjunto das arestas. Os vértices de uma partição representam os produtos e os vértices da outra partição os fornecedores. As arestas ligam os vértices. Note que, existem duas esferas de decisão envolvendo o problema. No contexto do fornecedor, a decisão sobre a capacidade da entrega dos produtos e quais produtos podem ser entregues. No contexto da planta, qual fornecedor é selecionado e a quantidade comprada de cada produto.

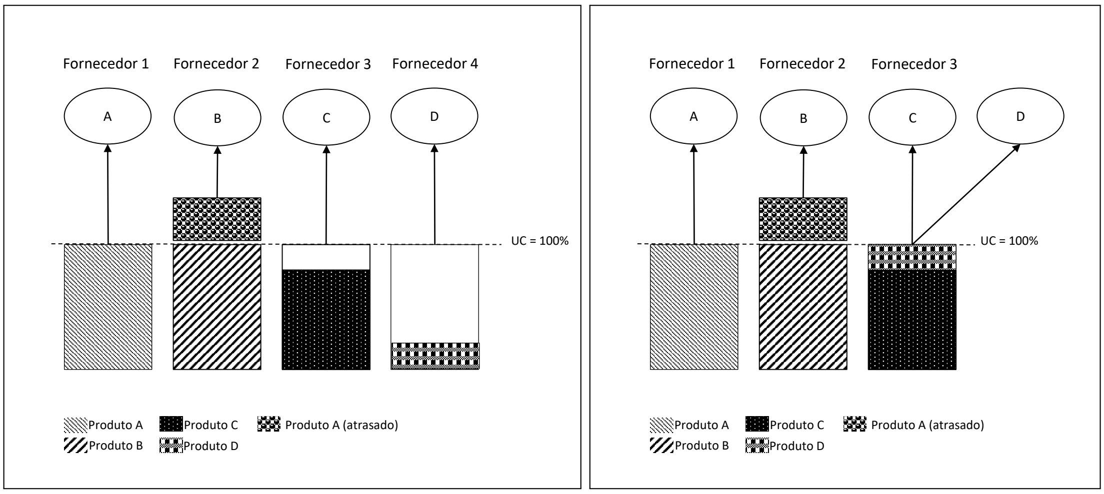


(a) Configuração dedicada. (b) Regra da cadeia. (c) Flexibilidade total.

Figura 3.1: Configurações para a flexibilidade de aquisição de produtos.

Um exemplo prático envolve os mercados com seus centros de distribuição. Nesse caso, os mercados são as plantas, que compram um conjunto de produtos dos fornecedores selecionados e então os estocam nos centros de distribuição (ou mesmo nos próprios fornecedores) até que os produtos são entregues para os centros de venda (que podem ser visto como os clientes). Observe que, portanto, existem fornecedores que são mais ou menos flexíveis, isso é, que podem entregar mais ou menos tipos de produtos. A intuição parece indicar que a melhor situação é aquela que o fornecedor pode entregar todos os itens, entretanto, dificilmente será possível encontrar esse fornecedor na prática. Será mostrado ao final da presente seção que não é preciso encontrar um fornecedor tão flexível.

Um dos benefícios da flexibilidade de aquisição de produtos consiste no uso eficiente da capacidade disponibilizada de cada fornecedor e a possibilidade de redução da quantidade de fornecedores selecionados. Observe que ao reduzir a quantidade de fornecedores ocorre uma diminuição de custos da gestão dos fornecedores, além de permitir o desenvolvimento de uma relação estável e duradoura com os fornecedores selecionados. A Figura 3.2 apresenta como é possível reduzir a quantidade de fornecedores selecionados ao adicionar flexibilidade. Observe na Figura 3.2(a) o cenário em que cada fornecedor pode entregar somente um produto. Os fornecedores 1 e 2 já entregaram para a planta tudo o que era possível, porém, a demanda do produto B ainda não foi totalmente atendida. Já os fornecedores 3 e 4 ainda podem entregar mais produtos C e D. Ao encontrar um fornecedor que possa entregar ambos os produtos C e D, é possível encerrar um contrato e diminuir a quantidade de fornecedores selecionados, simplificando o processo de gestão, como é mostrado na Figura 3.2(b).



(a) Configuração dedicada de flexibilidade.   (b) Configuração limitada de flexibilidade.

Figura 3.2: Valor da flexibilidade de aquisição de produtos na redução dos fornecedores selecionados.

Os experimentos computacionais foram pensados, primeiramente, para determinar os benefícios da flexibilidade de aquisição de produtos em diferentes contextos. Assim, foi proposto um modelo matemático que obtém o plano de compra ótimo (com menor custo) e quais fornecedores devem ser selecionados. Um dos objetivos desse trabalho consiste em demonstrar que ao adicionar flexibilidade é possível reduzir a quantidade de fornecedores selecionados, para isso, dada a solução ótima de cada problema, foi computada a quantidade de fornecedores selecionados e assim é possível concluir em quais situações é possível e vantajoso trabalhar com uma quantidade reduzida de fornecedores.

Para analisar o benefício da flexibilidade, um dos parâmetros do modelo é a quantidade de flexibilidade, isso é, a quantidade de arestas ativas no grafo G. Observe na Figura 3.1(a), a quantidade de ligações é 6 (6 produtos com 6 fornecedores, em que cada fornecedor pode entregar um único produto). Na Figura 3.1(b), a quantidade de ligações é $2*6 = 12$ (cada fornecedor pode entregar 2 produtos). Finalmente, na Figura 3.1(c), há $6*6 = 36$ ligações (cada fornecedor pode entregar todos os produtos). A intuição por trás do experimento é que se ao adicionar flexibilidade, mantendo todo o resto constante, for possível reduzir o custo em relação a um cenário sem flexibilidade, então a redução do custo advém da flexibilidade. A Tabela 3.1 apresenta o detalhamento da quantidade de ligações testadas.

Por outro lado, é conhecido na literatura que o benefício da flexibilidade está intimamente ligado à capacidade, por isso, 6 diferentes níveis de capacidade foram testados. Da mesma

Tabela 3.1: Quantidade de ligações de cada configuração de flexibilidade.

	# ligações (F_{max})	n	$n+n/2$	$2n$	n^2
# produtos		Dedicado	Flexibilidade 1	Flexibilidade 2	Flexibilidade Total
6		6	9	12	36
12		12	18	24	144
24		24	36	48	576

maneira, ao variar a capacidade (e mantendo todo o resto constante, inclusive a quantidade de ligações), busca-se compreender como o benefício da flexibilidade é afetado pela capacidade. Observe que a capacidade é uma decisão do fornecedor, porém, ao varia-la pode-se analisar o benefício da flexibilidade quando há fornecedores com capacidade mais apertada ou folgada.

Outra análise realizada envolve aspectos relacionados a homogeneidade dos dados. Primeiramente, foi testado um cenário em que os parâmetros são homogêneos, isso é, o preço de aquisição de produto é o mesmo para todos os fornecedores e a demanda não apresenta sazonalidade. O objetivo desse experimento é entender como o benefício da flexibilidade é afetado pelas variações nesses parâmetros. Note que quando o preço de aquisição é diferente de um fornecedor para o outro, isso significa que há fornecedores mais baratos do que outros, uma situação que ocorre recorrentemente na prática.

Os principais resultados obtidos indicam que com a flexibilidade de aquisição de produtos em um cenário homogêneo, é possível reduzir a quantidade de fornecedores selecionados nas capacidades intermediárias e altas. Para compreender, pense na seguinte situação, o contrato atual da planta contempla fornecedores com capacidade bastante apertada (o que significa que existem muitos itens em atraso). Ao adicionar flexibilidade, ou seja, ao encontrar um fornecedor que tenha o mesmo nível de capacidade, mas que possa entregar mais tipos de produtos, é possível reduzir a quantidade de demanda atrasada (como visto na Figura 3.2), mas não a quantidade de fornecedores (atraso alto). Já em um cenário em que a capacidade é intermediária ou alta, a quantidade de itens em atraso é menor (ou praticamente nula) e é possível reduzir a quantidade de fornecedores. Note que, em algumas situações, é preferível ter um pouco de itens em atraso, do que o contrato com um novo fornecedor.

Vale ressaltar que o benefício da flexibilidade advém da redução da quantidade de itens em atraso (como ilustrado na Figura 1.2, página 12), e também da redução da quantidade de fornecedores selecionados. Quando existem fornecedores mais baratos do que outros, o benefício da flexibilidade é ainda maior, já que a decisão do modelo será comprar dos fornecedores mais baratos. As conclusões obtidas no cenário homogêneo permaneceram nesse cenário. Já quando a demanda é sazonal, não foi possível reduzir a quantidade de fornecedores selecionados, isso porque é preciso manter todos os fornecedores para cobrir o pico da demanda, ainda assim, o benefício da flexibilidade é aparente e não é preciso que cada fornecedor entregue todos os produtos para se obter os mesmos benefícios da flexibilidade total. O artigo submetido com maiores detalhes do trabalho realizado se encontra no Anexo A.

3.2 A análise multicritério da flexibilidade de máquinas no problema de dimensionamento de lotes

Nessa seção é apresentado um estudo do benefício da flexibilidade a partir de uma abordagem multicritério. Diferente da seção anterior, a flexibilidade agora está associada a facilidade com a qual uma máquina pode ser adaptada para fabricar diferentes itens (SETHI; SETHI, 1990). Até o momento as análises do benefício da flexibilidade tem se concentrado em um único critério que é a função objetivo do problema, normalmente o custo, algumas vezes o lucro. Embora o custo seja um critério extremamente relevante, existem outros critérios que também devem ser analisados como a utilização da capacidade, a quantidade de flexibilidade, entre outros. Nesse contexto fica evidente a importância de se utilizar uma análise multicritério do benefício da flexibilidade.

O método ***T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution* (TOPSIS), é um método multicritério para ranqueamento de um conjunto de alternativas. A partir do conjunto de alternativas constrói-se uma alternativa fictícia ideal, isso é, tem o melhor desempenho em cada um dos critérios e uma alternativa fictícia anti-ideal, logo com o pior desempenho em cada um dos critérios. Note que essas alternativas não existem na realidade. Se a alternativa ideal estivesse disponível, ela deveria ser escolhida sem hesitação. O método TOPSIS ranqueia cada alternativa de acordo com a distância entre as duas alternativas construídas, isso é, a melhor alternativa é aquela que mais se aproxima da ideal e mais se distancia da anti-ideal. A Figura 3.3 apresenta a noção intuitiva do método TOPSIS para dois critérios (C_1 e C_2) e as alternativas A,B,C,D,E. Observe que ambos os critérios são de maximização. A alternativa fictícia ideal é composta pelo desempenho da alternativa E quanto ao critério C_1, e da alternativa A quanto ao critério C_2. Da mesma maneira, a alternativa anti-ideal é composta pelo desempenho da alternativa A frente ao critério C_1 e da alternativa E, frente ao critério C_2.

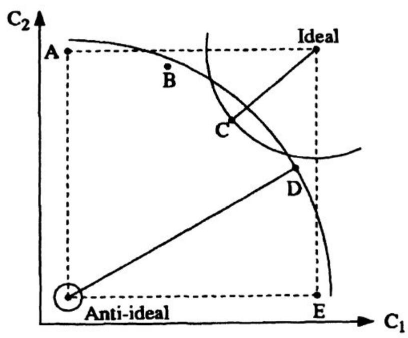


Figura 3.3: Ilustração quanto as distâncias no método TOPSIS. Extraído de Pomerol e Serio (2012).

Nesse trabalho, as alternativas são as diferentes configurações de flexibilidade, como pode ser visto na Tabela 3.2. Algumas dessas configurações foram apresentadas na Figura 1.1 (página 12), as demais não têm uma representação fixa. A configuração *aleatória* liga de maneira aleatória os itens às máquinas. A *melhor regra da cadeia* é uma configuração obtida pelo *solver* de modo que um caminho possa ser traçado à partir de qualquer item/máquina. Finalmente, as configurações $Fmax_1$ e $Fmax_2$ também são obtidas pelo *solver* dada uma limitação na quantidade de flexibilidade de $1.5n$ e $2n$ ligações, respectivamente, em que n é a quantidade de máquinas. Já os critérios são medidas operacionais do desempenho das configurações. Note que para cada critério existe um sentido de maximização ou minimização, isso é, para o custo a melhor alternativa é aquela que apresenta o menor custo. Para a utilização da capacidade, como existe atraso, deseja-se utilizar o máximo possível da capacidade. Cada preparação de máquina implica em tempo parado da máquina, logo, deseja-se minimizar a quantidade de preparações de máquinas. A quantidade de flexibilidade também deve ser minimizada já que máquinas mais flexíveis são mais caras (FIOROTTO; JANS; ARAUJO, 2018).

Observe que existe uma relação inversa entre alguns critérios, por exemplo, a flexibilidade total é aquela que apresenta o menor custo na solução ótima, porém, tem a maior quantidade de flexibilidade. A configuração dedicada apresenta o pior desempenho em custo, mas tem a menor quantidade de flexibilidade. O método TOPSIS gera um ranqueamento de cada uma dessas alternativas, se o método é capaz de representar as preferências de um tomador de decisão racional, então a primeira alternativa do ranking é preferível às demais (POMEROL; SERIO, 2012).

Tabela 3.2: Alternativas e critérios.

Alternativas	**Critérios**
Dedicado	Custo relativo (min)
Cluster	Utilização de capacidade (max)
Aleatório	Quantidade atrasada (min)
Regra da cadeia	Quantidade de preparações (min)
Melhor regra da cadeia	Quantidade de flexibilidade (min)
$Fmax_1$	
$Fmax_2$	
Total	

Outro parâmetro de entrada do método TOPSIS é o peso de cada critério. Note que, na prática, há critérios que são mais relevantes do que outros para um tomador de decisão. Primeiramente, considerou-se que cada critério havia o mesmo peso, e posteriormente essa suposição foi relaxada. Assim, foi realizada uma análise da sensibilidade do ranking quanto ao peso. Cada critério recebeu um peso em uma distribuição uniforme $U(0,1]$ e dez mil simulações foram feitas, cada uma gerando um novo ranking através do método TOPSIS, em seguida, analisou-se quantas vezes cada alternativa apareceu em cada posição do ranking. Para os experimentos computacionais utilizou-se os resultados apresentados por Fiorotto, Jans e Araujo (2018).

Em Fiorotto, Jans e Araujo (2018) os resultados foram divididos em cenários mais ou menos homogêneos. Os resultados mostram que a flexibilidade total ranqueou nas últimas posições em todos os cenários estudados por Fiorotto, Jans e Araujo (2018), o que é razoável já que a flexibilidade total apresenta uma quantidade de flexibilidade muito grande e outras configurações com menos ligações conseguem obter um comportamento bastante semelhante nos demais critérios. Note portanto que, embora a flexibilidade total seja conhecida na literatura pela melhor flexibilidade (na análise mono-critério), ao realizar uma análise multicritério isso se altera.

Além disso, a regra da cadeia que é conhecida pelo seu bom desempenho em termos da função objetivo (bastante próximo da flexibilidade total), ranqueou em posições intermediárias ao considerar diversos critérios. Na literatura, a regra da cadeia é apresentada como uma alternativa para se obter os mesmos benefícios da flexibilidade total (em termos da função objetivo), sem precisar de todas as ligações. Note portanto, que na análise multicritério, essa configuração é apenas mediana (isso é, existem outras configurações que são preferíveis no ranking).

Já configuração $Fmax_1$ com uma baixa quantidade de flexibilidade ranqueou nas primeiras posições em todos os cenários. Coerente com os resultados apresentados por Fiorotto, Jans e Araujo (2018), a configuração $Fmax_1$ definida pelo *solver* com apenas $1.5n$ ligações, teve um desempenho (em termos da função objetivo) tão bom quanto a flexibilidade total. Essa configuração, dentre todas as estudadas, apresenta a menor quantidade de ligações e bom desempenho nos demais parâmetros operacionais, e por isso, é preferível pelo tomador de decisão.

É interessante observar que a configuração dedicada apareceu nas primeiras posições quando as máquinas tem capacidade alta, mas nas últimas posições quando a capacidade é média ou baixa. A explicação reside no fato de que quando a capacidade é alta, existe (praticamente) empate entre diversos critérios operacionais (como quantidade atrasada e custo relativo), e os critérios que não estão em empate, como utilização de capacidade e quantidade de flexibilidade são vencidos pela configuração dedicada. Já nas capacidades médias e baixas, as demais configurações são preferíveis.

Finalmente, diferente de quando se considera somente a função objetivo, na análise multicritério já é vantajoso investir em flexibilidade desde as capacidades apertadas. Note que, como relatado em Fiorotto, Jans e Araujo (2018), quando a capacidade é apertada, não há (ou é muito pequeno) benefício da flexibilidade pois a quantidade de itens em atraso é muito grande e adicionar flexibilidade não consegue reduzir a quantidade atrasada (e logo a função objetivo). Entretanto, na análise multicritério, mesmo que o custo não seja reduzido, outros critérios operacionais sofrem impacto positivo pela flexibilidade e por isso é vantajoso investir em flexibilidade desde as capacidades mais apertadas. Para mais detalhes desse estudo, o artigo submetido se encontra no Anexo B.

3.3 A flexibilidade de máquinas no problema de dimensionamento de lotes não balanceado

Nessa seção é abordado a flexibilidade de máquinas no problema de dimensionamento de lotes quando a quantidade de itens é diferente da quantidade de máquinas. Há poucos estudos a respeito da flexibilidade de máquinas em sistemas não balanceados, todos eles consideram um contexto estocástico, normalmente, sobre a demanda.

Note que, na prática, é recorrente encontrar indústrias em que a quantidade de itens é muito maior do que a quantidade de máquinas, entretanto as configurações de flexibilidade para os sistemas não balanceado são mais difíceis de serem caracterizadas. O primeiro passo desse trabalho, portanto, foi definir uma maneira pela qual as configurações de flexibilidade já conhecidas podem ser representadas. A ideia da proposta é que cada máquina tenha seu conjunto de itens, (os itens não se repetem em cada conjunto) e é possível que uma máquina compartilhe a produção dos itens que pertencem a outra máquina. Formalmente, seja $N = \{1,...,n\}$ o conjunto dos itens e $M = \{1,...,m\}$ o conjunto das máquinas, S é uma partição de N, tal que $S_k \subseteq N$, $S_k \neq \emptyset$ e $S_k \cap S_j = \emptyset, \forall k, j \in M, k \neq j$. Portanto, S_k define quais itens i "pertencem" a máquina k. Ainda, seja λ_{kj} o parâmetro de intensidade que limita o compartilhamento de produção de itens entre as máquinas distintas k e j, ou seja, dado os itens em S_k que são produzidos pela máquina j, λ_{kj} é o máximo de itens em S_k que a máquina j pode produzir. Nesta abordagem, considera-se que cada entrada da diagonal principal de Λ é a cardinalidade do conjunto S_k, ou seja, $\lambda_{kk} = |S_k|$.

A Figura 3.4 ilustra diferentes perfis de flexibilidade e a organização do sistema não balanceado, sem nenhum padrão aparente de configuração de flexibilidade para perfis de flexibilidade já conhecidos. A Figura 3.4(a) não utiliza os conjuntos S_k e mostra claramente que o sistema é não balanceado. Em seguida, a Figura 3.4(b) mostra a relação entre os itens, o conjunto S_k e as máquinas. Note que a linha em negrito indica que a máquina pode fabricar todos os itens do conjunto que ela está ligada. As linhas mais claras indicam todas as possibilidades de ligações entre os conjuntos e as máquinas. A Figura 3.4(c) mostra um perfil de configuração de flexibilidade em que as máquinas não compartilham a produção de nenhum item. Em seguida, a Figura 3.4(d) mostra o perfil de configuração da regra da cadeia e finalmente, a Figura 3.4(e) mostra a configuração de flexibilidade total. Cada um desses perfis de configuração está associado à uma matriz Λ diferente. Note que, se por exemplo $\lambda_{23} = 2$, significa que a máquina $j = 3$ pode fabricar 2 itens do conjunto S_2. Ainda, perceba que ao variar o parâmetro λ_{kj} é possível obter diferentes configurações de flexibilidade com o mesmo perfil.

Note que na Figura 3.4(a) há 9 ligações (cada item é ligado à uma máquina uma única vez). Em seguida, na Figura 3.4(b) e Figura 3.4(c) ainda há 9 ligações, lembrando que as linhas mais claras indicam que podem haver muito mais. Na Figura 3.4(d) há as 9 ligações referente às linhas em negrito mais as ligações referentes ao compartilhamento da produção entre às máquinas, se cada máquina compartilhar um item, então haverá 9 mais 4 ligações, totalizando 13. Se cada máquina compartilhar todo o seu conjunto de itens, então haverá 18 vezes. Da mesma maneira para a Figura 3.4(e), há no mínimo 21 ligações e pode haver no máximo 36 (flexibilidade total de intensidade máxima). A Tabela 3.3 sintetiza os dados quanto a quantidade de ligações.

Note que a quantidade de ligações varia de acordo com a matriz Λ, já que a quantidade de ligações é dado pela soma de todas as entradas da matriz, por isso, para as duas últimas

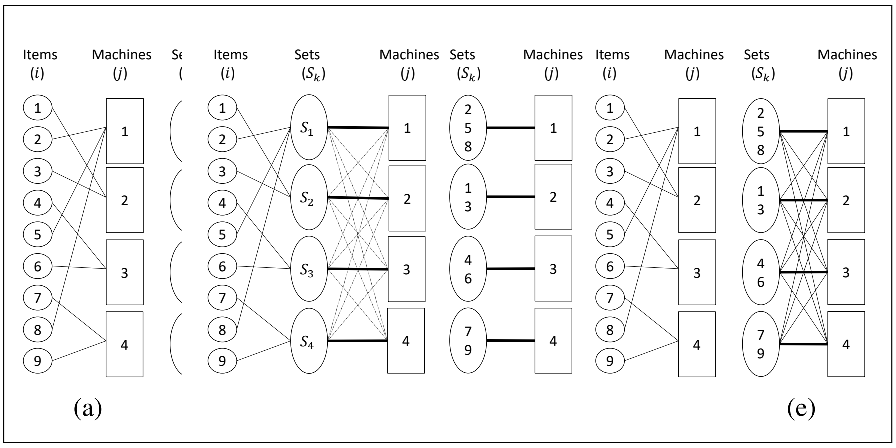


Figura 3.4: Perfis de configuração de flexibilidade de processo.

linhas da Tabela 3.3 a quantidade de ligações é apresentada dentro de um intervalo. Para as três primeiras linhas da Tabela 3.3, a matriz Λ é conhecida e somente os elementos da diagonal principal são não nulos ($\lambda_{kk} = |S_k|$).

Tabela 3.3: Quantidade de ligações na Figura 3.4.

	Quantidade de ligações
Figura 3.4(a)	9
Figura 3.4(b)	9
Figura 3.4(c)	9
Figura 3.4(d)	Entre 13 e 18
Figura 3.4(e)	Entre 21 e 36

Além da proposta de uma maneira de se representar as configurações de flexibilidade, foram apresentados novos indicadores de desempenho das configurações de flexibilidade. Não havia na literatura nenhum estudo de quantas ligações de fato, a solução ótima utiliza, dessa maneira, propôs-se o indicador de similaridade (IS) que mede a discrepância entre configuração permitida e a de fato utilizada na solução ótima. Um outro indicador apresentado é o índice de atendimento da demanda (IAD) que mede o quanto da demanda foi atendida dentro do horizonte de planejamento. Nesse trabalho deseja-se trazer a ideia de que o benefício da flexibilidade não é aparente somente na função objetivo do problema, mas afeta outros indicadores, como a utilização de capacidade, isso é, aumentar a flexibilidade permite que a capacidade das máquinas possa ser utilizada de maneira mais eficiente, outro impacto da flexibilidade é a melhoria da imagem da marca pela redução da quantidade de itens em atraso.

Para atingir os objetivos do trabalho, considerou-se três formulações para o problema de dimensionamento de lotes. A primeira permite estudar a flexibilidade quando a configuração já é conhecida e fixa. A segunda, utiliza a matriz Λ para fixar um perfil de configuração e a solução ótima da formulação apresenta a alocação ótima dos itens às máquinas. Note que com essa configuração é possível estudar o impacto da alocação dos itens às máquinas (ao comparar uma solução dessa formulação com mesmo perfil de flexibilidade e intensidade à solução da formulação 1) e também é possível estudar o efeito da intensidade (quantidade de flexibilidade) quando a alocação dos itens às máquinas é ótima. Finalmente, a terceira foi proposta por Fiorotto, Jans e Araujo (2018) em que, dada uma quantidade de ligações a solução ótima apresenta a configuração de flexibilidade (independente do perfil).

A Tabela 3.4 sintetiza os perfis de configuração estudados em relação à cada uma das formulações. Portanto, o perfil de configuração dedicado (aquele que não tem compartilhamento da produção entre as máquinas) é estudado na formulação 1 (com a configuração fixa) e na formulação 2 (com a designação dos itens às máquinas como decisão do *solver*). A mesma lógica se aplica para o perfil da regra da cadeia e da flexibilidade total. Finalmente, a formulação 3, como dito anteriormente, não apresenta nenhum perfil de configuração de flexibilidade. Um parâmetro da formulação 3 é a quantidade de ligações (representado pelo subíndice $\cdot_f$), foram estudados $f = n$ ligações, em que n é a quantidade de itens e $f = 1.5n$ ligações.

Tabela 3.4: Configurações de acordo com o compartilhamento e a formulação estudada.

	Perfil de configuração			
	Dedicado	Cadeia	Total	Sem perfil
Formulação 1	B1	C1	T1	
Formulação 2	B2	C2	T2	
Formulação 3				$L3_f$

Para melhor compreensão das conclusões obtidas, o termo teste, nos próximos parágrafos, indica todas as soluções oriundas de uma certa formulação e/ou configuração. Os resultados computacionais indicam que somente o teste $L3_{f=n}$ utilizou todas as ligações permitidas. Isso se justifica pela baixa quantidade de ligações, não há restrições quanto ao perfil da configuração de flexibilidade e o *solver* pode encontrar uma alocação que seja ótima. Vale ressaltar que para o teste $L3_{f=1.5n}$, nem todas as $1.5n$ ligações foram utilizadas. Outro resultado interessante é referente à formulação 1 e 2. Quando a capacidade é apertada, os testes da formulação 2 utilizam mais ligações, porém depois de uma certa capacidade intermediária ocorre uma inversão e os testes da formulação 1 utilizam mais ligações. A justificativa reside na liberdade das formulações. Como a formulação 1 tem uma configuração fixa, quando a capacidade é apertada, não é possível utilizar todas as ligações, já a formulação 2 pode encontrar uma alocação que seja melhor. Em contrapartida, quando a capacidade é intermediária a formulação 2 apresenta uma alocação que não precisa de todas as ligações, enquanto que a formulação 1 com mais capacidade precisa de mais ligações pois não é a alocação ótima.

Quanto à análise do benefício da flexibilidade, todos os experimentos mostraram que adicionar flexibilidade reduz o custo total em relação à um cenário sem flexibilidade. Para as configurações fixas, aumentar a intensidade melhora de maneira significativa o benefício da flexibilidade, enquanto que para a formulação 2 (em que a alocação é ótima) a melhoria é mais suave. É interessante notar que o teste $L3_{f=1.5n}$ atingiu o mesmo benefício da flexibilidade que a flexibilidade total de intensidade máxima enquanto quase todos os experimentos e a diferença com o teste $L3_{f=n}$ é de cerca de 1%. Essa informação associada com os resultados do índice de similaridade fornecem um forte indicativo de que a quantidade de ligações necessárias para se obter os mesmos benefícios da flexibilidade total está entre n e $1.5n$. Para mais detalhes, o artigo (em desenvolvimento) se encontra no Anexo C.

CAPÍTULO 4

Conclusão e propostas futuras

Este trabalho estudou aspectos da flexibilidade de máquinas no problema de dimensionamento de lotes em contexto determinístico. Primeiro, estudou-se uma extensão do problema de dimensionamento de lotes, ao considerar a integração com o problema de seleção de fornecedores. Em seguida, uma análise multicritério de diferentes configurações de flexibilidade de máquinas. Finalmente, a flexibilidade de máquinas no problema de dimensionamento de lotes não balanceado (quando a quantidade de itens é diferente da quantidade de máquinas). Cada um desses trabalhos obteve diferentes resultados sobre a flexibilidade de máquinas, todos eles concordam, como esperado, que não é necessário utilizar todas as ligações para se obter os benefícios da configuração de flexibilidade total. A análise multicritério da flexibilidade de máquinas mostrou que quando se considera outros critérios além da função objetivo do problema já é interessante investir em flexibilidade desde as capacidades mais apertadas, ao contrário do se foi observado na literatura. Isso acontece porque ao considerar outros critérios, os benefícios da flexibilidade de máquinas ficam aparentes nesses critérios mesmo que não tenham impacto evidente na função objetivo do problema. Outro aspecto interessante é o benefício da flexibilidade no problema de seleção de fornecedores. Nesse caso, é interessante trabalhar com fornecedores flexíveis, quando isso acontece é preciso uma quantidade reduzida de fornecedores em relação ao caso em que não há nenhuma flexibilidade. Um benefício de se utilizar fornecedores flexíveis é a simplificação no processo de gestão que tem impacto direto no custo dos produtos. Em relação à flexibilidade de máquinas para o problema de dimensionamento de lotes não balanceado, foi possível observar que os efeitos de aumentar a flexibilidade são maiores quando já existe uma configuração de flexibilidade conhecida, isso é, já se conhece quais produtos cada máquina pode fabricar, entretanto, mesmo quando essa decisão é tomada através do modelo, ou seja, a alocação dos itens às máquinas é ótima, aumentar a flexibilidade tem impacto positivo nos indicadores de desempenho.

Como propostas futuras, para o estudo de flexibilidade no problema de dimensionamento de lotes, sugere-se:

- Considerar outros aspectos práticos no problema de dimensionamento de lotes como a logística reversa e/ou a remanufatura de certos componentes do produto.
- Investigar os benefícios da flexibilidade quando existem diversas plantas, diversos produtos (considerando mais produtos do que plantas) e nem toda a planta pode enviar todos os produtos para todos os clientes;
  - Integrar com o problema de transporte;
  - Aplicar técnicas de otimização robusta.
- Estudar a flexibilidade em um contexto de incerteza, em que os parâmetros desconhecidos são intervalares (como na lógica fuzzy).
- Estudar métodos de solução para as formulações propostas nos artigos.
- Investigar novas formas de flexibilidade, como a flexibilidade de preparação de máquinas ou a flexibilidade na lista de materiais;
  - Integrar com o problema da mistura;
  - Aplicar técnicas de otimização robusta.

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ANEXO A

Impact analysis of flexibility on the integrated lot sizing and supplier selection problem

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Impact analysis of flexibility on the integrated lot sizing and supplier selection problem

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Abstract This paper addresses the integrated capacitated multi-period lot sizing and supplier selection problem considering backlog. The demand for the products is known over the planning horizon, and different products can be purchased from multiple suppliers to meet the order of a set of shortlisted customers. The problem involves a network connecting suppliers, products, and customers. Then, the decision-maker needs to decide what products to purchase in what quantities from the suppliers in each period. The paper analyzes the named value of product acquirement flexibility, where only certain types of products can be delivered by each one of the suppliers. Note that, in practice, it is unusual to use resources that have total (or complete) product acquirement flexibility, i.e., each resource can deliver all products. Therefore, it might be interesting to implement only a limited amount of product acquirement flexibility so that only certain types of products can be delivered by each supplier. Extensive computational experiments showed that it is possible to obtain benefits from the product acquirement flexibility by reducing the number of selected suppliers, that can decrease costs due to the supplier management process simplification, and establish a lasting partnership with the suppliers.

Keywords Lot sizing problem · Supplier selection problem · Process flexibility · Integrated problems

1 Introduction

The tight profit margins, high-quality product expectations by costumers, and accelerated lead-times have become the environment of companies highly competitive. Then they are forced to take advantage of any opportunity to optimize their business processes. In this respect, for a company to remain competitive, it has to get the best opportunities with its supply chain partners so that it refines the chain's total performance. In line with that, researchers have recognized that flexibility concepts are important for building sustainable manufacturing since it enables companies to achieve customization, even in large-scale production, without sacrificing cost-efficiency.

This paper aims to explore the alternative sources of products from different suppliers to improve the purchasing process for meeting the demand of customers. In some industries, such decisions about production activities are closely related to decisions about the lots of items that must be purchased and distributed. In this context, there is the integrated lot sizing and supplier selection problem that consists of determining the number of items to be purchased from each supplier in each period of a planning horizon. In the standard supplier selection problem, each supplier can deliver all products (complete

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product acquirement flexibility). In this study, a supplier can deliver only certain types of products in the planning horizon (limited product acquirement flexibility). Thus, the problem becomes more similar to what happens in many practical applications since it is not usual to have suppliers that deliver all products. The study of named product acquirement flexibility appears as an alternative to achieve a reduced configuration by linking some products to a minimal number of suppliers. It is expected to find that a reduced number of suppliers delivering a few types of products can provide a similar performance regarding total flexibility in terms of costs. Note that in practice this problem appears in markets that have distribution centers.

Since the seminar paper of Jordan and Graves (1995), several authors have studied the value of some flexibility configurations applied to different problems considering a stochastic production environment. The first and most studied flexibility configuration is the "chain" principle. A chain can be described as a group of products and plants, which are all connected by product assignment decisions. Recently, Fiorotto et al. (2018) have shown that process flexibility can have a significant value in a short-term deterministic planning environment. They pointed out that it happens, for example, in the semiconductor industry where machines must be qualified before being able to produce certain products, and these qualification decisions are periodically reevaluated. This qualification process hence constitutes the decision on the flexibility configuration (Johnzén et al., 2011; Rowshannahad et al., 2015). This study has opened the opportunity for further work on flexibility, considering a deterministic context.

Although the literature has presented some recent contributions to the concept of process flexibility in the context of the deterministic production environment, to the best of our knowledge, there are no studies analyzing the idea of product acquirement flexibility considering the integrated lot sizing and supplier selection problem with backlog in meeting the demand of customers. More specifically, the previous contributions have presented an analysis of the limited process flexibility to the lot sizing problem. They provided approaches to detect the total benefits obtained when trying to reduce the number of items that a machine can produce during the planning horizon. However, such particular aspects connecting suppliers, products, and customers, which only appear in the integrated lot sizing and supplier selection problem, still need to be investigated. Therefore, this study aims to cover this gap in the literature of lot sizing and supplier selection problems by proposing an analysis of product acquirement flexibility, considering a deterministic production environment.

This paper has the following contributions. First, we propose a new optimization model which gives the optimal flexibility configuration to the integrated lot sizing and supplier selection problem. Second, we analyze the effect of different levels of flexibility on the number of selected suppliers. Third, we analyze how different parameters, such as the purchase cost, and the demand distribution, have an impact on the total cost of purchase planning. Finally, computational results are performed to show the effect of the product acquirement flexibility configuration on the total cost.

Our computational results show that for the homogeneous setting and purchase cost heterogeneity, a very limited amount of flexibility is enough to reduce the total number of selected suppliers significantly. The computational experiments also indicate that, in terms of total costs, the reduced amount of selected suppliers delivering only a few products is enough to get similar benefits of each supplier delivering all products. However, for the case with demand heterogeneity, the total number of selected suppliers of all flexibility configurations is almost the same.

The organization of the paper is as follows. Section 2 presents a brief literature review on the lot sizing and supplier selection problems, and the process flexibility in lot sizing problem considering a deterministic context. Section 3 gives a formal description of the mathematical model for the integrated lot sizing and supplier selection problem with product acquirement flexibility. Section 4 presents the analysis of product acquirement flexibility and shows the computational results considering different scenarios and flexibility configurations. Finally, in Section 5, we give our conclusions and some ideas for future research.

2 Literature review

Although the product acquirement flexibility has not been addressed for the deterministic integrated lot sizing and supplier selection problem, the concepts of process flexibility have been recently studied on different production planning problems. Next, we review some lot sizing and supplier selection problems that are related to our approach and discuss the main insights from the literature related to process flexibility in a deterministic context.

2.1 Lot sizing and supplier selection problems

In the literature, there has been a broad effort to expand decision methods and procedures for the lot sizing and supplier selection problems. The supplier selection is crucial for companies since the cost of raw materials and components represents a significant portion of the total product cost. Moreover, purchasing is strategic since it provides possibilities to reduce costs and improves product quality. On the other hand, lot sizing is also important to find a suitable production and inventory management. Several factors have forced companies to obtain a competitive position by keeping the attention on their complete supply chain. Thus particular attention must be given on the decision aspects involving the lot sizing and supplier selection problems.

Several authors have studied the most important criteria for selecting suppliers. In line with that, Weber et al. (1991) identified that delivery time, quality, cost, and supplier capacity are the usual criteria addressed in the literature. They pointed out that often some criteria are conflicting, such as the cost and quality of a product. For this reason, it is usual to find studies that use multicriteria methods. Furthermore, the authors identified around ten papers that addressed the use of the linear programming model.

There are also some studies on the effect of combining the different products to be purchased from the suppliers. Hong and Hayya (1992) used some techniques to conclude that purchasing from a specific set of suppliers is beneficial in the just-in-time case in opposition to using a single supplier to acquire all products. The solution of the proposed model distributes the requests for the selected suppliers. While in the unique supplier case, a deterministic model finds the optimal number of requests.

Besides, the lot sizing problem can be presented as an extension of the supplier selection problem. Sometimes both problems appear as an individual integrated problem. Basnet and Leung (2005) solved the integrated uncapacitated multi-period lot sizing and supplier selection problem without constraints on the number of suppliers. The solution of the model links the suppliers and products in an integrated manner. They developed an enumeration procedure that is not fast for large size instances but proposed a search heuristic that is particularly well suited to these practical instances.

The review paper of Aissaoui et al. (2007) addressed the integrated lot sizing and supplier selection problem, and it emphasized the relevance of this integrated modeling for minimizing costs. The references therein highlight that the success in growing quality and decreasing costs involves maintaining close and lasting relationships with suppliers. Also, a significant number of papers reported to the case in which each supplier provides only one item, and there is no inventory of items over the planning horizon.

Some studies analyzed the trade-offs among the various criteria from multiobjective optimization approaches. Ustun and Demirtas (2008) proposed a multiobjective linear model that maximizes the total purchase value and minimizes the cost and the defect rate. The decision maker aims to obtain with this proposal the satisfaction of purchasing certain products from a preferred supplier. Computational tests conducted with a numerical example concluded that the Reservation Level Tchebycheff procedure (RLTP) was the most suitable. Also, RLTP can give flexible choices to the decision-maker from a proposed multiobjective model.

Hybrid algorithms have been also developed to lead with models concerning the lot sizing and supplier selection problems. Such algorithms comprise approximation and multicriteria methods, besides involving optimization models with discrete variables. Mendoza et al. (2008) introduced a framework shared into the following three stages. A comparison from a based-distance performance metric among suppliers provides a reduced initial list of suppliers. AHP method sorts this list to give the best suppliers.

Rezaei and Davoodi (2011) pondered that purchase costs reduce when increasing the stable relationship with the suppliers. Thus, they introduced two multiobjective mixed integer nonlinear programming models for which the purchase cost gradually reduces when the number of requests for a unique supplier increases. In this sense, it is interesting to have a few suppliers with multiple requests than several suppliers with a few requests.

Kilic (2013) applied a two-step framework to the production of air filters. The multicriteria Fuzzy-TOPSIS method provides a score for each available supplier that converts to a parameter of an integer linear model. Then the solution of this model gives a better match between items and suppliers to meet the demand. The constraints of the model involve the supplier capacity and limited use of suppliers, and due to technological limitations, each supplier could not produce all items.

Gonçalo and Alencar (2014) introduced a two-stage method to solve the supplier selection problem applied to a Brazilian distribution center. The multicriteria PROMETHEE II approach determines the most meaningful products and services. Then the solution of a mathematical formulation establishes the link among suppliers and items, with priority for those items with critical requests in the first stage.

Ghaniabadi and Mazinani (2017) studied a single item model with multiple suppliers and backlogging, for which it is applicable a discount on the total quantity of the product purchased, and it is free of constraints on the number of suppliers. A dynamic programming algorithm was refined to improve the solutions obtained from commercial software. A particular optimal solution of the model showed that it is possible to buy from a single supplier.

Interviews with managers performed by Alegoz and Yapicioglu (2019) confirmed that the use of various criteria is essential to select the suppliers and to allocate the quantity of purchased products from each of them. Thus, the authors proposed a hybrid framework composed of a trapezoidal type-2 fuzzy-TOPSIS and a goal programming model that considers as goals the capacity, quality, and discount. Computational experiments disclosed the trade-offs among the measured goals and produced solutions that satisfied all the requests and constraints.

It is known that for reducing costs, improving quality, and sharing profits along the supply chain requires to keep stable relationships with suppliers. In contrast, few studies examine the number of suppliers needed and, in particular, the connections among the suppliers and products in the direction of providing flexibility in the purchase of products. The next section outlines the reported papers concerning the aspects of the process flexibility in the lot sizing problem considering a deterministic context.

2.2 Process flexibility in the lot sizing problem

Several researchers have studied the value of flexibility configuration from the different stochastic point of views. However, only some recent studies have considered the value of the process flexibility, considering a deterministic context. Note that a limited amount of process flexibility among all flexibility configurations can significantly influence the manufacturing systems.

Jans and Degraeve (2004) planed the production of a tire industry through a lot sizing model. The conception of a type of tire requires rigorous management of processes. At the start of the operations, pre-heated molds receive the raw material. Next, powerful heaters are adjusted (machine setup) to accommodate the molds. In the addressed problem, there is a unique mold for each combination of heater and raw material, and each heater has a specific efficiency. As the different tires compete for the same sources, the limited combination of resources to produce these tires create a limited flexibility configuration. Due to the quality and capacity of the labor force, a new machine setup occurs only at the end of a cycle of operations (small bucket model). A proposed decomposition method solved the model, for which the solutions that generate the production plan for a type of tire incorporate some flexibility configurations.

Xiao et al. (2015) studied the capacitated lot sizing problem with parallel machines, setup dependent, and time window in the semiconductor industry. For which each machine may not produce all items. The machine setup time between different items is sequence-dependent, and the delivery beyond the due time may incur backlog penalty costs. Old machines may become obsolete for processing new items. At the same time, it continuously introduces new items and machines in the system. Thus, not all parallel machines in operation are eligible for every incoming item. A certain subset of eligible machines can only process each item. A proposed hybrid heuristic based on Lagrangian relaxation and simulated annealing method outperformed the numerical results observed in the literature. The solutions of the model connect the eligible machines to the items by satisfying a particular set of constraints that reducing the flexibility configurations.

Fiorotto et al. (2018) studied the process flexibility and the chaining principle in lot sizing problems by analyzing the value of the machine flexibility in balanced systems (the number of items and machines is equal). The complete flexibility configuration arises when each machine can produce all products. The limited flexibility configurations (e.g., clustered and different long chains) appear when the production of the machine restricts in some products (limited links). The comparison of different limited flexibility configurations concluded that the benefits of the best long chain and the complete flexibility configurations are practically the same. Also, when the flexibility value of the machine is a decision variable of model, it is possible to obtain a new configuration with a smaller amount of links than the best chain configuration that gets the same benefits of complete flexibility configuration. Finally, they also pointed out that the importance of flexibility value increases when the data are heterogeneous.

Catelan et al. (2020) also studied the machine flexibility in a deterministic context and presented three heuristics to determine the links between items and machines, i.e., to fix which items can be produced on each of the machines to an optimization model proposed by Fiorotto et al. (2018) which optimizes the flexibility configuration subject to an additional global budget. The quality of the solutions obtained by the heuristics is better for problems with many items and machines. They pointed out that although the model can be used to analyze the value of different flexibility configurations within a given limited budget, the computational results presented by Fiorotto et al. (2018) showed that this analysis might be impaired for medium and in particular for large size instances because the solutions of the model produced by a high-performance MIP software present a large optimality gap. Therefore, the proposed solution methods overcome the difficulties in the analysis of the value of flexibility presented in some computational results proposed by Fiorotto et al. (2018). Finally, these methods can be used to make some more consistent analysis of the value of flexibility for the deterministic lot sizing problem with backlog when considering large instances.

Teixeira et al. (2020) extended the analysis of machine flexibility proposed by Fiorotto et al. (2018) by considering the integrated lot sizing and transportation problem. More specifically, they look at this problem in the context of a network of existing plants that are (or can be) configured to make one or many different products. In this context, there is a network of customers and specific transportation costs between each plant and customer. The decision on which plants to upgrade and to which type(s) of product now also has to take into account the trade-off with the transportation cost and hence the geographical dispersion of the demand. The computational experiments with small instances showed that it is necessary approximately 50% of the total links to obtain the same benefits of the complete flexibility. Moreover, they observed that the model is very difficult to solve, and it disrupted the analysis for medium and large instances because the optimality gaps found for these instances were relatively large.

3 Problem formulation

This section presents a mathematical formulation to the integrated capacitated multi-period lot sizing and supplier selection problem with backlog in meeting the demand of customers. The planning horizon is finite and subdivided into macro-periods. The suppliers have predetermined delivery capacity and a limited amount of flexibility to deliver only certain types of products. Moreover, each customer has its particular demand that can be satisfied with a backlog. Furthermore, to purchase a certain amount of products from a supplier, an order cost to activate the supplier must be paid. The purchase planning must select the required quantity of suppliers respecting the delivery capacity of these suppliers to meet the customer demands per period. The goal of the problem is to find a purchase planning that meets all constraints by maximizing the total profits.

Figure 1 illustrates the studied problem with 6 suppliers, 6 products, $|K|$ customers, and three different flexibility configurations. The first case (case (a)) presents the dedicated configuration in which each supplier delivers only one product. Note that in this configuration, the number of links (amount of flexibility) in the system is equal to the number of products and suppliers. Case (b) shows the well known long-chain configuration. In this flexibility configuration, each supplier delivers 2 products, and the total number of links is equal to 12. Finally, in case (c), we have the complete flexibility where each supplier delivers all products. Therefore all links are present in the system (36 links).

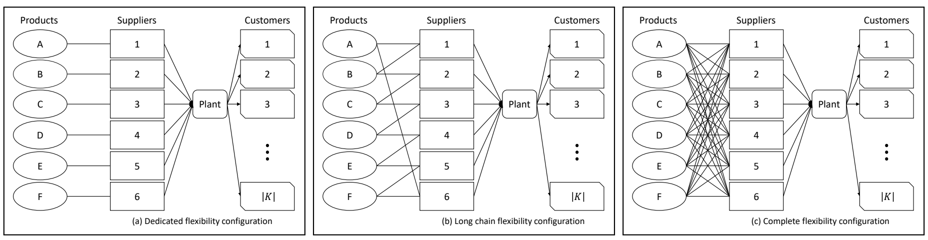
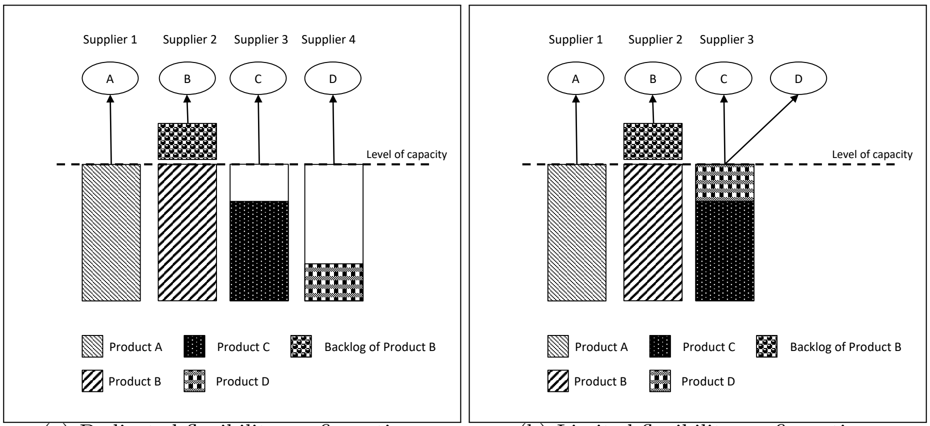


Fig. 1 Different configurations for the product acquirement flexibility

We highlight the different levels and environments involved in decision-making in this modeling. The supplier's decision environment deals with the available capacity and which items can be sold. On the other hand, the plant's decision environment chooses each supplier and the amount to be purchased. Note that different product acquirement flexibility configurations correspond to suppliers with different available products to be delivered. Moreover, although the supplier capacity is not decided by the plants, varying the supplier capacity allows analyzing and discussing different possibilities of supplier selection and to study the benefits of the product acquirement flexibility. A practical example of this problem in a typical organization is the markets that have distribution centers. Markets purchase a range of products from a selected set of suppliers and store them in their distribution centers until the products are delivered to the sales centers. Here, the sales centers can be seen as the customers in this problem.



(a) Dedicated flexibility configuration (b) Limited flexibility configuration

Fig. 2 Value of the product acquirement flexibility on the reduction of selected suppliers

It is also important to note that by adding flexibility the number of selected suppliers can be reduced by recombining the capacity utilization of suppliers that can improve the total cost. An advantage of the proposed study is to determine product acquirement flexibility configurations that perform with a decreasing number of selected suppliers while pondering the meeting of demand and total backlog in each period. Note, for example, that a reduced amount of suppliers offers savings from simplifying supplier management.

Figure 2 illustrates an example that combines the capacity utilization of two suppliers and gives a new product acquirement flexibility configuration from Case (a) to Case (b) of the figure. Case (a) illustrates a dedicated flexibility configuration (each supplier delivers a single product), where the plant uses all the available supplier capacity of the Suppliers 1 and 2. On the other hand, the Suppliers 3 and 4 have partial use of the capacities, 2/3, and 1/3 of them, respectively. Figure 2 also shows that adding the possibility of product acquirement flexibility (Supplier 3 can deliver product C and D) leads to a reduction on the number of selected suppliers (it is not necessary to use Supplier 4).

Next, the mathematical formulation for the integrated lot sizing and supplier selection problem is presented. We consider the following sets and input parameters used in the mathematical formulation.

I set of products (index i);
J set of suppliers (index j);
K set of customers (index k);
T set of time periods (index t);
p_{ikt} unit retail price of product type i for customer k in period t;
ac_{ijt} unit purchase cost for product type i from supplier j in period t;
oc_j order cost for products from supplier j;
hc_{it} unit inventory holding cost for product type i in period t;
w_{ikt} unit backlog cost of product type i for meeting the demand of customer k in period t;
d_{ikt} demand of product type i for customer k in period t;
Cap_{jt} capacity available from supplier j in period t;
F_{max} maximum amount of product acquirement flexibility;
M large number.

The decision variables are then defined as follows:

x_{ijt} purchase quantity of product type i from supplier j in period t;
y_{ikt} delivered quantity of product type i for customer k in period t;
z_{ikt} backlogged quantity of product type i for customer k in period t;
h_{it} inventory quantity of product type i in period t;
u_{ij} binary variable indicating if the supplier j has flexibility to deliver product type i;
s_j binary variable indicating if the supplier j is chosen.

The proposed mathematical formulation is as follows:

$$\text{Max} \sum_{i\in I}\sum_{t\in T}\left(\sum_{k\in K}(p_{ikt}y_{ikt} - w_{ikt}z_{ikt}) - \sum_{j\in J} ac_{ijt}x_{ijt}\right) - \sum_{i\in I}\sum_{t\in T} hc_{it}h_{it} - \sum_{j\in J} oc_j s_j \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in J} x_{ijt} + h_{i(t-1)} = \sum_{k\in K} y_{ikt} + h_{it} \quad i \in I, \quad t \in T; \tag{2}$$

$$y_{ikt} + z_{ikt} = d_{ikt} + z_{ik(t-1)}, \quad i \in I, \quad k \in K, \quad t \in T; \tag{3}$$

$$\sum_{i\in I}\sum_{t\in T} x_{ijt} \leq Ms_j, \quad j \in J; \tag{4}$$

$$\sum_{t\in T} x_{ijt} \leq Mu_{ij}, \quad i \in I, \quad j \in J; \tag{5}$$

$$\sum_{i\in I} x_{ijt} \leq Cap_{jt}, \quad j \in J, \quad t \in T; \tag{6}$$

$$\sum_{i\in I}\sum_{j\in J} u_{ij} \leq F_{max}; \tag{7}$$

$$u_{ij} \leq s_j, \quad i \in I, \quad j \in J; \tag{8}$$

$$z_{ik0}, \ h_{i0} = 0, \quad i \in I, \quad k \in K; \tag{9}$$

$$x_{ijt}, \ y_{ikt}, \ z_{ikt}, \ h_{it} \geq 0, \quad i \in I, \quad j \in J, \quad k \in K, \quad t \in T; \tag{10}$$

$$u_{ij}, \ s_j \in \{0,1\}, \quad i \in I, \quad j \in J. \tag{11}$$

The objective function (1) maximizes the total profit, which consists of the total retail price less the purchase, inventory, order, and backlog costs. Constraints (2) and (3) guarantee the inventory balance in each period. Demand that cannot be satisfied on time can be backlogged. Constraints (4) guarantee that there is purchase of products from a supplier j if this supplier is active. Next, Constraints (5) ensures that a supplier can deliver a specific product in a specific period only if this supplier has the flexibility to sell this product. Constraints (6) are the capacity constraints of each supplier. The amount of flexibility available is limited by (7). Constraints (8) includes the relations between the product acquirement flexibility and the suppliers. Constraints (9) fixes the initial values for the holding and backlogging levels. Finally, the sets of constraints (10) and (11) define the domain of the variables.

4 Computational experiments

4.1 Setup of the computational tests

In order to perform the computational experiments, we have adapted a standard data set proposed by Trigeiro et al. (1989). We used the problem sets F01–F20 and G51–G60 to create our instances. More specifically, we have used 5 instances from the set F01–F20 (with 6 products and 15 periods), 5 instances from the set G51–G55 (with 12 products and 15 periods) and 5 instances from the set G56–G60 (with 24 products and 15 periods). For each of the 15 problem instances, we created identical supplier problems, i.e., the capacities and order costs are the same for each supplier. Note that different from the problem instances proposed by Trigeiro et al. (1989), in this paper, we consider several costumers (with their particular demands). This study considers that the number of suppliers is equal to the number of products, and the demand for each customer is the same. In this way, it is possible to analyze the results with the dedicated configuration in which each supplier can deliver only one product (this flexibility configuration has n links). The other levels of flexibility analyzed are presented in Table 1. Observe that, in Constraint (7), F_{max} is equal to the number of links that are allowed to add (amount of product acquirement flexibility). It gives a limitation on the total number of links that can be used that imposes a limit on the number of supplier-product combinations that are allowed.

For the data sets F01–F20, G51–G55 and G56–60, the original capacities level are 728, 1456 and 2912, respectively. In this paper the choice of the supplier capacity levels was based on preliminary tests and varies from 500 to 4500 to have a broad range of problems. As such, by changing the supplier capacity level each test problem resulted in 6 different supplier selection test problems. As a result, 90 different test problems were created. Furthermore, the retail price of each product (p_{ikt}) was fixed to 300 for all

Table 1 Number of links in each flexibility configuration

	# links (F_{max})	n	$n + n/2$	$2n$	n^2
# products		Dedicated	Flexibility 1	Flexibility 2	Total flexibility
6		6	9	12	36
12		12	18	24	144
24		24	36	48	576

costumers and the purchase cost (ac_{ijt}) is 15% of this value, that is 45 per product. Finally, we set the backlog costs (w_{ikt}) for each product equal to $2000/|I|$.

The formulations were modeled in Python 3.6 using the concert technology and CPLEX 12.9 as solver. The tests were done on a computer with 2 Intel(R) Xeon(R) X5675 processors, 3.07 GHz with 96 GB of RAM and the Linux operating system. Moreover, when solving the formulations, we have limited the computational time for each instance to one hour (3600 seconds).

4.2 Analysis of product acquirement flexibility

In this section we analyzed the concept of product acquirement flexibility in the integrated lot sizing and supplier selection problem. The base case for comparison is the case in which each supplier is dedicated to deliver exactly one product. In the deterministic environment, the value of flexibility is apparent if, for this case, it is possible to find solutions with fewer suppliers (saving a supplier activation/order cost) or not all of the demand can be satisfied on time (leading with backorders). In such a case, adding product acquirement flexibility (i.e., some suppliers can deliver certain types of products instead of just one) can decrease the total cost. The objective of the computational experiments presented in this section is to analyze the benefits of different levels of product acquirement flexibility.

4.2.1 Base case experiments

In Table 2 we consider different levels of supplier capacity (Cap) and give the average of the relative total cost (columns RTC), the total number of selected suppliers (columns NS), the total backlog (columns Backlog), the optimality gaps measured in percentage (columns Gap) and the CPU times (columns T(s)) found for the instances and flexibility configurations. Moreover, we set the total cost of the dedicated configuration as the referential value (equal to 100%) and calculate the RTC of the other flexibility configurations related to these values.

Observe that throughout the discussion, the value of flexibility is measured as the percentage decrease in total cost compared to the dedicated configuration. Table 2 shows that, as expected and reported by Jordan and Graves (1995), the value of flexibility depends on the capacity levels. We see that considering flexibility allows to decrease the total number of selected suppliers significantly. Note that the reduction is on average approximately 16.7%, 13.3% and 20.4% for 6, 12 and 24 products and suppliers, respectively. Note that the total number of selected suppliers considering Flexibility 1 and Flexibility 2 is pretty much the same as Total flexibility. Moreover, it is important to note that different from the Total flexibility, considering Flexibility 1 and Flexibility 2 the selected suppliers deliver only a small amount of products. In practice, it allows a significant reduction of the supplier management costs. Figure 3 graphically illustrates the value of flexibility for the instances with 12 products and suppliers for the flexibility configurations. For all levels of flexibility used, we see that the value of product acquirement flexibility is the highest for low levels of flexibility where the solutions present a significant amount of backlog. It is also interesting to see that although for medium levels of supplier capacity the benefits of flexibility are very small, it increases again for high levels of supplier capacity. It happens because, for these supplier capacity levels, considering the possibility of flexibility configurations allows us to decrease the number of selected suppliers.

Regarding the total number of backlog, Table 2 shows that the Flexibility 1, Flexibility 2 and Total flexibility have similar levels for most instances. Observe that for some high supplier capacity levels the solutions of the dedicated configuration do not present backlog and the solutions of the flexibility configurations present backlog (for example, 24 products and supplier capacity equal to 4500). It occurs because a small amount of backlog is preferable rather than using a new supplier. In practice, this can occur when the supplier produces a customized product that becomes the selection of a new supplier an

Table 2 Numerical results for base case experiments

products	Cap	Dedicated			Flexibility 1			Flexibility 2			Total flexibility		
		RTC	NS	Backlog	RTC	NS	Backlog	RTC	NS	Backlog	RTC	NS	Backlog
6	500	100	6	96259	100	6	96259	100	6	96259	100	6	96259
	600	100	6	29675	89.73	6	26123	88.90	6	25883	88.90	6	25883
	700	100	6	2081	97.41	6	1811	97.28	6	1802	97.25	6	1800
	1000	100	6	-	98.19	5	-	98.19	5	-	98.19	5	-
	1100	100	6	-	96.37	4	-	96.37	4	-	96.37	4	-
	1450	100	6	-	94.56	3	-	94.56	3	-	94.56	3	-
Average		100	6	21336	96.04	5	20699	95.88	5	20657	95.88	5	20657
12	1150	100	12	30610	73.55	12	2802	67.40	12	322	67.21	12	317
	1250	100	12	2892	93.82	12	179	92.16	12	-	92.06	12	-
	1350	100	12	406	98.29	11.4	32	97.69	11	24	97.57	11	-
	1450	100	12	39	98.92	11	-	98.51	10.2	6	98.44	10.2	-
	1650	100	12	-	97.58	9.2	-	97.34	9	-	97.30	9	-
	1850	100	12	-	96.51	8.2	14	96.37	8	-	96.36	8	-
Average		100	12	5658	93.11	10.6	505	91.58	10.4	59	91.49	10.4	53
24	2000	100	24	613568	82.65	24	462614	77.43	24	422578	76.99	24	417864
	2400	100	24	66504	89.23	24	20621	83.03	24	883	82.95	24	840
	2850	100	24	1586	98.68	22	464	98.19	21	319	98.05	21	1
	3500	100	24	-	97.24	18.2	-	96.64	17	71	96.62	17	71
	4000	100	24	-	95.84	15.2	8	95.64	15	27	95.60	15	-
	4500	100	24	-	95.00	13.4	202	94.94	13.4	139	94.91	13.4	149
Average		100	24	113610	93.11	19.5	80652	90.98	19.1	70670	90.85	19.1	69821

products	Cap	Dedicated		Flexibility 1		Flexibility 2		Total flexibility	
		Gap	T(s)	Gap	T(s)	Gap	T(s)	Gap	T(s)
6	500	0	0.02	0	0.2	0	0.2	0	0.07
	600	0	0.01	0	26	0	0.8	0	0.4
	700	0	0.02	0	21	0	0.2	0	0.3
	1000	0	0.02	0	0.5	0	0.3	0	0.3
	1100	0	0.02	0	0.3	0	0.4	0	0.3
	1450	0	0.02	0	0.2	0	0.3	0	0.1
Average		0	0.02	0	8	0	0.4	0	0.3
12	1150	0	0.04	0.7	3600	0.02	2179	0	0.1
	1250	0	0.04	0.1	3600	0.01	69	0	0.3
	1350	0	0.04	0.04	3600	0.01	19	0	0.3
	1450	0	0.04	0.03	2881	0	15	0	0.6
	1650	0	0.04	0.02	2908	0	11	0	0.5
	1850	0	0.04	0.01	968	0	4	0	0.5
Average		0	0.04	0.1	2926	0.01	383	0	0.4
24	2000	0	0.1	1	3600	0.1	3600	0	0.4
	2400	0	0.1	0.7	3600	0	3511	0	0.7
	2850	0	0.1	0.04	3600	0	49	0	2
	3500	0	0.1	0.04	3600	0	20	0	3
	4000	0	0.1	0	2721	0	73	0	3
	4500	0	0.1	0	485	0	40	0	3
Average		0	0.1	0.3	2934	0.02	1216	0	2

expensive and difficult task. Moreover, our results show that the benefits of a very limited amount of flexibility (Flexibility 1) are close to the ones obtained by Total flexibility (the highest average performance difference is 2.26%). Considering the Flexibility 2 the performance difference compared to Total flexibility is almost 0 for all instances.

With respect to the optimality gaps, we observe that they are zero or very small for all configurations. In terms of the computational times, we see that the problem with dedicated configuration is much faster to solve than any other flexibility configuration. Furthermore, by increasing the level of flexibility the computational times decrease significantly.

4.3 Sensitivity analysis for the product acquirement flexibility

It is well known from the stochastic literature that the process flexibility has a better performance for homogeneous cases. In this section, we analyze the behavior of the product acquirement flexibility for the deterministic integrated lot sizing and supplier selection problem by varying the homogeneity in different ways. More specifically, we analyze separately the purchase cost and demand heterogeneity. Additionally, we analyze the case with these two factors together.

4.3.1 Purchase cost heterogeneity

In this section, we analyze the benefits of the flexibility by considering the purchase cost heterogeneity. In order to create a case with purchase cost heterogeneity, we use the same data sets considered in the previous section (F01–F20 and G51–G60) and set the purchase cost for each product equal to 15 times the inventory holding cost. In all the instances, each product has a different inventory holding cost, taken from a discrete uniform distribution between 1 and 5. As such, products and suppliers have different

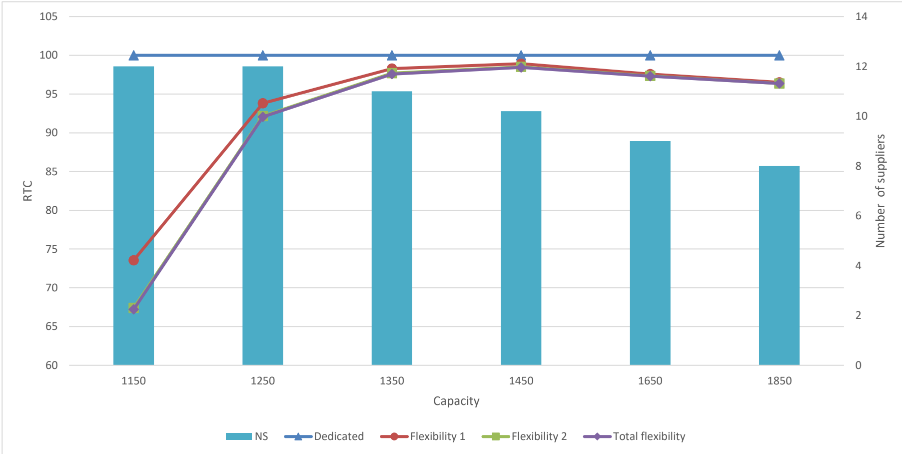


Fig. 3 Base case experiments: general results with 12 products and suppliers

purchase costs in the adapted instances. In this case there are cheaper suppliers than others which makes the problem more similar to what happens in practice.

Table 3 shows the average of the relative total cost, total number of selected suppliers, total backlog, optimality gaps and CPU times found for all flexibility configurations considering purchase cost heterogeneity. Note that in practice, although there are other used criteria, in general, the purchase costs have great importance to the selection of the suppliers. Moreover, when the products are very similar the purchase costs are the most important criteria to select the suppliers. The global analysis shows that different from the results with purchase cost homogeneity, the benefits of flexibility are highest for low and high levels of supplier capacity. For medium levels of supplier capacity the benefits of flexibility are still very significant (bigger than 7%, 11% and 26% for 6, 12 and 24 products and suppliers, respectively). It shows that the value of flexibility becomes more important in this scenario. It happens because with purchase cost heterogeneity there is the possibility to choose only the cheaper suppliers.

Regarding the number of selected suppliers, for the instances with 6 and 12 products and suppliers, there is a very similar trend compared to the case with purchase cost homogeneity. For the instances with 24 products and suppliers, we observe, however, a small difference. Considering the case with purchase cost homogeneity, the reduction is on average 20.4% for these instances, while with purchase cost heterogeneity the reduction is on average 22.9%. The reason is that for the instances with 24 products and suppliers, the backlog cost is lower than the instances with 6 and 12 products and suppliers (it is equal to $2000/|I|$). In such case it is better to focus on do not use expensive suppliers. Note that the level of backlog is much higher than the problem with purchase cost homogeneity for these instances.

Figure 4 illustrates clearly the general trend for the relative total cost for the instances with 12 products and 12 suppliers. This figure shows that for all supplier capacity levels the benefits of flexibility are bigger compared to the case with purchase cost homogeneity (Figure 3). Considering the case with purchase cost homogeneity the value of flexibility is smaller than 3% for the supplier capacity range of 1350 to 1650 (as shown in Figure 3), while with purchase cost heterogeneity (as shown in Figure 4), the value of flexibility is at least 11%. Furthermore, although there is a significant difference between the benefits of Total flexibility and Flexibility 1 specially for instances with low and medium supplier capacity levels, the benefits of Flexibility 2 are very similar compared to Total flexibility. It shows that with purchase cost heterogeneity, as in the base case experiments, a limit amount of flexibility is enough to find similar benefits of Total flexibility.

With respect to the optimality gaps, they are again zero or very small for all configurations. Regarding the computational times, the Dedicated and Total flexibility configurations are much faster to solve than the Flexibility 1 and Flexibility 2.

4.3.2 Demand heterogeneity

Using the ideas proposed by Fiorotto et al. (2018), in order to create a demand heterogeneity case, we have changed the demand distribution of the products. In the base case experiments, each product had the same average demand. In this new case, we increased the demand for the first half of the products

Table 3 Numerical results for purchase cost heterogeneity

products	Cap	Dedicated			Flexibility 1			Flexibility 2			Total flexibility		
		RTC	NS	Backlog	RTC	NS	Backlog	RTC	NS	Backlog	RTC	NS	Backlog
6	500	100	6	96259	100	6	96259	100	6	96259	100	6	96259
	600	100	6	29675	89.84	6	26123	89.03	6	25883	89.03	6	25883
	700	100	6	2081	93.77	6	1811	93.05	6	1800	93.00	6	1800
	1000	100	6	-	79.78	5	-	79.23	5	-	79.19	5	-
	1100	100	6	-	74.41	4	-	74.23	4	-	74.21	4	-
	1450	100	6	-	64.75	3	-	64.72	3	-	64.72	3	-
Average		100	6	21336	83.76	5	20699	83.38	5	20657	83.36	5	20657
12	1150	100	12	30610	71.99	12	2496	65.91	12	325	65.70	12	317
	1250	100	12	2892	90.48	12	236	87.59	12	-	87.46	12	-
	1350	100	12	406	90.65	11.2	31	88.86	11	-	88.69	11	-
	1450	100	12	39	88.01	11	-	85.92	10.2	-	85.83	10.2	-
	1650	100	12	-	79.42	9.2	36	78.02	9	-	77.92	9	-
	1850	100	12	-	72.13	8.0	24	71.51	8	-	71.42	8	-
Average		100	12	5658	82.11	10.5	470	79.63	10.4	54	79.50	10.4	53
24	2000	100	24	845109	82.7	23	697136	79.07	22.8	681248	78.84	22.8	680486
	2400	100	24	348806	83.41	23	175762	77.25	22.8	166752	76.64	22.8	166738
	2850	100	24	303181	79.02	21	16854	74.01	20	12341	73.77	20.4	12214
	3500	100	24	301906	63.83	17	1483	61.28	17	-	61.16	17	-
	4000	100	24	301906	55.82	15	298	54.58	15	6	54.50	15	-
	4500	100	24	301906	50.12	13	149	49.64	13	7	49.62	13	-
Average		100	24	400469	6915	18.6	148614	65.97	18.5	143392	65.75	18.5	143240

products	Cap	Dedicated		Flexibility 1		Flexibility 2		Total flexibility	
		Gap	T(s)	Gap	T(s)	Gap	T(s)	Gap	T(s)
6	500	0	0.02	0	0.1	0	0.1	0	0.07
	600	0	0.01	0	356	0	0.5	0	0.07
	700	0	0.01	0	328	0	0.4	0	0.09
	1000	0	0.02	0	72	0	0.3	0	0.1
	1100	0	0.02	0	9	0	0.2	0	0.1
	1450	0	0.02	0	0.1	0	0.1	0	0.09
Average		0	0.02	0	128	0	0.3	0	0.09
12	1150	0	0.04	0.7	3600	0	2857	0	0.08
	1250	0	0.04	0.2	3600	0	756	0	0.2
	1350	0	0.04	0.1	3600	0	762	0	0.2
	1450	0	0.04	0.1	3600	0	674	0	0.2
	1650	0	0.04	0.08	3600	0	6	0	0.2
	1850	0	0.04	0.04	3599	0	4	0	0.2
Average		0	0.04	0.2	3600	0	843	0	0.2
24	2000	0	0.1	0	2883	0	2881	0	0.5
	2400	0	0.1	0.5	3600	0	2904	0	0.5
	2850	0	0.1	0.5	3600	0	2930	0	0.7
	3500	0	0.1	0.2	3600	0	1379	0	0.7
	4000	0	0.1	0.09	3600	0	73	0	0.8
	4500	0	0.1	0.03	3600	0	33	0	0.7
Average		0	0.1	0.2	3481	0	1700	0	0.7

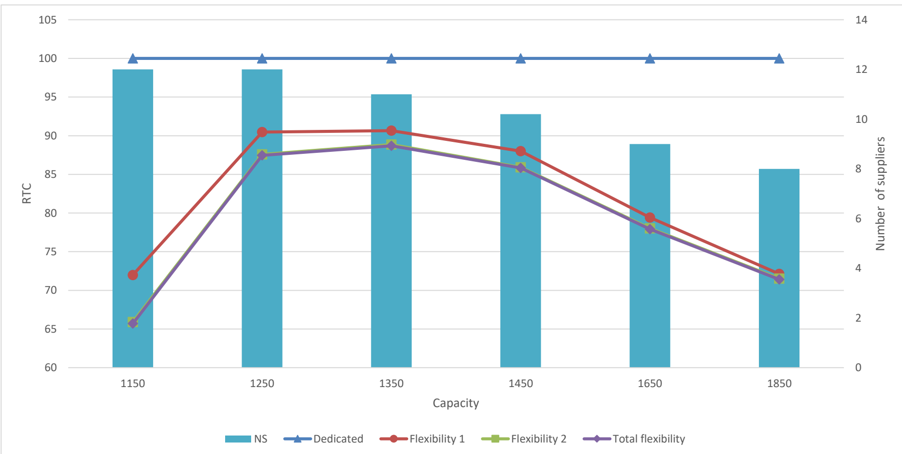


Fig. 4 Purchase cost heterogeneity: general results with 12 products and suppliers

by 50% and decreased the other half by 50%. Note that the purchase costs for all products are fixed to 45 as in the base case experiments. This case can happen in practice when the products have a seasonal demand.

Table 4 shows the average of the relative total cost, total number of selected suppliers, total backlog, optimality gaps and CPU times found for all flexibility configurations considering demand heterogeneity. In terms of the total number of selected suppliers, we see that the reduction considering flexibility is much smaller compared to the base case experiments (it is on average only 3.3%, 0% and 4%). It occurs because the high concentration of the demand in the first periods forces a selection of almost all suppliers in order to try to avoid very high backlog costs. This means that in scenarios containing seasonal demand, it is interesting to keep contracts with a more substantial number of suppliers compared to scenarios in which the demand has not high variations.

Table 4 Numerical results for demand heterogeneity

products	Cap	Dedicated			Flexibility 1			Flexibility 2			Total flexibility		
		RTC	NS	Backlog	RTC	NS	Backlog	RTC	TS	Backlog	RTC	NS	Backlog
6	500	100	6	324389	100	6	324389	100	6	324389	100	6	324389
	600	100	6	252389	100	6	252389	100	6	252389	100	6	252389
	700	100	6	180389	100	6	180389	100	6	180389	100	6	180389
	1000	100	6	11110	88.24	6	8252	87.54	6	8126	87.54	6	8126
	1100	100	6	-	99.99	6	-	99.99	6	-	99.99	6	-
	1450	100	6	-	97.75	5	-	97.75	5	-	97.75	5	-
Average		100	6	128046	97.66	5.8	127570	97.55	5.8	127549	97.55	5.8	127549
12	1150	100	12	708838	98.35	12	698573	98.31	12	698314	98.31	12	698314
	1250	100	12	569644	97.26	12	555710	97.02	12	554488	97.02	12	554488
	1350	100	12	433138	95.34	12	414360	94.83	12	412409	94.82	12	412394
	1450	100	12	302902	91.48	12	276683	90.44	12	273762	90.43	12	273757
	1650	100	12	91124	72.52	12	41465	67.22	12	35656	67.10	12	35633
	1850	100	12	7143	91.13	12	606	88.84	12	-	88.74	12	-
Average		100	12	352131	91.01	12	331233	89.44	12	329105	89.40	12	329098
24	2000	100	24	3452174	97.02	24	3390269	96.99	24	3389832	96.99	24	3389832
	2400	100	24	2357688	94.05	24	2262014	92.57	24	2238367	92.54	24	2237861
	2850	100	24	1210966	86.15	24	1061981	82.62	24	997799	82.19	24	991109
	3500	100	24	143338	85.75	24	47734	78.63	24	7920	78.27	24	7301
	4000	100	24	11562	96.54	23	4012	95.20	22.2	536	95.08	22.2	23
	4500	100	24	-	97.63	21	432	96.79	20	50	96.73	20	-
Average		100	24	1195955	92.86	23.3	1127740	90.47	23.03	1105751	90.30	23.03	1104354

products	Cap	Dedicated		Flexibility 1		Flexibility 2		Total flexibility	
		Gap	T(s)	Gap	T(s)	Gap	T(s)	Gap	T(s)
6	500	0	0.02	0	0.2	0	0.1	0	0.08
	600	0	0.02	0	0.1	0	0.1	0	0.07
	700	0	0.02	0	0.1	0	0.1	0	0.07
	1000	0	0.02	0	214	0	0.3	0	0.1
	1100	0	0.01	0	0.2	0	0.2	0	0.1
	1450	0	0.02	0	0.4	0	0.3	0	0.2
Average		0	0.02	0	36	0	0.2	0	0.1
12	1150	0	0.05	0	948	0	1	0	0.08
	1250	0	0.05	0	2161	0	349	0	0.07
	1350	0	0.04	0.2	3600	0	42	0	0.08
	1450	0	0.04	0.3	3600	0	182	0	0.07
	1650	0	0.04	0.7	3600	0	1531	0	0.07
	1850	0	0.04	0.2	3600	0	38	0	0.2
Average		0	0.04	0	2918	0	357	0	0.1
24	2000	0	0.2	0	1730	0	2	0	0.4
	2400	0	0.1	0.8	3600	0	1455	0	0.4
	2850	0	0.2	1.3	3600	0	3045	0	0.4
	3500	0	0.1	0.9	3600	0	3532	0	0.6
	4000	0	0.1	0.1	3600	0	275	0	1
	4500	0	0.1	0.1	3600	0	27	0	2
Average		0	0.1	0.5	3288.3	0	1389.4	0	0.8

The results show that the benefits of flexibility are highest for high and medium supplier capacity levels. Figure 5 presents the relative total cost information for various supplier capacity levels for the case with 12 products and suppliers. Note that the value of the flexibility is quite different compared to the base case (Figure 3) in which for the highest supplier capacity level, Total flexibility configuration leads to a decrease of only 1.7% compared to the dedicated configuration. Furthermore, the benefits of flexibility are less than 5% for the supplier capacity levels from 1150 to 1350. This occurs because in this scenario all flexibility configurations present substantial levels of backlog for these supplier capacity levels because of the very high demand for the first half of the products. The supplier capacity level for which the highest value of flexibility is reached has shifted to the right (1650). At this level, we observe a decrease of more than 30% in total cost compared to the dedicated configuration.

The results also show that with demand heterogeneity considering Flexibility 1 and Flexibility 2 the selected suppliers deliver only a small amount of products, and it is enough to find similar benefits of

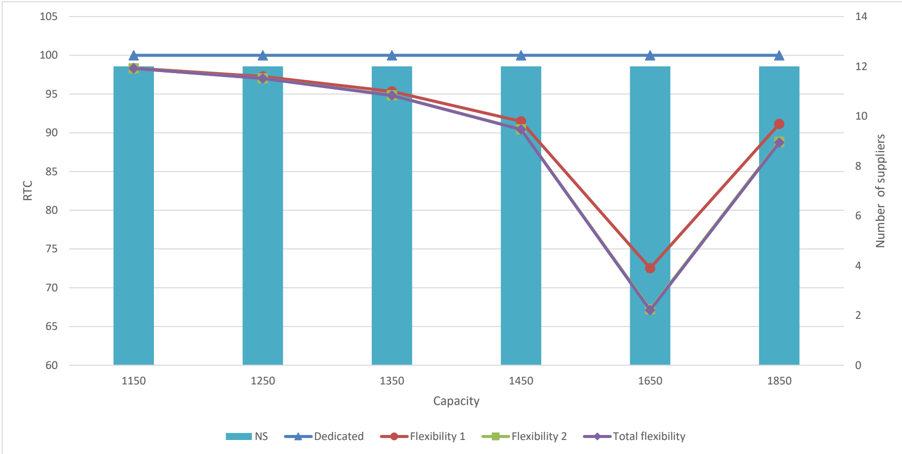


Fig. 5 Demand heterogeneity: general results with 12 products and suppliers

Total flexibility. The highest performance difference between Flexibility 1 and Total flexibility is 7.5% and considering Flexibility 2 the highest performance difference compared to Total flexibility is only 0.4%.

We also see that the total backlog is significantly larger than the total backlog in the base case experiments, especially for the instances with low and medium supplier capacity levels for all flexibility configurations. However, as in the base case experiments, Flexibility 1, Flexibility 2 and Total flexibility present similar levels of backlog for most instances.

Finally, the results show that the gaps and computational times of this scenario are similar to the base case experiments for almost all instances and flexibility configurations.

4.3.3 Purchase cost and demand heterogeneous

In order to create a case with high level of heterogeneity, we consider the two factors that have been analyzed (purchase cost and demand heterogeneity) together. Table 5 shows the average of the relative total cost, total number of selected suppliers, total backlog, optimality gaps and CPU times found for all flexibility configurations.

The results show that the benefits of flexibility are highest for high supplier capacity levels. As expected, the benefits of the flexibility is again very different compared to the base case. Note that for the highest supplier capacity level, Total flexibility configuration presents a decrease of approximately 20%, 15% and 24%, respectively for the instances with 6, 12 and 24 products and suppliers. These values were only approximately 5.5%, 3.7% and 5.1%, respectively for the base case. Furthermore, Table 5 indicates that the benefits of flexibility are on average 3.3% bigger than the base case. Regarding the total number of selected suppliers, the reduction is significantly smaller compared to the base case experiments. Upon further analysis, the results show that the highest average performance difference between Flexibility 1 and Total flexibility is 3.1% for the instances with 24 products and suppliers. It is important to note that for these instances the amount of flexibility presented by Flexibility 1 and Total flexibility are quite different (Flexibility 1 has only 36 links and Total flexibility has 576 links). Comparing Flexibility 2 and Total flexibility, the highest average performance difference decrease to only 0.24%. It shows that even for a very heterogeneous case, a very limited amount of flexibility is enough to find almost the same benefits as Total flexibility. We also see that the total backlog is much larger than the base case experiments for instances with low and medium supplier capacity levels. The gaps and computational times of this scenario are similar to the base case experiments for almost all instances and flexibility configurations.

Figure 6 presents the average relative total cost information for various supplier capacity levels for the case with 12 products and suppliers. We see that the shape of Figure 6 is similar to Figure 5 (demand heterogeneity). However, there are some important differences when comparing all the results of these two scenarios specially in terms of the average relative total costs for medium and high supplier capacity levels. More specifically, the benefits of the flexibility of the purchase cost and demand heterogeneity (Table 5) are bigger than the benefits of the demand heterogeneity (Table 4). This difference is on average 4.4%, 0.8% and 3.9% for the instances with 6, 12 and 24 products and suppliers, respectively. Note that

Table 5 Numerical results for purchase cost and demand heterogeneous

products	Cap	Dedicated			Flexibility 1			Flexibility 2			Total flexibility		
		RTC	NS	Backlog	RTC	NS	Backlog	RTC	TS	Backlog	RTC	NS	Backlog
6	500	100	6	324389	100	6	324389	100	6	324389	100	6	324389
	600	100	6	252389	100	6	252389	100	6	252389	100	6	252389
	700	100	6	180389	100	6	180389	100	6	180389	100	6	180389
	1000	100	6	11110	87.26	6	8253	86.18	6	8126	86.14	6	8126
	1100	100	6	-	93.25	6	-	92.39	6	-	92.36	6	-
	1450	100	6	-	80.76	5	-	80.23	5	-	80.18	5	-
Average		100	6	128046	93.54	5.8	127570	93.13	5.8	127549	93.11	5.8	127549
12	1150	100	12	708838	98.33	12	698542	98.30	12	698314	98.30	12	698314
	1250	100	12	569644	97.22	12	555602	97.00	12	554492	97.00	12	554488
	1350	100	12	433138	95.30	12	414385	94.79	12	412416	94.78	12	412394
	1450	100	12	302902	91.42	12	276827	90.34	12	273762	90.33	12	273757
	1650	100	12	91124	71.44	12	40421	66.37	12	35639	66.24	12	35633
	1850	100	12	7143	88.61	12	421	84.90	12	-	84.72	12	-
Average		100	12	352131	90.39	12	331033	88.62	12	329104	88.56	12	329098
24	2000	100	24	3730220	97.92	23	3678034	97.92	22.8	3677832	97.92	22.8	3677832
	2400	100	24	2683099	95.04	23	2590800	94.67	22.8	2583806	94.65	22.8	2583461
	2850	100	24	1585937	87.90	23	1408825	86.10	23	1392215	85.94	22.8	1391170
	3500	100	24	580920	91.05	22.8	314557	83.44	22.8	291733	82.74	22.8	291127
	4000	100	24	462924	85.76	21.8	104137	82.09	21	97132	81.71	21.2	97250
	4500	100	24	452858	79.54	20	3470	75.84	19.6	-	75.70	19.6	-
Average		100	24	1582660	89.54	22.2	1349971	86.68	22.03	1340453	86.44	22	1340140

products	Cap	Dedicated		Flexibility 1		Flexibility 2		Total flexibility	
		Gap	T(s)	Gap	T(s)	Gap	T(s)	Gap	T(s)
6	500	0	0.02	0	0.11	0	0.1	0	0.08
	600	0	0.02	0	0.10	0	0.1	0	0.08
	700	0	0.02	0	0.12	0	0.1	0	0.08
	1000	0	0.01	0	757	0	0.6	0	0.09
	1100	0	0.01	0	207	0	0.4	0	0.1
	1450	0	0.01	0	105	0	0.2	0	0.1
Average		0	0.02	0	178	0	0.2	0	0.09
12	1150	0	0.05	0	743	0	0.3	0	0.08
	1250	0	0.05	0	2172	0	8.4	0	0.08
	1350	0	0.05	0.2	3600	0	200	0	0.08
	1450	0	0.04	0.3	3600	0	110	0	0.07
	1650	0	0.04	0.6	3600	0	1750	0	0.07
	1850	0	0.04	0.2	3600	0	2096	0	0.1
Average		0	0.05	0.2	2886	0	694	0	0.08
24	2000	0	0.1	0	1441	0	2	0	0.5
	2400	0	0.1	0	2250	0	1389	0	0.4
	2850	0	0.1	0	2881	0	2881	0	0.4
	3500	0	0.2	0.8	3600	0	2985	0	0.5
	4000	0	0.1	0.3	3600	0	2982	0	0.5
	4500	0	0.1	0.5	3600	0	2830	0	0.6
Average		0	0.1	0.03	2895	0	2178	0	0.5

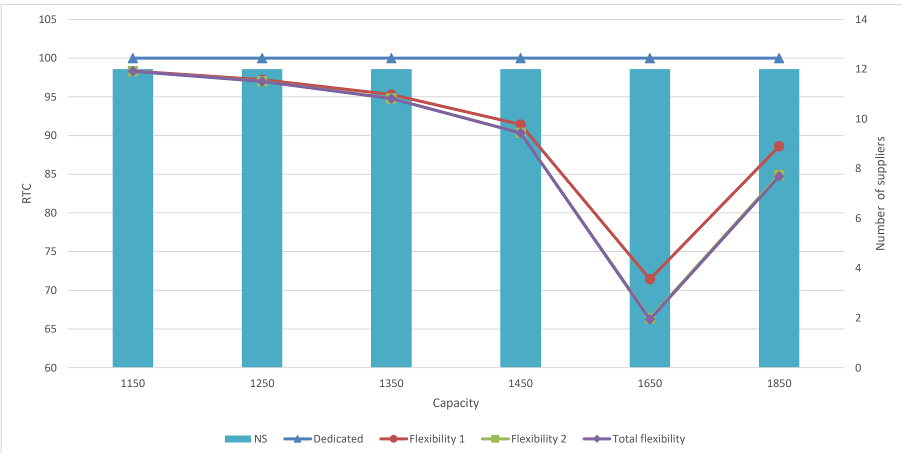


Fig. 6 Purchase cost and demand heterogeneous: general results with 12 products and suppliers

these differences come from the possibility of purchasing products from cheaper suppliers (purchase cost heterogeneity).

5 Conclusions

In this paper the integrated capacitated multi-period lot sizing and supplier selection problem with backlog in meeting the demand of customers in a deterministic context was studied. Different from the standard supplier selection problem, the studied problem considers a limited amount of product acquirement flexibility so that each supplier can deliver only certain types of products. In order to analyze the benefits of the flexibility we proposed a new optimization model. This model considers the possibility to study the benefits of choosing suppliers with some level of flexibility. Our computational experiments show that for the homogeneous setting and purchase cost heterogeneity, a very limited amount of flexibility is enough to reduce the total number of selected suppliers compared to the dedicated configuration significantly. However, for the case with demand heterogeneity, the number of suppliers of all flexibility configurations is the same as the dedicated configuration. The computational experiments also indicate that, in terms of total costs, a very limited amount of flexibility obtains almost all benefits of the total flexibility for all instances and scenarios. Moreover, we observe that the optimality gaps are zero or very small for all flexibility configurations and scenarios. Finally, by increasing the level of flexibility the computational times decrease significantly.

There are some interesting issues that can be explored as further research, for example, to extend the study to unbalanced systems in which the number of products and suppliers is not the same. It would also be interesting to focus on devising specific heuristics to add flexibility, i.e., given a level of flexibility (links between products and suppliers) the heuristic would determine a good way to distribute these links. Another possibility that can be explored is to study the two-level of flexibility with different plants. In this case each supplier and plant can distribute only certain types of products. The objective would be to analyze the value of flexibility and develop solution methods for this case. A final extension could be to study this problem considering a multi-criteria context. In such case, there is no enough supplier capacity to satisfy the demand of all costumers and different criteria can be used to determine the preferred costumers and what proportion of the demands will be satisfied.

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ANEXO B

Evaluating process flexibility in lot sizing problems: an approach based on multicriteria decision making

Evaluating process flexibility in lot sizing problems: an approach based on multicriteria decision making

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Abstract This paper presents a multicriteria analysis of the process flexibility in the context of the lot sizing problem with parallel machines. In the standard design for lot sizing problems, each machine can manufacture all products (total or complete flexibility). However, for several practical applications, it can be costly to install machines with complete flexibility. Therefore, it becomes interesting to implement only a limited amount of machine flexibility, where each machine can produce only a small number of different products. Recently, some works presented analyses of process flexibility by considering only the production cost as a criterion. However, the literature lacks a more comprehensive analysis that considers other essential criteria regarding the problem to compute the value of a flexibility configuration. Thus, we provide a detailed multicriteria analysis based on the TOPSIS method that produces a ranking of alternatives for the flexibility configurations. Extensive computational experiments and sensitivity analyses for different scenarios of the lot sizing problem compare individual flexibility configurations and evaluate its advantages in manufacturing planning.

Keywords Lot sizing problems · process flexibility · multicriteria analysis · TOPSIS method

1 Introduction

The intense competition for consumer markets has been placing increasing pressure on the manufacturing process of companies to produce quickly and customized products. Therefore, the optimization of the process becomes paramount to face a more complex and volatile environment, including a more diversified product portfolio, shorter product life cycles, and higher demand volatility (**?**). Indeed, the company can increase its competitiveness by implementing the operation strategy known as process flexibility to better match supply with uncertain demand.

In general, the process flexibility results from a company's ability to build different types of products in the same manufacturing plant or production facility at the same time (**?**). This adapted manufacturing process appears in several practical production planning problems. We consider the case of a deterministic multiperiod production planning where the products can be manufactured on different resources (plants or machines). In this situations, the problems are treated as deterministic lot sizing models.

The lot sizing problem with parallel machines involves determining the number of products to be manufactured from each machine in each period of a planning horizon. In the standard design for lot sizing problems, each machine can manufacture all products (total or complete flexibility). However, for several practical applications, it can be expensive to implement machines that have total flexibility. Thus, it becomes interesting to implement only a limited amount of machine flexibility so that each machine can produce a reduced part of products.

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Multiple authors addressed some approaches to analyze the effects of flexibility in the lot sizing problem by considering only the production total cost (or profit) as a criterion. However, the literature still lacks a more consistent analysis that considers other essential criteria of the lot sizing problem to determine whether a flexibility configuration is suitable to be implemented. A flexibility configuration can be defined as a distribution of links in a bipartite graph between machines/plants and products (see Figure 1). Some of the flexibility configurations, such as the long-chain configuration, present successful results in terms of the production cost. However, although the production cost is a fundamental operational criterion, other measures are relevant such as the capacity utilization and the setup time of the machine. Therefore, it is important to study the performance of different flexibility configurations from multicriteria methods.

This paper extends the results of **?** by analyzing the process flexibility for the lot sizing problem based on a ranking of alternatives. The technique for order preference by similarity to ideal solution (TOPSIS) produces a ranking for the different alternatives of flexibility configurations, allowing to examine the process flexibility. The TOPSIS method has been successfully used in several studies applied to industrial sectors (**?**) and, to the best of our knowledge, this is the first study that takes multicriteria approach into account for evaluating the advantages of implementing a particular flexibility configuration.

This paper aims to provide a detailed multicriteria analysis for flexibility configurations in different scenarios of the lot sizing problem with parallel machines. The main contributions include: i) the use of the TOPSIS method and the proposition of a multicriteria analysis on the process flexibility; ii) the examination of different flexibility configurations to determine the most suitable configuration in a multicriteria perspective; iii) the investigation of the ranking of alternatives by performing a sensitivity analysis by varying different parameters of the model; and iv) the performing of an extensive computational study to determine the advantages of different flexibility configurations. The computational results showed that limited flexibility configurations outperform the total flexibility in all scenarios. Moreover, different from the studies considering only the total cost as criterion, there are advantages of investing in flexibility for all capacity levels.

The remainder of this paper is organized as follows. Section 2 presents a brief literature review on the process flexibility in lot sizing problem considering a deterministic context and explains some of the aspects of multicriteria decision analysis and TOPSIS method. Section 3 provides a formal description of the steps to obtain the ranking in TOPSIS method. Section 4 gives the mathematical formulations for the lot sizing problem containing a set of constraints that allows setting the different flexibility configurations. Section 5 presents the proposed multicriteria analysis for the process flexibility through computational experiments for different scenarios and flexibility configurations. The last section provides a summary of our findings and some clues for future research.

2 Literature review

2.1 Capacited lot-sizing with multiple resources

In the literature, there has been a broad effort to expand decision methods and procedures for the lot sizing problems. Recently, a particular attention has been given on the decision aspects involving the lot sizing with multiple resources and process flexibility. The class of relevant problems with multiple resources for this investigation is the lot sizing problems with multiple machines or multiple plants.

There are several studies on the lot sizing problem with multiple machines considering total flexibility. **?** proposed a heuristic for a lot sizing problem with identical machines, setup times, and constant demand. **?** developed a reformulation based on the shortest path for the same problem and proposes new symmetry break constraints. Considering the problem with unrelated parallel resources, **?** relaxed the capacity constraints and proposed a Lagrangian heuristic to solve the problem. **?** proposed an iterative process to generate production plans that consider scheduling constraints. **?** applied a Lagrangian heuristic based on the relaxation of the demand constraints in order to find good feasible solutions. **?** proposed hybrid methods using Lagrangian relaxation and Dantzig-Wolfe decomposition and found better lower and upper bounds compared to **?** and **?**. **?** proposed different mathematical formulations for this problem and analyzed the items and periods decomposition for these formulations. Recently, **?** developed a meta-heuristic based on the relaxation of the capacity constraints in order to explore the set of solutions to obtain more feasible solutions.

Other relevant studies examine multiple plants with total flexibility. In general, it considers that each plant has its own demand and there is a possibility of transferring production among the plants, with a due cost (**?**). **?** presented one of the first studies on production environment composed of multiple plants, in which the objective is to coordinate the production plans in all plants so that the company performance is improved. In **?** the lot sizing decisions were integrated to the transport of the items among the plants of an industry so that some plants produce intermediate products and others the final products. **?** presented a formulation for a problem with multiple plants in a beverage industry. The authors studied the planning operations that define the scheduling and size of the production, in which the objective is to satisfy the demand by minimizing production, overtime and transfer costs. **?** addressed this problem considering that all plants produce the same items (each one of the plants with a single machine) and that the demands must be satisfied without backlog. Considering the problem with multiple plants and setup carryover, **?** applied a meta-heuristic approach to find feasible solutions. The authors pointed out that the set of feasible solutions becomes significantly bigger considering the possibility of setup carryover.

? was the first paper taking into account the efficiency of a limited amount of flexibility for a manufacturing system with stochastic demand for an automotive industry. After the studies presented by **?**, several works were proposed to analyze the value of resource flexibility in a context of stochastic demand. **?** developed theoretical principles to build measures to quantify the concept of flexibility. **?** extended the work of **?** and proposed new strategies to implement the concepts of flexibility. **?** presented a literature review on the concepts of flexibility and discuss three characteristics of flexibility for the supply chain. **?** showed that almost all the benefits with an increase in sales can be found with the chain principle. Moreover, the level of inventory is significantly reduced as more flexibility is added to the system. Therefore, if the inventory costs are high, add flexibility efficiently makes the production process more economical. **?** pointed out that the long chain configuration does not present good results for non-homogeneous scenarios. **?** presented computational results considering a problem with queuing systems. The results showed that a non-chained configuration performed better than the long chain in asymmetric systems. **?** also confirmed that although the long chain configuration is desirable in homogeneous scenarios, it does not perform well when the data are heterogeneous.

Although the lot sizing problem with multiple machines with a limited amount of flexibility considering a deterministic environment is a natural and more realistic extension of the standard assumption (each machine can produce all items), there are few studies on this topic. Considering a tire industry, **?** discussed a problem where not all types of tires can be produced on all types of heaters. **?** studied the capacity lot sizing problem with parallel resources in the semiconductor industry where not all resources are eligible to produce all items. A proposed hybrid heuristic based on Lagrangian relaxation and simulated annealing outperformed the numerical results obtained by the standalone Lagrangian relaxation algorithm and the standalone simulated annealing algorithm most of the time.

Recently, **?** addressed the process flexibility and the chaining principle in lot sizing problems by analyzing the value of the resource flexibility in balanced systems (the number of items and resources is equal). The comparison of different limited flexibility configurations concluded that the benefits of the best long-chain and the total flexibility configurations are practically the same. Finally, they also pointed out that the importance of flexibility value increases when the data are heterogeneous.

This research adds to the literature of lot sizing problems with multiple machines and limited flexibility by considering a multicriteria perspective.

2.2 Aspects of multicriteria decision analysis

The field of multiple criteria decision analysis (MCDA) or multiple criteria decision making (MCDM) is a full-grown segment of operations research that explicitly evaluates multiple criteria in decision making (both in everyday life and in particular settings such as business, industrial engineering, and medicine). MCDA is a generic term for all methods that can assist the decision-maker according to its preferences in problems with more than one conflicting criterion. Researches in MCDA have produced many applied and theoretical papers and books consisting of several approaches (see **?**, **?**, and the references therein). **?** described the early history of initial conceptions for MCDA that separately tracked the origins of decision utility theory and mathematical programming with multiple objectives.

The MCDA approaches, methods and techniques are diverse and based on the following straightforward essential ingredients: a finite or an infinite set of actions (alternatives, solutions), at least two criteria, and at least one decision-maker. Given these essential elements, it regards the designing of mathematical and computational tools for solving problems regarding the choice of preferred alternatives, the classification of alternatives in a small number of categories, and the ranking of alternatives in a subjective judgment order (**?**). These methods can also be viewed as a way of dealing with intractable problems by splitting them into smaller parts. After weighing some criteria and making previous judgments about the smaller parts, these are regrouped to present a broad overview to assist the decision-maker.

? emphasized that the most crucial success of the multicriteria methods is their capability of addressing the problems that are identified by incompatible interests. From these techniques, actors can solve problems that are not possible to solve by using conventional optimization models. Moreover, **?** highlighted that one of the difficulties in using a specific MCDA method is selecting an aggregation technique for solving the decision problem since there is no single and well-structured methodology that one could follow step-by-step from the beginning to the end of a decision aiding process.

? stated that an aggregation method is compensatory when the increase in the value of one alternative, relative to one criterion, can compensate for the decrease relative to another criterion. In contrast, in the non-compensatory methods, the performance of one specific criterion does not influence the performance of another criterion (**??**). Some of the compensatory methods are based on referent points that evaluate a relative distance from an “ideal” alternative. Note that the decision-maker would choose the ideal alternative without hesitation. However, in general, this fictitious “ideal” does not figure among the possible choices, and the decision-maker must look for an alternative that is as close as possible to the ideal alternative (**?**). Indeed, one of the most popular procedures among those based-referent points is the TOPSIS method.

2.3 Brief review of the TOPSIS method

? and **?** proposed and improved the TOPSIS method to assist in choosing the most desirable alternative with a finite number of criteria that makes full use of attribute information and provides a ranking of alternatives. As a popular MCDA method, TOPSIS has received much interest from researchers and practitioners. **?** performed a state-of-the-art literature survey to categorize and interpret the studies on TOPSIS applications and methodologies. The classification scheme for this review contains a set of scholarly papers of years 2000-2012 distributes into nine application areas, which revealed successful applications for the TOPSIS method in a wide range of areas and industrial sectors with varying terms and subjects. These studies make the TOPSIS method workable in handling practical and theoretical problems, such as supply chain management and logistics; design, engineering and manufacturing systems; business and marketing management; health, safety, and environment management. And other fields such as agriculture, education, and sports.

? presented the Hellinger-TOPSIS method to compare the performances of evolutionary algorithms. The method was used as a statistical analysis tool to rank algorithms of the stochastic nature, whose simulation results showed the effectiveness of the well-established and reliable TOPSIS methodology in handling the stochastic nature of evolutionary algorithms. **?** reviewed the developments of TOPSIS techniques from 2000 to 2015 and observed that some key advantages of TOPSIS are its ability to deal with different types of values and in addressing rank reversal issue. **?** presented an approach that uses the performance metrics of the supply chain operations reference (SCOR) model to evaluate the suppliers regarding cost and delivery performance. The proposed approach combined two fuzzy-TOPSIS models for indicating the needs of improvements of suppliers. From an illustrative application based on a manufacturing environment, the authors described the several advantages of the combination between the SCOR and fuzzy-TOPSIS.

? analyzed the effects of a series of data normalization approaches on the integrated Entropy and TOPSIS method. It found that normalization can affect the decision result by strongly affecting the diversity of attribute data (DAD) since DAD affects the contribution of attributes to each alternative's distance from the ideal and negative-ideal alternative. **?** proposed variants of the TOPSIS method for sorting problems that prevent ranking reversal. The named TOPSIS-Sort-B improved the previous version of TOPSIS-Sort for sorting problems, which includes a step for determining a domain for each criterion and a normalized interval addressed to problems with boundary profiles. A numerical application for the proposed methods estimated the degree of economic freedom of 180 countries and assigned them to five pre-defined ordered classes.

These reported successful applications are examples that justify the choice of the TOPSIS method as a useful tool to assist in the interpretation of diverse conflicting objectives of lot sizing problems in terms of the process flexibility.

3 Formulation for TOPSIS method

TOPSIS is a ranking method in conception and application whose standard approach searches for alternatives that should have the shortest distance from the ideal alternative and the farthest distance from the negative-ideal alternative (**?**). The ideal alternative maximizes the benefit criteria and minimizes the cost criteria, whereas the negative-ideal alternative maximizes the cost criteria and minimizes the benefit criteria (**?**). To apply the TOPSIS method each attribute value takes either monotonically increasing or monotonically decreasing utility.

Let $D = (x_{ij})$ be the standard decision matrix, where each row i of D is part of the set of alternatives, $i = 1, \ldots, m$, each column j of D belongs to the set of criteria, $j = 1, \ldots, n$, and x_{ij} registers the rating of the alternative A_i according to criterion C_j. And let w_j be the individual weight for each criterion C_j, $j = 1, \ldots, n$, satisfying $\sum_{j=1}^{n} w_j = 1$.

In general, the criteria are classified into *benefit* and *cost*, where the benefit criterion indicates that a higher value is better while for cost criterion is valid the reverse. Since, in general, the entries of the decision matrix D originate from different sources, it is necessary to create the normalized decision matrix. This procedure attempts to transform the various attribute dimensions into nondimensional attributes, which allows comparison across the attributes. One approach is to take the rating of each criterion divided by the norm ℓ_2 of the total rating vector of this criterion.

Let $R = (r_{ij})$ be the normalized decision matrix, where $r_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{m} x_{ij}^2}$ for each $i = 1, \ldots, m$, and $j = 1, \ldots, n$. The matrix R represents the relative rating of the alternatives. And, let $P = (p_{ij})$ be the weighted normalized decision matrix, where $p_{ij} = w_j x_{ij}$ for each $i = 1, \ldots, m$, and $j = 1, \ldots, n$. Similarly with **?** and **?**, the TOPSIS method is described in the following steps.

Step 1. Identify the positive ideal alternative A^+ (benefits) and the negative ideal alternative A^- (costs) as follows.

$$A^+ = (p_1^+, \ldots, p_n^+)^T, \text{ where } p_j^+ = \{(\max_{i \in I} p_{ij},\ j \in J_1),\ (\min_{i \in I} p_{ij},\ j \in J_2)\}; \tag{1}$$

$$A^- = (p_1^-, \ldots, p_n^-)^T, \text{ where } p_j^- = \{(\min_{i \in I} p_{ij},\ j \in J_1),\ (\max_{i \in I} p_{ij},\ j \in J_2)\}, \tag{2}$$

where $I = \{1, \ldots, m\}$, and J_1 and J_2 represent benefit and cost criteria, respectively.

Step 2. Calculate the separation measures from the positive ideal alternative A^+ and the negative ideal alternative A^- for each alternative A_i, respectively as follows.

$$d_i^+ = \sqrt{\sum_{j=1}^{n}(p_{ij} - p_j^+)^2}, \quad i \in I; \tag{3}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}(p_{ij} - p_j^-)^2}, \quad i \in I. \tag{4}$$

Step 3. Calculate the relative closeness coefficient ε_i of each alternative A_i with respect to the positive ideal alternative as follows.

$$\varepsilon_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{5}$$

Step 4. Rank the alternatives according to the relative closeness coefficients.

Note that $\varepsilon_i = 1$ if $A_i = A^+$, and $\varepsilon_i = 0$ if $A_i = A^-$. The best alternatives are those that have higher value ε_i, and therefore should be chosen.

4 Mathematical formulations for the lot sizing problem with process flexibility

? proposed some formulations for the lot-sizing problem with process flexibility in which multiple resources have a limited amount of flexibility to be used. In this section, we first present a formulation of the lot sizing problem with unrelated parallel machines considering a limited amount of flexibility where a specific machine can produce only small number of types of products. This formulation allows us to find the optimal total cost given a specific flexibility configuration. Next, we present a formulation that considers the possibility of investing in flexibility and determines the optimal flexibility configuration for a given number of links or a limited budget.

For the first mathematical formulation of the problem, consider the following parameters and variables:

$I = \{1,...,n\}$: set of items;
$J = \{1,...,r\}$: set of machines;
$T = \{1,...,m\}$: set of periods;
I_j: set of items i that can be produced on machine j;
J_i: set of machines j that can produce item i;
d_{it}: demand of item i in period t;
$sd_{it\tau}$: sum of the demand for item i, from period t until period τ $(\tau \geq t)$;
hc_{it}: unit inventory cost of item i in period t;
bc_{it}: unit backlog cost of item i in period t;
sc_{ijt}: setup cost for item i on machine j in period t;
vc_{ijt}: production cost of item i on machine j in period t;
st_{ijt}: setup time for item i on machine j in period t;
vt_{ijt}: production time of item i on machine j in period t;
Cap_{jt}: capacity (in units of time) of machine j in period t.

The decision variables are then defined as follows:

x_{ijt}: quantity (lot-size) of item i to be produced on machine j in period t;
y_{ijt}: binary setup variable, indicating if machine j is configured for production or not of item i in period t;
s_{it}: inventory of item i at the end of period t;
b_{it}: backlog of item i at the end of period t.

The mathematical formulation of the problem is then as follows:

$$\text{Min} \quad \sum_{t\in T}\sum_{j\in J}\sum_{i\in I_j}(sc_{ijt}y_{ijt} + vc_{ijt}x_{ijt}) + \sum_{t\in T}\sum_{i\in I}(hc_{it}s_{it} + bc_{it}b_{it}) \tag{6}$$

$$\text{s.t.} \quad s_{i(t-1)} - b_{i(t-1)} + \sum_{j\in J_i} x_{ijt} = d_{it} + s_{it} - b_{it}, \quad i \in I, \quad t \in T; \tag{7}$$

$$x_{ijt} \leq \min\{(Cap_{jt} - st_{ijt})/vt_{ijt},\ sd_{i1m}\}\, y_{ijt}, \quad i \in I_j, \quad j \in J, \quad t \in T; \tag{8}$$

$$\sum_{i\in I_j}(st_{ijt}y_{ijt} + vt_{ijt}x_{ijt}) \leq Cap_{jt}, \quad j \in J, \quad t \in T; \tag{9}$$

$$y_{ijt}, \quad x_{ijt} \geq 0, \quad i \in I_j, \quad j \in J, \quad t \in T; \tag{10}$$

$$s_{it},\ b_{it} \geq 0, \quad s_{i0} = s_{im} = 0, \quad b_{i0} = 0, \quad i \in I, \quad t \in T. \tag{11}$$

The objective function (6) minimizes the total costs, which consists of production, setup, inventory and backlog costs. Constraints (7) ensure that demand is met for each period. Demand that cannot be satisfied on time can be backlogged. The setup constraints (8) do not allow any production unless a setup is done. The capacity constraints (9) limit the sum of the total setup and production times. Finally, constraints (10) and (11) define the variable domains.

The first formulation finds the optimal cost from a fixed flexibility configuration. However, the next formulation also gives the flexibility configuration. Therefore, it includes the chance to invest in different flexibilities by upgrading a machine for a specific product. The structure of a flexibility configuration derives from various links (levels of the global budget), each choice is a binary variable, and there is a global budget on the investment decisions.

For the second mathematical formulation, consider the following additional parameters and variables:
fc_{ij}: flexibility investment cost for producing item i on machine j;
F_{max}: global budget (number of links) to invest in flexibility.

The additional decision variables are then defined as follows:
z_{ij}: binary variable, indicating that machine j can produce item i or not.

The mathematical formulation consists of the objective function (6) and the constraints (7)–(11) of the previous formulation replacing the sets I_j and J_i by I and J, respectively. In addition, we have the following constraints:

$$y_{ijt} \leq z_{ij}, \quad i \in I, \quad j \in J, \quad t \in T; \tag{12}$$

$$\sum_{j\in J}\sum_{i\in I} fc_{ij} z_{ij} \leq F_{max}; \tag{13}$$

$$z_{ij} \in \{0,1\}, \quad i \in I, \quad j \in J; \tag{14}$$

Constraints (12) ensure that a machine can be set up to produce a specific item in a specific period only if this machine has the flexibility to produce this item. Constraint (13) limits the budget (number of links) available for investing in flexibility. We observe that in this formulation, the flexibility investment is model as part of a budget constraint, instead of putting it in the objective function. This will allow us to trace the trade-off between the level of flexibility and the operational costs. Companies can put restrictions on the number of machine-product combinations, which can be modeled as a special case of our global budget constraint. Observe that if $F_{max} = n^2$, then we have the total flexibility configuration.

5 Multicriteria analysis of flexibility configurations

We analyze the concept of process flexibility in a deterministic lot sizing context considering a multicriteria perspective. The objective of the experiments presented in this section is to analyze the effect of consider several criteria when analyzing the value of different flexibility configurations compared to the study proposed by **?** in which only one criterion (total cost) is used to determine the value of the flexibility configurations.

5.1 Setup of the computational tests

For the computational experiments, we use the same instances and results presented by **?**. In general, a total of 360 different problems were created separated into instances with 6, 12, and 24 items and machines. The backlog costs for each item is equal to 300, which is 100 times the average inventory holding cost, and the setup times are equal to zero. Furthermore, the capacity levels vary from 40 to 140 to have a broad range of problems so that the solutions have different levels of backlog. We refer to **?** in order to find more details about the generated instances.

In order to apply the TOPSIS method we consider eight alternatives and five criteria. More specifically, for each instance, we ranked eight alternatives of flexibility configurations named Dedicated, Cluster, Random, Long chain, Best chain, F_{max1}, F_{max2}, and Total flexibility (Figure 1 illustrates these alternatives). The criteria are the total cost, capacity utilization, total backlog, the total number of setups, and amount of flexibility considered on each one of the eight alternatives of flexibility configurations.

Figure 1 illustrates the flexibility configurations analyzed from TOPSIS method in this section. The figure refers to an instance containing six items and six machines. The Dedicated configuration is a pattern as Figure 1a), where each machine can produce only one item (the number of links of this configuration is equal to the number of items). The Cluster configuration appears in Figure 1b), where the pattern contains 3 clusters of 2 machines (the number of additional links on the Dedicated configuration is equal to the number of items). The pattern in Figure 1c) is the Long chain configuration, for which excepting Machine 1 that produces Item 1 and $n = 6$, each Machine i produces Items i and $i-1$ (the number of additional links of this configuration is equal to the Cluster configuration in Figure 1b)). The Total flexibility configuration is as Figure 1f), where all the flexibility links inform that each machine can produce all items. The configurations in Figure 1d) and 1e) do not follow a typical pattern. Instead, they are the configurations obtained by

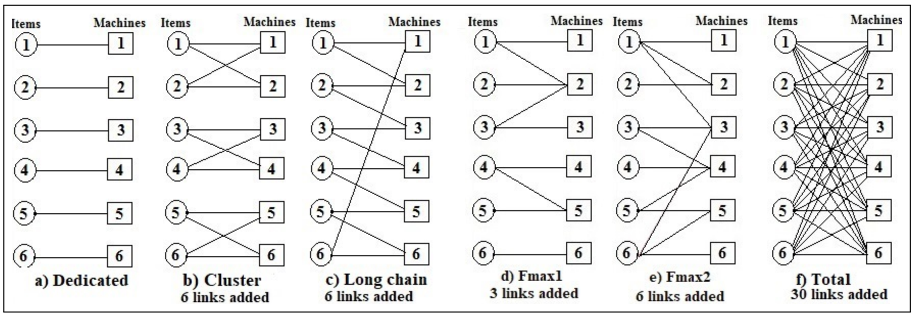


Fig. 1 Flexibility configurations for an example with 6 items and 6 machines.

solver at the optimal solution when the total number of allowed links (parameter F_{max}) is $n+n/2$ and $2n$, respectively. The parameter F_{max} will lend its name to these configurations, i.e., Configuration F_{max1} refers to $F_{max} = n+n/2$, and Configuration F_{max2} refers to $F_{max} = 2n$. Here, the solver gives the optimal flexibility configurations related to the fixed total number of links. In the case of 12 and 24 machines this figure is extended in a straightforward way.

We also consider the Random flexibility configuration in which there is no pattern, and the links are added randomly. In the Random flexibility configuration, the number of additional links on the Dedicated configuration is again equal to the number of items. We also observe that there are many possible Long chain configurations by changing the sequence of the items. Therefore, the performance of the best of them is also analyzed, i.e., the Best chain configuration.

We observe that the formulation (6)–(11) with the appropriate configuration of the links is used to analyze the following cases: Dedicated, Cluster, Random, Long chain, Best chain and Total flexibility and the formulation (6)–(11) and (12)–(14) is used to analyze the case where process flexibility is a decision variable (configurations F_{max1} and F_{max2}). Note that in constraints (13) the set fc_{ij} is fixed to one, and F_{max} equal to the number of links that are allowed to add. It gives a limitation on the total number of links that can be used, i.e., imposing a limit on the number of machine-product combinations that is allowed. Finally, The tests were done on a computer with 2 Intel(R) Xeon(R) X5675 processors, 3.07GHz with 96GB of RAM and the Linux operating system. Moreover, when solving the formulations, we have limited the computational time for each instance to 3 hours (10800 seconds).

5.2 Computational Results

Table 1 presents the decision matrix for the instances with 12 items and machines (the decision matrices for 6 and 24 items and machines are presented in Appendix A). In this table, we consider different levels of capacity and give the average of the relative upper bounds (columns UB), capacity utilization (columns CU), the total backlog (columns Backlog), the total number of setups (columns NS) and the total number on links allowed (columns NL) found for the instances and flexibility configurations. Note that, we set the total cost of the Dedicated configuration as the referential value (equal to 100%) and calculate the upper bounds of the other flexibility configurations related to these values. Moreover, we also give the standard deviation (S.E.) and the coefficient of variation (C.V.) of each criterion.

Table 1 shows that some criteria present the same value (are tied) for all flexibility configurations and the amount of ties varies according to the capacity levels. More specifically, for low capacity levels, only one criterion presents equal values (CU). However, for high levels of capacity (equal to 140) three of the five criteria have the same values for all flexibility configurations. It is also important to note that, as expected, in terms of total cost (column UB) the Total flexibility presents the best values (lowest values) for all capacity levels. And, the Total and Dedicated configurations are opposite each other in total number of links.

Table 2 presents the position obtained for each flexibility configuration applying the TOPSIS method for all instances and capacity levels. Note that for the instances with 6 items and machines and the capacity level equal to 140, the Dedicated configuration is ranked in the first position, followed by F_{max1} and F_{max2} configurations.

The results show that the Total flexibility stayed in the last positions for all size of instances and capacity levels. It occurs mainly due to the large number of links (n^2) presented in this flexibility configuration compared to the total number of links presented by the other flexibility configuration which is smaller than $2n$. We observe that it is in line with practice application, since it can be very expensive and/or usually impossible to install machines with Total flexibility. Moreover, although the Total flexibility obtained the best upper bounds for all instances, other flexibility configurations such as the Long chain, F_{max1} and F_{max2} presented values in the decision matrix very similar to the Total flexibility for this criterion. Therefore, the upper bound is not a criterion that highlights the Total flexibility. Considering the Cluster and Random flexibility we see that they ranked between the 5th and 7th position for almost all instances with 6 and 12 items and machines. We observe that the instances with 24 items and machines are extremely difficulty to solve and the results presented large optimality gaps for most of flexibility configurations and it has a big influence on the conclusions for these instances. It is also interesting to see that the Dedicated configuration ranked first for very low (40 and 60) and high capacity levels (140). Note that for very low and high capacity levels, the benefits of investing in flexibility is almost 0

Table 1 Decision matrix for the instances with 12 items and machines for different configurations.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	73349	180.0	12	100	100	44851	180.0	12	100	100	17526	180.0	12
Cluster	99.7	100	73214	180.9	24	99.2	100	44535	190.8	24	95.0	100	16668	190.3	24
Random	99.7	100	73219	181.1	24	99.0	100	44462	194.1	24	92.1	100	16182	198.1	24
Long	99.7	100	73209	181.2	24	98.9	100	44431	194.3	24	91.0	100	16001	201.4	24
Best	99.6	100	73212	180.4	24	98.8	100	44402	188.4	24	90.8	100	15965	197.6	24
F_{max1}	99.6	100	73222	181.4	18	98.8	100	44409	186.6	18	92.0	100	16169	189.6	18
F_{max2}	99.6	100	73227	180.4	24	98.8	100	44407	188.2	24	90.9	100	15975	199.4	24
Total	99.6	100	73195	200.4	144	98.8	100	44395	203.0	144	90.7	100	15964	201.2	144
S.E.	0.1	0	48.7	7.0	43.6	0.4	0	154.3	6.7	43.6	3.2	0	546.3	7.5	43.6
C.V. (%)	0.1	0	0.1	3.8	118.5	0.4	0	0.3	3.5	118.5	3.5	0	3.4	3.8	118.5

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	94.1	980	176.4	12	100	78.9	5	161.0	12	100	67.6	0	144.8	12
Cluster	47.4	94.5	255	184.1	24	99.1	78.9	1	161.3	24	100	67.6	0	144.6	24
Random	33.3	94.7	31	184.9	24	98.9	78.9	0	161.0	24	100	67.6	0	144.7	24
Long	31.0	94.7	0	184.5	24	98.9	78.9	0	160.9	24	100	67.6	0	144.7	24
Best	31.0	94.7	0	184.4	24	98.8	78.9	0	160.8	24	100	67.6	0	144.6	24
F_{max1}	33.0	94.7	25	181.4	18	98.8	78.9	0	160.8	18	100	67.6	0	144.8	18
F_{max2}	31.3	94.7	0	185.0	24	98.9	78.9	0	160.8	24	100	67.6	0	144.6	24
Total	30.7	94.7	0	184.0	144	98.8	78.9	0	160.8	144	100	67.6	0	144.6	144
S.E.	24.0	0.2	342.0	2.9	43.6	0.4	0	1.8	0.2	43.6	0	0	0	0.1	43.6
C.V. (%)	56.9	0	211.9	1.6	118.5	0.4	0	233.7	0.1	118.5	0	0	-	0.1	118.5

Table 2 Position obtained for the flexibility configurations using TOPSIS method.

		Dedicated	Cluster	Random	Long	Best	F_{max1}	F_{max2}	Total
6	140	1	7	6	5	4	2	3	8
	120	8	6	5	2	4	1	3	7
	100	8	6	5	4	3	1	2	7
	80	1	7	5	6	4	2	3	8
	60	1	5	6	7	3	2	4	8
	40	1	4	6	5	3	2	7	8
12	140	1	5	7	6	4	2	3	8
	120	8	6	5	4	2	1	3	7
	100	8	6	5	3	2	1	4	7
	80	2	3	5	7	4	1	6	8
	60	1	5	6	7	4	2	3	8
	40	1	5	6	7	3	2	4	8
24	140	1	4	3	2	6	5	8	7
	120	1	4	2	3	8	5	6	7
	100	8	5	3	1	2	4	6	7
	80	3	2	4	5	6	1	7	8
	60	1	5	6	7	3	2	4	8
	40	1	4	5	6	3	2	7	8

(the upper bounds presented by all flexibility configurations are close to 100%). Finally, Table 2 shows that F_{max1} ranked in the first positions for almost all size of instances and capacity levels.

Figure 2 summarizes the frequency that a flexibility configuration appears in the top positions (1st or 2nd position) and bottom positions (7th or 8th position). F_{max1} is in the top positions for most instances and is the only one that does not appear in the last positions. On the other hand, Total flexibility appears in the bottom positions for all experiments. Moreover, Dedicated configuration is the second that appears most in the top positions. Note also that the frequency of Long chain configuration in the bottom positions is higher than Best chain configuration, but they have the same frequency in the top positions.

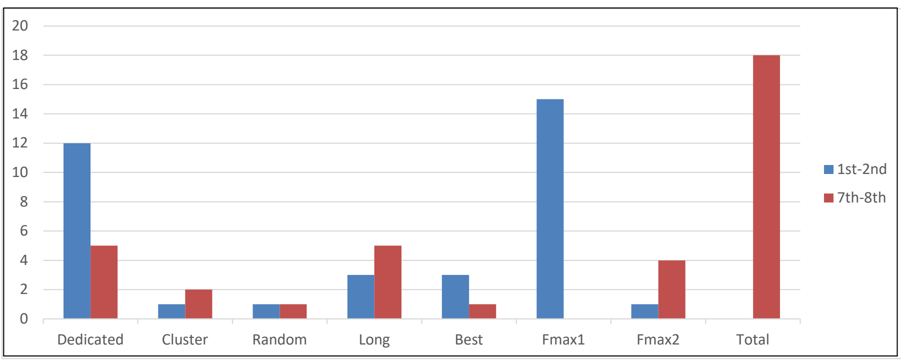


Fig. 2 Top (1st and 2nd positions) and bottom (7th and 8th positions) positions.

Table 3 Position obtained for the flexibility configurations using TOPSIS method with backlog heterogeneity.

		Dedicated	Cluster	Random	Long	Best	F_{max1}	F_{max2}	Total
6	140	1	7	6	5	4	2	3	8
	120	8	6	5	4	3	1	2	7
	100	8	6	5	4	3	1	2	7
	80	7	6	5	4	2	1	3	8
	60	2	7	6	5	4	1	3	8
	40	2	6	5	4	3	1	7	8
12	140	1	6	5	4	3	2	7	8
	120	8	6	5	4	3	1	2	7
	100	8	6	5	3	2	1	4	7
	80	7	6	4	2	5	1	3	8
	60	7	6	5	4	2	1	3	8
	40	7	5	6	4	2	1	3	8
24	140	1	4	3	2	7	5	6	8
	120	1	5	4	3	8	2	6	7
	100	8	5	2	1	3	6	4	7
	80	7	6	4	1	3	5	2	8
	60	7	6	5	4	2	1	3	8
	40	7	6	5	4	1	2	3	8

Finally, these results show some significant differences compared to the conclusions presented by **?** that considered only the total cost as a criterion. More specifically, **?** showed that the Total flexibility and Long chain configurations are very similar and outperform the other flexibility configurations. However, our multicriteria study shows that the Total flexibility configuration frequently ranked in the bottom positions, and the Long chain configuration regularly ranked in intermediate positions.

5.3 Sensitivity analysis for the process flexibility

It is well known from literature that the process flexibility has a better performance for homogeneous cases. In this section, we analyze the behavior of the process flexibility for the deterministic lot sizing context considering a multicriteria approach by varying the homogeneity in different ways. More specifically, using the ideas proposed by **?** we analyze separately the backlog cost and demand heterogeneity. Additionally, we analyze the case where setup times are present. Finally, we also address the case considering all these three factors together.

5.3.1 Backlog cost heterogeneity

In order to create a case with backlog heterogeneity, the same data sets considered in the previous section are used and the backlog cost for each item is equal to 100 times the inventory holding cost. In all instances, each item has a different inventory holding cost, taken from a discrete uniform distribution between 1 and 5. As such, items will have different backlog costs in the adapted instances. This allows us to check the influence of the level of the backlog cost.

In Table 3 we present the position obtained for each flexibility configuration applying the TOPSIS method for all instances and capacity levels considering the backlog cost heterogeneity. The tables with the decision matrices considering the backlog cost heterogeneity used to build the Table 3 are presented in Appendix A. The global analysis shows that the Total flexibility is ranked again in the last positions for all size of instances and capacity levels. However, different from the results with backlog homogeneity, the Dedicated configuration ranked in the last positions for many size of instances and capacity levels. It occurs because in this scenario there are items with backlog cost more expensive than others (what is common in practice) and the Dedicated configuration does not allow readjustment capacity among the machines (each machine produce only one item). It is interesting to observe that, just like in the backlog homogeneity, the configuration F_{max1} ranked first and second for 15 from 18 size of instances and capacity levels (see Figures 2 and 3). Considering the configurations Long chain, Best chain and F_{max2} we see a small improvement compared to the previous scenario specially for the instances with medium and high capacity levels. On the other hand, the Cluster configuration performed worse. It makes sense that the since Long and Best chain are well connected configurations, they perform better in heterogeneous scenarios while the Dedicated and Cluster configurations should perform worse.

In Figure 3 we present the results summarized by top and bottom positions considering the backlog cost heterogeneity. We see that while the flexibility configuration F_{max1} is in the top positions for most instances and does not appear in the bottom positions, the Total flexibility is in the bottom positions for all size of instances and level of flexibility. Moreover, different from Figure 2, the Dedicated configuration ranked seventh or eighth for 12 from 18 size of instances and levels of capacity. Finally, note that the Long and Best chain configurations are in the top positions for some instances.

Table 4 Position obtained for the flexibility configurations using TOPSIS method with setup times.

		Dedicated	Cluster	Random	Long	Best	F_{max1}	F_{max2}	Total
6	140	8	6	5	4	3	1	2	7
	120	7	6	5	4	2	1	3	8
	100	1	7	6	5	4	2	3	8
	80	1	7	6	5	4	2	3	8
	60	1	7	5	6	4	2	3	8
	40	1	5	3	4	6	2	7	8
12	140	8	6	5	4	3	1	2	7
	120	7	6	5	2	3	1	4	8
	100	4	7	6	5	2	1	3	8
	80	6	7	5	4	3	1	2	8
	60	7	6	5	4	3	1	2	8
	40	5	3	2	4	7	1	6	8
24	140	8	6	3	1	2	5	4	7
	120	7	5	2	1	3	6	4	8
	100	7	6	1	4	3	2	5	8
	80	7	6	5	4	2	1	3	8
	60	7	6	5	4	3	1	2	8
	40	7	2	3	1	6	4	5	8

5.3.2 Considering setup times

In this section we consider the setup times which have a uniform distribution between 10 and 50. Note that the backlog costs for all items are fixed to 300 as in the base setting. Table 4 shows the position obtained for each flexibility configuration applying the TOPSIS method for all instances and capacity levels considering setup times. The tables with the decision matrices considering setup times used to build the Table 4 are presented in Appendix A. A global analysis of the results (Table 4 and Figure 4) indicates that the configuration F_{max1} performed very well as in the previous scenarios. However, although the Dedicated configuration ranked first for the instances with 6 items and machines, and medium and high capacity levels, it ranked in the last positions for all other size of instances and capacity levels. Moreover, the configuration F_{max2} presents a small ascent in the ranking compared to the base case and the backlog heterogeneity scenarios.

Table 4 also provides information on the other flexibility configurations. Comparing the performance of the Cluster and Random configurations, we see that the Random ranked best for all size of instances and capacity levels. Moreover, the long chain ranked first for some instances with 24 items and machines and it does not rank in the last positions for any instance. Comparing the Long and Best chain configurations, we see that in general, the Long chain appears in intermediate positions while the Best chain is at the top of the ranking what is expected since the Best chain configuration is the Best long chain in terms of total cost. Finally, the Total flexibility ranked in the last positions as in the previous scenarios.

Figure 4 summarizes the results for the top and bottom positions. Compared to the base case in Figure 2, we observe some differences with respect to the ranking of some flexibility configurations. Considering setup times, the Dedicated and Cluster configurations ranked considerably more times in the bottom positions compared to the base case. We also see that the number of times that the Random and F_{max2} appears in the top positions increase significantly. Moreover, the Dedicated configuration ranked in the bottom positions for 11 from 18 size of instances and capacity levels (this configuration ranked in the first position for 12 size of instances in base case (Figure 2)).

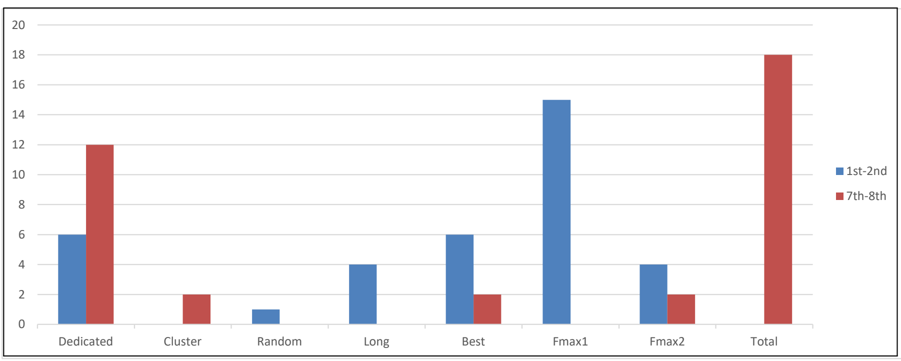


Fig. 3 Top (1st and 2nd positions) and bottom (7th and 8th positions) positions - backlog heterogeneity.

Table 5 Position obtained for the flexibility configurations using TOPSIS method with demand heterogeneity.

		Dedicated	Cluster	Random	Long	Best	F_{max1}	F_{max2}	Total
6	140	8	6	5	4	3	1	2	7
	120	8	6	5	4	3	1	2	7
	100	8	6	5	4	2	1	3	7
	80	7	6	5	4	2	1	3	8
	60	2	7	5	6	3	1	4	8
	40	1	7	6	5	3	2	4	8
12	140	8	6	5	3	4	1	2	7
	120	8	6	5	4	2	1	3	7
	100	8	6	5	4	2	1	3	7
	80	7	6	5	4	2	1	3	8
	60	7	6	5	4	3	1	2	8
	40	1	7	6	5	3	2	4	8
24	140	8	6	3	1	2	4	5	7
	120	8	6	4	1	2	5	3	7
	100	7	6	3	2	5	1	4	8
	80	1	5	6	7	3	2	4	8
	60	1	7	5	4	3	2	6	8
	40	1	5	4	6	3	2	7	8

5.3.3 Demand heterogeneity

In order to create a demand heterogeneity case, the demand distribution of the items have been changed. In the base setting, each item had the same average demand. In this new case, the demand of the first half of the items is increased by 50% and the other half is decreased by 50%. Note that the backlog costs for all items are fixed to 300 as in the base setting and there is no setup time in this scenario.

Table 5 presents the position obtained for each flexibility configuration applying the TOPSIS method for all instances and capacity levels considering demand heterogeneity. The tables with the decision matrices considering demand heterogeneity used to build the Table 5 are presented in Appendix A.

The results show that the Total flexibility remain in the last positions as in the previous scenarios. Moreover, although the Cluster configuration ranked between 5th and 7th position as in the base setting, the Random configuration ranked between 3rd and 5th positions. Considering the Dedicated configuration, it still ranked first for low capacity levels. However, different from the base setting, for medium and high capacity levels, the Dedicated configuration ranked in the last positions. This occurs because in this scenario the Dedicated configuration presents substantial levels of backlog for

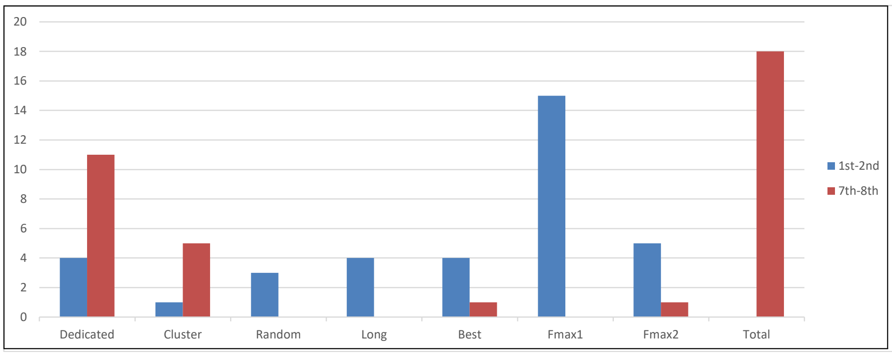


Fig. 4 Top (1st and 2nd positions) and bottom (7th and 8th positions) positions - considering setup times.

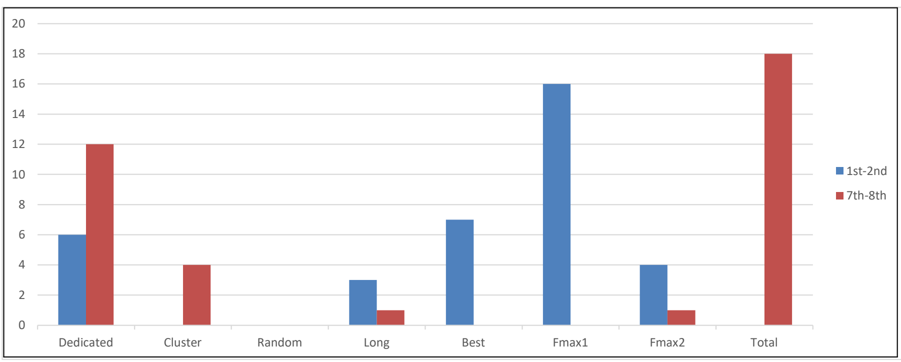


Fig. 5 Top (1st and 2nd positions) and bottom (7th and 8th positions) positions - considering demand heterogeneity.

Table 6 Position obtained for the flexibility configurations using TOPSIS method with backlog and demand heterogeneity considering setup times.

		Dedicated	Cluster	Random	Long	Best	F_{max1}	F_{max2}	Total
6	140	8	6	5	4	3	1	2	7
	120	8	6	5	4	2	1	3	7
	100	7	6	4	5	3	1	2	8
	80	7	6	5	4	3	1	2	8
	60	2	7	6	5	4	1	3	8
	40	1	5	3	4	7	2	6	8
12	140	8	6	5	4	2	1	3	7
	120	8	6	5	4	2	1	3	7
	100	7	6	5	4	3	1	2	8
	80	7	6	5	4	3	1	2	8
	60	7	6	5	4	3	1	2	8
	40	7	5	1	4	6	2	3	8
24	140	7	6	3	1	2	5	4	8
	120	7	6	5	3	2	4	1	8
	100	7	6	5	4	3	1	2	8
	80	7	6	5	4	3	1	2	8
	60	7	6	5	4	3	1	2	8
	40	7	1	3	2	6	4	5	8

these capacity level because of the very high demand of the first half of the items. It is also interesting to see that the Best chain performed much better than the Long chain configuration for this scenario. The reason is that the Best long chain can always combine one item with high and one with low demand distribution on one machine and it does not always happen for a fixed long chain. Finally, Table 5 shows that F_{max1} remain ranked in the first positions for almost all size of instances and capacity levels.

Figure 5 presents the results summarized by top and bottom positions considering demand heterogeneity. We observe that different from the base setting, the Cluster and Random configurations do not rank in the top positions for any instance. Moreover, the Cluster configuration ranked in the bottom positions more times. It shows that these configurations had a bad performance for this scenario. We also see that the Best chain is in the top positions for a significantly amount of instances while the Long chain configuration presents a similar trend compared to the base setting.

5.3.4 Backlog and demand heterogeneity considering setup times

In order to create a case with a very high level of heterogeneity, the three factors that have been analyzed (backlog and demand heterogeneity and setup times) are considered together. Table 6 presents the position obtained for each flexibility configuration. The tables with the decision matrices considering used to build the Table 6 are presented in Appendix A. The results show that the Total flexibility and Dedicated configuration ranked in 8th and 7th position for most instances, respectively. It is interesting to see that the configuration F_{max2} , different from the other scenarios, ranked better than the Long and Best chain configuration for most instances. Finally, the configuration F_{max1} remains in the first positions for most instances.

Figure 6 summarizes the results for the top and bottom positions considering the backlog and demand heterogeneity with setup times. We see that while the flexibility configuration F_{max1} is in the top positions for most instances followed by the configuration F_{max2} . We also see that the Total flexibility and Dedicated configuration concentrate the bottom positions for almost all instances. Therefore, for the most heterogeneous scenario, the flexibility configurations with very limited amount of links (F_{max1} and F_{max2}) are the most efficient ones. It is also interesting to observe that the well-known chain configurations present an intermediate performance from a multicriteria point of view.

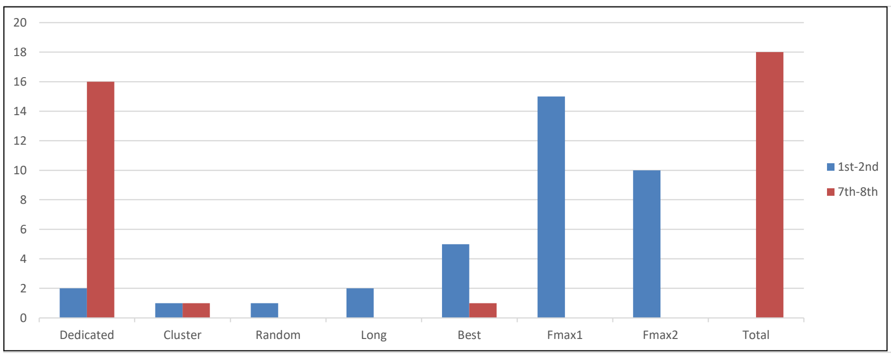


Fig. 6 Top (1st and 2nd positions) and bottom (7th and 8th positions) positions - considering backlog and demand heterogeneity with setup times.

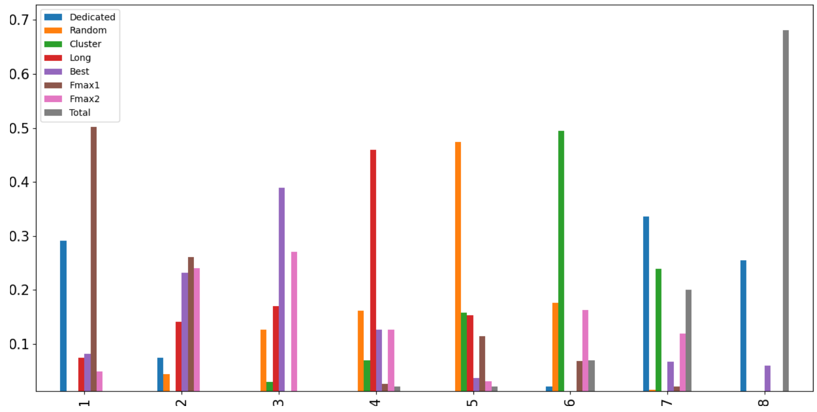


Fig. 7 Proportion that each flexibility configuration ranked in a specific position.

5.4 Sensitivity analysis for the weights of the criteria

In the previous sections when analyzing the ranking position of each flexibility configuration, we have considered that each criterion had the same weight. In order to make a more consistent analysis, in this section we analyze the behavior of the process flexibility for the lot sizing problem considering a multicriteria approach by varying the weight of the criteria. More specifically, each weight obtains a value from a uniform distribution $U(0,1)$ and these values are normalized by the size of the weight vector. After weighting each criterion, the TOPSIS method ranks the studied flexibility configurations (alternatives). Moreover, in order to have a wide range of results, we have generated 50000 experiments.

Figure 7 shows the proportion that each flexibility configuration ranked in a specific position. Note that the configuration F_{max1} ranked 50% in the first position. Moreover, F_{max1} has the highest proportion in top positions. It means that even varying the weights, configuration F_{max1} is robust and can be appropriated to the decision-maker. We also observe that the Best and Long chain configurations are most ranked in the third and fourth positions, respectively. Finally, the Total flexibility ranked around 70% in the last position, and the Dedicated configuration is the second of them that most appears both in the first and last positions.

Figures 8 and 9 present a series of the box plot charts to compare the variability of the positions for each flexibility configuration in the ranking. The bullet points of the charts are outliers in the data. Thus, Figure 8 shows that each configuration ranked at least once in each such position. Note that the Cluster, Random and Long chain configurations ranked mostly in the intermediate positions, while the Total flexibility ranked mostly in the 7th or 8th positions. Moreover, although the Dedicated configuration ranked around 30% in the 1st position (see Figure 7), it ranked 50% in the bottom positions. Furthermore, F_{max1} configuration highlighted by ranking 75% from 1st to 3rd positions.

Figure 9 aims to examine the sensibility of the ranking of alternatives by varying capacity levels (40, 80, and 140). Note that the spreading of the Dedicated configuration from 1st to 8th positions in the ranking (Figure 8) manly occurs because of the results for low capacity level (Figure 9(a)). We also observe that it regularly ranked in the 7th and 1st positions for medium and high capacity levels, respectively (Figures 9(b) and 9(c)). This last observation is in line with the literature, considering that there is no (or only a little) benefit of investing in flexibility for systems with very high capacity levels. Moreover, Figure 9 provides two significant insights of this research. It shows that regarding a multi-criteria approach to implement a limited flexibility configuration has advantages even for low capacity levels (F_{max1} and Best chain configurations frequently ranked in the top positions). Finally, although the Total flexibility configuration is

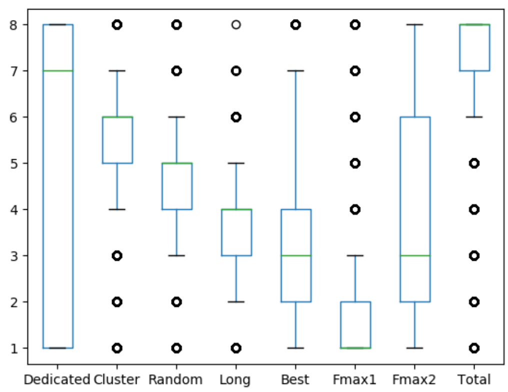
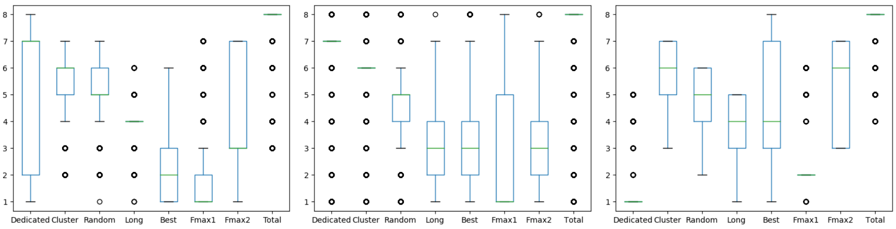


Fig. 8 Variability of the positions for each flexibility configuration.



(a) Boxplot cap 40 (b) Boxplot cap 80 (c) Boxplot cap 140

Fig. 9 Variability of the positions for each flexibility configuration separated by capacity levels.

known for having best values for total production cost, Figure 9 shows that concerning multicriteria approach, the Total flexibility configuration mostly ranked in the last position for all capacity levels and should not be chosen.

As a final remark, we apply the Borda's method (**?**) as an aggregated method to verify where each flexibility configuration stands out in the ranking of alternatives. We score each flexibility configuration according to its position in the ranking as follows. If the flexibility configuration ranked in the first position, it scores 8 points; in the second position, it scores 7 points; and so on in the eighth position, it scores 1 point. Figure 10 presents the total scored points by each flexibility configuration, in which the F_{max1} and Total flexibility configurations presented the best and worst performance, respectively. Moreover, except for the F_{max1} configuration, the Best chain and Long chain configurations score better than other configurations. Furthermore, the Dedicated configuration outperforms only the Cluster and Total flexibility configurations.

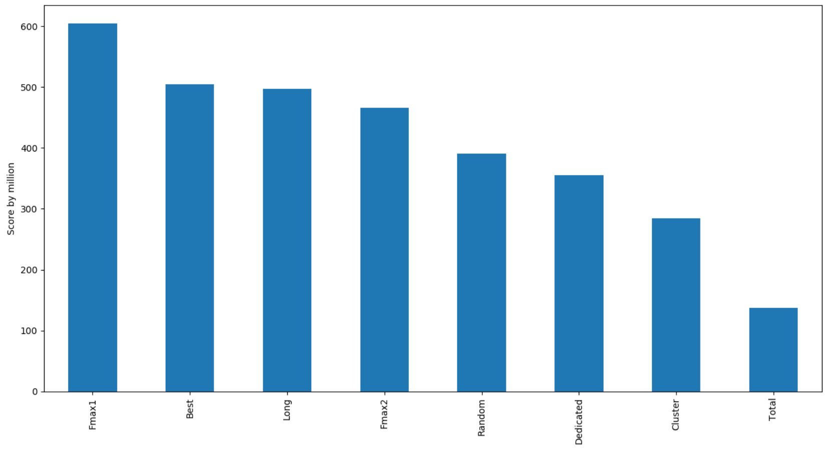


Fig. 10 Performance of the flexibility configurations aggregated by scored points.

6 Conclusions

In this paper, the process flexibility was studied in the context of a deterministic lot sizing problem. Different from the standard lot sizing problem with parallel machines, the studied problem considers a limited amount of machine flexibility so that each machine can produce only certain types of products. In order to fill a gap of the literature, a multicriteria analysis is proposed in order to analyze the efficiency of several flexibility configurations. Our computational experiments showed that, the Total flexibility configuration ranked in the last positions for all proposed scenarios. Moreover, the well known Long chain configuration had in general an intermediate behavior for most instances. The configuration F_{max1} (configuration with $n+n/2$ links) showed that only a small amount of flexibility obtain the best results for all scenarios analyzed. The computational experiments also indicate that the Dedicated configuration ranked in the first positions for very high capacity levels. However, it ranked in the last positions for medium capacity levels. Finally, different from the studies considering only the total cost as criterion, there are advantages of investing in flexibility for all capacity levels.

There are several interesting issues that can be explored as further research, for example, to extend the study to unbalanced systems in which the number of products and machines is not the same. It would also be interesting to focus on devising specific heuristics to add flexibility, i.e., given a level of flexibility (links between products and machines) the heuristic would determine a good way to distribute these links. Another possibility that can be explored is to study the two level of flexibility with different plants and customers. In this case each plant can deliver its production for only

Table 7 Decision matrix for the instances with 6 items and machines for different configurations.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	35948	90.0	6	100	100	21786	90.0	6	100	100	8403	90.0	6
Cluster	99.5	100	35837	90.8	12	98.9	100	21566	95.4	12	93.4	100	7887	94.8	12
Random	99.5	100	35824	91.0	12	98.4	100	21481	97.0	12	89.8	100	7599	97.2	12
Long	99.5	100	35820	91.0	12	98.2	100	21447	97.4	12	87.4	100	7408	99.3	12
Best	99.4	100	35809	90.5	12	98.1	100	21430	94.8	12	87.3	100	7405	98.3	12
$F_{max}1$	99.4	100	35813	91.1	9	98.2	100	21441	93.6	9	88.2	100	7471	95.1	9
$F_{max}2$	99.4	100	35812	91.9	12	98.1	100	21431	94.8	12	87.3	100	7405	97.7	12
Total	99.4	100	35803	98.2	36	98.1	100	21425	100.8	36	87.3	100	7405	99.4	36
S.E.	0.2	0	47.5	2.6	9.2	0.7	0	124.3	3.1	9.2	4.5	0	356.7	3.1	9.2
C.V. (%)	0.2	0	0.1	2.9	66.3	0.7	0	0.6	3.3	66.3	5.0	0	4.7	3.2	66.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	93.5	488	87.5	6	100	78.4	1	80.4	6	100	67.2	0	72.5	6
Cluster	54.5	93.9	146	91.6	12	99.8	78.4	0	80.5	12	100	67.2	0	72.5	12
Random	42.4	94.0	35	92.2	12	99.7	78.4	0	80.4	12	100	67.2	0	72.5	12
Long	38.1	94.1	0	91.9	12	99.7	78.4	0	80.3	12	100	67.2	0	72.5	12
Best	37.9	94.1	0	91.8	12	99.7	78.4	0	80.4	12	100	67.2	0	72.5	12
$F_{max}1$	39.5	94.0	13	90.9	9	99.7	78.4	0	80.4	9	100	67.2	0	72.5	9
$F_{max}2$	37.9	94.1	0	91.2	12	99.7	78.4	0	80.4	12	100	67.2	0	72.4	12
Total	37.9	94.1	0	91.9	36	99.7	78.4	0	80.4	36	100	67.2	0	72.3	36
S.E.	21.6	0.2	170.2	1.5	9.2	0.1	0	0.4	0.1	9.2	0	0	0	0.1	9.2
C.V. (%)	44.4	0.2	199.6	1.7	66.3	0.1	0	282.8	0.1	66.3	0	0	-	0.1	66.3

Table 8 Decision matrix for the instances with 24 items and machines for different configurations.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	146320	360.0	24	100	100	89377	360.0	24	100	100	34641	360.0	24
Cluster	99.7	100	146030	360.8	48	99.2	100	88761	383.3	48	94.5	100	32737	378.4	48
Random	99.7	100	146028	361.2	48	99.0	100	88543	389.8	48	91.6	100	31716	393.6	48
Long	99.7	100	146001	361.8	48	98.9	100	88497	390.2	48	91.0	100	31511	395.3	48
Best	99.6	100	145978	360.2	48	98.7	100	88378	374.2	48	90.6	100	31365	396.6	48
$F_{max}1$	99.6	100	145996	364.4	36	99.2	100	88780	371.2	36	93.3	100	32370	375.0	36
$F_{max}2$	99.6	100	145991	365.8	48	98.7	100	88368	378.8	48	90.8	100	31417	397.0	48
Total	99.6	100	145947	417.2	576	98.7	100	88353	408.2	576	90.3	100	31279	399.0	576
S.E.	0.1	0	117.6	19.6	188.7	0.4	0	344.7	14.6	188.7	3.3	0	1140.4	14.1	188.7
C.V. (%)	0.1	0	0.1	5.3	172.3	0.4	0	0.4	3.8	172.3	3.5	0	3.5	3.6	172.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	92.5	2177	346.2	24	100	77.5	3	317.4	24	100	66.4	0	285	24
Cluster	50.0	92.9	635	364.4	48	99.8	77.5	0	317.6	48	100	66.4	0	284.6	48
Random	30.9	93.0	53	363.9	48	99.7	77.5	0	317.2	48	100	66.4	0	284.6	48
Long	29.0	93.0	1	361.7	48	99.7	77.5	0	317.3	48	100	66.4	0	284.6	48
Best	30.8	93.0	5	371.4	48	339.8	77.0	1609	314.6	48	114.1	66.4	37	287.0	48
$F_{max}1$	48.7	92.8	570	361.2	36	195.3	77.2	468	315.8	36	107.6	66.4	14	284	36
$F_{max}2$	69.3	92.5	1225	363.6	48	214.3	77.3	567	316.0	48	142.7	66.4	224	284.0	48
Total	28.7	93.0	1	360.0	576	99.7	77.5	0	317.2	576	100	66.4	0	284.4	576
S.E.	25.3	0.2	779.0	7.1	188.7	88.3	0.2	567.4	1.1	188.7	14.9	0	77.7	1.0	188.7
C.V. (%)	52.3	0.2	133.5	2.0	172.3	56.6	0.2	171.5	0.3	172.3	13.8	0	226.1	0.3	172.3

certain number customers. The objective would be to analyze the value of flexibility and develop solution methods for this case. A final extension could be to study this problem considering a multicriteria context. In such case, there is no enough capacity to satisfy the demand of all costumers and different criteria can be used to determine the preferred costumers and what proportion of the demands will be satisfied.

Conflict of interest

The authors declare that they have no conflict of interest.

A Matrix decisions for all instances and scenarios

Table 9 Decision matrix for the instances with 6 items and machines for different configurations with backlog heterogeneity.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	35948	90.0	6	100	100	21786	90.0	6	100	100	8403	90.0	6
Cluster	79.0	100	36102	90.6	12	71.8	100	21570	121.5	12	68.6	100	7894	114.9	12
Random	78.5	100	36010	90.6	12	61.9	100	21805	111.0	12	49.7	100	7709	123.5	12
Long	78.5	100	35977	90.6	12	60.5	100	21706	111.3	12	39.0	100	7734	121.8	12
Best	73.0	100	36021	90.6	12	55.3	100	21770	114.2	12	38.7	100	7711	121.7	12
$F_{max}1$	69.7	100	35877	101.9	9	55.4	100	21914	114.6	9	45.7	100	8086	115.5	9
$F_{max}2$	69.7	100	35847	103.8	12	54.9	100	21718	117.3	12	38.6	100	7703	121.9	12
Total	69.7	100	35839	107.1	36	54.9	100	21711	117.2	36	38.6	100	7699	120.8	36
S.E.	10.1	0	93.1	7.3	9.2	15.5	0	98.9	9.6	9.2	21.8	0	255.8	11.1	9.2
C.V. (%)	13.1	0	0.3	7.6	66.3	24.1	0	0.5	8.5	66.3	41.6	0	3.3	9.5	66.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	93.5	488	87.5	6	100	78.4	1	80.4	6	100	67.2	0	72.5	6
Cluster	51.0	93.9	147	93.5	12	99.6	78.4	0	80.5	12	100	67.2	0	72.5	12
Random	42.5	94.0	36	92.9	12	99.6	78.4	0	80.4	12	100	67.2	0	72.5	12
Long	39.4	94.1	0	91.8	12	99.5	78.4	0	80.4	12	100	67.2	0	72.5	12
Best	39.2	94.1	0	91.8	12	99.5	78.4	0	80.4	12	100	67.2	0	72.5	12
$F_{max}1$	40.9	94.1	21	90.9	9	99.5	78.4	0	80.4	9	100	67.2	0	72.5	9
$F_{max}2$	39.2	94.1	0	91.7	12	99.5	78.4	0	80.4	12	100	67.2	0	72.5	12
Total	39.2	94.1	0	91.8	36	99.5	78.4	0	80.4	36	100	67.2	0	72.5	36
S.E.	21.0	0.2	169.7	1.8	9.2	0.2	0	0.4	0.0	9.2	0	0	0	0.0	9.2
C.V. (%)	43.0	0.2	196.2	2.0	66.3	0.2	0	282.8	0.0	66.3	0	0	-	0.0	66.3

Table 10 Decision matrix for the instances with 12 items and machines for different configurations with backlog heterogeneity.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	73349	180.0	12	100	100	44851	180.0	12	100	100	17526	180.0	12
Cluster	78.1	100	73761	181.2	24	71.8	100	44541	242.9	24	70.4	100	16680	233.8	24
Random	80.6	100	73469	181.7	24	64.6	100	44877	220.6	24	49.9	100	16560	246.4	24
Long	77.9	100	73514	181.6	24	59.9	100	44959	222.2	24	41.4	100	16550	251.8	24
Best	70.5	100	73559	180.8	24	48.3	100	45392	221.4	24	39.6	100	16207	264.8	24
$F_{max}1$	66.8	100	73394	208.0	18	47.9	100	45565	220.0	18	45.8	100	17747	224.8	18
$F_{max}2$	66.5	100	73263	205.8	24	47.3	100	45291	224.0	24	39.6	100	16244	263.2	24
Total	66.5	100	73259	217.4	144	47.3	100	45153	228.8	144	39.3	100	16168	260.2	144
S.E.	11.3	0	168.1	15.5	43.6	18.4	0	334.1	17.8	43.6	21.6	0	604.1	28.3	43.6
C.V. (%)	15.0	0	0.2	8.1	118.5	30.2	0	0.7	8.1	118.5	40.5	0	3.6	11.8	118.5

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	94.1	980	176.4	12	100	78.9	5	161.0	12	100	67.6	0	144.8	12
Cluster	46.6	94.5	256	187.3	24	99.5	78.9	1	161.2	24	100	67.6	0	144.7	24
Random	37.1	94.7	32	185.1	24	99.3	78.9	0	160.9	24	100	67.6	0	144.7	24
Long	35.4	94.7	1	184.4	24	99.3	78.9	0	160.9	24	100	67.6	0	144.6	24
Best	35.4	94.7	0	184.2	24	99.2	78.9	0	160.8	24	100	67.6	0	144.6	24
$F_{max}1$	38.2	94.6	62	183.2	18	99.2	78.9	0	160.8	18	100	67.6	0	144.8	18
$F_{max}2$	35.8	94.7	11	184.6	24	99.2	78.9	0	160.8	24	100	67.6	0	144.8	24
Total	35.0	94.7	1	184.2	144	99.2	78.9	0	160.8	144	100	67.6	0	145.0	144
S.E.	22.4	0.2	339.2	3.2	43.6	0.3	0	1.8	0.1	43.6	0	0	0	0.1	43.6
C.V. (%)	49.2	0	202.1	1.7	118.5	0.3	0	233.7	0.1	118.5	0	0	-	0.1	118.5

Table 11 Decision matrix for the instances with 24 items and machines for different configurations with backlog heterogeneity.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	146320	360.0	24	100	100	89377	360.0	24	100	100	34641	360.0	24
Cluster	78.6	100	146921	361.6	48	71.1	100	88776	482.7	48	68.6	100	32759	467.6	48
Random	78.4	100	146627	361.4	48	61.8	100	89378	442.4	48	42.2	100	32553	496.7	48
Long	78.2	100	146561	361.7	48	59.9	100	89406	444.3	48	38.5	100	32581	498.1	48
Best	67.5	100	146862	360.8	48	49.9	100	89349	457.2	48	34.6	100	32429	512.6	48
$F_{max}1$	62.9	100	146507	410.2	36	49.7	98.9	91895	439.6	36	55.4	97.0	43181	403.0	36
$F_{max}2$	62.7	100	146360	415.4	48	47.0	100	89108	475.8	48	34.2	100	32442	507.8	48
Total	62.7	100	146342	431.0	576	46.6	100	88737	493.4	576	33.3	100	31848	516.6	576
S.E.	12.8	0	231.1	30.5	188.7	18.0	0.4	1004.1	41.3	188.7	23.4	1.1	3776.8	58.0	188.7
C.V. (%)	17.4	0	0.2	8.0	172.3	29.7	0.4	1.1	9.2	172.3	46.0	1.1	11.1	12.3	172.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	92.5	2177	346.2	24	100	77.5	3	317.2	24	100	66.4	0	285.0	24
Cluster	44.2	92.9	639	372.2	48	99.8	77.5	1	317.3	48	100	66.4	0	284.6	48
Random	31.3	93.0	54	365.0	48	99.7	77.5	1	317.1	48	100	66.4	0	284.6	48
Long	29.6	93.0	1	361.7	48	99.7	77.5	1	317.1	48	100	66.4	0	284.6	48
Best	32.0	93.0	16	373.4	48	171.9	77.5	509	319.8	48	131.7	66.4	239	282.4	48
$F_{max}1$	49.0	92.7	765	360.2	36	103.9	77.5	4	319.2	36	104.2	66.4	15	282.0	36
$F_{max}2$	38.8	92.9	270	368.0	48	103.7	77.5	5	320.4	48	116.0	66.4	108	286.6	48
Total	29.4	93.0	0	360.4	576	99.7	77.5	1	317.0	576	100	66.4	0	284.6	576
S.E.	23.6	0.2	745.1	8.6	188.7	25.2	0	179.2	1.4	188.7	12	0	86.7	1.5	188.7
C.V. (%)	53.4	0.2	152.0	2.4	172.3	22.9	0	273.0	0.4	172.3	11	0	191.6	0.5	172.3

Table 12 Decision matrix for the instances with 6 items and machines for different configurations with setup times.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	81.7	56111	73.5	6	100	100	42434	90.0	6	100	100	28182	90.0	6
Cluster	94.3	94.7	52927	85.2	12	92.1	100	39038	90.0	12	93.6	100	26370	90.0	12
Random	94.1	95.5	52783	85.0	12	91.6	100	38823	90.0	12	91.3	100	25721	90.2	12
Long	94.4	94.3	52993	84.8	12	91.9	100	38983	90.0	12	91.1	100	25641	90.1	12
Best	92.2	98.2	51730	88.4	12	89.1	100	37744	90.0	12	89.8	100	25295	90.2	12
$F_{max}1$	89.7	100	50340	90.0	9	88.4	100	37435	90.0	9	89.9	100	25317	90.5	9
$F_{max}2$	89.7	100	50337	90.0	12	88.4	100	37424	90.2	12	89.7	100	25252	90.4	12
Total	89.7	100	50336	90.0	36	88.4	100	37427	90.3	36	89.7	100	25249	90.4	36
S.E.	3.5	6.1	1979.1	5.5	9.2	3.9	0	1686.3	0.1	9.2	3.5	0	1005.2	0.2	9.2
C.V. (%)	3.8	6.4	3.8	6.4	66.3	4.3	0	4.4	0.1	66.3	3.8	0	3.9	0.2	66.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	99.9	14371	90.0	6	100	97.3	3818	89.2	6	100	86.3	267	83.3	6
Cluster	95.0	100	13653	90.0	12	72.9	98.8	2789	90.5	12	71.6	87.0	51	84.2	12
Random	92.0	100	13228	90.3	12	66.8	99.6	2459	90.8	12	65.4	87.1	14	84.5	12
Long	90.4	100	13008	90.4	12	45.9	99.9	1836	91.0	12	63.0	87.2	0	84.8	12
Best	90.0	100	12947	90.8	12	45.3	99.9	1818	91.5	12	62.9	87.2	0	84.8	12
$F_{max}1$	91.7	100	13185	90.4	9	49.0	99.8	1950	90.7	9	63.0	87.2	0	84.7	9
$F_{max}2$	90.0	100	12943	90.7	12	45.4	99.9	1819	91.4	12	62.9	87.2	0	84.8	12
Total	90.0	100	12942	90.7	36	45.3	99.9	1817	91.6	36	62.9	87.2	0	84.9	36
S.E.	3.5	0	500.5	0.3	9.2	19.9	0.9	717.4	0.8	9.2	13	0.3	93	0.5	9.2
C.V. (%)	3.8	0	3.8	0.3	66.3	33.8	0.9	31.4	0.9	66.3	18.7	0.4	223.7	0.6	66.3

Table 13 Decision matrix for the instances with 12 items and machines for different configurations with setup times.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	68.3	110923.0	123.0	12	100	100	84367.0	180.0	12	100	100	55874.0	180.0	12
Cluster	91.1	91.0	101030.0	163.8	24	88.2	99.9	74406.0	180.0	24	91.3	99.9	50952.0	180.0	24
Random	91.8	89.8	101773.0	161.7	24	88.2	99.9	74327.0	180.0	24	88.3	99.9	49293.0	180.3	24
Long	91.1	91.5	101004.0	164.7	24	87.6	99.9	73876.0	180.0	24	87.6	99.9	48901.0	180.1	24
Best	88.3	100	97841.0	180.0	24	83.9	100	70671.0	180.0	24	84.5	100	47090.0	180.4	24
$F_{max}1$	82.7	99.8	91596.0	180.0	18	83.2	100	70081.0	180.6	18	84.5	99.9	47073.0	180.2	18
$F_{max}2$	82.7	99.9	91598.0	180.0	24	83.1	99.9	70040.0	181.2	24	84.2	100	46947.0	181.2	24
Total	82.7	99.8	91594.0	180.0	144	83.1	100	70038.0	181.8	144	84.2	100	46939.0	182.2	144
S.E.	6.1	10.8	6772.8	19.5	43.6	5.7	0.1	4831.2	0.7	43.6	5.5	0.1	3090.1	0.8	43.6
C.V. (%)	6.8	11.7	6.9	11.7	118.5	6.5	0.1	6.6	0.4	118.5	6.2	0.1	6.3	0.4	118.5

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	99.7	28701	180.0	12	100	95.5	9067	176.8	12	100	86.0	881	166.2	12
Cluster	93.2	99.9	26769	180.0	24	71.8	97.6	6635	179.7	24	56.3	86.8	214	168.0	24
Random	88.2	99.9	25349	180.9	24	49.9	99.1	4769	182.8	24	45.3	87.2	33	169.9	24
Long	86.8	99.9	24923	180.9	24	40.0	99.9	3962	183.9	24	42.9	87.3	3	170.2	24
Best	85.2	100	24443	181.8	24	40.1	99.9	3969	184.6	24	42.7	87.3	1	170.2	24
$F_{max}1$	87.3	100	25006	180.6	18	46.6	99.4	4458	181.0	18	42.9	87.2	1	169.8	18
$F_{max}2$	85.1	99.9	24410	182.4	24	41.2	99.8	4051	183.8	24	42.7	87.3	0	170.4	24
Total	84.9	100	24347	183.2	144	38.8	99.9	3842	186.8	144	42.6	87.2	0	169.8	144
S.E.	5.3	0.1	1516.1	1.1	43.6	21.7	1.6	1846.7	3.1	43.6	20.0	0.5	308	1.5	43.6
C.V. (%)	5.9	0.1	5.9	0.6	118.5	40.5	1.6	36.3	1.7	118.5	38.5	0.5	217.2	0.9	118.5

Table 14 Decision matrix for the instances with 24 items and machines for different configurations with setup times.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	55.0	227639	198.0	24	100	100	174795	360.0	24	100	100	117670	360.0	24
Cluster	91.7	79.5	208622	286.2	48	88.6	100	154888	360.0	48	91.3	100	107440	360.0	48
Random	92.2	78.3	209724	282.0	48	88.1	100	153938	360.0	48	88.0	100	103527	360.5	48
Long	91.4	80.6	207847	290.1	48	87.6	100	153044	360.0	48	87.8	100	103238	360.6	48
Best	86.7	100	197077	360.0	48	81.9	100	143106	360.0	48	84.7	100	99603	361.8	48
$F_{max}1$	81.2	100	184643	360.2	36	80.7	99.9	140997	360.0	36	86.7	98.8	101839	359.0	36
$F_{max}2$	81.2	100	184618	360.0	48	80.6	100	140753	361.2	48	84.8	100	99663	361.6	48
Total	81.2	100	184598	360.4	576	80.5	100	140567	363.4	576	84.0	100	98758	366.8	576
S.E.	6.8	16.3	15628.5	58.9	188.7	6.7	0.1	11747.2	1.2	188.7	5.2	0.4	6208.2	2.4	188.7
C.V. (%)	7.7	18.8	7.8	18.9	172.3	7.8	0.1	7.8	0.3	172.3	5.9	0.4	6.0	0.7	172.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	99.6	63107	359.2	24	100	95.5	21414	350.4	24	100	86.2	2220	332.0	24
Cluster	93.2	99.9	58760	360.5	48	77.8	97.6	16634	358.5	48	52.8	87.0	845	336.2	48
Random	88.7	99.9	55909	326.2	48	51.9	99.7	11250	364.9	48	29.6	87.4	93	339.5	48
Long	88.3	99.9	55690	362.1	48	47.0	99.9	10269	364.4	48	26.9	87.5	6	340.4	48
Best	87.8	99.7	55243	362.0	48	56.6	99.6	12199	364.8	48	30.0	87.3	74	336.8	48
$F_{max}1$	93.1	99.4	58589	359.2	36	97.5	95.9	20194	354.8	36	48.9	87.2	445	337.4	36
$F_{max}2$	88.5	99.7	55725	361.8	48	61.1	98.6	13349	358.0	48	37.5	87.4	212	340.4	48
Total	85.8	100	54036	366.4	576	45.2	100	9872	370.4	576	26.6	87.5	0	339.8	576
S.E.	4.6	0.2	2907.7	12.7	188.7	22.0	1.8	4491.2	6.5	188.7	24.7	0.4	756.2	2.9	188.7
C.V. (%)	5.0	0.2	5.1	3.6	172.3	32.7	1.9	31.2	1.8	172.3	56.1	0.5	155.3	0.9	172.3

Table 15 Decision matrix for the instances with 6 items and machines for different configurations with demand heterogeneity.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	36534	90.0	6	100	89.3	27272	85.0	6	100	79.4	20153	79.4	6
Cluster	99.1	100	36222	96.3	12	85.5	97.2	23320	94.3	12	56.5	94.5	11413	92.0	12
Random	98.9	100	36165	96.0	12	80.3	100	21901	96.4	12	44.1	99.2	8878	98.7	12
Long	98.9	100	36158	95.9	12	80.1	100	21866	98.6	12	44.5	98.4	8986	97.0	12
Best	98.8	100	36148	91.3	12	79.8	100	21784	94.1	12	38.3	100	7748	98.5	12
$F_{max}1$	98.7	100	36138	91.4	9	79.7	100	21777	93.2	9	38.8	100	7843	98.7	9
$F_{max}2$	98.7	100	36137	92.2	12	79.6	100	21762	94.6	12	38.2	100	7732	99.0	12
Total	98.7	100	36131	99.3	36	79.6	100	21757	102.0	36	38.2	100	7732	100.0	36
S.E.	0.4	0	136.4	3.3	9.2	7.1	3.8	1930.2	4.9	9.2	21.2	7.1	4265.3	6.9	9.2
C.V. (%)	0.4	0	0.4	3.5	66.3	8.6	3.8	8.5	5.2	66.3	42.6	7.4	42.4	7.3	66.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	73.6	13154	75.1	6	100	69.6	6570	69.6	6	100	65.7	1673	65.9	6
Cluster	28.0	88.7	3557	91.9	12	26.0	76.3	1577	78.0	12	28.4	67.1	234	70.7	12
Random	12.4	91.7	1528	92.1	12	2.9	78.7	4	80.7	12	18.6	67.5	0	71.1	12
Long	7.1	92.7	832	93.2	12	2.8	78.7	1	80.1	12	18.5	67.5	0	70.9	12
Best	1.5	94.4	1	94.3	12	2.8	78.7	0	79.7	12	18.5	67.5	0	70.7	12
$F_{max}1$	1.7	94.4	30	95.4	9	2.8	78.7	0	79.7	9	18.5	67.5	0	70.8	9
$F_{max}2$	1.5	94.4	1	94.5	12	2.8	78.7	0	79.6	12	18.5	67.5	0	70.7	12
Total	1.5	94.4	1	94.7	36	2.8	78.7	0	79.6	36	18.5	67.5	0	70.7	36
S.E.	33.9	7.1	4521.2	6.7	9.2	34.2	3.2	2309.8	3.6	9.2	29	0.6	585.4	1.7	9.2
C.V. (%)	176.5	7.9	189.3	7.3	66.3	191.3	4.2	226.7	4.6	66.3	95	0.9	245.6	2.5	66.3

Table 16 Decision matrix for the instances with 12 items and machines for different configurations with demand heterogeneity.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	74713	180.0	12	100	89.2	56139	167.6	12	100	79.4	41847	154.2	12
Cluster	99.5	100	74362	194.0	24	91.1	94.3	51137	187.8	24	73.0	89.0	30479	174.3	24
Random	99.3	100	74224	191.8	24	83.9	98.6	47043	195.6	24	56.0	95.4	23304	188.9	24
Long	99.2	100	74190	190.0	24	82.0	99.8	45956	196.3	24	51.3	96.9	21332	194.3	24
Best	99.2	99.9	74190	181.8	24	80.9	100	45377	188.0	24	40.9	100	16965	199.4	24
$F_{max}1$	99.1	100	74182	181.8	18	80.8	100	45377	186.6	18	41.4	100	17170	197.8	18
$F_{max}2$	99.1	99.9	74194	182.4	24	80.8	100	45381	187.2	24	40.8	100	16929	202.4	24
Total	99.1	100	74165	197.4	144	80.8	100	45365	206.4	144	40.7	100	16902	203.0	144
S.E.	0.3	0	186.6	6.7	43.6	7.0	4.0	3930.1	11.1	43.6	21.2	7.4	8931.2	17.0	43.6
C.V. (%)	0.3	0	0.3	3.6	118.5	8.2	4.1	8.2	5.9	118.5	38.2	7.8	38.6	9.0	118.5

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	73.6	27728	145.4	12	100	69.6	14193	135.6	12	100	66.0	3316	130.6	12
Cluster	53.9	83.0	14764	169.9	24	51.8	74.2	7190	149.5	24	38.6	67.2	1151	138.4	24
Random	25.8	89.1	6340	181.3	24	11.7	78.5	1361	160.0	24	15.1	68.0	224	141.2	24
Long	17.6	91.0	4611	184.2	24	3.3	79.3	149	161.6	24	9.2	68.1	0	141.9	24
Best	1.4	95.4	0	191.2	24	2.4	79.5	0	160.2	24	9.2	68.1	0	142.0	24
$F_{max}1$	2.2	95.2	207	196.6	18	2.4	79.5	1	160.6	18	9.2	68.1	0	141.4	18
$F_{max}2$	1.5	95.4	0	194.8	24	2.4	79.5	0	160.4	24	9.2	68.1	0	141.8	24
Total	1.4	95.4	0	190.0	144	2.4	79.5	0	160.6	144	9.2	68.1	0	141.6	144
S.E.	35.3	7.9	9917.7	17.0	43.6	35.8	3.7	5203.0	9.1	43.6	32.0	0.8	1172	3.9	43.6
C.V. (%)	138.4	8.8	147.9	9.3	118.5	162.3	4.7	181.8	5.9	118.5	128.1	1.1	199.9	2.8	118.5

Table 17 Decision matrix for the instances with 24 items and machines for different configurations with demand heterogeneity.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	100	193468	360.0	24	100	100	136176	360.0	24	100	100	79701	359.8	24
Cluster	99.7	100	193137	361.3	48	99.5	100	135688	375.6	48	98.5	100	78515	387.1	48
Random	99.7	100	193137	361.0	48	99.4	100	135595	372.7	48	98.1	100	78182	391.8	48
Long	99.7	100	193108	361.5	48	99.4	100	135577	372.2	48	98.0	100	78144	393.2	48
Best	99.7	100	193070	360.0	48	99.3	100	135471	366.2	48	97.6	100	77873	373.8	48
$F_{max}1$	99.7	99.8	193111	360.4	36	99.3	100	135494	363.8	36	97.9	99.9	78042	367.6	36
$F_{max}2$	99.8	99.5	193157	360.4	48	99.3	100	135466	373.0	48	97.6	100	77863	375.8	48
Total	99.6	100	193043	396.2	576	99.3	100	135443	424.8	576	97.6	100	77843	397.8	576
S.E.	0.1	0.2	132.3	12.6	188.7	0.2	0	241.7	20.4	188.7	0.8	0.1	619.8	13.6	188.7
C.V. (%)	0.1	0.2	0.1	3.4	172.3	0.2	0	0.2	5.4	172.3	0.8	0.1	0.8	3.6	172.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	92.2	31758	343.2	24	100	77.5	7815	299.2	24	100	66.5	842	268.4	24
Cluster	88.6	92.8	28057	363.8	48	61.6	77.5	4592	319.9	48	60.1	66.5	234	279.6	48
Random	81.0	93.0	25575	379.1	48	27.1	77.5	1633	344.6	48	46.2	66.5	35	280.8	48
Long	79.6	93.0	25156	376.9	48	16.4	77.5	673	358.3	48	43.5	66.5	1	280.7	48
Best	78.3	93.0	24697	389.4	48	15.0	77.5	522	372.4	48	46.6	66.5	21	282.4	48
$F_{max}1$	84.2	92.8	26579	359.2	36	53.2	77.5	3785	322.4	36	50.5	66.5	72	283.2	36
$F_{max}2$	79.6	93.0	25119	385.4	48	26.6	77.5	1578	354.8	48	51.6	66.5	93	281.8	48
Total	77.9	93.0	24611	379.0	576	13.8	77.5	490	350.6	576	43.4	66.5	0	277.2	576
S.E.	7.5	0.3	2433.1	15.4	188.7	30.4	0	2591.8	24.2	188.7	18.9	0	285.0	4.8	188.7
C.V. (%)	9.0	0.3	9.2	4.1	172.3	77.4	0	98.3	7.1	172.3	34.2	0	175.7	1.7	172.3

Table 18 Decision matrix for the instances with 6 items and machines for different configurations with backlog and demand heterogeneity considering setup times.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	81.4	56439	73.3	6	100	99.9	42893	90.0	6	100	96.1	31032	88.7	6
Cluster	92.9	93.6	54327	84.6	12	87.7	99.8	41487	90.1	12	79.2	98.9	28582	89.6	12
Random	92.5	94.0	53985	84.8	12	85.7	100	41076	90.1	12	72.5	100	28017	90.2	12
Long	92.6	93.9	54210	84.6	12	85.4	100	41225	90.1	12	71.9	100	28066	90.3	12
Best	90.9	97.3	53590	87.9	12	82.8	100	40625	90.3	12	68.3	100	27844	90.4	12
$F_{max}1$	84.0	99.1	52370	89.3	9	74.2	100	39877	90.3	9	60.9	100	27876	90.6	9
$F_{max}2$	84.0	99.1	52367	89.5	12	74.1	100	39911	90.3	12	60.6	100	27687	90.7	12
Total	84.0	99.1	52427	89.3	36	74.1	100	39913	90.4	36	60.5	100	27642	90.8	36
S.E.	5.7	5.9	1384.3	5.4	9.2	8.9	0	1036.7	0.1	9.2	13.3	1.3	1125.0	0.7	9.2
C.V. (%)	6.4	6.2	2.6	6.3	66.3	10.8	0	2.5	0.2	66.3	18.5	1.3	4.0	0.8	66.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	86.0	23169	82.0	6	100	78.4	16111	76.1	6	100	72.9	9351	70.5	6
Cluster	57.5	95.2	16830	87.7	12	32.0	92.9	6896	86.6	12	22.0	83.2	2583	80.5	12
Random	45.1	99.8	15011	90.8	12	19.6	96.7	4598	90.3	12	5.7	86.5	503	83.9	12
Long	45.8	99.5	15199	90.8	12	17.5	97.3	4058	90.8	12	3.2	87.2	114	85.0	12
Best	39.4	99.8	15209	90.8	12	10.1	99.8	2643	92.9	12	2.5	87.5	3	85.4	12
$F_{max}1$	39.0	99.7	15631	90.4	9	11.7	99.5	2963	91.3	9	2.7	87.4	32	85.1	9
$F_{max}2$	37.7	99.8	15091	91.5	12	9.7	99.8	2653	93.1	12	2.5	87.5	3	85.2	12
Total	37.6	99.9	15089	91.7	36	9.6	99.8	2634	93.6	36	2.5	87.5	3	85.2	36
S.E.	21.2	4.9	2798.4	3.3	9.2	30.7	7.3	4599.6	5.8	9.2	33.9	5.1	3263.7	5.2	9.2
C.V. (%)	42.1	5.0	17.1	3.6	66.3	117.0	7.7	86.5	6.5	66.3	192.4	6.0	207.3	6.2	66.3

Table 19 Decision matrix for the instances with 12 items and machines for different configurations with backlog and demand heterogeneity considering setup times.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	68.3	111915	123.0	12	100	99.1	86504	179.6	12	100	93.2	64582	172.2	12
Cluster	92.0	88.9	105055	161.4	24	87.8	99.7	83664	180.1	24	77.1	97.1	59617	176.5	24
Random	92.0	88.9	81957	160.5	24	86.6	99.8	81957	180.3	24	73.4	99.1	57168	180.0	24
Long	91.6	90.5	104371	163.5	24	86.1	99.8	82497	180.3	24	71.5	99.8	57320	181.3	24
Best	89.0	98.4	101479	177.8	24	82.1	100	82025	180.2	24	63.4	100	58363	181.4	24
$F_{max}1$	78.9	100	97814	180.2	18	70.0	100	78063	180.8	18	52.7	100	58107	181.0	18
$F_{max}2$	78.9	99.9	97794	180.0	24	69.9	100	78082	181.2	24	52.4	100	57915	182.2	24
Total	78.9	100	97809	180.0	144	69.9	100	78065	181.2	144	52.4	100	57936	182.2	144
S.E.	7.9	10.8	8688.3	19.4	43.6	10.9	0.3	3078.3	0.6	43.6	16.4	2.4	2423.3	3.5	43.6
C.V. (%)	9.0	11.7	8.7	11.7	118.5	13.4	0.3	3.8	0.3	118.5	24.2	2.4	4.1	2.0	118.5

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	84.6	48074	157.8	12	100	77.4	33902	147.8	12	100	72.4	20303	140.2	12
Cluster	62.9	91.4	39819	168.0	24	46.8	86.3	22468	161.1	24	38.6	79.3	11079	153.1	24
Random	52.3	96.9	33597	178.3	24	29.7	92.4	14976	173.9	24	13.8	85.1	3392	165.4	24
Long	49.0	98.5	32734	181.3	24	23.2	94.6	12523	177.3	24	6.7	86.3	1931	167.9	24
Best	36.4	99.8	31601	183.0	24	9.3	99.8	6448	188.2	24	2.1	88.3	8	171.2	24
$F_{max}1$	32.1	99.4	33112	181.0	18	11.2	99.0	7815	184.4	18	2.4	88.6	62	175.8	18
$F_{max}2$	30.7	99.8	32402	184.4	24	9.0	99.6	6724	189.4	24	2.2	88.7	0	176.0	24
Total	30.5	99.9	32320	187.0	144	8.6	100	6382	195.6	144	2.1	88.5	0	172.6	144
S.E.	23.7	5.5	5715.0	9.8	43.6	31.4	8.1	9835.2	16.0	43.6	34.3	5.9	7379.0	12.5	43.6
C.V. (%)	48.1	5.7	16.1	5.5	118.5	105.7	8.7	70.7	9.0	118.5	163.3	6.9	160.5	7.6	118.5

Table 20 Decision matrix for the instances with 24 items and machines for different configurations with backlog and demand heterogeneity considering setup times.

	40					60					80				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	55.0	274743	198.0	24	100	100	221911	360.0	24	100	100	164652	360.0	24
Cluster	92.9	79.5	257402	286.2	48	86.4	100	219812	360.0	48	78.7	100	161906	361.4	48
Random	93.2	78.3	258783	282.0	48	86.6	100	208647	360.0	48	77.6	100	160445	360.7	48
Long	92.7	80.6	256720	290.1	48	86.1	100	208933	360.1	48	76.9	100	160759	360.7	48
Best	89.7	100	244629	360.0	48	81.1	100	206351	359.8	48	68.5	100	162433	362.2	48
$F_{max}1$	79.3	98.2	239837	354.8	36	75.2	99.3	207276	359.4	36	65.3	97.7	165336	355.2	36
$F_{max}2$	79.1	100	239435	359.8	48	74.9	100	205035	361.0	48	64.7	98.6	164613	359.0	48
Total	79.1	100	239213	360.4	576	74.9	100	205912	361.2	576	63.9	100	164104	364.0	576
S.E.	8.1	16.1	12733.2	58.3	188.7	8.6	0.2	6561.1	0.6	188.7	12.0	0.8	1892.4	2.6	188.7
C.V. (%)	9.1	18.6	5.1	18.7	172.3	10.4	0.2	3.1	0.2	172.3	16.2	0.8	1.2	0.7	172.3

	100					120					140				
	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL	Min UB	Max CU	Min Back	Min NS	Min NL
Dedicated	100	99.3	108294	358.4	24	100	95.1	57818	347.4	24	100	85.7	21793	320.4	24
Cluster	73.7	99.8	109736	362.9	48	70.9	97.3	56521	360.0	48	63.8	86.8	18377	332.0	48
Random	66.3	99.7	109007	365.6	48	53.0	98.3	56893	370.5	48	37.2	87.3	16181	338.7	48
Long	65.1	99.7	109495	364.8	48	50.6	98.5	56748	373.5	48	33.4	87.4	15636	339.7	48
Best	54.0	99.4	110850	364.8	48	44.5	98.5	58324	372.2	48	32.2	87.7	16492	344.2	48
$F_{max}1$	54.4	94.8	114227	348.2	36	50.6	92.0	64720	340.4	36	48.4	85.7	20390	324.6	36
$F_{max}2$	52.5	96.9	112640	356.6	48	44.5	95.7	60332	358.6	48	38.5	86.6	18497	334.0	48
Total	50.7	100	110389	376.4	576	40.0	99.9	57225	391.4	576	26.4	88.2	15204	362.4	576
S.E.	16.5	1.8	1972.8	8.2	188.7	19.8	2.5	2769.5	16.2	188.7	24.2	0.9	2367.1	12.9	188.7
C.V. (%)	25.5	1.9	1.8	2.3	172.3	34.9	2.6	4.7	4.4	172.3	50.9	1.0	13.3	3.8	172.3

ANEXO C

Machine flexiblity for the lot-sizing problem in unbalanced systems

Machine flexibility for the lot-sizing problem in unbalanced systems

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Abstract

In this work, we seek to analyze the benefits of machine flexibility for the problem of unbalanced lot-sizing (i.e., number of items greater than the number of machines) in a deterministic context, in which a new methodology is proposed to represent the configurations of flexibility. In this approach the notion of sharing intensity between machines is considered, which allows different flexibility configurations to be analyzed. To analyze the developed methodology, a new mathematical formulation for the problem that considers the profile of the flexibility configuration is proposed. In addition, some performance indicators of the configurations were proposed, among them, the similarity index that allows to analyze the discrepancy between the studied configuration profile and the configuration obtained in the optimal solution. The computational results for the similarity indicators show that most of the configurations in the optimal solution of the problem are constrained configurations.

Keywords: lot-sizing, machine flexibility, process flexibility, unbalanced systems, integer programming.
2010 MSC: 90B05, 90B30, 90C11.

1. Introdução

The great competitiveness imposed by the globalized market has forced the development of industrial processes to face more complex environments, in which new strategies have been produced to improve decision making. In this highly competitive environment, companies need to optimize their services and products to achieve better margins and stay in the market. Research on lot sizing problems is in line with this trend in the evolution of the decision process. The lot-sizing problem consists of finding a production plan in order to minimize production costs and meet the demand of the products. This paper deals with the lot-sizing problem with flexible parallel machines and the possibility of backlog in meeting demand. While considering flexible systems Sethi e Sethi (1990) defines flexibility as the ability that a company has to respond to recurring variations in industry daily life. This ability can come in many forms, such as the ability to change the production line inexpensively to manufacture a new product, or change the production planning to deal with

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changes in demand. In this context, machine flexibility can be defined as the ability of a machine to produce different products. This aptitude is usually represented by a flexibility configuration, an idea that involves the use of a bipartite graph, whose vertices in each partition represent the products or machines exclusively and the edges connect the vertices between the partitions (representing the production options). As such, flexibility configuration can be understood as a designation of a certain set of itens to a machine (Jordan e Graves, 1995). conf flex

The amount of flexibility is defined as the amount of existing connections (active edges) in the graph. One can observe that with the same amount of connections it is possible to obtain different flexibility configurations. Another important aspect of flexibility configuration is related to the amount of items and machines. In general practice, the amount of items is greater than the amount of machines, thus the importance of studying flexibility configurations in these so-called unbalanced systems. qt flex ñ baln

Note that, in general, it is difficult to make decisions about machine flexibility, as the number of setup for the production of a product grows exponentially according to the number of items and machines and, moreover, since the strategy used by the majority of papers that consider parallel machines to use total machine flexibility (all machines can produce all products) can be very expensive, or even impractical, it is important to study ways to implement a limited amount of flexibility (which products should be produced in each of the machines) to balance the costs and benefits.

In the classical formulation of the lot-sizing problem with parallel machines(Fiorotto et al., 2015), the machine is considered to produce all products; such configuration is called *total flexibility*. It is worth observing that, the more items a machine can produce (i.e., the more flexible it is), the greater is the necessary investment in technology. As such, it is necessary to seek configurations that might be efficient and economical. Note that, in general, in practice, the configurations found have *limited flexibility* , that is, each machine can produce a small part of the products.

Studies of the benefits of machine flexibility have been proposed in the literature for both stochastic and deterministic contexts. In general, in stochastic contexts, the machine flexibility makes it possible to respond to variations in demand by redistributing production across machines. In a deterministic context, when there is no flexibility and it is not possible to meet all the demand, adding flexibility allows to decrease the backlog by using machine capacity more efficiently. benefit flex

Although they appear in most practical applications, there are still few papers in the literature on flexibility configurations for unbalanced systems (all for stochastic contexts) and there are no studies so far on flexibility configurations in unbalanced systems applied to the lot-sizing problem in a deterministic context. In this paper, we present a way to build limited flexibility configurations for unbalanced systems. Furthermore, through various computational experiments the performance of these configurations is analyzed. Thus, the main contributions of this work are i) to generalize the concepts of flexibility configurations, proposing an approach that encompasses unbalanced systems; ii) to develop a tool for constructing and analyzing flexibility; iii) to propose mathematical formulations that allow studying the benefits of flexibility configurations; iv) to propose new measures to calculate the benefits of flexibility; and v) to investigate, through computational testing, the benefits of flexibility in the lot-sizing problem in unbalanced systems.

The other sections of the article are organized as follows. In Section 2 the literature review of machine flexibility studies will be presented, as well as the frameworks already proposed. In Section 3 the proposed generalization of flexibility configurations to unbalanced systems will be presented, as well as some performance indicators for the configurations. Furthermore, in Section 3 the mathematical formulations of the problem will also be presented. In the Section 4 the computational results of the work will be presented and finally, in Section 5 we will present the conclusions and future proposals.

2. Literature review

This section presents a literature review of the developments that have been made in studying machine flexibility in production planning problems. It can be observed that the works proposed so far consider the study of flexibility in balanced systems (that is, when the number of items and machines is the same). No articles were found regarding machine flexibility in the lot sizing problem in unbalanced systems.

The notion of flexibility is broad, so some authors have classified different types of flexibility. In this line of studies, for example, Gupta e Goyal (1989) revisited the literature to analyze how each type of flexibility was defined and what metrics were proposed to analyze the impact of flexibility. In Gupta e Somers (1992), the authors expanded on the previous work by conducting a survey of managers in several companies in order to list the main metrics used in practice, for each type of flexibility.

Koste e Malhotra (1999) proposed a framework by classifying the flexibility metrics into four major classes: range-number (R-N) related to the number of flexibility options; range-heterogeneity (R-H) related to the degree of heterogeneity among the flexibility options; mobility (M) related to the switching costs among the options; and uniformity (U) related to the degree of similarity of the outputs. In addition, the authors proposed a hierarchical classification of flexibility types according to planning levels and showed that the different flexibility types communicate to each other and often interfere with the same metrics.

For stochastic contexts, in their seminal work, Jordan e Graves (1995) inaugurated studies of the benefit cadeia
of flexibility in stochastic contexts. The authors proposed a flexibility configuration known in the literature as the chain. In this configuration, each item can be produced on two machines and each machine can produce two items, and a path can be drawn between any item and machine (according to graph theory). One of the great advantages of this configuration is the high connectivity between the machines and the items. According to the way in which the chain was defined by Jordan e Graves (1995), it could only be constructed in a balanced system. With the approach proposed in this paper, the unbalanced systems encompass the balanced one, and therefore a chain can be obtained with this approach.

Studies involving the performance of the chain configuration have developed even further. Several authors have demonstrated the efficiency of the chain configuration in several productive systems. For example, Muriel et al. (2006) proposed an analytical model to analyze the impact of chain on operational indicators such as regra cadeia lost sales and total stocked. The results showed that a small increase in the amount in flexibility is able to generate an increase in sales and reduce the amount in stock. On the other hand, some studies have concluded that the chain does not perform well in heterogeneous scenarios. Along these lines, Mak e Shen (2009) developed a full stochastic model considering the cost of investing in flexibility. The computational results showed that when the coefficient of variation of demand is high, the chain did not perform satisfactorily. Andradóttir et al. (2013) showed that for the queueing problem, the chain gets good performance when the scenario is homogeneous, but it is possible for others to get configurations with superior performance for the heterogeneous scenarios.

Fiorotto et al. (2018) conducted the first study showing the importance of machine flexibility in deterministic context in the lot-sizing problem. In this context, the flexibility configuration decision becomes a tactical/operational decision that is made at the beginning of the planning horizon when the demand is known. The authors compared different flexibility configurations and noted that the chain was able to achieve the same benefits as total flexibility. In addition, the authors have shown that an even smaller amount of flexibility than is required for the chain can already achieve almost the same benefits as total flexibility.

Although most practical applications have unbalanced systems, that is, the number of items is different from the number of machines, the number of works on these systems is still little in the literature (all that exist consider stochastic contexts). This is mainly due to the difficulty in characterizing flexibility configurations when the system is unbalanced.

Tanrisever et al. (2012) presented a capacity management model where the system was unbalanced. The authors compared the performance of total flexibility with two other limited flexibility configurations. The first is an adaptation of the chain of Jordan e Graves (1995) and the second, a configuration removing some arcs from the first configuration. The computational results showed that the performance, in terms of total backlog, of the adapted chain was the same as the total flexibility configuration. In contrast, when considering the cost of each existing link, the partial chain performed slightly better.

Deng e Shen (2013) proposed a new chain configuration adapted for unbalanced systems, considering a circular view of the problem, that is, items and machines are arranged in a circular fashion and then, the authors proposed some guidelines to obtain a chain. In addition, the authors studied a flexibility configuration with $3n$ links, n being the quantity of items. The authors showed that constructing the chain using their proposed orientations produced a solution with superior performance to the configuration constructed using the orientations proposed by Jordan e Graves (1995).

Feng et al. (2017) conducted a study of machine flexibility in an unbalanced stochastic production planning problem and proposed a two-stage model. In the first, the items were assigned to the plants, and in the second, the production planning was done. To approximate the problem to what occurs in practice, the authors considered the heterogeneity of plants and products, i.e., some plants are more technological and therefore more adept at producing some products. Product heterogeneity, on the other hand, deals with the importance of the product to the plant. The results showed that when the plant efficiency decreases, the amount of links needed in the system increases, and when the plant efficiency increases, more products are assigned to the plant.

Fügener et al. (2018) studied the problem of multifunctional nurses. At work, each care unit has its nurses, and some of them are multifunctional, that is, they can work in more than one care unit. It is interesting to note, that the number of nurses is greater than the number of care units; thus, it is an unbalanced system. Several rules must be met to achieve the *scheduling* of nurses, such as restricting the amount of hours in each unit, minimum rest time, among others. The authors proposed a flexibility configuration called *one-to-each*, in which each care unit assigns a nurse to each of the other units. This flexibility configuration performed well in terms of savings compared to the scenario without multi-functional workers.

3. Building the configurations and formulations for the lot-sizing problem with machine flexibility

This section introduces the notation used and proposes a new idea for constructing the flexibility configurations. It can be seen that in the way that the construction of these configurations is proposed, it is possible to generalize the way in which unbalanced and balanced systems are approached. That is, the flexibility configurations known in the literature for balanced systems can be obtained for the unbalanced case (since the former is a particular case of the latter), which makes it possible to analyze the benefits of classical flexibility configurations that were proposed for the balanced case in an unbalanced production environment. In addition, three lot-sizing models that consider limited flexibility configurations will be proposed. The first is associated with a fixed flexibility configuration, which occurs in organizations that already own the machinery with its productive possibilities. The second and third models are able to obtain the optimal flexibility configuration given some parameters. Finally, some measures for analyzing the performance of the flexibility configurations are proposed.

3.1. Construction of the flexibility configurations

In this section a new idea for constructing machine flexibility configurations is proposed. Consider N the set of items and M the set of machines, such that the quantity of items is at least equal to the quantity of machines. The idea associated with this construct is that it is possible to cover the set of items, that is, for each machine some items can be assigned, so that each item is assigned to only one machine. Therefore, a case of no production sharing would be obtained. It is important to note that this assignment does not imply production of the item by the machine. If a machine can also produce an item that has not been assigned to it, it is a case of sharing the production of the items. The formal definition of the concepts are described below.

Be $N = \{1, \dots, n\}$ e $M = \{1, \dots, m\}$ the set of indices such as $n \geq m$. The set $S = \bigcup_{k \in M} S_k$ is the union of m subsets of N in which S is a coverage of N, that is, $S_k \subseteq N$, $\bigcup_{k \in M} S_k = N$. In particular, consider that $S_k \neq \emptyset$, $S_j \cap S_k = \emptyset$, $j, k \in M$ and $j \neq k$. Furthermore, the entry $c_{ij} \in \{0, 1\}$ of the binary matrix $C = (c_{ij})$

assumes value one if there is a link between element $i \in N$ and element $j \in M$, and value zero otherwise. One way to represent flexibility configurations is through the incidence matrix C and every configuration can be represented in this way.

Figure 1 illustrates the case in which $|S_j| > 1$, in which $S_1 = \{2,5,8\}$, $S_2 = \{1,3\}$, $S_3 = \{4,6\}$ and $S_4 = \{7,9\}$. The bold line in the figure represents that the machine is able to produce all items within that subset it is linked to. From the configuration presented in Figure 1(a) that does not show any apparent pattern of configuration, the subsets S_k were used to obtain a known configuration pattern. Figure 1(b) demonstrates the transformation representation of an unbalanced system into a new system. The items are designated to sets S_k and it is the relation between sets S_k with the machines that define the profile of the flexibility configuration. In Figure 1(b), the weaker links between the sets S_k and the machines indicate that these are possible links, and that depending on the profile of the flexibility configuration, they might be active or not. The relation between sets S_k and the machines is defined by matrix Λ, as follows.

Let λ_{kj} be the intensity parameter that limits production sharing of items between distinct machines k and j, that is, given the items in S_k that are produced by machine k, λ_{kj} is the maximum amount of items in S_k that the machine j can produce. This is related to the number of active links in the incidence matrix C, that is, $\sum\limits_{i \in S_k} c_{ij} \leq \lambda_{kj}$. As such, each binary matrix $C = (c_{ij})$ is related with a intensity square matrix $\Lambda = (\lambda_{kj})$, whose lines and rows refer to the machines.. In this approach, it is worth considering that each entry in the main diagonal of Λ is the cardinality of the set S_k, that is, $\lambda_{kk} = |S_k|$.

By observing the new system in Figure 1(c), one can note that there is no item sharing between machines. Note that it presents the same links that occur in Figure 1(a). This configuration produces a matrix Λ where only the main diagonal is nonzero (see Figure 2(a)). In a balanced system, that is, when $|S_k| = 1 \ \forall k \in M$, the configuration shown in Figure 1(c) is known as a dedicated configuration, since each machine is dedicated to one item and each item is dedicated to one machine. Then, the Figure 1(d), shares a certain amount of produced items (λ_{kj}) between machine k and j.. It is worth noticing that in the matrix of Figure 2(b), in addition to the main diagonal, there are non-zero entries that represent the sharing intensity between the machines. Importantly, this configuration presents the profile of the chain rule in the balanced system, as proposed by Jordan e Graves (1995). Finally, the Figure 1(e) configuration represents the case where any two machines share production. The Λ matrix of this configuration can be seen in Figure 2(c); it is important to note that all entries are non-null.

Note that by varying the intensity λ_{kj}, one obtains different configurations with the same matrix pattern Λ, that is, it is possible to obtain the same flexibility configuration profile with different intensities. It is important to remember that the null and non-null positions in the Λ matrix are important, since they profile the flexibility configuration. It is also important to note that the balanced system configurations are a particular case of the unbalanced one. Note that by taking $n = m$, the matrices C and Λ are equivalent, so all the configurations known in the literature can be obtained.

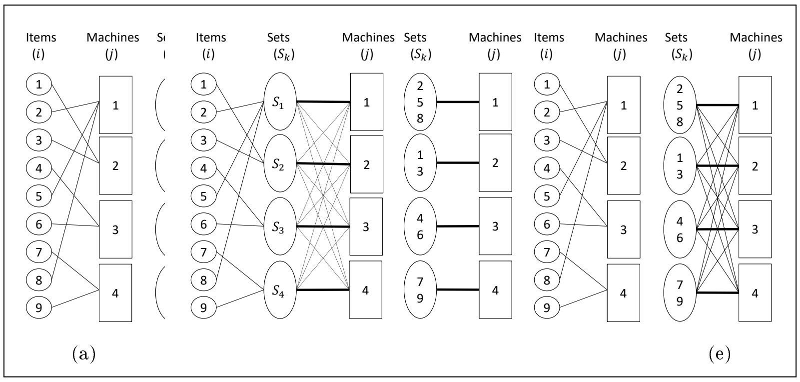


Figure 1: Process flexibility configuration profiles.

$$\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 3 & \lambda_{12} & 0 & 0 \\ 0 & 2 & \lambda_{23} & 0 \\ 0 & 0 & 2 & \lambda_{34} \\ \lambda_{41} & 0 & 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 3 & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & 2 & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \lambda_{32} & 2 & \lambda_{34} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & 2 \end{pmatrix}$$

(a) Matrix Λ referring to the Figure 1(c). (b) Matrix Λ referring to the Figure 1(d). (c) Matrix Λ referring to the Figure 1(e).

Figure 2: Matrices Λ for the configurations of Figure 1.

3.2. Mathematical formulation for a fixed flexibility setting

This section presents a mathematical formulation for the lot-sizing problem where the flexibility configuration is fixed. The flexibility configuration is passed to the formulation through the parameter c_{ij}, as described in Section 3.1. The parameters and variables of the model are as follows.

Sets

$N = \{1, \dots, n\}$	set of items;
$M = \{1, \dots, m\}$	set of machines
$T = \{1, \dots, \tau\}$	set of periods.

Parameters

$C = (c_{ij})$	binary matrix indicating if machine j has the flexibility to produce item i;
d_{it}	demand of the item $i$in the period$t$;
sd_{itp}	sum of the demand of item i, in the period$t$ to the period p $(p \geq t)$;
hc_{it}	cost of stocking item i in the period$t$;
bc_{it}	cost of the delay of item i in the period$t$;
sc_{ijt}	preparation cost of the machine for the item i in the machine j in the period$t$;
vc_{ijt}	production cost of the item i in the machine$j$ in the period$t$;
st_{ijt}	preparation time of the machine for the item$i$ in the machine j in the period t;

vt_{ijt}	production time of the item i in the machine j in the period$t$;
Cap_{jt}	capacity (in units of time) of the machine$j$ in the period t.
Variables	
x_{ijt}	number of produced units of the item i in the machine j in the period t;
y_{ijt}	binary variable, indicating if there is production or not of the item i in the machine j in the period t
s_{it}	quantitiy of stock of the item i at the end of the period t;
b_{it}	quantity of items $i$delayed at the end of period$t$.

The first mathematical formulation (Model 1) for the problem is presented below.

$$\text{Min} \quad \sum_{j \in J} \sum_{i \in N} \sum_{t \in T} c_{ij}(sc_{ijt}y_{ijt} + vc_{ijt}x_{ijt}) + \sum_{i \in N} \sum_{t \in T}(hc_{it}s_{it} + bc_{it}b_{it}) \tag{1}$$

$$\text{s.a} \quad s_{i(t-1)} + b_{it} + \sum_{j \in M} c_{ij}x_{ijt} = d_{it} + s_{it} + b_{i(t-1)}, \quad i \in N, \quad t \in T; \tag{2}$$

$$x_{ijt} \leq \min\{(Cap_{jt} - st_{ijt})/vt_{ijt}, sd_{i1\tau}\}c_{ij}y_{ijt}, \quad i \in N, \quad j \in M, \quad t \in T; \tag{3}$$

$$\sum_{i \in N} c_{ij}(st_{ijt}y_{ijt} + vt_{ijt}x_{ijt}) \leq Cap_{jt}, \quad j \in M, \quad t \in T; \tag{4}$$

$$y_{ijt} \in \{0,1\}, \ x_{ijt} \geq 0, \quad i \in N, \quad j \in M, \quad t \in T; \tag{5}$$

$$s_{it} \geq 0, \ s_{i0} = 0, \ s_{i\tau} = 0, \ b_{it} \geq 0, \ b_{i0} = 0, \quad i \in N, \quad t \in T. \tag{6}$$

The objective function (1) minimizes the total cost, composed of production cost, machine setup, stocking, and delay. Equations (2) perform stock balancing, (3) block production of item i in period t on machine j if machine setup is not performed. In (4) production is limited due to the machine's capability. Finally, (5) and (6) present the domain of the variables.

This formulation allows the analysis of the flexibility benefit when the company already has its machinery with defined flexibility, that is, it is already known which items each machine can produce. It is noteworthy that when the flexibility of the machines is known, the decisions made are at the tactical level. From the model solution, taken as a basis, it is possible to add new links, i.e., add flexibility and check the savings in terms of total cost. Therefore, in practice, if it is possible to adapt the machine to produce a new item, and the cost associated with this change is less than the flexibility benefit, it is advantageous to make the change.

3.3. Mathematical formulation for optimal configuration given a flexibility profile

This section presents the mathematical formulation for the lot-sizing problem where the solution of the model obtains the optimal flexibility configuration and production planning. One of the required parameters is the profile of the desired flexibility configuration, which in turn is represented by the Λ matrix as described in Section 3.1. It is important to remember that in addition to the flexibility profile, the Λ matrix contains the information about the intensity of sharing between two distinct machines. The new parameters and variables are described below.

Parâmetros	
ℓ	Maximum difference between the cardinality of sets S_k;
Variáveis	
w_{ik}	binary variable indicating if item i belongs to set S_k;
z_{ikj}	binary variable indicating if item i belonging to set S_k can be produced in machine j.

The second mathematical formulation (Model 2) for the problem is presented below.

$$\text{Min} \quad \sum_{j\in J}\sum_{i\in N}\sum_{t\in T}(sc_{ijt}y_{ijt} + vc_{ijt}x_{ijt}) + \sum_{i\in N}\sum_{t\in T}(hc_{it}s_{it} + bc_{it}b_{it}) \tag{7}$$

$$\text{s.a} \quad s_{i(t-1)} + b_{it} + \sum_{j\in M} x_{ijt} = d_{it} + s_{it} + b_{i(t-1)}, \quad i \in N, \quad t \in T; \tag{8}$$

$$x_{ijt} \leq \min\{(Cap_{jt} - st_{ijt})/vt_{ijt}, sd_{i1\tau}\}y_{ijt}, \quad i \in N, \quad j \in M, \quad t \in T; \tag{9}$$

$$\sum_{i\in N}(st_{ijt}y_{ijt} + vt_{ijt}x_{ijt}) \leq Cap_{jt}, \quad j \in M, \quad t \in T; \tag{10}$$

$$y_{ijt} \leq \sum_{k\in M} z_{ikj} \quad \forall i \in N, \quad \forall j \in M, \quad \forall t \in T; \tag{11}$$

$$\sum_{i\in N} z_{ikj} \leq \lambda_{kj} \quad \forall k, j \in M, \quad k \neq j; \tag{12}$$

$$z_{ikj} \leq w_{ik} \quad \forall i \in N, \quad \forall k, j \in M; \tag{13}$$

$$\sum_{k\in M} w_{ik} = 1 \quad \forall i \in N; \tag{14}$$

$$\sum_{i\in N} w_{ik} - \sum_{i\in N} w_{ij} \leq \ell \quad \forall k, j \in M; \tag{15}$$

$$\sum_{i\in N} w_{ik} - \sum_{i\in N} w_{ij} \geq -\ell \quad \forall k, j \in M; \tag{16}$$

$$y_{ijt} \in \{0,1\}, \; x_{ijt} \geq 0, \quad i \in N, \quad j \in M, \quad t \in T; \tag{17}$$

$$s_{it} \geq 0, \; s_{i0} = 0, \; b_{it} \geq 0, \; b_{i0} = 0, \quad i \in N, \quad t \in T; \tag{18}$$

$$w_{ik}, z_{ikj} \in \{0,1\} \quad \forall i \in N, \quad \forall k, j \in M. \tag{19}$$

In which the objective function (7) and the restrictions (8)-(19) are the same in (1)-(6) when the parameter c_{ij} is removed. The restrictions (11) block the *setup* if the machine j lacks flexibility to produce the item i. The restrictions(12) limit the amount of items shared between machines k and j and fix the flexibility configuration profile. The restrictions(13) block the possibility that the machine j manufactures an item that does not belong to the set S_k. The restrictions(14) force each item to be in unique set S_k.. The restrictions(15) and (16) control the uniformity between the S_k sets. Finally, the restrictions (19) present the domain of the variables.

Figure 3 presents the logic of the second model. It is noteworthy that in the left side of the figure, the variable w_{ik} relates the items to the sets S_k, thus creating a configuration in which each item "belongs" to a machine. The benefit of flexibility will be apparent if this system is unable to meet the demand without delay.. On the right side of the figure, the variable z_{ikj} relates the items i belonging to the set S_k to the machine j ($\neq k$), allowing the sharing of the items' production. Furthermore, there is a direct relationship between the items on the left side and the machines on the right side. This relationship is defined by the variable y_{ijt} which indicates, if in fact, item i was produced by machine j. It is important to note that in Section 3.1 it has been defined that λ_{kk} is the cardinality of the set S_k. Note that in the restrictions (12) it is not considered $k = j$. The cardinality of the set is controlled in the restrictions(15) and (16), where the difference between the cardinality of any two sets S_k cannot be greater than ℓ. It is worth mentioning that the interest of this model is precisely for organizations that do not yet have the machinery with the defined flexibility and can make a decision based on future demand, such as, for example, back orders.

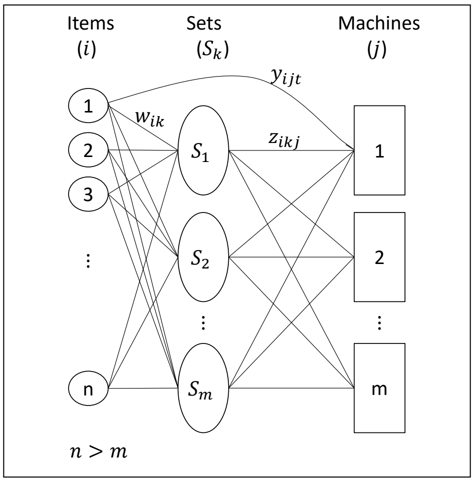


Figure 3: Logic of the mathematical formulation given a flexibility profile.

3.4. Mathematical formulation for optimal configuration given a flexibility budget

Taking inspiration from the ideas proposed by Fiorotto et al. (2018), in this section we use their model in which given a number of links, the solution of the model presents the flexibility configuration and the optimal production planning. The new parameters and variables of the models are as described below.

Parâmetros	
f	Quantity of permitted links;
Variáveis	
u_{ij}	binary variable indicating whether item i can be produced on machine j.

The third mathematical formulation (Model 3) for the problem is composed of (7)–(10), (17), (18) and the following restrictions.

$$y_{ijt} \leq u_{ij} \quad \forall i \in N, \quad j \in M, \quad t \in T; \tag{20}$$

$$\sum_{i \in N} \sum_{j \in M} u_{ij} \leq f; \tag{21}$$

$$u_{ij} \in \{0,1\} \quad \forall i \in N, \quad j \in M. \tag{22}$$

In which restrictions (20) block machine setup if machine j does not have the flexibility to produce item i. The constraint (21) limits the amount of links for the flexibility configuration. The restrictions (22) represent the domain of variables.

Thus, this model deals with the flexibility configuration directly in the unbalanced system, regardless of the profile of the flexibility configuration, so that the model is freer than the previous one. It is worth noting that the flexibility quantity restriction can be understood as a budget restriction, where each additional connection has a cost. Note that given the optimal configuration obtained by the model, it is entirely possible to group the items in search of a flexibility configuration profile.

3.5. Indicators of performance

This section introduces some new indicators associated with flexibility settings, which can be used to analyze the benefits of flexibility. Furthermore, with these indicators a post-optimization analysis of the flexibility configurations can be performed, both in terms of the sensitivity of the solution and in terms of the performance of the proposed configurations. It is observed that most works on machine flexibility use only cost to assess the benefit of flexibility, however, it is important to note that the benefit of flexibility is not only apparent in terms of the production planning objective function. By reducing the amount of delayed demand there is a positive impact on brand image; by using machines with limited flexibility instead of full flexibility, you can reduce the cost of machinery; more flexible machines allow more efficient use of machine capacity, among other benefits. For the calculation of the indicators, the optimal production planning is represented by $(x^*_{ijt}, y^*_{ijt}, h^*_{it}, b^*_{it})$.

The first proposed indicator is the Similarity Index (SI) that measures the discrepancy between the allowed configuration for the model, and the configuration actually used in the optimal solution. This discrepancy is measured in terms of the number of links, which allows you to analyze whether there are links that are not used. Thus, when the similarity index equals one, it indicates that all allowed links were used at least once over the planning horizon. On the other hand, the closer the SI approaches zero, the more discrepant the configuration used in the optimal solution is from the allowed configuration. Let Γ be the amount of bonds allowed, equation (24) presents how the SI is calculated.

$$\gamma_{ij} = \begin{cases} 1, & \text{if there is } t \in T, \text{so that, } y^*_{ijt} = 1 \\ 0, & \text{c.c.} \end{cases} \tag{23}$$

$$SI = 1 - \frac{\Gamma - \sum_{i \in N} \sum_{j \in M} \gamma_{ij}}{\Gamma}. \tag{24}$$

SI can be used as a sensitivity analysis. Note that if the links allowed for a configuration are not used in the optimal solution, they can be removed without detriment to the solution.

The second indicator compares the performance in terms of the objective function of the problem relative to a baseline configuration. The relative total cost (RTC) is calculated as follows, the total cost (TC) of the base configuration equals 100% and the cost of the analyzed configuration is calculated relative to the total cost of the base configuration. Thus, the complement of the RTC of the analyzed configuration is the savings, in terms of total cost, generated by the configuration. The equation representing the RTC calculation can be seen at (25).

$$RTC = \frac{CT(conf, cap, intensidade, instancia)}{TC(base, cap, intensidade, instancia)}. \tag{25}$$

The third indicator is the Demand Index (DI). This key figure measures how much of the demand has not been met, that is, how much at the end of the planning horizon there were still undelivered items. This indicator provides indications that an improvement action plan is needed, for example, in terms of flexibility it is interesting to look for new configurations, or increase the amount of hours worked to expand capacity, or outsource part of the production, among others, so that it is possible to meet all the demand within the planning horizon. The formula for the calculation can be seen in equation (26).

$$DI = \frac{\sum_{i\in N}\sum_{j\in M}\sum_{t\in T} x^*_{ijt}}{\sum_{i\in N}\sum_{t\in T} d_{it}}. \tag{26}$$

4. Numerical experiments

This section describes the examples used in the computational experiments and presents a detailed discussion of the numerical results obtained from comparing a set of flexibility configurations. The goal of the experiments is to analyze the benefit of using machine flexibility for unbalanced systems from different configurations built according to the methodology described in the Section 3. The models were coded in Python 3.6 language and solved via *Solver* CPLEX 12.10 on a computer with 2 Intel Xeon Six Core 5680 3.33GHz processors with 36GB of RAM. Each execution of the *Solver* was performed using a single *Thread* and limited to 3600 seconds.

4.1. Description of the examples and parameter setting

The data set used to generate the examples is based on the literature for lot-sizing problems. More specifically, it is an adaptation of some examples proposed by Toledo e Armentano (2006). Each example class is characterized by a combination of different quantities of items (6, 12 or 24) and machines (4 or 6). For all classes the number of periods was fixed at 12. For the 6- and 24-item examples, 6 capacities were generated, while for 12 items, 10 capacities. As for intensity, all possible intensities for each example were used, that is $1, \ldots, \lceil \frac{m}{n} \rceil$. Thus, a total of 610 examples were generated. In addition, each example was chosen with high machine preparation cost because they are the most difficult example types to solve according to Toledo e Armentano (2006) and Fiorotto et al. (2015). The number of experiments is described in the Section 4.2 along with the detail of the settings.

The values for each parameter were randomly generated on an interval $[a, b]$ using a uniform distribution as follows.

- Production cost: $vc_{ijt} \in [15, 25]$;
- Machine setup cost: $sc_{ijt} \in [50, 950]$;
- Stocking cost: $hc_{it} \in [2, 4]$;
- Delay cost: $bc_{it} = 100 \times hc_{it}$;
- Production time: $vt_{ijt} \in [1, 5]$;
- Machine setup time: $st_{ijt} \in [15, 75]$;
- Demand: $d_{it} \in [0, 180]$.

The values for the capacities were chosen over several experiments so that it was possible to obtain solutions with many delayed items and solutions with few delayed items, as follows.

- Instances of 6 items: 6 capacities equally spaced between 270 and 840, including the ends.
- Instances of 12 items: 10 capacities equally spaced between 425 and 1277, including the ends.
- Instances of 25 items: 6 capacities equally spaced between 1000 and 2660, including the ends.

The original examples do not have values for delay cost, so we choose $bc_{it} = 100 \times hc_{it}$. More details about the generation of the examples can be found at Toledo e Armentano (2006). Next we present the description of eight flexibility configurations used in the analyses, and for the computational experiments five examples were selected for each configuration.

4.2. Flexibility settings description

The first flexibility configuration, called basic configuration 1 (B1), is a flexibility configuration related to a Model 1 solution where there is no sharing of item production. It is made up as follows: Machine 1 can produce Item 1, Machine 2 can produce Item 2, and so on until the last machine. However, since we have $n > m$, the distribution of the remaining items is done as follows: Machine 1 can also produce the first remaining item, Machine 2 can also produce the second remaining item, and so on until each item is assigned to some machine. As illustrated in Figure 1(c), notice that $S_1 = \{1, 5, 9\}$, $S_2 = \{2, 6\}$, $S_3 = \{3, 7\}$, $S_4 = \{4, 8\}$ is an example for this type of flexibility configuration considering four machines and nine items. A second flexibility configuration, called basic configuration 2 (B2), is obtained with the same rule as above that assigns items to machines, but as a solution from Model 2. Note, therefore, that intensity has no influence on these settings.

Unlike the basic rule that generates configurations B1 and B2, all other flexibility configurations are obtained by considering that machines can now diversify the production sharing of items. The configurations called Chain 1 (C1) and Chain 2 (C2) are associated with the solutions of Models 1 and 2, respectively. As is illustrated in Figure 1(d), they feature limited sharing of item production between machines and, according to the nomenclature known from the literature, they form a long chain of links between items and machines. On the other hand, the configurations named Total 1 (T1) and Total 2 (T2), associated with the solutions of Models 1 and 2, respectively, exhibit total item production sharing as is illustrated in Figure 1(e).

Finally, the last two flexibility configurations are associated with the solutions of Model 3 and are formed according to two choices for the value of the parameter f, i.e., the total amount of connections allowed between items and machines ($f = n$ and $f = 1.5n$). These two flexibility configurations also feature limited sharing of item production between machines. However, they show no apparent pattern of flexibility profile when compared to the previous cases. We shall express by $L3_f$ the bounded flexibility configuration associated with a solution of Model 3 considering a given value for the parameter f.

The Table 2 separates the flexibility configurations that have been described according to the level of item production sharing between machines and the mathematical model studied.

Table 2: Configurations according to item production sharing between machines and mathematical model studied

	Sharing		
	Basic	Limited	Total
Model 1	B1	C1	T1
Model 2	B2	C2	T2
Model 3		$L3_f$	

Table 3 presents the quantity of experiments carried out. It is important to note that the $B1$ and $B2$ configurations do not use intensity, so the total number of experiments for these configurations is given by the × amount of instances amount of capacities. Instances C1, C2, T1, and T2 use intensity, so the total number of experiments equals the × amount of instances × amount of capabilities × amount of intensities. Finally, for $L3_f$ the total number of experiments is given by the amount of instances × amount of capacities × 2 ($f = n$ and $f = 1.5n$). In total, 3200 experiments were performed.

4.3. Computation results

This section presents the results of the computational experiments. In the Table 4 considering different intensity levels the average Similarity Indicator (SI) found for all flexibility configurations discussed in the previous section considering all capabilities is presented. The tables with the results of all the instances are

Table 3: Quantity of experiments per configuration

				B1	C1	T1	B2	C2	T2	$L3_f$	Total
	Instances	**Capacities**	**Intensities**								
6 items 4 machines	5	6	2	30	60	60	30	60	60	60	
12 items 4 machines	5	10	3	50	150	150	50	150	150	100	
12 items 6 machines	5	10	2	50	100	100	50	100	100	100	
25 items 4 machines	5	6	6	30	180	180	30	180	180	60	
25 items 6 machines	5	6	4	30	120	120	30	120	120	60	
Total				190	610	610	190	610	610	380	3200

available at the following repository. The formula for calculating SI can be seen in equation (24). Note that this indicator measures the discrepancy between an allowed flexibility configuration and the one actually used in the optimal solution through the number of links. The Table 4 shows that most configurations did not use all the allowed links. This is mainly due to two factors, firstly, for very tight capacity levels it is not possible to use all the allowed links, secondly, in some cases the demand has already been met and so it is not necessary to use all the allowed links. In this way, a relevant question that arises is: how many links are needed? When looking at the T1 configuration (with up to nm connections allowed) the SI was, on average, 50.6%. For the T2 configuration (also with up to nm connections allowed), the SI was, on average, 45.1%. That is, on average, almost 50% of the allowed connections were not used. When analyzing the C1 configuration, where up to $2n$ links are allowed, the SI averaged 77.3%; and for C2, it averaged 80.9%, so it is concluded that the $2n$ links are not necessary. Analyzing Model 3 with only $1.5n$ links ($L3_{f=n}$ configuration), it is observed that the SI was on average 95.6%. Note that using Model 3 with the amount of links allowed equal to n ($L3_{f=n}$ configuration) was the only configuration that managed to achieve 100% similarity, across all intensities and sizes. It can be seen that the configurations obtained by Model 3 do not have any fixed flexibility configuration profile. With these results presented by Model 3 one can conclude that the amount of connections needed is between n and $1.5n$, provided that the allocation of the items to the machines is done in the right way, which is reinforced by the SI of configurations B1 and B2 that are close to 100%.

Figure 4 graphically illustrates the similarity index for the instances with 12 items for all capability levels considering the average across all configurations for each proposed model. Note that, in general, higher SI values occur in the intermediate capacities. This fact reinforces the explanation of why the links are, or are not, used, i.e. at tight capacities it is not possible to use the links as the machines are being overworked. On the other hand, when the capacity increases, new connections start to be used, until no more new connections need to be used because the demand has been met. Figure 4 further shows that for all capacity levels the SI found by Model 3 is significantly higher than those found by Models 1 and 2. It is also interesting to compare the SI related to the fixed configurations of Model 1 with the configurations defined by Model 2. In general, when capacity is tight, the similarity index obtained by Model 2 is higher than that of Model 1. This is justified by the allocation of Model 2, which seeks to minimize delay by allowing more links to be used. On the other hand, when capacity is high, more links allowed in Model 1 need to be used to meet demand, while

Table 4: Similarity Index by intensity

Size	Intensity	B1	C1	T1	B2	C2	T2	$L3_{f=n}$	$L3_{f=1.5n}$
6i12p4m	**1**	97.8	83.3	66.1	100	89.0	53.2	100	98.1
	2	97.8	76.7	49.4	100	67.6	31.2	100	98.1
	Avg.	97.8	80.0	57.8	100	78.3	42.2	100	98.1
12i12p4m	**1**	96.5	85.6	71.1	98.8	93.8	69.8	100	96.6
	2	96.5	79.0	52.8	98.8	84.2	47.0	100	96.6
	3	96.5	70.6	43.0	98.8	72.0	34.3	100	96.6
	Avg.	96.5	78.4	55.6	98.8	83.3	50.4	100	96.6
12i12p6m	**1**	100	87.8	44.7	100	84.1	42.1	100	97.2
	2	100	74.2	28.2	100	67.7	24.2	100	97.2
	Avg.	100	81.0	36.5	100	75.9	33.2	100	97.2
25i12p4m	**1**	94.8	88.2	77.1	99.1	96.6	82.0	100	93.1
	2	94.8	82.1	63.2	99.1	93.0	65.3	100	93.1
	3	94.8	75.6	56.0	99.1	86.7	51.1	100	93.1
	4	94.8	70.4	47.8	99.1	80.9	42.4	100	93.1
	5	94.8	69.2	44.4	99.1	74.1	36.0	100	93.1
	6	94.8	64.4	41.1	99.1	68.8	31.1	100	93.1
	Avg.	94.8	75.0	54.9	99.1	83.3	51.3	100	93.1
25i12p6m	**1**	99.1	86.2	60.2	100	92.9	63.2	100	96.5
	2	99.1	78.0	45.7	100	83.4	40.5	100	96.5
	3	99.1	74.0	37.6	100	74.4	29.6	100	96.5
	4	99.1	68.1	31.2	100	66.5	23.6	100	96.5
	Avg.	99.1	76.6	43.7	100	79.3	39.2	100	96.5
Avg.		97.1	77.3	50.6	99.5	80.9	45.1	100	95.6

Model 2 is able to obtain an allocation that does not necessarily use all the links allowed.

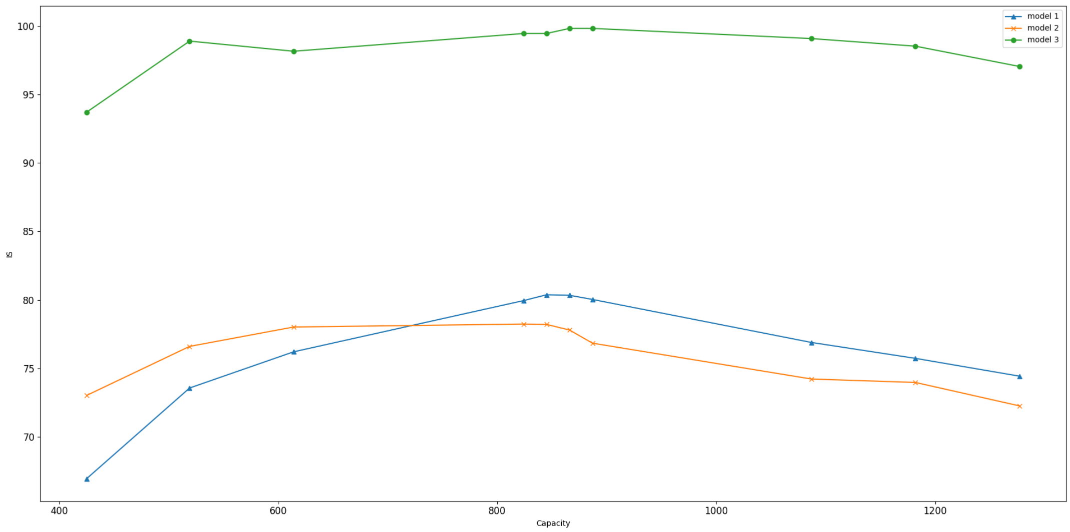


Figure 4: Similarity Index by ability

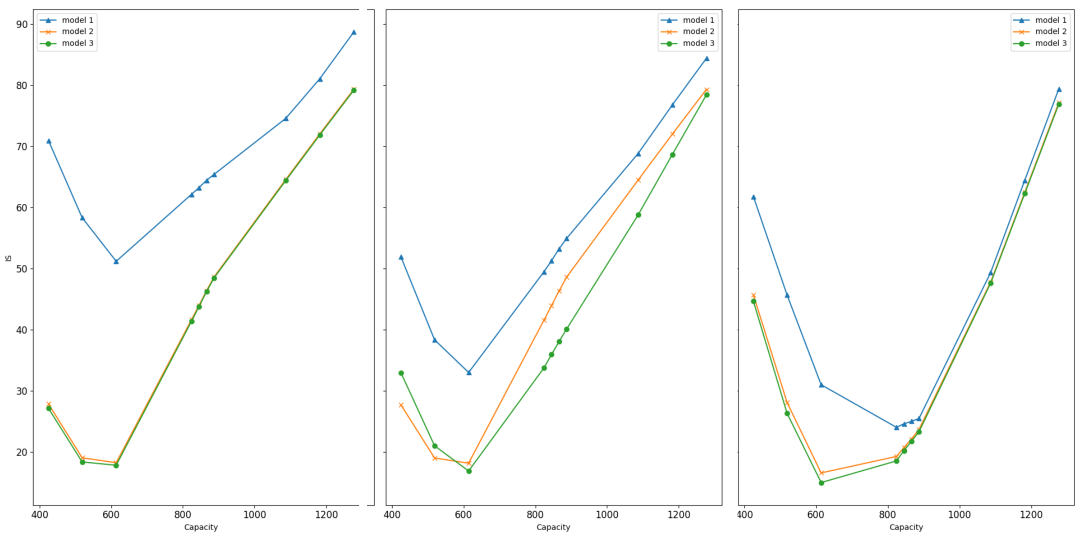
The Table 5 displays the average RTC for all sizes, configurations and intensities. It can be seen that the RTC is the ratio of the objective function value of a flexibility configuration over the baseline configuration, so it is possible to measure the savings generated in the objective function by changing certain characteristics of the flexibility configuration. Before analyzing the RTC, it is noted that the average gap of instances that did not reach optimality within 1 hour was 2.1% (the Table A.7 of Appendix A presents the gap by capacity and instance size). First, we shall look at the configurations of flexibilities obtained with Model 1. Configuration C1 obtained, on average, a RTC equal to 68%. This means that given a fixed configuration and no sharing of production of the items between machines and items (baseline configuration), savings of 32% can be achieved by adapting the machines to be flexible using configuration C1. When looking at the T1 configuration, on average, it was possible to obtain a RTC of about 51%, i.e., with the addition of a few more links, there was a reduction of approximately 15% of the RTC compared to the C1 configuration. Next, we shall observe the settings for Model 2. Configuration B2 (which does not share production between machines) averaged a RTC equal to 47%, that is, better performance than configurations C1 and T1 for Model 1. The explanation lies in the fact that Model 2 is free to choose the allocation of items to machines given a flexibility configuration profile. Still, note that the C2 configuration averaged a RTC equal to 44.9%. The T2 configuration averaged a RTC equal to 44.5%, a difference of only 0.4% compared to C2. Note that the T2 configuration has many more connections allowed than the C2 configuration. It is interesting to remember that the T2 configuration obtained the similarity of about 45%, i.e., about 55% of the allowed links were not used (see Table 4). This information allows us to state that with a very small number of connections it is already possible to achieve the savings allowed by the full maximum intensity configuration. Next, the Model 3 configurations reinforce this conclusion. On average, the $L3_{f=n}$ configuration achieved a RTC equal to 46%, slightly better than the B2 configuration. Finally, the $L3_{f=1.5n}$ configuration, with only $1.5n$ links, achieved a RTC, on average, equal to 44.6%, a difference of only 0.1% from the T2 configuration. Therefore, one can conclude that an amount of links between n and $1.5n$ is sufficient to achieve the same performance as the full maximum intensity configuration.

The Figure 5 graphically illustrates the RTC of the 12-item instances, separated by intensity and capacity for each of the proposed models. Note that Model 1 (considering C1 and T1 only), shows the highest RTC, while for Model 2 and 3, the curves were close. It is interesting to note the effect of intensity in Model 1. Increasing the intensity (therefore the amount of flexibility) allows for savings compared to the scenario without flexibility. Note that the greatest savings occurred at the intermediate capacities (between 600 and 900). Note also that at intensity 3 (maximum intensity for the 12-item instances), the fixed flexibility configurations (Model 1) were close to the Model 2 and 3 curves (at the high capacities), which is reasonable since the Model 1 curve contemplates the total configuration at its maximum intensity, which is the best case in terms of the RTC.

The Table 6 presents Demand Index for all sizes, configurations and intensities. The formula for the calculation is described in equation <u1 Note that the DI measures how much of the total demand was met within the planning horizon, that is, if at the end of the planning there was still unmet demand.

Table 5: Relative Total Cost - RTC

Size	**Intensity**	**B1**	**C1**	**T1**	**B2**	**C2**	**T2**	$\boldsymbol{L3_{f=n}}$	$\boldsymbol{L3_{f=1.5n}}$
6i12p4m	**1**	100	50.9	37.4	37.1	32.8	32.1	36.7	32.3
	2	100	43.5	32.1	37.1	32.7	32.1	36.7	32.3
	Avg.	100	47.2	34.7	37.1	32.8	32.1	36.7	32.3
12i12p4m	**1**	100	78.6	50.5	37.8	35.2	34.8	36.5	34.9
	2	100	59.5	39.8	37.8	35.1	34.8	36.5	34.9
	3	100	51.4	34.8	37.8	35.0	34.8	36.5	34.9
	Avg.	100	63.2	41.7	37.8	35.1	34.8	36.5	34.9
12i12p6m	**1**	100	78.8	64.1	57.8	56.0	55.4	56.6	55.5
	2	100	70.1	55.4	57.8	55.8	55.4	56.6	55.5
	Avg.	100	74.5	59.8	57.8	55.9	55.4	56.6	55.5
25i12p4m	**1**	100	83.6	70.4	44.9	44.2	44.0	44.6	44.0
	2	100	77.2	61.0	44.9	44.1	44.0	44.6	44.0
	3	100	74.0	59.2	44.9	44.1	44.0	44.6	44.0
	4	100	69.8	51.5	44.9	44.1	44.0	44.6	44.0
	5	100	62.2	47.2	44.9	44.1	44.0	44.6	44.0
	6	100	59.5	45.8	44.9	44.1	44.0	44.6	44.0
	Avg.	100	71.1	55.8	44.9	44.1	44.0	44.6	44.0
25i12p6m	**1**	100	84.1	69.9	57.0	56.6	56.3	57.0	56.3
	2	100	74.8	61.8	57.0	56.4	56.3	57.0	56.3
	3	100	71.2	58.4	57.0	56.4	56.3	57.0	56.3
	4	100	66.1	57.2	57.0	56.4	56.3	57.0	56.3
	Avg.	100	74.1	61.8	57.0	56.5	56.3	57.0	56.3
Avg.		100	68.0	52.7	47.1	45.5	45.2	46.6	45.2



(a) Intensity 1.

(b) Intensity 2.

(c) Intensity 3.

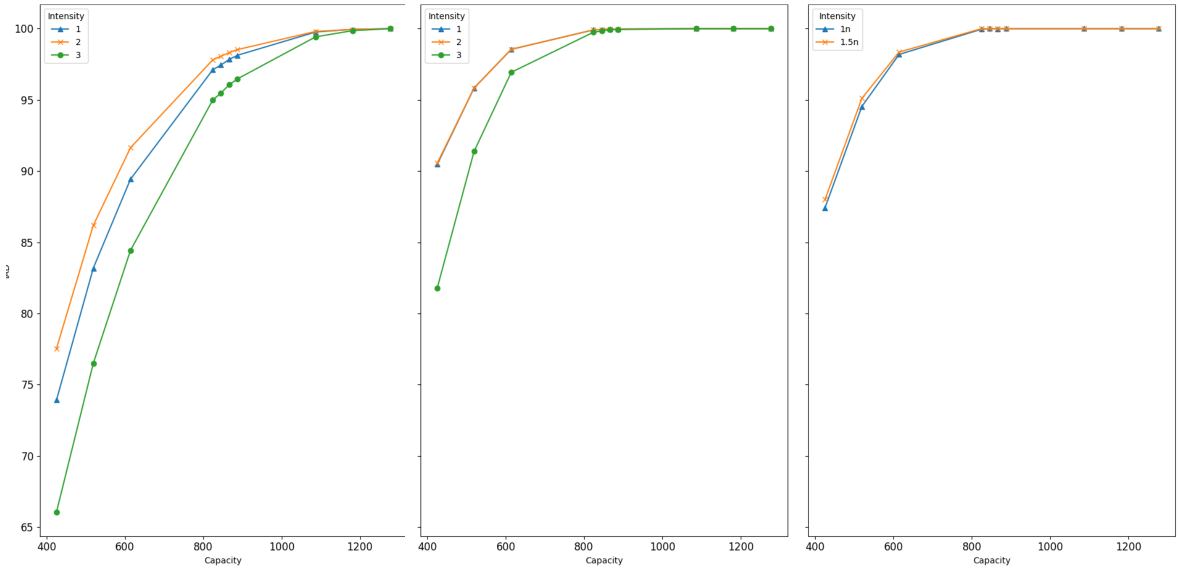
Figure 5: Relative Total Cost (RTC).

Note that virtually all configurations struggled to get all the demand met (since the DI $<$ 100% for most instances). Configuration B1, as expected, had the lowest DI (91.3% on average), then configuration C1 had an average DI of 94.7%, and configuration T1 had 97.2% for DI. For Model 2, where the allocation of items to machines is a model decision, the DI of configuration B2 reached 98.2%, and by adding a little more flexibility, configuration C2 reached 98.5%. Note that configuration T2 (with a larger amount of links compared to configuration C2) showed the same DI. Finally, the $L3_{f=n}$ configuration obtained 98.3% for DI, almost equal to the B2 configuration, while the $L3_{f=1.5n}$ configuration, with a greatly reduced amount of links, obtained the same DI as the C2 and T2 configurations.

Table 6: Demand Index - DI

Size	Intensity	B1	C1	T1	B2	C2	T2	$L3_{f=n}$	$L3_{f=1.5n}$
6i12p4m	1	86.7	93.4	96.5	97.0	97.8	97.9	96.8	97.9
	2	86.7	94.8	97.9	97.0	97.9	97.9	96.8	97.9
	Avg.	86.7	94.1	97.2	97.0	97.9	97.9	96.8	97.9
12i12p4m	1	86.8	88.6	93.7	96.7	97.2	97.2	97.0	97.2
	2	86.8	91.5	95.6	96.7	97.2	97.2	97.0	97.2
	3	86.8	92.9	97.2	96.7	97.2	97.2	97.0	97.2
	Avg.	86.8	91.0	95.5	96.7	97.2	97.2	97.0	97.2
12i12p6m	1	96.1	97.8	99.1	99.7	99.9	100	99.9	100
	2	96.1	98.6	100	99.7	100	100	99.9	100
	Avg.	96.1	98.2	99.5	99.7	100	100	99.9	100
25i12p4m	1	89.7	91.3	92.8	97.8	97.9	97.9	97.9	97.9
	2	89.7	91.9	94.2	97.8	98.0	97.9	97.9	97.9
	3	89.7	92.3	94.8	97.8	97.9	97.9	97.9	97.9
	4	89.7	93.0	96.3	97.8	97.9	97.9	97.9	97.9
	5	89.7	94.3	97.3	97.8	97.9	97.9	97.9	97.9
	6	89.7	94.9	97.6	97.8	97.9	97.9	97.9	97.9
	Avg.	89.7	92.9	95.5	97.8	97.9	97.9	97.9	97.9
25i12p6m	1	96.9	97.7	99.2	100	100	100	100	100
	2	96.9	98.6	99.8	100	100	100	100	100
	3	96.9	99.1	100	100	100	100	100	100
	4	96.9	99.6	100	100	100	100	100	100
	Avg.	96.9	98.7	99.8	100	100	100	100	100
Avg.		91.3	94.7	97.2	98.2	98.5	98.5	98.3	98.5

It is worth noting the effect of intensity on the settings of each model. The Figure 6 presents the DI for the 12-item instances, separated by capability for with one of the models. The Figure 6(a) shows that for Model 1, increasing intensity allows for improved demand fulfillment. At tight capacity, increasing from intensity 1 to maximum intensity reduced the amount of unmet demand within the planning horizon by approximately 10%. Whereas for the loose capacities, all the demand had already been met within the horizon and so there was no effect of intensity. Figure 6(b) shows that for Model 2, there was no effect when increasing the intensity from 1 to 2, however, when reaching the maximum intensity, there was an increase of about 10% of the demand met within the planning horizon. It is interesting to note that Model 2 is able to achieve 100% on DI already at intermediate capacities and at the first intensity, which demonstrates the importance of a good allocation of items to machines. Finally, the Figure 6(c) shows that the effect of the amount of flexibility is practically zero in Model 3. It is worth noting that at tight capacities Model 3 was able to meet more demand within the planning horizon than Models 1 and 2.



(a) Model 1.

(b) Model 2.

(c) Model 3.

Figure 6: Demand Index (DI)

5. Conclusion

In this paper, the benefits of machine flexibility in the lot sizing problem in unbalanced systems were studied. First a new methodology was proposed to build the flexibility configurations considering that each machine can manufacture a certain set of items. By adding the idea of the intensity of sharing between the machines it is possible to represent flexibility configurations known in the literature, such as the chain rule in unbalanced systems. Then, we proposed new mathematical formulations for the studied problem in which the desired configuration profile and sharing intensity are input parameters. Finally, some performance indicators were proposed such as the similarity index that measures the discrepancy between the studied configuration profile and the configuration obtained in the optimal solution. As for the computational results obtained, it was possible to show that most configurations do not use all the flexibility allowed and that the decision of allocating items to machines has a substantial impact on the indicators. Moreover, even when the allocation is optimal, increasing flexibility (intensity) can improve system performance. Finally, there is evidence that the amount of links used in the optimal solution does not exceed 1.5 times the total number of items.

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Appendix A. Tables of the 6- and 25-item instances.

Table A.7: Gap

	Capacity	base	base2	chain	chain2	m3f1	m3f1.5	total	total2
6i12p4m	270	0	0	0	0	0	0	0	0
	384	0	0	0	0	0	0	0	0
	498	0	0	0	0	0	0	0	0
	612	0	0	0	0	0	0	0	0
	726	0	0	0	0	0	0	0	0
	840	0	0	0	0	0	0	0	0
	Avg.	0	0	0	0	0	0	0	0
12i12p4m	425	0	0	0	0.5	0	0.5	0.6	0.7
	519	0	0	0.6	1.1	0	1.1	0.9	1
	614	0	0	0.5	1.7	1.2	2.9	1.2	2
	824	0	0	1.2	0.5	7.6	0.8	0.7	0.6
	845	0	0	1	0.5	6.8	0.9	0.6	0.5
	866	0	0	0.8	0.5	0	0.6	0.5	0.4
	887	0	0	0.6	0.4	0	0.8	0.5	0.4
	1087	0	0	0.3	0.3	0	0.5	0.2	0.3
	1182	0	0	0.1	0.1	0	0.3	0.2	0.2
	1277	0	0	0	0.1	0	0.1	0.1	0.1
	Avg.	0	0	0.6	0.6	5.2	0.9	0.6	0.6
12i12p6m	425	0	58.5	0.3	4.3	24.2	5.1	2	3.3
	519	0	41.6	1.5	1.7	23.8	1	1	0.4
	614	0	0	0.3	0.8	6.8	0.3	0.5	0.1
	824	0	0	0	0	0	0	0.3	0
	845	0	0	0	0	0	0	0	0
	866	0	0	0	0	0	0	0.1	0
	887	0	0	0	0	0	0	0	0
	1087	0	0	0	0	0	0	0	0
	1182	0	0	0	0	0	0	0	0
	1277	0	0	0	0	0	0	0	0
	Avg.	0	50.1	0.7	2.3	18.3	2.1	0.6	1.3
25i12p4m	1000	0	1.4	0.4	1.6	1.8	1.5	0.8	1.5
	1332	0	11	1	2.7	6.1	2.8	1.9	2.4
	1664	0	1.3	1	0.7	2	0.6	1	0.5
	1996	0	0	0.2	0.2	0.1	0.2	0.1	0.2
	2328	0	0	0.1	0	0	0	0	0
	2660	0	0	0	0	0	0	0	0
	Avg.	0	3.4	0.5	1.1	2.5	1.3	0.8	0.9
25i12p6m	1000	0	9.5	2.5	2.6	20.3	1.5	2.2	1.2
	1332	0	2	0.5	0.7	1.8	0.5	0.5	0.4
	1664	0	0.3	0.1	0.2	0.2	0.1	0.1	0.1
	1996	0	0	0	0.1	0	0.1	0	0.1
	2328	0	0	0	0	0	0	0	0
	2660	0	0	0	0	0	0	0	0
	Avg.	0	3.9	1	0.9	7.4	0.6	0.7	0.5