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A First Approach Toward Diagnosing Mood Disorders from Incomplete Data By Means of Fuzzy Natural Logic

Uma Primeira Abordagem para o Diagnóstico de Transtornos de Humor por meio da Lógica Fuzzy Natural

Campinas

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A First Approach Toward Diagnosing Mood Disorders from Incomplete Data By Means of Fuzzy Natural Logic

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Dissertação apresentada ao Instituto de Matemática, Estatística e Computação Científica da Universidade Estadual de Campinas como parte dos requisitos exigidos para a obtenção do título de Mestre em Matemática Aplicada.

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Resumo

A dissertação de mestrado tem como objetivo estabelecer um método de detecção de um episódio depressivo maior presente no transtorno bipolar e no transtorno depressivo maior considerando, apenas, a atividade motora e o tempo total de sono por dia medidos com o procedimento da actigrafia. Usualmente atividade motora e tempo total de sono são medidos subjetivamente segundo relato do paciente ou percepção do profissional. Entretanto, a partir de uma medição objetiva, conseguimos descrever, linguisticamente, melhor a situação do paciente.

Criamos regras implicativas a partir do conhecimento já estabelecido de como os transtornos de humor funcionam e utilizamos técnicas da Lógica Fuzzy Natural como método de classificação, pois ela é capaz de lidar com uma base de dados pequena e com a imprecisão presente nos transtornos psiquiátricos. A Lógica Fuzzy Natural é uma área da Lógica Fuzzy que tenta modelar, matematicamente, expressões imprecisas da linguagem natural. O texto é escrito de forma que o entendimento seja fácil para pesquisadores de diversas áreas.

Para testar o método, utilizamos uma base de dados de domínio público com 23 pessoas possuindo alguns dos dois transtornos de humor supracitados (grupo condição) e 32 pessoas do grupo controle. A base de dados nos fornece os dados de actigrafia de cada participante, minuto a minuto e durante alguns dias, além de algumas informações básicas como idade e sexo.

Nossos resultados foram satisfatórios porque, em um dos métodos, menos de 5% do grupo condição foi considerado “bem de saúde”, enquanto menos de 19% do grupo controle foi considerado “mal de saúde”. Concluímos que novos estudos a partir deste serão importantes para melhorar o método, uma vez que com regras simples, já conseguimos tal resultado.

Palavras-Chave: Lógica Fuzzy. Transtorno de Humor. Diagnóstico Médico.

Abstract

This master's thesis aims to establish a method for detecting major depressive episodes experienced by people with bipolar disorder and major depressive disorder. The method only considers motor activity and total sleep time per day. Both symptoms were measured using actigraphy. Motor activity and total sleep time are usually measured subjectively according to the patient's report or according to a professional's perception. However, we can better describe the patient's situation linguistically from an objective measurement.

We defined implicative rules based on already established knowledge about mood disorders. In addition, we used techniques of Fuzzy Natural Logic (FNL) as a classification method since FNL is capable of dealing with a small database and the inaccuracy present in diagnoses of psychiatric disorders. FNL is an area of fuzzy logic that attempts to mathematically model imprecise expressions of natural language.

To test the method, we used a public domain database of 23 people having either one of the two aforementioned mood disorders (condition group) and 32 people from the control group. The database provides us with each participant's actigraphy data, minute by minute, and over a few days. Moreover, it contains basic information such as age and sex.

Our results are satisfactory because, in one of the methods, less than 5% of the condition group were considered "well", while less than 19% of the control group were considered "unwell". We conclude that new studies based on this one will be important to improve the method, since with simple rules, we have already achieved reasonable results.

Keywords: Fuzzy Logic. Mood Disorder. Medical Diagnostics.

List of abbreviations and acronyms

MDD	Major Depressive Disorder
BD	Bipolar Disorder
FNL	Fuzzy Natural Logic
DSM-V-TR	Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition, Text Revision.
TST	Total Sleep Time
TMA	Total Motor Activity
MADRS	Montgomery-Asberg Depression Rating Scale

List of symbols

μ_A	Membership function of a fuzzy set A.
$A \subsetsim B$	A is a fuzzy subset of B.
$\mathscr{F}(A)$	Set of all fuzzy subsets of B.
$\|A\|$	Cardinality of the fuzzy set A.
A^c	Complement of the fuzzy set A.
$\otimes$	Łukasiewicz T-norm.
$\oplus$	Łukasiewicz T-conorm.
$\wedge$	Minimum T-norm or logical conjunction.
$\vee$	Maximum T-conorm or logical disjunction.
$\neg$	Logical negation.
$\rightarrow$	Łukasiewicz implication or logical implication.
$\cap$	Łukasiewicz intersection.
$\cup$	Łukasiewicz union.
$\Rightarrow$	Łukasiewicz residuum.
$Ext_w(\mathscr{A})$	Extension of the evaluative expression $\mathscr{A}$ in the context w.
$\ll$	Ordering of evaluative expressions.
$< v_L, v_S, v_R >$	A context defined by v_L, v_S and v_R.
$w^{(-1)}$	The extended inverse of a context w.
$\nu_{a,b,c}$	Function that models linguistic hedges.
$LPerc^K$	Local perception function according to the finite set of evaluative predications K.
$F_R(B)(A)$	Degree of subsethood of A in B.
$Q^{\forall}_{ExBi}(B, A)$	Almost all B is A.
$Q^{\forall}_{VeBi}(B, A)$	Most B is A.

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Introduction

Medicine is an imprecise science. Therefore, in recent years, the number of articles that use fuzzy logic in research into the diagnosis of various diseases has increased (AHMADI et al., 2018). Psychiatry is the branch of medicine that identifies, diagnoses and treats mental and behavioral disorders. As a field of knowledge, it has had difficulties in distinguishing between normality and pathology, especially in borderline cases of a disorder (ODA; DALGALARRONDO; BANZATO, 2022). Interviewing patients is the main part of the psychiatric diagnosis process that can result in imprecision and uncertainties when obtaining information for decision-making. However, such interviews are necessary to resolve communication problems and understand the various possible causes of the reported symptoms. These inaccuracies cause underdiagnosis or overdiagnosis (HOFMANN, 2022). The former is the reason for the lack of adequate treatment, while the latter is responsible for putting the patient under risks associated with treatment and unnecessary costs (FILHO; VIEIRA, 2022). The proportion of psychiatrists relative to the general Brazilian population was around 0.0065% in 2021 (SCHEFFER; et al., 2023). Mental health professionals are poorly distributed across the country. Especially in the public sector and in some private medical insurance plans, the main difficulties faced by these professionals are excessive workload and consultations being done with an insufficient amount of time (ODA; DALGALARRONDO; BANZATO, 2022). The latter is one of the main causes of erroneous diagnoses (AHMADI et al., 2018).

The objective of this master's thesis is to implement a mathematical tool that helps detect a major depressive episode that is present in both major depressive disorder (MDD) and bipolar disorder (BD). To test our methodology, we used a database consisting of 55 participants to distinguish between healthy participants and participants with MDD or BD (GARCIA-CEJA et al., 2018). Garcia-Ceja et al. (2018) achieved an accuracy of 72.7% using linear kernel support vector machine (SVM) for the classification of the same database.

There are other papers in which fuzzy neural networks have been used to achieve an accuracy of 95% (CHATTOPADHYAY, 2017) and 85% (KUMAR et al., 2022) in detecting depressive episodes. While Chattopadhyay (2017) considers all the symptoms of a major depressive episode based on the perception of a group of psychiatrists, Kumar et al. (2022) consider only what can be measured with actigraphy. We achieved an accuracy[1] accuracy of 72.7% in the extended abstract entitled "A First Approach Toward Diagnosing

[1] Our method is not a binary classification. We calculated accuracy by considering condition group's participants who were classified as "unwell" to be true positives. We considered control group's participants who were not classified as "unwell" to be true negatives.

Mood Disorders from Incomplete Data by Means of Fuzzy Natural Logic", presented at FSTA[2] (OLIVEIRA; SUSSNER, 2024). This value improved to 76.4% in this master's thesis, after some parameter adjustments.

Unfortunately, the database used in this master's thesis only has a few participants. The small size of the database precludes the use of neural networks. Based on the assumption that the method must be comprehensible by both psychiatry and psychology professionals, we believe that methods of fuzzy logic appear to be a good choice. These methods resemble our conception of linguistic expressions and human reasoning, seeking logical conclusions that apply to complicated real-life problems. Therefore, methods of fuzzy logic can allow experts to have more confidence in making decisions in a short time frame.

While our work uses methods from Fuzzy Natural Logic, Kumar et al. (2022) use a hybrid of Convolutional Neural Networks and Adaptive Neuro Fuzzy Inference System. Their work resembles this thesis the most because it uses the same database (GARCIA-CEJA et al., 2018) than us. However, we did not train our model, whereas Kumar et al. (2022) increased the numbers of instances to train their model by splitting each participant's actigraphy data in 48-hour periods of time. The potential problem in doing that is that data from a single person can be used for training and testing. Although different time frames are considered, this may lead to biased results due to the ongoing nature of the symptoms of BD and MDD. Furthermore, somebody's organism will change throughout the day, but usually not so much to show meaningful changes. There will be patterns. Using different people's data for training and testing would lead to less biased results.

BD and MDD are mood disorders that affect individuals' personal and social functioning. Sleep problems and abnormal psychomotor activity are possible symptoms of a major depressive episode. However, in most cases these symptoms are measured subjectively by mental health professionals (AMERICAN PSYCHIATRIC ASSOCIATION, 2014). The main procedure for objectively measuring sleep is called polysomnography, in which the patient sleeps in a medical center equipped with multiple sleep sensors. Moreover, another method that stands out is called actigraphy, which presents many advantages. Despite not being as precise in characterizing sleep as polysomnography, actigraphy records uninterrupted measurements for days. It also comes at a lower cost and allows for the description of the patient's circadian cycle throughout everyday life.

In the revised text of the fifth edition of Diagnostic and Statistical Manual of Mental Disorders (DSM-V-TR) (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), a possible symptom of a major depressive episode is hypersomnia (sleeping excessively). The meaning of the word "excessively" is vague because it requires a particular context

[2] The 17^{th} International Conference of Fuzzy Set Theory and Applications. Slovak Republic. 2024.

of comparison to determine what is considered normal. Furthermore, throughout the DSM-V-TR text, how many minutes of sleep must be prolonged to be considered a definitive “excessively” is not strictly specified. Nevertheless, specialists can understand vague concepts such as this one, due to the reasoning they acquired during their training and clinical experience. To interpret this data, we propose the use of fuzzy logic because it can be effective in supporting the diagnosis of complex diseases (AHMADI et al., 2018). It is able to model and solve real-world problems in which there is imprecision and uncertainty. Fuzzy logic originates from the judgment of premises according to their degree of truth (ZADEH, 1965). A branch of fuzzy logic is Fuzzy Natural Logic (FNL), which is a set of theories that seek a mathematical model of human reasoning through expressions used in natural language, which has inaccuracies and intrinsic uncertainties (NOVÁK, 2015).

In addition to mood disorders, there are many other factors that can cause sleep issues. Therefore, we do not have exact information on whether a certain deviation in the sleep pattern is caused by one of the two disorders mentioned before (BD or MD). We hope that the result of this study will be a resource to help develop diagnostic reasoning, and not necessarily replace it. To this end, we analyzed the data obtained by actigraphy because this data provides us with information about an individual’s motor activity and sleep time.

Part I

Theoretical Foundations

1 Imprecision

This chapter defines imprecision and how it presents itself in language. Moreover, it differentiates imprecision from the concept of uncertainty. We explain the importance of fuzzy logic to deal with vague propositions.

1.1 Vagueness and Imprecision

Imprecise terms and vague terms can be confused with one another because both express indeterminacy. As a matter of fact, some authors call imprecision a type of vagueness (SOLT, 2015). Owing to this and the purpose of this text, vagueness, and imprecision are treated as synonyms henceforth. There is no unanimity in characterizing imprecision, but there are three main characteristics that must be taken into account: the presence of borderline cases, ill-defined borders, and susceptibility to the Sorites Paradox (VÁSCONEZ, 2006).

1.1.1 Sorites Paradox

The Sorites Paradox is often cited in texts that aim to characterize vagueness from a logical point of view because it elucidates how the imprecision of a concept may lead to incorrect logical conclusions. History also plays an important role in popularizing this paradox because it was an argument used to discredit classical logic (HYDE; RAFFMAN, 2018). However, it also led to vague and imprecise propositions not being objects of classical logic studies (HYDE; RAFFMAN, 2018). The Sorites Paradox can be described as follows:

"One grain of wheat does not make a heap. If n grains do not make a heap, then $n+1$ grains do not make a heap either. Therefore, heaps of wheat do not exist."

The hypothesis appears to be true because (according to the contrapositive) one less grain of wheat does not turn a heap into anything else (VÁSCONEZ, 2006). It is a paradox because the conclusion is incorrect since heaps of wheat exist. The erroneous conclusion arises due to the premise using the term "heap", which is vague for logical deductions (HYDE; RAFFMAN, 2018).

1.1.2 Ill-Defined Boundary

Vague expressions have ill-defined boundaries. Identifying all objects, beings, or situations described by these expressions is confusing. In the case of the Sorites paradox, for example, we know that 2 grains of sand do not form a heap. Nevertheless, a billion grains of sand grouped do form a heap, depending on the spatial arrangement of these grains. Therefore, how many grains are used to make a heap? There is no answer to this question, as we cannot define the boundary between being and not being a heap.

1.1.3 Borderline Cases

Borderline cases of a vague expression are cases in which it is not possible to define whether an object is described or not by the expression. We know that by grouping a billion individual grains of wheat we make a heap. But with just two grains of wheat, we do not. However, what if there were a hundred grains? Or five thousand grains? Maybe a million grains? The set of borderline cases is larger for a person who has never counted grains of wheat, compared to someone who has experience in doing so. Such a person could still have doubts but they would be able to narrow the boundary further according to their perception.

Example 1. *"She is tall" is a vague sentence. The term "tall" is vague because various heights can be considered "tall" depending on the interlocutor's perception and the context of the sentence (SOLT, 2015). If we suppose this person is 2 meters tall, she is tall indeed. However, what if she is 1.77 meters tall or 1.70 meters tall? In these cases, there could be doubts about whether she is tall or not. In other words, cases in which there are doubts are called* ***borderline cases****. The boundaries of the term "tall" are* ***ill-defined*** *because we do not know at what height precisely someone is considered tall. Thus, the term is susceptible to the* ***Sorites paradox*** *because we may think that somebody being just one millimeter taller does not make that person tall.*

1.1.4 Intension and Extension of Vague Expressions

Two important concepts in semantics are *intension* and *extension*. The extension of a predicate[1] is the class of objects described by it. Whereas the intension is the set of characteristics of an object in order to permit its description in terms of a particular predicate.

Example 2. *Given the expression "even", the intension includes the properties: "is an integer" and "does not leave a remainder when divided by two". Furthermore, the extension is the set* $2\mathbb{Z}$.

Before explaining how to deal with vagueness, we must briefly list some points of view on the topic of vagueness: an ontological, an epistemological, and a semantic perspective. Roughly speaking, ontology is a branch of philosophy that seeks to define entities. On the other hand, epistemologic thinking states that it is not possible to know

[1] A predicate is a property that is affirmed or denied about a subject. For our purpose, a predicate will be a linguistic expression.

the essence of an entity. Therefore, it seeks to understand characteristic phenomena for a better understanding of the matter. From an ontological point of view, imprecision is an intrinsic characteristic of an entity. In other words, the world is imprecise, and consequently, language is vague because of its use to describe the world(VÁSCONEZ, 2006). In epistemology, the world is precise, so imprecision is a type of ignorance about an entity or a type of communication problem (KEEFE, 2000). On an epistemological approach, every extension of an expression is precise but we do not know how to determine it (KöLBEL; BARCELONA, 2010).

In this text, we assume that **vagueness is a semantic phenomena**. The meaning of an expression determines an intension depending on the context in which the expression is evaluated. An intension, in a specific context, determines an extension. Semantically, vague expressions have indeterminate extensions, i.e. intensions of vague predicates do not determine exact extensions (KöLBEL; BARCELONA, 2010).

Example 3. *According to the Example 1, "tall" is a vague expression. Its intension may include "person whose height is well above the average height of a specific group". The extension is indeterminate because as explained in Example 1, we cannot define whether people with heights that fall under borderline cases belong to the set of tall people or not.*

1.1.5 Vagueness and Many-Valued Logic

Consider a borderline case in the grains of wheat, for example. If we added a single grain of wheat, we would still have a borderline case. Nevertheless, our confidence in saying that this is a heap increases (even if only by a little). Furthermore, ill-defined boundaries cause extensions that cannot be described as classical sets. Based on the Sorites paradox, it is intuitive to think there are premises in which it is not possible to use the law of the excluded middle. Therefore, a many-valued logic is of interest for problems with vague and imprecise terms.

It is possible to describe well-defined objects using vague concepts. This phenomenon of swapping existing terms for less precise ones is common in natural language. For instance, instead of using the term "meter", language uses more vague expressions such as "near" or "far" instead of indicating a more precise distance. This may happen either due to ignorance of the real precise distance or to facilitate communicating a message (KEEFE, 2000). This is an important feature because it is a tool to group and classify elements according to our perception and context.

Therefore, by being able to model imprecise extensions (sets) we can group elements that can be described with a vague expression. One way to model these sets is through the use of many-valued logic, more specifically fuzzy logic. Finally, One may use the theory of fuzzy sets, which will be explained in the second chapter.

1.2 Uncertainty and Imprecision

Uncertainty is often confused with the concept of imprecision because it is common for the former to be a consequence of the latter. However, these concepts have important differences that affect mathematical modeling. Although imprecision and uncertainty are associated with indeterminacy, uncertainty arises from lack of information, either total or partial, about an object or an event (SMETS, 1997). Imprecision is linked to information, while uncertainty is linked to our knowledge of the world (SMETS, 1997).

Example 4. *Consider the following sentences*

1. *She arrived around three o'clock.*

2. *I guess she arrived at three o'clock sharp.*

The first sentence is a case of imprecision, while the second sentence is a case of uncertainty. Suppose she arrived at 3:01. Then, the first sentence is true and the second (not considering "I guess") is false.
If you know that sentence 1 is true, then you can guess any exact time that is close to 3 o'clock. However, if you know that sentence 2 is true (considering "I guess"), you cannot be sure that the person arrived around 3 o'clock.

Most quantitative models, such as probability theory and possibility theory, concern uncertainty. In contrast, fuzzy set theory deals with imprecision (SMETS, 1997).

2 Fuzzy Logic and Fuzzy Set Theory

This chapter briefly describes fuzzy set theory and presents Fuzzy Natural Logic, explaining how to model linguistic expressions.

2.1 Fuzzy Sets

A fuzzy set is a quantitative imprecision model that seeks to generalize the model of classical sets, where an element either completely belongs to a set or not, with no middle ground.

Definition 1. *(ZADEH, 1965) Let U be a universal set according to the definition of classical sets. A fuzzy set A in U is characterized by a membership function*

$$\mu_A : U \to [0, 1],$$

such that $\mu_A(x)$ represents the membership degree[1] *of x in A, $\forall x \in U$.*

Example 5. *Let $U = \mathbb{R}^+ = (0, \infty)$ be the set of all possible heights in meters. Let A be the set of all heights at which a person is tall. A person who is 2 meters tall is definitely tall, thus $\mu_A(2) = 1$. On the other hand, a person who is 1 meter tall is definitely not tall, thus $\mu_A(1) = 0$. As in Example 1, a 1.77 meter tall person is in a borderline case, so $\mu_A(1.77) \in (0, 1)$. Considering that a person with this height tends to be considered tall rather than not tall, we can choose the membership degree $\mu_A(1.77) = 0.7$.*

Membership degrees are not measured objectively. They are values between 0 and 1 assigned based on the evaluator's perception. However, there are models for the membership functions of fuzzy sets that can yield good results. Some of those models will be presented throughout the text.

When $\mu_A(x) = 0$, x indeed does not belong to A. When $\mu_A(x) = 1$, x completely belongs to A. From this, we can define the concepts of kernel and support of a fuzzy set:

Definition 2. *The support of a fuzzy set A is defined by*

$$Supp(A) = \{x | x \in U, \mu_A(x) > 0\},$$

and the kernel of A is defined by

$$Ker(A) = \{x | x \in U, \mu_A(x) = 1\}.$$

[1] The notation $A(x)$ is an alternative to $\mu_A(x)$, and both represent the membership degree of the element x in A.

Moreover, a fuzzy set is called normal *if there is a non-empty kernel.*

Definition 3. *Given two fuzzy sets A and B, we say that A is a* subset *of B if, and only if,*

$$\mu_A(x) \leqslant \mu_B(x), \forall x \in U.$$

We use the notation $A \subsetneqq B$ to represent that A is a subset of B.

$\mathscr{F}(B)$ indicates the set of all fuzzy subsets of B, i.e., $\mathscr{F}(B) = \{A | A \subsetneqq B\}$.

Remark 1. *The notation $A \subseteq B$ represents that A is a subset of B and both sets are classical sets.*

Definition 4. *A classical set is also called a* crisp set. *A crisp set may be taken as a case of fuzzy sets. Let $B \in \mathscr{F}(U)$ be a crisp set. The membership function of $B \in \mathscr{F}(U)$ is:*

$$\mu_B : U \rightarrow \{0, 1\}$$

such that:

$$\mu_B(x) = \begin{cases} 1, & \text{if } x \in B \\ 0, & \text{if } x \notin B \end{cases}$$

Example 6. *The set of real numbers* **close to zero** *may be modeled as the fuzzy set $A \in \mathscr{F}(\mathbb{R})$ such that:*

$$\mu_A(x) = \begin{cases} 1 - x, & 0 \leqslant x \leqslant 1 \\ x + 1, & -1 \leqslant x < 0 \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, $Ker(A) = 0$ and $Supp(A) = (-1, 1)$, which is the open interval from -1 to 1.

The set of real numbers **greater than zero** *may be the crisp set $B \subsetneqq \mathbb{R}$ such that:*

$$\mu_B(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leqslant 0 \end{cases}$$

Figure 1 shows the membership function of the fuzzy set A. Figure 2 shows the membership function of the crisp set B.

Definition 5. *Let U be a finite universal set. The cardinality of $A \in \mathscr{F}(U)$ is defined as follows:*

$$|A| = \sum_{x \in U} \mu_A(x)$$

Figure 1 – Membership function of A

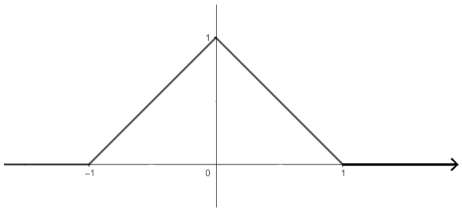

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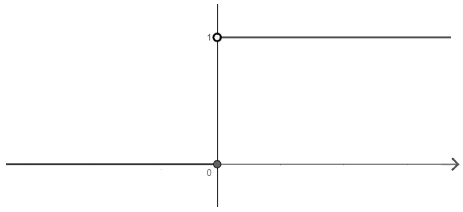
Figure 2 – Membership function of B



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2.1.1 Operations on Fuzzy Sets

It is possible to associate fuzzy logic with fuzzy set theory. The objects of interest in fuzzy logic are propositions that can have their veracity judged with values between 0 and 1 (CINTULA; FERMULLER; NOGUERA, 2023). A truth degree of 0 means that the proposition is "completely false", while 1 means "completely true". This way, we can describe a fuzzy set A by judging the degree of truth of propositions of the form "x belongs to A", $\forall x \in U$. For this reason, when characterizing a logical operation, we obtain an operation between fuzzy sets. An important operation between fuzzy sets is the complementarity:

Definition 6. *Let p be any proposition and let $\mu(p)$ be its degree of truth. The negation of p is $\neg p$, on which $\mu(\neg p) = 1 - \mu(p)$. Let A be a fuzzy set, A^c is called the complement of the fuzzy set A, such that: $\mu_{A^c}(x) = 1 - \mu_A(x), \forall x \in U$.*

For the intersection and union operators of fuzzy sets, we will start with an auxiliary function and then define the concept of t-norm and t-conorm.

Definition 7. *Let $\star : [0,1] \times [0,1] \to [0,1]$ be a function with the following properties:*

1. *Symmetry: $\star(a,b) = \star(b,a)$*
2. *Associativity: $\star(a,\star(b,c)) = \star(\star(a,b),c)$*
3. *Monotonicity: If $a \leqslant b$ then $\star(a,c) \leqslant \star(b,c)$*
4. *Existence of a neutral element e: $\exists e \in [0,1]$ such that $\star(e,a) = a$*

If $e = 1$, then $T(\cdot,\cdot)$ is called a t-norm *and if $e = 0$ then $S(\cdot,\cdot)$ is called a* t-conorm *or an* s-norm.

A t-norm returns the degree of truth of a proposition composed with a logical conjunction, whereas the t-conorm does so with a logical disjunction. Let p and q be logical propositions, we can see from the truth table that this definition is valid for classical logic:

Table 1 – Truth Table.

$\mu(p)$	$\mu(q)$	$T(\mu(p), \mu(q))$	$S(\mu(p), \mu(q))$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Source: Own authorship.

Two commonly used t-norms are:

- Minimum t-norm: $T(a, b) = a \wedge b = min\{a, b\}, \forall a, b \in [0, 1]$.
- Łukasiewicz t-norm : $T(a, b) = a \otimes b = max\{0, a + b - 1\}, \forall a, b \in [0, 1]$.

For $A, B \in \mathscr{F}(U)$, a t-norm may determine an intersection between fuzzy sets as follows:

$$\mu_{A \cap B}(x) = T(\mu_A(x), \mu_B(x)), \forall x \in U.$$

To calculate the t-norm, it is common to extend De Morgan's Laws to fuzzy logic, i.e.: $\neg S(\mu(p), \mu(q)) = T(\neg\mu(p), \neg\mu(q))$. Thus, the union between fuzzy sets may be calculated as:

$$\mu_{A \cup B}(x) = 1 - T(1 - \mu_A(x), 1 - \mu_B(x)), \forall x \in U$$

$$\mu_{A \cap B}(x) = S(\mu_A(x), \mu_B(x)), \forall x \in U$$

Two of the most commonly used t-conorms are:

- Maximum t-conorm: $S(a, b) = a \vee b = max\{a, b\}, \forall a, b \in [0, 1]$
- Łukasiewicz t-conorm: $S(a, b) = a \oplus b = min\{1, a + b\}, \forall a, b \in [0, 1]$

Example 7. *Let A and B be the fuzzy sets from Example 6. Let $A \cap B$ and $A \cup B$ be the Łukasiewicz intersection and Łukasiewicz union respectively. Thus $\mu_{A \cap B}(x) = \mu_A(x) \otimes \mu_B(x), \forall x \in \mathbb{R}$ and $\mu_{A \cup B}(x) = \mu_A(x) \oplus \mu_B(x), \forall x \in \mathbb{R}$:*

$$\mu_{A \cap B}(x) = \begin{cases} 1 - x, & 0 < x \leqslant 1 \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{A \cup B}(x) = \begin{cases} 0, & x \leqslant -1 \\ x + 1, & -1 < x \leqslant 0 \\ 1, & x > 0 \end{cases}$$

Figure 3 shows the membership function of $A \cup B$ while Figure 4 shows the membership function of $A \cap B$.

Figure 3 – Membership of $A \cup B$

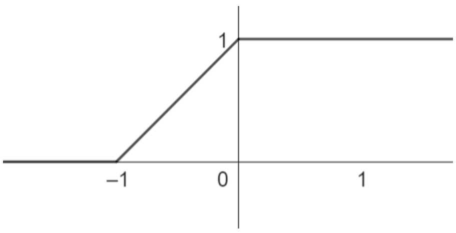

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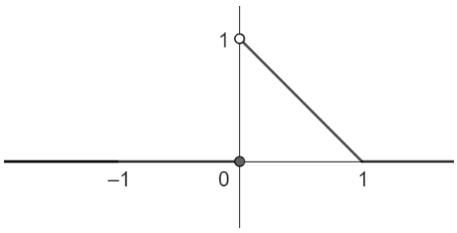
Figure 4 – Membership of $A \cap B$



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Another very common operation in classical logic is implication. An important propositional equivalence in classical logic is: $p \to q \equiv \neg p \vee q$. From this, we obtain a fuzzy implication as follows:

$$\mu(p \to q) = \mu(\neg p \vee q) = S(1 - \mu(p), \mu(q)).$$

Remark 2. *It is important to highlight that when symbols $\wedge, \vee, \to$ are linking logical propositions, they stand for conjunction, disjunction and logical implication. On the other hand, when they are between membership values they mean minimum t-norm, maximum t-norm and fuzzy implication, respectively.*

The fuzzy implications arising from the aforementioned t-conorms are

- Kleene-Dienes implication: $a \to b = max\{1 - a, b\}, \forall a, b \in [0, 1]$;
- Łukasiewicz implication: $a \to b = min\{1, 1 - a + b\}, \forall a, b \in [0, 1]$.

An operation between fuzzy sets $A, B \in \mathscr{F}(U)$ is residuum that is defined as:

$$\text{If } A \implies B, \text{ then } \mu_{A \implies B}(x) = \mu_A(x) \to \mu_B(x), \forall x \in U.$$

Finally, we define $\longleftrightarrow$ as a fuzzy bi-implication, such that

$$a \longleftrightarrow b \equiv (a \to b) \wedge (b \to a).$$

Remark 3. *Throughout the text, we* **only** *use Łukasiewicz implication as well as the corresponding union, intersection and residuum.*

Example 8. *Using the Example 6 sets again, we obtain:*

$$\mu_{A\Rightarrow B}(x) = \begin{cases} -x, & -1 \leqslant x < 0 \\ 1, & \text{otherwise.} \end{cases}$$

$$\mu_{B\Rightarrow A}(x) = \begin{cases} 1, & x < 0 \\ 1-x, & 0 \leqslant x < 1 \\ 0, & x \geqslant 1 \end{cases}$$

Figure 5 shows the membership function of $A \Rightarrow B$ while the Figure 6 shows the membership function of $B \Rightarrow A$.

Figure 5 – Membership of $A \Rightarrow B$

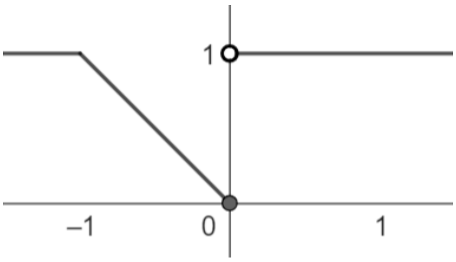

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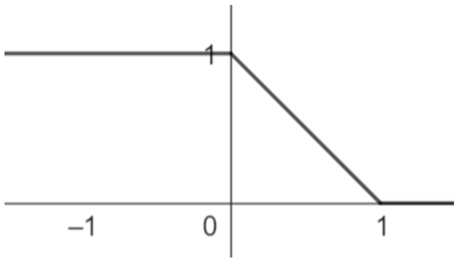
Figure 6 – Membership of $B \Rightarrow A$



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2.2 Fuzzy Natural Logic

This section is heavily based on (NOVÁK, 2015) and (NOVÁK; PERFILIEVA; DVOŘÁK, 2016). We used such texts as the theoretical basis for applications in psychiatric cases.

One way to characterize an object is to describe it using linguistic expressions. If the information given by a linguistic expression can be evaluated and classified on an ordered scale, we will call the expression **Evaluative Linguistic Expressions**. Fuzzy Natural Logic is a field of study that seeks to model evaluative expressions of *natural language* mathematically, regardless of the language, using semantic concepts together with the structure of fuzzy logic (NOVÁK; PERFILIEVA; DVOŘÁK, 2016).

For the purposes of this master thesis, we will focus only on the grammatical structure of a pure evaluative linguistic expression of the following form[2]:

<adverb><adjective>.

[2] When we refer to "adjective" and "adverb" we can also refer to adjectival phrases and adverbial phrases respectively.

Alternatively, of a pure evaluative linguistic predication whose form is:

<noun or pronoun><verb to be><adverb><adjective>.

However, for an evaluative linguistic expression "to be" of our interest, we need specific types of adjectives and adverbs. Such types are specified throughout the text.

Example 9. $\underbrace{\text{She}}_{\text{pronoun}}$ is $\underbrace{\underbrace{\text{very}}_{\text{adverb}}\ \underbrace{\text{tall}}_{\text{adjective}}}_{\text{evaluative expression}}$.

Due to the fact that the verb "to be" is a linking verb, the evaluative expression directly characterizes the subject of the sentence. The adjective "tall" is the most important word in the evaluative expression and "very" is an adverb that intensifies it. This expression is an evaluative linguistic expression because we can classify a person as "tall" according to a height scale in a given unit of measurement.

Example 10. $\underbrace{\text{Two}}_{\text{noun}}$ is $\underbrace{\underbrace{\text{an even number}}_{\text{adjectival phrase}}}_{\text{evaluative expression}}$.

According to the first definition presented, this expression is an evaluative linguistic expression because we can classify a number as "even" base on the scale of integers.

2.2.1 Linguistic Variable

A linguistic variable is a variable in which values are expressed by words and not numeric values. Originally defined as a quintuplet (ZADEH, 1975), this concept was later refined to a sextuplet (NOVÁK; PERFILIEVA; DVOŘÁK, 2016).

Definition 8. *(ZADEH, 1975)(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) A* linguistic variable *is a sextuplet* $(X, T(X), U, G, W, M)$*, such that:*

- X *is the name of the variable.*
- *T(X) is the set of evaluative linguistic expressions that describe this variable.*
- U *is the universal set.*
- G *is the syntactic rule by which linguistic expressions are formed.*
- W *is the set of all possible contexts.*
- M *is the semantic rule in which each linguistic expression is assigned its meanings.*

Example 11. *The linguistic variable* X *in the sentence "X is short" can be replaced by a noun such as "height" or a pronoun. Considering* $X =$ *"height", we obtain:*

- $T(X) = \{$"very short", "short", "somewhat short", ...$\}$
- $U = (0, \infty)$. *Possible heights in a given unit of measurement.*
- *The sentence "X is very today" does* **not** *follows the syntactic rule G, because there are only adverbs after a linking verb. On the other hand, "X is tall" follows the syntactic rule even with the absence of an adverb.*
- W *={height of people in Brazil in 2024, height of people in China in the 15th century, height of buildings in the interior of the state of São Paulo in the 19th century,...}*
- *The sentence "X is happy" does not follow the semantic rule M, as "happy" is not a term that characterizes "height". The term "happy"* $\notin T(X)$.

2.2.2 Intension and Extension

As seen in subsection 1.1.4, intension and extension are two important concepts from semantic theory.

Definition 9. *(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) Let W be a set of contexts and U be a universal set. Let* $\mathscr{A}$ *be an evaluative linguistic expression. The* intension *of* $\mathscr{A}$ *(and "X is* $\mathscr{A}$*") is a function*

$$Int(\mathscr{A}) : W \to \mathscr{F}(U),$$

which assigns to each context $w \in W$ *a fuzzy set contained in U. This fuzzy set is called the* extension *of the expression* $\mathscr{A}$ *in a context* $w \in W$*:*

$$Ext_w(\mathscr{A}) = Int(\mathscr{A})(w) \in \mathscr{F}(U).$$

The *extension* is a fuzzy set that encompasses all objects that an expression can describe. Meanwhile, the *intension* relates an expression in all possible contexts to all fuzzy subsets of the universal set. Consequently, the extension will be very useful for modeling evaluative expressions and predications.

In the next subsections, we will define the concept of pure evaluative linguistic expression and of linguistic context. Then we can present an example of a fuzzy extension.

2.2.3 TE-Adjectives

Adjectives are words that characterize nouns, describing an object of interest. For fuzzy natural logic, the interest lies in evaluative and gradual adjectives.

Definition 10. Evaluative adjectives *characterize the noun based on the interlocutor's perception, whereas* gradual adjectives *can be intensified when accompanied by an appropriate adverb.*

Example 12. *In the sentence "he is single", we do not have an evaluative adjective because "being single" does not depend on the interlocutor's perception. We also do not have a gradual adjective, because we cannot intensify the term "single" as "very single" or "a little single".*

Example 13. *In the sentence"she is tall", we have an evaluative adjective because "being tall" can change depending on the interlocutor's perception. The adjective is also gradual, because "very tall" and "extremely tall" are valid expressions.*

Another important notion is that we can classify evaluative and gradual adjectives according to the behavior of their antonym pair. Complementary adjectives are pairs of antonymous adjectives in which one is equivalent to the negation of the other. However, for non-complementary adjectives, the pair of antonyms do not encompass all possible descriptions of that scale.

Example 14. *An example of non-complementary adjectives are "small" and "big" because*

$$\text{"X is not small"} \neq \text{"X is big".}$$

Thus, on a "size" scale, objects that are not described by either of these adjectives are called "medium".

Definition 11. *(NOVÁK, 2015)* **TE-adjectives**[3] *are evaluative and gradual adjectives that participate in a fundamental evaluative trichotomy. It consists of a pair of non-complementary antonyms and an adjective that lies between them on the scale of the universe in which the discourse is inserted.*

Example 15. *"Small", "medium" and "big" form a fundamental evaluative trichotomy, so these three adjectives are TE-adjectives.*

The adjectives: "small", "medium" and "big" are called canonical TE-adjectives. We do so because we can interchange them with another evaluative trichotomy without changing the mathematical structure of the expression. Thus, the TE-adjective to the left of the scale will be called "small", the one to the right may be called "big" and the central one "medium".

2.2.4 Linguistic Hedge

Adverbs are words that modify the meaning of a verb, adjective or another adverb. Hedging is a linguistic phenomenon to express the vagueness of a topic of interest. Often, but not always, this phenomenon is caused by either an adverb or adverbial phrase.

[3] "TE-adjective" is an abbreviation of "trichotomous evaluative adjective".

Definition 12. *In the case of evaluative expressions, we will consider* linguistic hedges *only as intensifying adverbs or adverbial phrases that have the effect of narrowing or widening the meaning of a TE-adjective. That is, decreasing or increasing the extension of the expression, respectively (NOVÁK, 2015).*

Example 16. *"Very" and "extremely" are linguistic narrowing hedges because they reduce the number of objects that the expression describes:*

This font is very large but not extremely large.

This font is very large and extremely large as well.

An example of a widening hedge is "somewhat". It describes objects that are somewhat large as well as objects that are large. Therefore, it increases the number of objects that can be described by the expression that does not contain the adverb.

Remark 4. *As the adverb may or may not appear, we identify its absence as "empty hedge".*

2.2.5 Pure Evaluative Linguistic Expressions

From this subsection onwards, a pure evaluative linguistic expression and a pure evaluative linguistic predication will be defined as follows:

Definition 13. *A* pure evaluative linguistic expression *is an expression of the form*

$$< \text{linguistic hedge} >< \text{TE-adjective} > . \tag{2.1}$$

A pure evaluative linguistic predication *is a predication of the form:*

$$\text{X is } < \text{linguistic hedge} >< \text{TE-adjective} > . \tag{2.2}$$

We denote the set of all pure evaluative linguistic expressions and all pure evaluative linguistic predications by $EvExpr$.

The symbol denotes that given $\mathscr{A} \in EvExpr$, a pure evaluative linguistic expression, we can form an evaluative predication of the form "X is $\mathscr{A}$" $\in EvExpr$. We made this distinction so that there would be no doubt about what an expression and a predication mean. However, in our modeling, the model of an expression is the same as the model of the corresponding predication. In subsection 2.2.6 we will see that when modeling an evaluative expression, the linguistic variable will already be taken into account in the linguistic context. Therefore, modeling an expression is equivalent to modeling a corresponding predication[4].

Definition 14. *One way to order the pure evaluative linguistic expressions (and predications) of a fundamental evaluative trichotomy is to compare first the TE-adjectives and then the linguistic hedges (lexicographical order). The canonical TE-adjectives are ordered as follows:*

$$\text{small} \ll \text{medium} \ll \text{big}$$

If hedge_1 *has a greater narrowing effect than* hedge_2*, the linguistic hedges will be ordered as follows:*

$$\text{hedge}_1 \ll \text{hedge}_2$$

That is, $\ll$ *means that the hedge on the left will cause a smaller extension than the one on the right.*

Example 17. *In Example 16 we saw examples of linguistic hedges and their widening or narrowing effect. Below there is an ordering of some pure evaluative linguistic expressions:*

$$\text{very small} \ll \text{small} \ll \text{somewhat small} \ll \text{medium}$$

$$\ll \text{extremely big} \ll \text{very big} \ll \text{somewhat big}$$

This ordering will be useful when we use the Local Perception Function from Definition 20.

2.2.6 Linguistic Context

A linguistic context is related to the circumstances in which expressions are inserted. For example, a person who is considered tall among the general population may not be considered tall on a professional basketball team. It is necessary to know context, so that we can find the extension of an evaluative expression or evaluative predication. Considering that evaluative expressions are necessarily composed of TE-adjectives, the fundamental evaluative trichotomy divides the universe, imprecisely, into three parts.

Example 18. *Considering the height of people in Brazil in the 21st century, we can evaluate someone who is 1.60 meters tall or less as short. Meanwhile, someone who is 1.85 meters tall or more can be classified as tall. In addition, people between these heights will either be medium-height or there will be doubts to choose which expression describes their height.*

[4] There are some differences from the predication and expression model in (NOVÁK; PERFILIEVA; DVOŘÁK, 2016), in which the model of an expression is always in a specific context. However, we will consider that the models are the same because we will always inform the context.

Definition 15. *(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) Let v_L, v_S and $v_R \in \mathbb{R}$ be distinct numerical points with appropriate units of measurement of the universal set (associated with a pure evaluative linguistic expression) such that $v_L < v_S < v_R$, in which v_L is the least value that makes sense in the situation, v_R is the greatest value that makes sense in the situation and v_S is a value between them, normally the most common value in the situation. A linguistic context is a strictly increasing bijection*

$$w : [0, 1] \to [v_L, v_S] \cup [v_S, v_R] = [v_L, v_R].$$

Where $w(0) = v_L, w(0.5) = v_S$ and $w(1) = v_R$. We denote the context as $w =< v_L, v_S, v_R >$.

When it is said that v_L is the most significant lower limit, it expresses the fact that this value (or a smaller one) from the ordered scale is small with degree of membership one. That is, it is chosen as a value that is undoubtedly small. Choosing v_R is analogous. For v_S, we choose the most common value of that scale (usually the mode instead of the mean between v_L and v_S).

Defining a context is a challenging task. If we choose a very high v_L value, we may end up judging values that are not small as small. Similarly for v_R. On the other hand, if we choose a very low v_L or a very high v_R, we end up judging almost all values as medium. As a consequence, the model will not be useful.

Example 19. *Take the context of sports balls in Brazil. A ball smaller than a marble is extremely small and a ball larger than a Pilates ball is extremely large. A marble is approximately* 1.5 *cm in diameter and a Pilates ball is approximately* 65 *cm in diameter. Therefore, we can choose $v_L = 1.5$ and $v_R = 65$. When we think about sports balls in Brazil, the most common is a soccer ball. It is approximately* 22 *cm in diameter. As a result, we choose $v_S = 22$. Therefore*

$$w =< 1.5, 22, 65 > .$$

Example 20. *The context of Example 19 may not be significant. If something more specific is wanted, such as the context of the most* **common** *sports balls in Brazil, a handball , soccer ball, basketball and volleyball would be considered medium-size balls. In this case, we can take $v_L = 17$ (value smaller than the diameter of a handball ball in centimeters) and $v_R = 25$ (value greater than the diameter of a basketball ball in centimeters). So*

$$w =< 17, 22, 25 > .$$

This way, we can better differentiate among the sizes of the balls using linguistic expressions.

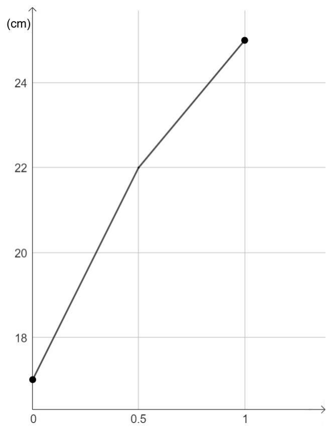
When choosing the values of v_L and v_R, we look for the greatest and lowest value that make sense according to our perception. However, in real life there may be values that will be to the left of v_L or to the right of v_R. Therefore (even though w is a function) instead of saying $x \in U$, we say $x \in w$ so that it is clear what context we are using.

We define the extended inverse of w as a function $w^{(-1)} : \mathbb{R} \to [0,1]$, so that we can work with elements that fall outside $[v_L, v_R]$:

$$w^{(-1)}(x) = \begin{cases} w^{-1}(x), & \text{if } x \in [v_L, v_R] \\ 0, & \text{if } x < v_L, \\ 1, & \text{if } x > v_R \end{cases} \tag{2.3}$$

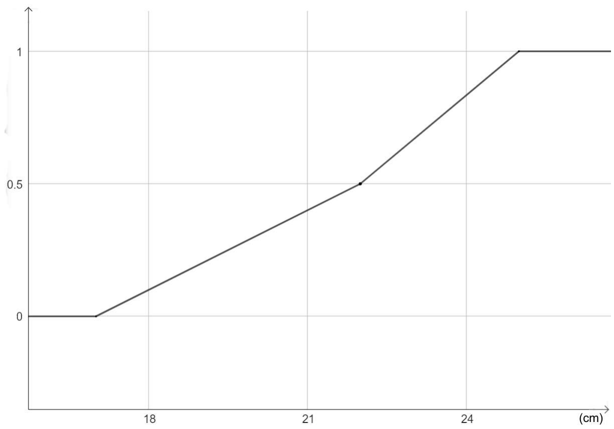
Example 21. *For the context $w =< 17, 22, 25 >$ from Example 20, we obtain Figure 7 and Figure 8:*

Figure 7 – Context w from Example 20



Source: Own authorship

Figure 8 – Extended inverse of context w from Example 20



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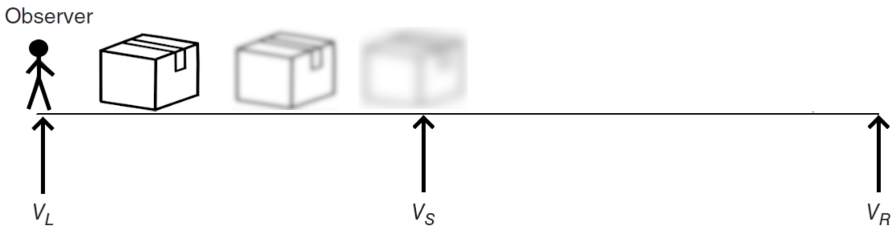
2.2.7 Model for Pure Evaluative Linguistic Expressions

We will refer to pure evaluative linguistic expressions simply as an evaluative expressions. Novák, Perfilieva e Dvořák (2016), based on the ideas of Vopěnka (1979), proposed the use of the concept of *horizons* to understand the source of vagueness of the extensions of evaluative expressions. In other words, to find the extensions of evaluative expressions, we model the TE-adjectives with the help of the concept of horizon. As the adverb modifies the adjective, we deform the horizon according to the linguistic hedge of the expression.

A possible meaning of "horizon" is a person's field of visibility. This concept of horizon is based on an observer's perception. The sharpness of an object decreases as the object moves away from the observer. Therefore, the horizon changes depending on the circumstances. Given a context $w =< v_L, v_S, v_R >$, suppose it is possible to position in v_L

an observer whose visual field does not exceed the point v_S. The further away the object, the less clearly the observer sees it. Figure 9 illustrates the concept of a real-world horizon.

Figure 9 – Schematic illustration of a real-world horizon.



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The idea for defining the horizon in the case of pure evaluative expressions is similar. Given a fuzzy set of small objects and an ordered scale of object sizes, we can evaluate whether an object is small or not in the context w. The further to the right the object is from v_L, the lower the membership degree of the object to the fuzzy set of small objects. When the object is to the right of v_S, its membership degree will be 0 to the fuzzy set of small objects.

Definition 16. *(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) Given a context* $w = < v_L, v_S, v_R >$, *the* **left horizon** *is the fuzzy set associated with the adjective "small". The membership function of the left horizon is* $LH : w \to [0,1]$, *such that*

$$LH(x) = \left(\frac{v_S - x}{v_S - v_L}\right)^* .$$

The **right horizon** *is the fuzzy set associated with the adjective "big". The membership function of the right horizon is* $RH : w \to [0,1]$, *such that*

$$RH(x) = \left(\frac{x - v_S}{v_R - v_S}\right)^* .$$

Furthermore, the **middle horizon** *is defined as the fuzzy set that is neither small nor big. The membership function of the middle horizon is* $MH : w \to [0,1]$, *such that*

$$MH(x) = \neg LH(x) \otimes \neg RH(x)$$

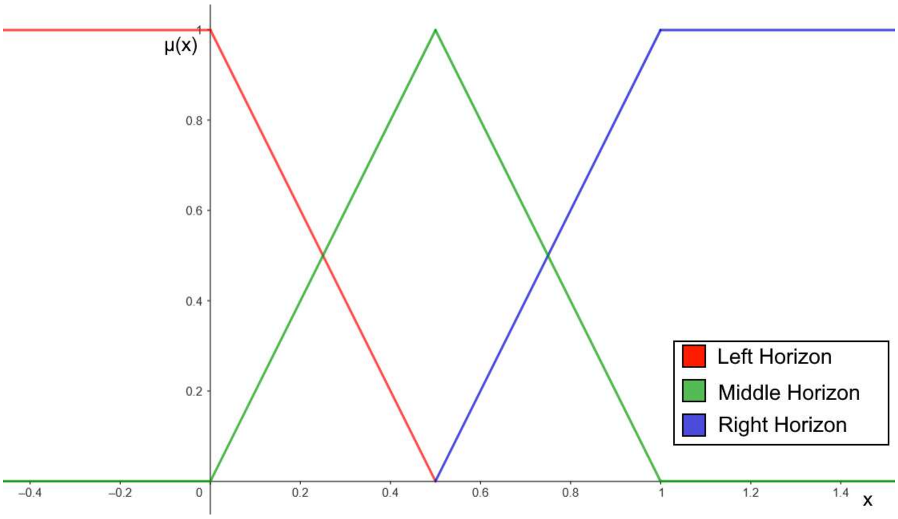
$$MH(x) = \left(\frac{x - v_L}{v_S - v_L}\right)^* \otimes \left(\frac{v_R - x}{v_R - v_S}\right)^* ,$$

where $x \in w$ *and the asterisk stands for*

$$f^*(x) = min\{1, max\{f(x), 0\}\}.$$

The use of the asterisk operator occurs due to the extended context defined above, since values below v_L completely belong to the left horizon and values above v_R completely belong to the right horizon. Figure 10 shows the three horizons of the context $w =< 0, 0.5, 1 >$.

Figure 10 – Horizons of the context $w =< 0, 0.5, 1 >$



Source: Own authorship

Definition 17. *(NOVÁK, 2015) Let* $\nu_{a,b,c} : [0, 1] \to [0, 1]$ *be a class of nondecreasing functions that mathematically models the deformation that a linguistic hedge causes on a TE-adjective. These functions are determined by three parameters*[5] a, b *and* c*, such that*

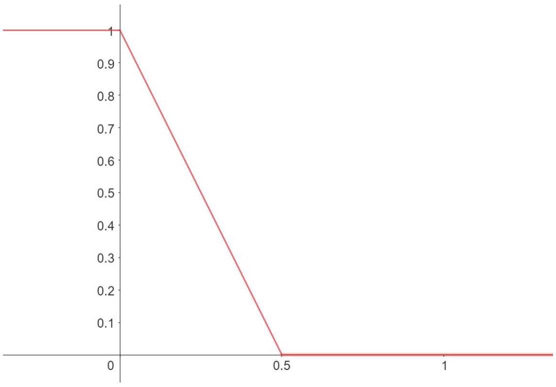
- $\nu_{a,b,c}(y) = 0$*, for* $y \leqslant a$*;*
- $\nu_{a,b,c}(b) = \dfrac{b-a}{c-a}$*;*
- $\nu_{a,b,c}(y) = 1$*, for* $c \leqslant y$*.*

Where $0 < a < b < c < 1$*.*

Example 22. *Let* $a = 0.2$*,* $b = 0.56$ *and* $c = 0.8$*. The deformation of the left horizon with* $w =< 0, 0.5, 1 >$ *by the function* $\nu_{a,b,c}$ *is schematized below:*

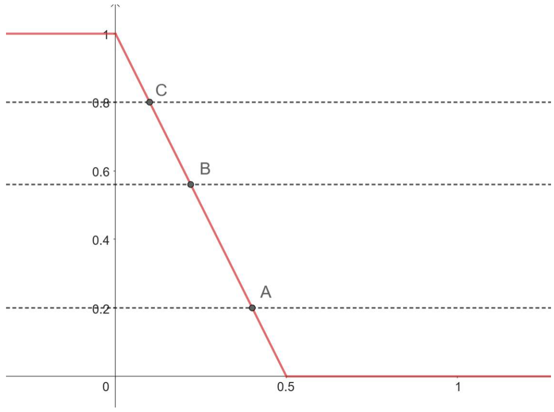
[5] These parameters are obtained empirically according to the specific hedge.

Figure 11 – Step 1.



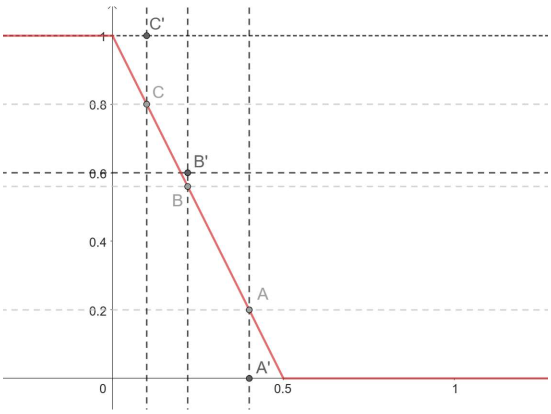
Source: Own authorship

Figure 12 – Step 2.



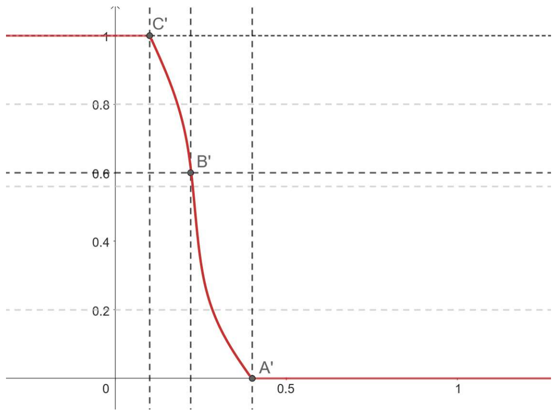
Source: Own authorship

Figure 13 – Step 3.



Source: Own authorship

Figure 14 – Step 4.



Source: Own authorship

- *In Figure 11, the left horizon is represented.*
- *In Figure 12, we marked the points A, B and C where the horizon curve has a membership degree equal to a, b and c respectively.*
- *In Figure 13, we mark the points where the resulting curve will pass. From point A we find A', from point B we find B' and from C we find C'. The membership degree of A', B' and C' are 0, $\dfrac{b-a}{c-a} = 0.6$ and 1 respectively. B' is the point where the curves' concavity changes.*
- *In Figure 14, we find a curve that passes through A', B' and C'. The curve has not been defined yet.*

Looking for a function that was simple but non-linear, a quadratic function was chosen that satisfied the above characteristics (NOVÁK; PERFILIEVA; DVOŘÁK, 2016):

$$\nu_{a,b,c}(y) = \begin{cases} 1, & y \geqslant c \\ 1 - \dfrac{(c-y)^2}{(c-b)(c-a)}, & b \leqslant y < c \\ \dfrac{(y-a)^2}{(b-a)(c-a)}, & a \leqslant y < b \\ 0, & y < a \end{cases} \tag{2.4}$$

Remark 5. *In Equation 2.4, $y \in [0,1]$. This occurs because we apply values from the image of the membership function and not from the domain.*

Let $\mathscr{A} =<$ linguistic hedge $><$ TE-adjective $> \in EvExpr$. In context $w =< v_L, v_S, v_R >$, the extension of $\mathscr{A}$ is the function $\nu_{a,b,c}$ associated with linguistic hedging and applied to the horizon of the respective TE-adjective, at a certain point in the context. Considering the canonical TE-adjectives, the extension of $\mathscr{A}$ in w is of the form:

$$Ext_w(<\text{ linguistic hedge }>\text{ small})(x) = \nu_{a,b,c}(LH(x))$$

$$Ext_w(<\text{ linguistic hedge }>\text{ medium})(x) = \nu_{a,b,c}(MH(x))$$

$$Ext_w(<\text{ linguistic hedge }>\text{ big})(x) = \nu_{a,b,c}(RH(x))$$

$\forall x \in w$.

Example 23. *Given $x = 19$, $a = 0.2$, $b = 0.6$, $c = 0.8$ and $w =< 17, 22, 25 >$ (from Example 20). We may obtain*

$$LH(19) = \frac{22-19}{22-17} = 0.6,$$

since $LH(19) = b = 0.6$, then

$$\nu_{a,b,c}(0.6) = \frac{0.6-0.2}{0.8-0.2} = \frac{2}{3}.$$

Therefore

$$Ext_w(<\textit{ linguistic hedge }>\textit{ small})(19) = \nu_{a,b,c}(LH(19)) = \frac{2}{3}.$$

The extension of the expression means the fuzzy set formed by the most common sports balls in Brazil that are <linguistic hedge> small. When we apply this extension to $x = 19$, we are checking the truth degree that a ball with a diameter of 19 cm is <linguistic hedge> small. Comparing with the notation of Definition 1, we can call $P = Ext_w(<$ linguistic hedge $>$ small) and we would obtain $\mu_P(19) \approx 0.67$.

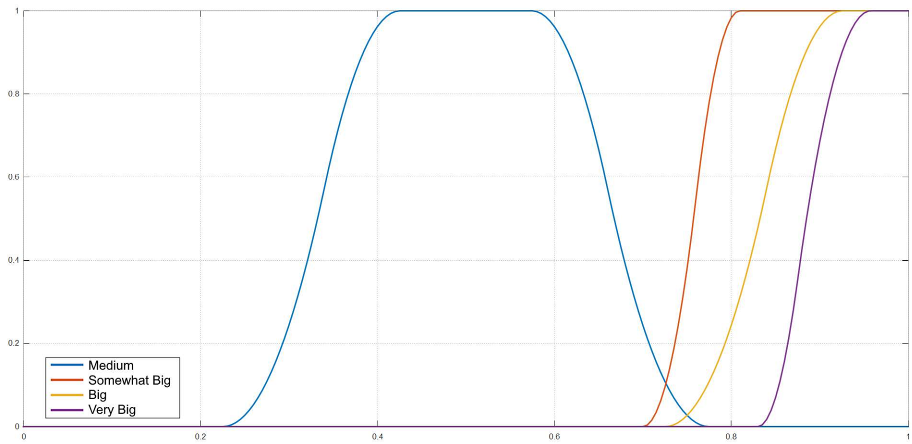
Table 2 associates the parameters a, b and c with some common linguistic hedges. Figure 15 shows extensions of some evaluative expressions in context $w =< 0, 0.5, 1 >$.

Table 2 – Experimental values of parameters a, b and c for linguistic hedges.

Linguistic hedge	a	b	c
Somewhat[6]	0.4	0.52	0.619
Empty hedge	0.45	0.68	0.851
Very	0.66	0.79	0.915

Source: (NOVÁK; PERFILIEVA; DVOŘÁK, 2016).

Figure 15 – Extensions of Some Evaluative Expressions in Context $< 0, 0.5, 1 >$, with Parameters from Table 2.



Source: Own authorship.

Remark 6. *The first definitions of a fuzzy model for a linguistic hedge (ZADEH, 1972) states that a condition for modeling the deformation that the adverb causes in the adjective is that the kernel and the support of the fuzzy sets of the models with different hedges would not change. However, this model is unsatisfactory (LAKOFF, 1973) as cited by Novák, Perfilieva e Dvořák (2016). As seen in Example 16, there are objects that can be described with one linguistic hedge but not with another, which is one of the reasons why Lakoff (1973) did not consider invariant kernel and invariant support.*

2.2.8 Linguistic Descriptions

Let

$$\mathscr{R} : \text{If X is } \mathscr{A}, \text{ then Y is } \mathscr{B} \tag{2.5}$$

be an If-Then rule. We can calculate intension and extension as follows (NOVÁK; PERFILIEVA; DVOŘÁK, 2016):

[6] In (NOVÁK; PERFILIEVA; DVOŘÁK, 2016) the correspondent word is “roughly”.

$$Int(\mathscr{R}) := W \times W \to \mathscr{F}(\mathbb{R}) \times \mathscr{F}(\mathbb{R})$$

$$Ext_{<w,w'>}(\mathscr{R}) = Ext_w(\mathscr{A}) \Rightarrow Ext_{w'}(\mathscr{B})$$

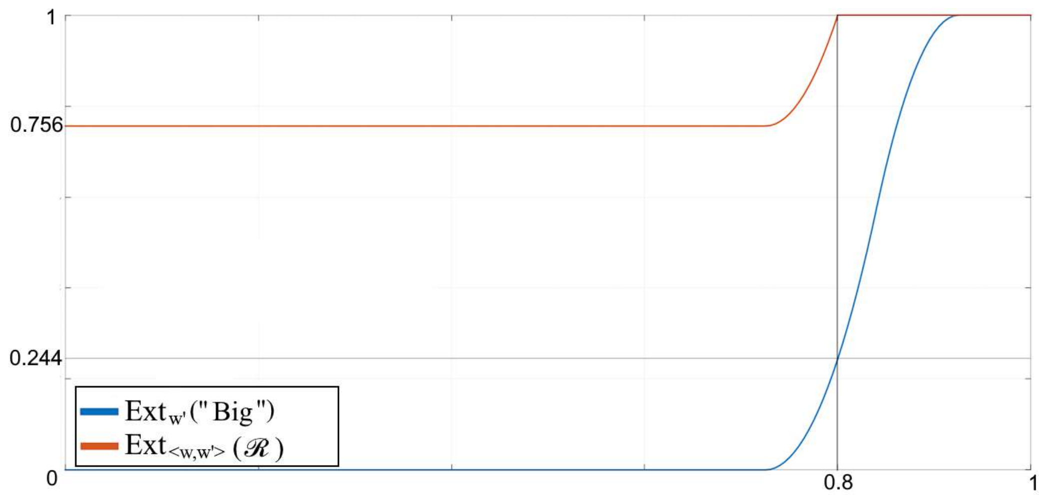
$$Ext_{<w,w'>}(\mathscr{R})(x,y) = Ext_w(\mathscr{A})(x) \to Ext_{w'}(\mathscr{B})(y)$$

Example 24. *Given the rule $\mathscr{R}$:"If X is $\mathscr{A}$ then Y is $\mathscr{B}$", suppose that $Ext_w(\mathscr{A})(x_0) = 0.244$ for some $x_0 \in w$. Note that the value of the rule's extension will depends on the value of the consequent's extension:*

$$\begin{aligned} Ext_{<w,w'>}(\mathscr{R})(x_0,y) &= Ext_w(\mathscr{A})(x_0) \to Ext_{w'}(\mathscr{B})(y) \\ &= min\{1, 1 - 0.244 + Ext_{w'}(\mathscr{B})(y)\} \\ &= \begin{cases} 1, & \text{if } Ext_{w'}(\mathscr{B})(y) \geqslant 0.244 \\ 0.756 + Ext_{w'}(\mathscr{B})(y), & \text{otherwise.} \end{cases} \end{aligned}$$

If $\mathscr{B}$ ="big" and $w' =< 0, 0.5, 1 >$, then we obtain the graph in Figure 16.

Figure 16 – Extensions of the rule and the consequent, given a value for the antecedent.



Source: Own authorship

Proposition 1. *Note that $Ext_{<w,w'>}(\mathscr{R})(x,y) = 1$ when $Ext_{w'}(\mathscr{B})(y) \geqslant Ext_w(\mathscr{A})(x)$. As a consequence, we will consider that given x_0 and a rule[7] $\mathscr{R}$ as in Equation 2.5, $Ext_{w'}(\mathscr{B})(y(x_0)) = Ext_w(\mathscr{A})(x_0)$. We considered it because $Ext_w(\mathscr{A})(x_0)$ is the lowest value that the membership function of $Ext_{w'}(\mathscr{B})$ must take at $y(x_0)$ for the rule to be completely true at (x_0, y).*

[7] Notice there will exist a dependence of y on x_0.

In this case, we will not know the value that $y(x_0)$ takes. If you wish to estimate this value, you can simply apply some defuzzification methods, but this is not our objective in this text.

Let $n \in \mathbb{N}$. We can increase the number of predications in the antecedent of each rule, so that

$$\mathscr{R} : \text{If } X_1 \text{ is } \mathscr{A}^1 \text{ and/or } X_2 \text{ is } \mathscr{A}^2 \text{ and/or ... and/or } X_n \text{ is } \mathscr{A}^n\text{, then Y is } \mathscr{B}. \tag{2.6}$$

Assuming that all logical connectives of the antecedent are "and", we obtain

$$Int(\mathscr{R}) := \underbrace{W \times ... \times W}_{n+1} \to \underbrace{\mathscr{F}(\mathbb{R}) \times ... \times \mathscr{F}(\mathbb{R})}_{n+1}$$

$$Ext_{<w_1,..,w_n,w'>}(\mathscr{R}) = \left[\bigcap_{i=1}^{n} Ext_{w_i}(\mathscr{A}^i)\right] \Rightarrow Ext_{w'}(\mathscr{B})$$

$$Ext_{<w_1,..,w_n,w'>}(\mathscr{R})(x_1, ..., x_n, y) = \left[\bigotimes_{i=1}^{n} Ext_{w_i}(\mathscr{A}^i)(x_i)\right] \to Ext_{w'}(\mathscr{B})(y)$$

If the logical connective is "or", simply replace the t-norm with a t-connorm.

Definition 18. *A* linguistic description *of an object or situation is the set of rules* $LD = \{\mathscr{R}_1, ..., \mathscr{R}_m\}$, *such that*

$\mathscr{R}_i =$ If X_1 is $\mathscr{A}_i^1$ *and/or* X_2 *is* $\mathscr{A}_i^2$ *and/or ... and/or* X_n *is* $\mathscr{A}_i^n$, then Y is $\mathscr{B}_i$

for $i \in \{1, 2, ..., m\}$ *where* $m, n \in \mathbb{N}$.

Example 25. *Suppose we want to describe the risk of a surgery based on its difficulty. We can describe this situation as follows:*

- $\mathscr{R}_1$*: If the difficulty is medium, then the risk is low.*
- $\mathscr{R}_2$*: If the difficulty is somewhat high, then the risk is somewhat low.*
- $\mathscr{R}_3$*: If the difficulty is high, then the risk is medium.*
- $\mathscr{R}_4$*: If the difficulty is very high, then the risk is very high.*

Here, $LD = \{\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4\}$. *We know that the risk of a surgery is not only related to its difficulty. There are other factors that impact it such as the patient's health status, the surgeon's professional experience, etc. Therefore, we can either increase the components of each rule or even increase the number of rules.*

Note that there is no description of "low difficulty". In a linguistic description we do not need to describe all possibilities. In cases where the difficulty is not described, the risk is undefined.

We will only consider linguistic descriptions that are *consistent*, that is, in which the rules do not contradict each other and in which no two rules are the same.

Example 26. *Let* $LD = \{\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3\}$ *be such that:*

- $\mathscr{R}_1$*: If* X *is medium, then* Y *is small.*
- $\mathscr{R}_2$*: If* X *is big, then* Y *is small.*
- $\mathscr{R}_3$*: If* X *is medium, then* Y *is medium.*

If X *is considered medium, then* Y *would be medium and small at the same time. That is, the rules* $\mathscr{R}_1$ *and* $\mathscr{R}_3$ *are contradictory, because they have the same antecedent but different consequents. Therefore, this* LD *is inconsistent.*

We say that the logical connectives of a linguistic description are the same if all the logical connectives of the antecedent of every rule are “or” or all are “and’.

Definition 19 is a new definition introduced in this master’s thesis, as follows:

Definition 19. *Let* $LD = \{\mathscr{R}_1, ..., \mathscr{R}_m\}$ *be defined as in Definition 18. Let* $K_j \subset EvExpr$ *be the set of all evaluative expressions used to describe* X_j *in this description, with* $j \in \{1, 2, ..., n\}$*. Let* k_j *also be the number of elements in the set* K_j*. We say that* LD *a* complete linguistic description *if it is consistent, if all logical connectives are the same*[8] *and if there are* $\prod_{j=1}^{n} k_j$ *different rules.*

Corollary 1. *A linguistic description formed by rules with a single predication in the antecedent* $(n = 1)$ *is always complete.*

Example 27. *Let* $LD = \{\mathscr{R}_1, \mathscr{R}_2\}$ *be a linguistic description:*

- $\mathscr{R}_1$*: If* X_1 *is small and* X_2 *is big, then* Y *is small.*
- $\mathscr{R}_2$*: If* X_1 *is big and* X_2 *is medium, then* Y *is big.*

Since $K_1 = \{$“small”,“big”$\}$ and $K_2 = \{$“medium”,“big”$\}$, we have that $k_1 = 2$ and $k_2 = 2$. LD is not complete because $|LD| = 2 < 4 = k_1 k_2 = 4$.

Example 28. *Let* K_1 *and* K_2 *be the same as in Example 27. An example of a complete linguistic description is composed of the following rules:*

- $\mathscr{R}_1$*: If* X_1 *is small and* X_2 *is big, then* Y *is small.*

[8] Although it is possible to create rules where the logical connectives differ, Definition 19 does not account for different logical connectives in the antecedent.

- $\mathscr{R}_2$: *If* X_1 *is small and* X_2 *is medium, then* Y *is medium.*
- $\mathscr{R}_3$: *If* X_1 *is big and* X_2 *is big, then* Y *is big.*
- $\mathscr{R}_4$: *If* X_1 *is big and* X_2 *is medium, then* Y *is very big.*

Note that consequent expressions do not affect whether a linguistic description is complete or not. Note also that we do not describe X_2 *as small at any point. But if we had described it as small,* k_2 *would be equal to* 3.

2.2.9 Perception-Based Logical Deduction

When there are rules describing the same problem, more than one rule could be fired. This happens because the extensions of the same horizon are contained within each other. Consequently, from the same observation we could reach two or more different conclusions.

Example 29. *Let* $LD = \{\mathscr{R}_1, \mathscr{R}_2\}$ *be a linguistic description as follows:*

- $\mathscr{R}_1$: *If* X *is small, then* Y *is big.*
- $\mathscr{R}_2$: *If* X *is very small, then* Y *is medium.*

Let the observation be "X is very small". From this observation, $\mathscr{R}_1$ *and* $\mathscr{R}_2$ *would be fired because* $Ext_w(\text{“}X \text{ is very small”}) \subsetneqq Ext_w(\text{“}X \text{ is small”})$.

Therefore, perception-based logical deduction becomes a necessary tool. The local perception function (which we will define in Definition 20) is necessary in a set of implicative rules because it indicates only one rule that will be fired.

Definition 20. *(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) Let* W *be a set of contexts and* $x \in \mathbb{R}$*. Let* $K \subset EvExpr$ *be a finite set of evaluative predications, such that,* $(K, \ll)$ *is a linearly ordered set. Let* P *be the set*

$$P = \{\text{“X is }\mathscr{B}\text{”} \in K | Ext_w(\text{“X is }\mathscr{B}\text{”})(x) > 0\},$$

and $P' \subseteq P$ *such that "X is* $\mathscr{B}$*"* $\in P'$ *if, and only if,* $Ext_w(\text{“X is }\mathscr{B}\text{”})(x)$ *is maximal in* P.

Lastly, let

$$P'' = \{\text{“X is }\mathscr{B}\text{”} \in K | Ext_w(\text{“X is }\mathscr{B}\text{”})(x) \geqslant a_0\},$$

where $a_0 > 0$ *is some threshold.*

The local perception function is $LPerc^K : \mathbb{R} \times W \to K$*, such that*

$$LPerc^K(x,w) = \text{“X is } \mathscr{A}\text{”} = \begin{cases} minP'', \text{ if } P'' \neq \varnothing \\ minP', \text{ if } P'' = \varnothing \text{ and } P \neq \varnothing \\ \text{Undefined otherwise.} \end{cases}$$

Example 30. *Let $K = \{$“X is very big”, “X is big”, “X is somewhat big”, “X is medium”$\}$ in a context $w =< 0, 0.5, 1 >$, as described in Figure 15. If $x = 0.75$ then:*

- Ext_w(“Medium”)$(0.75) = 0.27$
- Ext_w(“Somewhat big”)$(0.75) = 0.38$
- Ext_w(“Big”)$(0.75) = 0.27$
- Ext_w(“Very big”)$(0.75) = 0$

We are able to construct the sets P, P' and P'' from this as in Definition 20. Taking $a_0 = 0, 9$,

- $P = \{$“X is big”, “X is somewhat big”, “X is medium”$\}$
- $P' = \{$“X is somewhat big”$\}$
- $P'' = \varnothing$

$$LPerc^K(0.75, w) = \text{“X is somewhat big”.}$$

Example 31. *Let $K = \{$“X is very big”, “X is big”, “X is somewhat big”, “X is medium”$\}$ in a context $w =< 0, 0.5, 1 >$, as in the previous example. Insert $x = 0.9$:*

- Ext_w(“Medium”)$(0.9) = 0$
- Ext_w(“Somewhat big”)$(0.9) = 1$
- Ext_w(“Big”)$(0.9) = 0.96$
- Ext_w(“Very big”)$(0.9) = 0.68$

Taking $a_0 = 0.9$,

- $P = \{$“X is very big”, “X is big”, “X is somewhat big”$\}$
- $P' = \{$“X is somewhat big’$\}$
- $P'' = \{$“X is big”, “X is somewhat big”$\}$

As "big"≪ "somewhat big", we obtain $minP'' =$ "X is big", *and consequently*

$$LPerc^K(0.9, w) = \text{“X is big”}.$$

Proposition 2. *Let LD be a complete linguistic description with rules of the form of Equation 2.6. Then, to compute the local perception function (from Definition 20) for antecedents with more than one predication, we define* $LPerc^{LD} : \mathbb{R}^n \times W^n \to K_1 \times K_2 \times ... \times K_n$*, such that*

$$LPerc^{LD}((x_1, ..., x_n), (w_1, ..., w_n)) = (LPerc^{K_1}(x_1, w_1), ..., LPerc^{K_n}(x_n, w_n)).$$

Remark 7. *Note that by increasing the number of predications in the antecedent connected with "and", the membership degree of the extension of the consequent tends to decrease. As a result, the value of* a_0 *of the local perception function of Definition 20 must be well chosen. Usually,* $a_0 = 0.9$ *or* $a_0 = 1$*.*

Proposition 3. *(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) Given a linguistic description* $LD = \{\mathscr{R}_1, ..., \mathscr{R}_m\}$ *where its rules have the form:*

$$\mathscr{R}_i := \text{If } X \text{ is } \mathscr{A}_i \text{ then } Y \text{ is } \mathscr{B}_i.$$

Let w *be the context of the antecedent,* w' *be the context of the consequent of the rules of* LD*, and let* $x \in w$ *and* $y \in w'$*. If the i-th rule is fired with a given* x^0*, that is,*

$$LPerc^{LD}(x^0, w) = \text{“If } X \text{ is } \mathscr{A}_i\text{”}$$

then there exists $\hat{y} \in w'$ *such that* $Ext_{<w,w'>}(\mathscr{R}_i)(x^0, \hat{y}) = 1$*.*

Remark 8. *We will denote by* $\hat{y}_0 \in w'$ *a value*[9] *of* $\hat{y}$ *that minimizes* $Ext_{w'}(\mathscr{B}_i)(\hat{y})$*.*

As described in Proposition 1, $Ext_{w'}(\mathscr{B}_i)(\hat{y}_0) = Ext_{w'}(\mathscr{A}_i)(x_0)$.

Example 32. *Let* $w = w' =< 0, 0.5, 1 >$ *be the context of the antecedent and consequent, such that* $x \in w$ *and the linguistic description is from Example 25. Recall that the rules are as follows:*

- $\mathscr{R}_1$*: If the difficulty is medium, then the risk is low.*
- $\mathscr{R}_2$*: If the difficulty is somewhat high, then the risk is somewhat low.*
- $\mathscr{R}_3$*: If the difficulty is high, then the risk is medium.*
- $\mathscr{R}_4$*: If the difficulty is very high, then the risk is very high.*

[9] Note that $\hat{y}_0$ may not be unique. In this text, our objective is not estimating $\hat{y}_0$. However, in (NOVÁK; PERFILIEVA; DVOŘÁK, 2016), the authors present some defuzzifications functions to estimate $\hat{y}_0$.

Suppose that difficulty is measured with a value in $[0, 1]$.

If the difficulty is $x_0 = 0.75$*, then by Example 30,*

$$LPerc^{LD}(0.75, w) = LPerc^{K}(0.75, w) = \text{“X is somewhat high”}.$$

Correspondingly, the rule that will be fired is $\mathscr{R}_2$*, where* X *is the linguistic variable that indicates "difficulty". We have seen in Example 30 that*

$$Ext_w(\text{“Somewhat high”})(0.75) = 0.38.$$

Therefore:

$$Ext_{<w,w'>}(\mathscr{R}_2) = 0.38 \rightarrow Ext_{w'}(\text{“Somewhat low”})(\hat{y}).$$

The truth degree of the second rule is 1 when $Ext_{w'}(\text{“Somewhat low”})(\hat{y}) \geqslant 0.38$*. Thus, for a* $\hat{y}_0$ *(as denoted by Remark 8), we will consider that* $Ext_{w'}(\text{“Somewhat low”})(\hat{y}_0) = 0.38$*. The proposition "The risk is somewhat low" has a degree of truth* 0.38*. Note that we do not calculate the degree of risk but rather the degree of truth of the proposition*[10]*. The degree of risk would be the value of* $\hat{y}_0$*.*

If the difficulty is $x = 0.9$*, then by Example 31 we have that*

$$LPerc^{LD}(0.9, w) = \text{“X is high”}.$$

In this case, the fired rule is $\mathscr{R}_3$*, with*

$$Ext_{w'}(\text{“medium”})(\hat{y}_0) = 0.96.$$

If the difficulty is $x = 0.1$ *then no rule will be fired because the extensions of all antecedents will be equal to zero (since we did not define any rule with <linguistic hedge> small difficulty). In this case we say that the degree of risk is* ***undefined****.*

Proposition 4 is a version of Proposition 3 but with more than one predication in the antecedent based on Novák's work (NOVÁK; PERFILIEVA; DVOŘÁK, 2016).

Proposition 4. *Given a complete linguistic description* $LD = \{\mathscr{R}_1, ..., \mathscr{R}_m\}$ *where its rules has the form*

$$\mathscr{R}_i := \text{If } X_1 \text{ is } \mathscr{A}_i^1 \text{ and/or ... and/or if } X_n \text{ is } \mathscr{A}_i^n \text{ then } Y \text{ is } \mathscr{B}_i,$$

let $\{w_1, ..., w_n\}$ *be the contexts of the antecedents of the rules of* LD*,* w' *be the context of the consequent of the rules of* LD *and let* $x_1 \in w_1, ..., x_n \in w_n$ *and* $y \in w'$*. If the i-th rule is fired with a given* $x^* = (x_1^0, ..., x_n^0)$*, that is,*

$$LPerc^{K_j}(x_j^0, w_i) = \text{“If } X_j \text{ is } \mathscr{A}_i^j\text{”}$$

$\forall j \in \{1, 2, ..., n\}$*, then there exists* $\hat{y} \in w'$ *such that* $Ext_{<w_1,...,w_n,w'>}(\mathscr{R}_i)(x^*, \hat{y}) = 1$.

[10] This can generate confusion because we are using the context $w =< 0, 0.5, 1 >$ and the domain values (for such a context) tend to be in $[0, 1]$.

Remark 9. *If the linguistic description is not complete, then we would not be able to fire a valid rule in some cases. In Example 27 we could fire the predications "X_1 is small" and "X_2 is medium". However, there would be no rule with these two predications.*

Example 33. *For an example with two predications in the rule antecedents, let's consider $k_1 = k_2 = 2$ (where k_1 and k_2 are according to Definition 19), so we only have four rules. Let $LD = \{\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4\}$ be a complete linguistic description where $w_1 = w_2 = w' =< 0, 0.5, 1 >$, $K_1 = \{\text{“Medium”}, \text{“Somewhat big”}\}$ and $K_2 = \{\text{“Big”}, \text{“Very big”}\}$.*

- *$\mathscr{R}_1$: If X_1 is medium and X_2 is big, then Y is small.*
- *$\mathscr{R}_2$: If X_1 is medium and X_2 is very big, then Y is medium.*
- *$\mathscr{R}_3$: If X_1 is somewhat big and X_2 is big, then Y is big.*
- *$\mathscr{R}_4$: If X_1 is somewhat big and X_2 is very big, then Y is very big.*

For $x^* = (0.75, 0.9)$ we already know the extensions from Example 30 and Example 31. Thus

$$LPerc^{K_1}(0.75, w_1) = \text{“}X_1 \text{ is somewhat big”},$$
$$LPerc^{K_2}(0.9, w_2) = \text{“}X_2 \text{ is big”}.$$

Therefore, the fired rule is $\mathscr{R}_3$ and

$$Ext_{<w,w,w'>}(\mathscr{R}_3) = [0.38 \otimes 0.96] \to Ext_{w'}(\text{“Big”})(\hat{y}).$$

Finally, for $\hat{y}_0$ we have

$$Ext_{w'}(\text{“Big”})(\hat{y}_0) = 0.34.$$

2.3 Intermediate Quantifiers

Quantifiers are expressions that indicate the quantity of something. In classical logic, the quantifiers are the universal (∀) and existential (∃). On the other hand, in natural language there are quantifiers that generate imprecision. Such quantifiers can be called fuzzy quantifiers. Fuzzy quantifiers can be divided into *absolute* and *relative* (ZADEH, 1983). While absolutes express complete meaning, relative ones require comparison between two sets, often implicitly, for their understanding.

Example 34. *"About (a number)", "many more than (a number)", "approximately (a number)" and "at least (a number)" are examples of absolute fuzzy quantifiers (KLIR; YUAN, 1995). For example, the sentence "many more than 3" has the imprecision caused by "many". At the same time, it indicates that the closer a number larger than 3 gets to 3, the lower the truth degree of the proposition. Moreover, if the number is equal to or less than 3, then the proposition is false.*

Example 35. *"Most", "almost all" and "many". are examples of relative fuzzy quantifiers (KLIR; YUAN, 1995). For example "many people are tall" only makes sense if we compare it to a larger group of people. We can rewrite the sentence as "Of all the people in the world, many people are tall".*

For this text, we will only consider relative fuzzy quantifiers.

Relative fuzzy quantifiers are so called because we relate two sets by measuring how much a subset fills a set in which it is inserted. According to Novák (2008), it is better to speak of intermediate quantifiers, due do the fact that they mean something between "All" ($\forall$) and "None" ($\nexists$), the representations of the most extreme quantities present in classical logic. We think about intermediate quantifiers as follows:

$$\mathscr{Q}X \text{ is } \mathscr{A}. \tag{2.7}$$

Where $\mathscr{Q}$ is a intermediate quantifier.

Example 36. $\underbrace{\text{Almost all}}_{\mathscr{Q}}\ \underbrace{\text{balls}}_{X}\ \text{are}\ \underbrace{\text{small}}_{\mathscr{A}}.$

Another way to write Equation 2.7 is to think of sets A and B, where $A \subsetneqq B$, and

$$\mathscr{Q}B \text{ is } A. \tag{2.8}$$

Example 37. $\underbrace{\text{Almost all}}_{\mathscr{Q}}\ \underbrace{\text{balls}}_{B}\ \text{are}\ \underbrace{\text{small balls}}_{A}.$

In Example 36, "small" is characterizing "almost all balls", however it is not so evident that there is a comparison between two sets. In Example 37, it is easier to see that we have a set B of all balls and the subset A of small balls. "Almost all" tells us that the subset A fills the set B almost completely.

We will use the theory proposed by Novák (2008) to model intermediate quantifiers. This theory is based on the work of Peterson (2000). Below, there is the definition of a function that will help us measure how much a set fills another in which it is contained. This function will yield a number in $[0, 1]$ that will be useful for modeling intermediate quantifiers.

Definition 21. *(NOVÁK, 2008) Let R be the Łukasiewicz bi-implication and U be a universal set. Let $B \neq \varnothing$ be a finite crisp set, $A \subsetneqq B$ and $F_R : 2^U \times \mathscr{F}(U) \to [0, 1]$ given by*

$$F_R(B)(A) = \frac{1}{|B|} \sum_{x \in B} 1 \leftrightarrow \mu_A(x). \tag{2.9}$$

The function[11] $F_R(B)(A)$ measures the "*degree of subsethood*" (KLIR; YUAN, 1995) of A in B , by comparing the membership degree of each element in A with that in B. Simplifying Equation 2.9, we obtain

$$F_R(B)(A) = \frac{1}{|B|}\sum_{x\in B} 1 - |1 - \mu_A(x)| = \frac{1}{|B|}\sum_{x\in B} \mu_A(x)$$

$$F_R(B)(A) = \frac{|A|}{|B|}.$$

Example 38. *Let U be the set of all sports balls, B be the set of sports balls that are sold in a store and $A \subsetneqq B$ be the set of sports balls sold there that are considered small. Suppose $B = \{x_1, x_2, x_3, x_4\}$, ($|B| = 4$) where $\mu_A(x_1) = 0.57$, $\mu_A(x_2) = \mu_A(x_3) = 0.75$ and $\mu_A(x_4) = 1$[12]:*

$$|A| = 1 + 0.75 + 0.75 + 0.57 = 3.07;$$

$$F_R(B)(A) = \frac{3.07}{4} = 0.7675.$$

We may say that the degree of subsethood of A in B is 0.7675.

At this point, the main idea is to choose the degree of truth of a quantifier according to the subsethood degree of one set in another.

Definition 22. *(NOVÁK, 2008) Let $Q^{\forall}_{ExBi}(B, A)$ be the truth degree of propositions of the type*

Almost all B is A.

The degree of truth of this proposition is calculated by

$$Q^{\forall}_{ExBi}(B, A) \equiv Ext_w(\text{“Extremely Big”})(F_R(B)(A)),$$

where $w =< 0, 0.5, 1 >$ and extension of "Extremely big" according to Table 3.

For the use of intermediate quantifiers, the parameters a, b and c chosen are different from those used for evaluative predications as described in Table 2. Figure 17 shows the extensions of the intermediate quantifiers with the linguistic hedges of Table 3.

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[11] In the case where A is a crisp set, the function $F_R(B) : 2^U \to [0, 1]$ is a fuzzy measure where 2^U is a σ-algebra. For more details on fuzzy measure, see (SUGENO, 1974) and (KLIR; YUAN, 1995) and (BASSANEZI; BARROS, 2010).

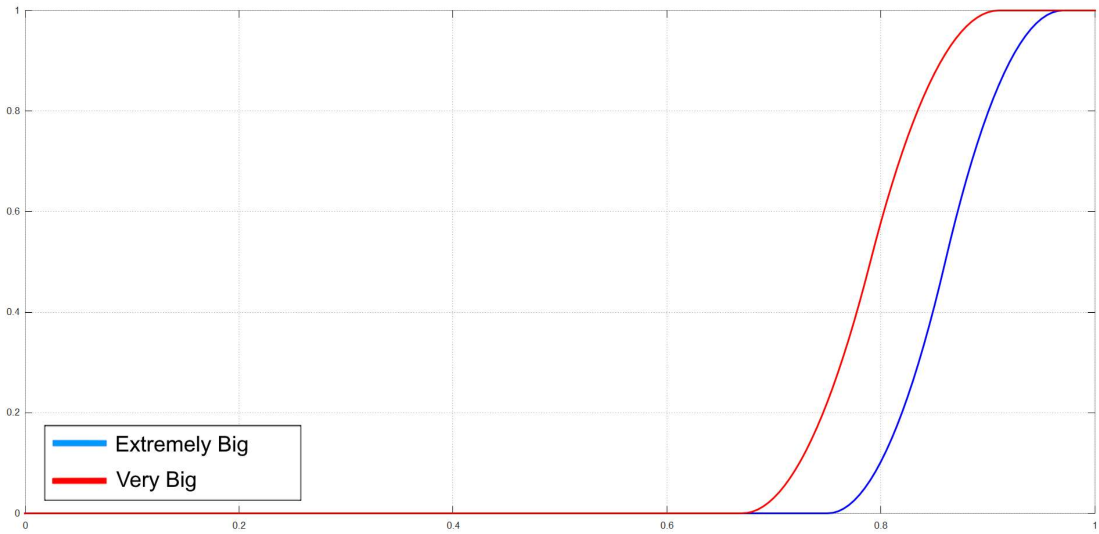
[12] Notice that $\mu_A(x_i) = Ext_w(\text{“small”})(x_i)$, for $i \in \{1, 2, 3, 4\}$ in a context w, as Example 20.

Table 3 – Experimental values of parameters a, b and c for linguistic hedges applied to intermediate quantifiers.

Linguistic Hedge	a	b	c
Very	0.34	0.58	0.82
Extremely	0.5	0.72	0.94

Source: (MURINOVÁ; NOVÁK, 2015).

Figure 17 – Extensions of intermediate quantifiers with the linguistic hedges of Table 3.



Source: Own authorship

Definition 23. *(NOVÁK, 2008) Let $Q^{\forall}_{VeBi}(B, A)$ be the degree of truth of propositions of the type*

Most of B is A.

The truth degree of propositions of this type is calculated by

$$Q^{\forall}_{VeBi}(B, A) \equiv Ext_w(\text{“Very Big”})(F_R(B)(A)),$$

where $w =< 0, 0.5, 1 >$ and extension of “Very” according to Table 3.

Remark 10. *In cases where A and B are sets formed by evaluative predications in a context w' such that A and B depend on a vector $\boldsymbol{x} = (x_1, x_2, ..., x_n)$ where $n = |B|$, $x_i \in w'$ and $i \in \{1, 2, .., n\}$, we use the notations:*

$$(Q^{\forall}_{ExBi}\boldsymbol{x})(B, A)$$

$$(Q^{\forall}_{VeBi}\boldsymbol{x})(B, A)$$

Remark 11. *Formally, the quantifier in Definition 22 is defined as follows (NOVÁK, 2008):*

$$(Q^{\forall}_{ExBi}x)(B, A) \equiv (\exists z)(\Delta(z \subseteq B) \wedge (\text{"Extremely Big"})(F_R(B)z) \wedge (\forall x)(zx \Rightarrow Ax)).$$

This means[13] *that* $(Q^{\forall}_{ExBi}x)(B, A)$ *is equal to the degree of truth in which there is a fuzzy set* $z \underset{\sim}{\subset} B$ *such that* $\forall x \in B$, $\mu_A(x) \leqslant \mu_z(x)$ *and the value of* $F_R(B)(z)$ *is extremely large in the context* $w =< 0, 0.5, 1 >$. *(MURINOVÁ; NOVÁK, 2015)*

Example 39. *Let* $w' =< 17, 22, 25 >$ *be the context of the most common sports balls in Brazil, as described in Example 20. Let* $B = \{x_1, x_2, x_3, x_4\}$ *the set of sports balls sold in a store and let* $\boldsymbol{x} = \{17.5, 18.4; 18.4, 18.6\}$ *be the set of diameters (in centimeters) of the elements of* B. *Let* $A \underset{\sim}{\subset} B$ *be the set of sports balls sold there that are small. Taking parameters from Table 2, we may calculate:*

- $Ext_{w'}(\text{"Small"})(17.5) = 1$
- $Ext_{w'}(\text{"Small"})(18.4) = 0.75$
- $Ext_{w'}(\text{"Small"})(18.6) = 0.57$

Therefore, following Example 38, we know that $F_R(B)(A) = 0.7675$. *According to Table 3, we may obtain:*

- $(Q^{\forall}_{ExBi}\boldsymbol{x})(B, A) = Ext_w(\text{"Extremely big"})(0.7675) = 0.01$
- $(Q^{\forall}_{VeBi}\boldsymbol{x})(B, A) = Ext_w(\text{"Very big"})(0.7675) = 0.33$

Thus, we can say that the degree of truth of :"Almost all balls are small" is 0.01. And that the degree of truth of "Most balls are small" is 0.33.

[13] See (NOVÁK, 2005) if the reader is interested in better understanding the formula and fuzzy type theory.

3 Mood Disorders and Fuzzy Modeling

In this chapter, we describe the actigraphy method and its usefulness in detecting a major depressive episode. How the state-of-health of an individual may be modeled is also introduced.

It is necessary to understand the characteristics of the episode as well as the main results found in the literature to develop a good rule model for detecting depressive episodes. We summarize how major depressive disorder (MDD) and bipolar disorder (BD) are characterized. Then, we introduce MADRS test that assesses the severity of an individual's depressive symptoms. Throughout the chapter, we describe the actigraphy method and its usefulness in diagnosing MDD and BD.

3.1 Mood Disorders

3.1.1 Manic Episode and Hypomanic Episode

According to the revised text of the fifth edition of the Diagnostic and Statistical Manual of Mental Disorders (DSM-V-TR) (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), a manic and hypomanic episode are periods (which have not been induced by a substance) of expansive or irritable mood in which there is a high increase in energy for at least seven days for the former and four days for the latter. These episodes are also characterized by at least three of the following symptoms: inflated self-esteem, reduced need for sleep, exacerbated communication, difficulty expressing ideas, distractability, psychomotor agitation, and carrying out potentially harmful activities. Manic episodes cause social or professional impairment due to the severity of the symptoms, while hypomanic episodes do not. Unlike hypomanic episodes, manic episodes may also lead to hospitalization. Furthermore, if there are psychotic characteristics, the episode is necessarily manic.

3.1.2 Major Depressive Episode

According to the DSM-V-TR (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), a major depressive episode is a period of at least two weeks in which there is significant distress or impairment in important areas of an individual's life. The episode is not a consequence of the effects of any substance or other medical or emotional conditions such as grief. In addition, at least five of the following symptoms are present: depressed mood or loss of pleasure in almost all areas of life, changes in appetite, insomnia or hypersomnia, agitation or psychomotor retardation, fatigue, excessive guilt (which may be delusional), difficulty to concentrate and suicidal ideation. More precisely, at least one of the symptoms is either having a depressed mood or losing interest or pleasure.

3.1.3 Bipolar Disorder (BD)

According to DSM-V-TR (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), type 2 BD is determined by a course of recurrent mood episodes, consisting of one or more major depressive episodes and at least one hypomanic episode during life. The patient cannot have previously suffered from any manic episodes. Otherwise, they are considered to have type 1 BD[1] (AMERICAN PSYCHIATRIC ASSOCIATION, 2022). Type 2 BD is not considered a “milder” condition than type 1 BD because those with type 2 BD spend more time in the depressive phase, which can be severe and/or disabling (AMERICAN PSYCHIATRIC ASSOCIATION, 2022).

3.1.4 Major Depressive Disorder (MDD)

According to DSM-V-TR (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), a MDD is defined by at least one major depressive episode involving clear changes in affect and cognition, as well as no manic or hypomanic episodes during life.

3.1.5 Montgomery and Åsberg Depression Rating Scale (MADRS)

MADRS is a measure applied by healthcare professionals that assesses the severity of a depressive episode ranging from 0 to 60 points. The higher the score, the greater the severity of the depressive episode at the time the test was carried out. The severity of the depressive episode is considered mild with a score ranging from 7 to 19, moderate from 20 to 34, and severe from 35 onwards (QUILTY et al., 2013).The MADRS score is based on perceived sadness, reported sadness, inner tension (undefined feeling, restlessness or agitation), change in sleep, change in appetite, difficulty concentrating, lassitude, apathy, as well as pessimistic and suicidal thoughts (QUILTY et al., 2013). The MADRS scale does not help in differentiating between MDD and BD (HIRSCHFELD, 2014).

3.2 Actigraphy

Actigraphy is a procedure that analyzes a person’s circadian cycle by recording the individual’s degrees of motor activity over the course of a number of days using a device called actigraph. It consists of an accelerometer that identifies the intensity, quantity and duration of movement.

[1] There are other characteristics to be taken into account for the diagnosis, but the idea of this chapter is just a brief introduction to MDD and BD.

Actigraphy allows the assessment of sleep over periods of days and is indicated for characterizing circadian rhythmic patterns or sleep disorders in individuals with insomnia, including insomnia associated with MDD (MORGENTHALER et al., 2007). It provides a valid estimate of total sleep time (TST) (MARTIN; HAKIM, 2011), in addition to counting the individual's motor activity over the course of the day. It is important to highlight that actigraphs of different models must be analyzed with separate measurement scales due to non-uniformity between actigraphs (CRESCENZO et al., 2017). The meta-analysis of article conducted by Smith et al. (2018) suggests that actigraphy is a useful and relatively low-cost method for measuring sleep patterns in children and adults when executed using validated algorithms with attention to sensitivity settings and standardized scoring procedures.

Although the effectiveness of actigraphy in assessing sleep-wake patterns in individuals with major health problems is arguable (SADEH, 2011), it is more convenient for the patient, it costs less, and it is capable of capturing data for several uninterrupted days. In addition, actigraphy can be used to evaluate a participant's daily motor activity (ANCOLI-ISRAEL et al., 2003).

According to DSM-V-TR (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), fatigue and sleep disturbance are present during depressive episodes in a high proportion of cases. Although motor agitation is a possible symptom of a depressive episode, total motor activity (TMA) is normally lower in patients with MDD and BD, in depressive episodes and in euthymic periods, regardless of age or biological sex (CRESCENZO et al., 2017)(WUTHRICH et al., 2022).

3.3 Fuzzy Linguistic Model for Characterizing State of Health

Health is a state of well-being, not simply the absence of diseases (SADEGH-ZADEH, 2000). As much as there is no consensus on the definition of a disease (AMZAT; RAZUM, 2014), we will consider a disease to be an abnormal condition of the body that can be diagnosed on the basis of a number of symptoms, albeit with some uncertainty. There are diagnosis that can be made with less uncertainty, such as those of diseases caused by microorganisms because their presence in somebody's system indicate a possible cause for the disease. In this scenario, a doctor analyzes both causes and symptoms when diagnosing. Usually, this is not the case for MDD and BD, which diagnoses are made by a qualified professional by subjectively evaluating only the symptoms (ODA; DALGALARRONDO; BANZATO, 2022).

Generally, diagnosing a disease is an uncertain process, but we are only dealing with imprecision issues in this text. Therefore, we will not propose a diagnosis of diseases, but rather an assessment of state of health based on TST and TMA. However, understanding health status is an effective tool to help professionals define a patient's diagnosis. In other words, the worse the state of health of an individual, the greater the chance of being ill. Nevertheless, even a person who has a very high chance of being ill is not necessarily sick. Still, by combining this information with other symptoms, the professional will be able to make decisions more confidently, reducing cases of erroneous diagnosis.

When modeling a person's state of health with evaluative expressions, we follow two steps (SADEGH-ZADEH, 2000). First, we define a person's state of health as a value belonging to $[0, 1]$ such that the higher the value, the healthier the person is considered to be. Sadegh-Zadeh (2000) interprets this value as a person's membership degree to the group of healthy people:

Proposition 5. *(SADEGH-ZADEH, 2000) Let P be a fuzzy set of healthy people in a given context. Let $x \in \Omega$ be an element of all people in the same context. The* state of health *of a person $x \in \Omega$ is $\mu_P(x)$.*

Classifying the degree of a person's state of health is a very difficult task as there are many factors that influence it. This leads to an issue of not knowing how to calculate $\mu_P(x)$. Therefore, we may overcome this issue by describing $\mu_P(x)$ linguistically. Taking "state of health" as a linguistic variable, we may define the evaluative expressions "well", "borderline" and "unwell" with respect to the factors we are analyzing and not to all circumstances that affect state of health. These linguistic labels may be calculated based on implicative rules.

Let H_L, H_M and H_R be the horizons referring to "well", "borderline" and "unwell". In this case:

$$H_L(x) = 1 - H_R(x)$$

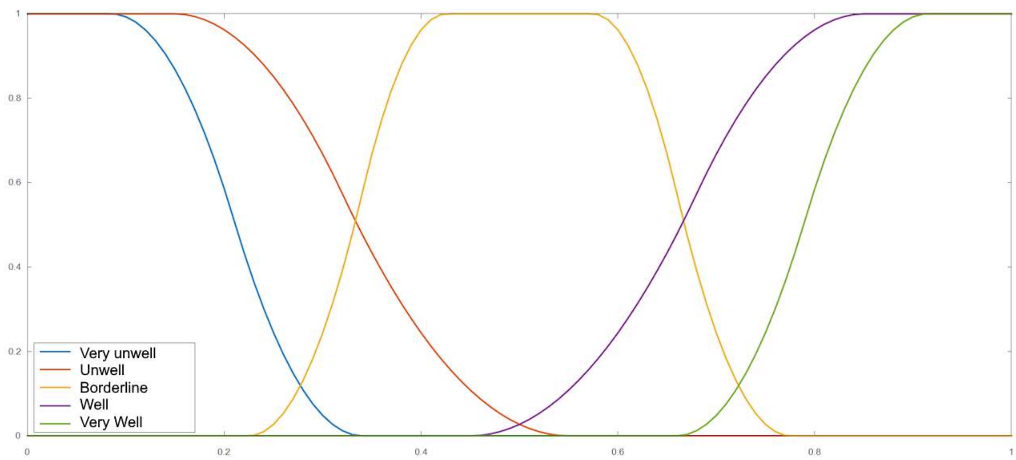
This way, we cannot define the horizons as an evaluative trichotomy, but rather an evaluative dichotomy. The horizon of borderline cases is defined as that which is neither on the horizon of healthy people nor on the horizon of unhealthy people:

$$H_M = ((H_R^c \cap H_L) \cup (H_L^c \cap H_R))^c$$

$$H_M(x) = \begin{cases} 0, & \text{if } x \geqslant 1 \\ 2 - 2x, & \text{if } 1 > x \geqslant 0.5 \\ 2x, & \text{if } 0.5 > x \geqslant 0 \\ 0, & \text{if } 0 > x \end{cases}$$

However, the deformation of hedges in these horizons will be carried out in the same way as described in Chapter 2. A sketch of the graph of the extensions of evaluative expressions that describe the state of health is presented in Figure 18. We call it a sketch because we do not know which values to choose for the parameters a, b and c of the Definition 17.

Figure 18 – Extensions of states of health.



Source: Own authorship

Let $\mathscr{C} \in$ {"Very unwell", "Unwell", "Borderline", "Well", "Very well"} and P be the fuzzy set of healthy people. If only TST and TMA are evaluated, given w' as the context of a person's state of health at a given time, then $Ext_{w'}(\mathscr{C})(\mu_P(x))$ is the truth degree of the predication "the person x is $\mathscr{C}$". On the other hand, if we consider $y = \mu_P(x)$, then $Ext_{w'}(\mathscr{C})(y)$ is the truth degree of the predication "the state of health y is $\mathscr{C}$". Where y stands for the state of health of the person x.

Our objective is to classify people according to the evaluative expressions of their state of health through implicative rules. Subsequently, we do not need to know the value of y, we only need to know which evaluative expression is the best to describe the state of health of a person. However, the truth degrees will be useful for interpreting the results.

Part II

Application

4 Materials and Methods

4.1 Materials

4.1.1 Database

The Depresjon Dataset (GARCIA-CEJA et al., 2018) was originally collected by Berle et al. (2010) for the study of the motor activity of patients with schizophrenia, MDD and BD. The researchers made information available from the control group and part of the condition group that has MDD or TB. The participant's motor activity was monitored with an actigraph worn on each participant's right wrist. The Actwatch AW4 model actigraph (Actiwatch, Cambridge Neurotechnology Ltd, England, model AW4), used for data collection measured activity per minute through an accelerometer that identifies the intensity, quantity and duration of movement in all directions. The Actwatch AW4 manual (CAMNTECH, 2008) also provides an algorithm for detecting the beginning and end of sleep.

The database comprises 55 signals obtained using actigraphy that correspond to 55 participants, with 32 belonging to control group and 23 to condition group. More precisely, condition group comprises:

- 15 participants having MDD;
- 7 participants having BD type II;
- 1 participant having BD type I.

For the condition group, the Montgomery-Asberg Depression Rating Scale (MADRS) score is reported at the beginning and end of data recording, in addition to gender and age for both groups.

The diagnosis of the database's condition group was based on the criteria defined by the DSM-IV (AMERICAN PSYCHIATRIC ASSOCIATION, 1994)(BERLE et al., 2010). However, in this master's thesis, the rules were created heavily based on the criteria defined by DSM-V-TR. The core symptoms of the diagnosis of a major depressive episode and the required duration of at least two weeks have not changed from DSM-IV to DSM-V-TR. However, in DSM-V-TR the criteria for classifying manic and hypomanic episodes include an emphasis on changes in activity and energy, as well as mood (AMERICAN PSYCHIATRIC ASSOCIATION, 1994) (AMERICAN PSYCHIATRIC ASSOCIATION, 2014) (AMERICAN PSYCHIATRIC ASSOCIATION, 2022)(AMERICAN PSYCHIATRIC ASSOCIATION, 2013).

In addition to being a CC0 public domain, the main reason for selecting The Depresjon Dataset (GARCIA-CEJA et al., 2018) is the presented psychiatric data that could be analyzed mathematically. In fact, the choice of the database predates the choice of FNL as an aid to solve this problem.

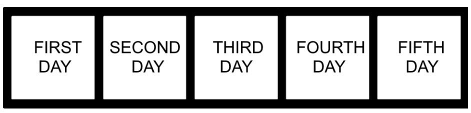
4.1.2 Extraction of Actigraph Information

The data from the actigraphy devices is a vector for each participant in which each entry relates to the motor activity per minute. Entries are measured in counts. Each research participant used the device for a different number of days. We decided to use only the first 7200 entries of each vector since all participants used the device for at least five days. This way, we constructed five 1440-dimensional vectors so that we could analyze each participant's day separately.

To calculate TST, we implemented the sleep scoring algorithm according to the Actiwatch manual (CAMNTECH, 2008). The algorithm identifies in minutes when the individual is asleep and when they are awake. Then, we followed the steps below:

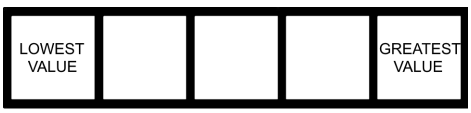
- **Step 1:** We counted how many minutes a person was considered to be sleeping and inserted the TST in minutes each day into a vector with five entries, as illustrated in Figure 19.
- **Step 2:** We reordered the vector from the lowest to the greatest value of TST, as in Figure 20.
- **Step 3:** We eliminated the greatest and lowest values of the vector, which can be seen in Figure 21.
- **Step 4:** The final vector to be evaluated will have only three entries, as in Figure 22.

Figure 19 – Step 1.



Source: Own authorship

Figure 20 – Step 2.



Source: Own authorship

Figure 21 – Step 3.

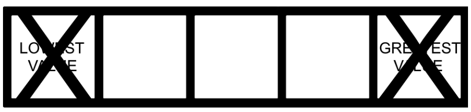


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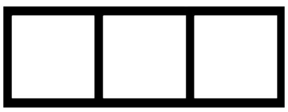
Figure 22 – Step 4.



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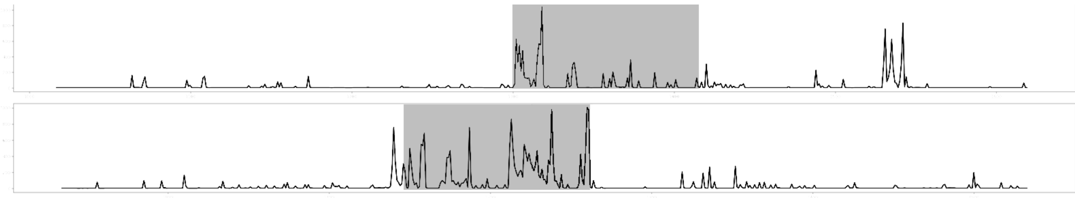
The reasoning for following these steps was to try to avoid discrepant values and to take into account that people do not necessarily experienced symptoms every single day (AMERICAN PSYCHIATRIC ASSOCIATION, 2022). We calculated the arithmetic average among the values of the three entries and the resulting value was used to choose the best expression to describe a person's TST according to Definition 20.

The process was a little different for the vector of TMA. For each vector that represented a day (from the actigraph), we searched the 240-minute interval in which the sum of the activity of all minutes was the largest possible, as illustrated in Figure 23. Then, the interval was discarded.

Figure 23 – Step 3.



Source: Own authorship

It is important to emphasize that we did not discard the 240 minutes of greatest activity, but rather the interval of greatest activity. We did this to eliminate possible periods of intense and purposeful motor activity. We did not remove moments of sleep since activity is relatively insignificant compared to the rest of the day. In the end, we added the activity at each minute for each day, obtaining a vector with five entries of TMA per day. Then, we followed the steps illustrated in Figure 19, Figure 20, Figure 21 and Figure 22 for TMA instead of TST. We calculated the arithmetic average among the values of the three entries and the resulting value was used to choose the best expression to describe a person's TMA according to Definition 20.

4.2 Methods

4.2.1 Fuzzy Set Modeling

We used five evaluative predications as labels for the TST value: “TST is low”, “TST is somewhat low”, “TST is medium”, “TST is somewhat high” and “TST is high”. We characterize “somewhat” as a linguistic hedge, and “low”, “medium”, and “high” as TE-adjectives. The chosen context was $w_Y = < 225, 340, 510 >$ (in minutes).The first attempts were to place the value of v_S close to seven hours. However, the results were poor because almost all participants were being classified with the label “TST is low”. We decided to choose an approximate value in which the majority of participants in the control group were characterized by average sleep, which was $v_S = 340$. The values of 225 and 510 were chosen so that $v_S \approx 1.5 \times v_L$ and $v_R \approx 1.5 \times v_S$[1].

To model TMA, we used only the evaluative predications without linguistic hedges: "TMA is low", "TMA is medium" and "TMA is high". We chose the context $w_X =< 130, 190, 280 >$ (in kilocounts). To define the context of TMA, we performed a procedure similar to that of TST. The v_L and v_S were chosen so that $1.5v_L \approx v_S$ and $1.5v_S \approx v_R$.

To model state of health we use the model described in section 3.3. Finally, to choose the verbal label for TST and TMA, we use the Local Perception function $LPerc^K$:$\mathbb{R} \times W \to K$, as described in Definition 20:

1. For TMA, we used
$$K_X = \{\text{"low"}, \text{"medium"}, \text{"high"}\};$$
2. For TST, we used
$$K_Y = \{\text{"low"}, \text{"somewhat low"}, \text{"medium"}, \text{"somewhat high"}, \text{"high"}\};$$
3. For State of health, we used
$$K_Z = \{\text{"very well"}, \text{"well"}, \text{"borderline"}, \text{"unwell"}, \text{"very unwell"}\}.$$

Let $\mathscr{A}$ be a generic element of K_X, $\mathscr{B}$ a generic element of K_Y and $\mathscr{C}$ a generic element of K_Z.

4.2.2 Implicative Rules for the First Method

The Depresjon Dataset (GARCIA-CEJA et al., 2018) does not contain information on which mood episode the person in the condition group is experiencing. Since the majority of participants has MDD and depressive episodes last longer than hypomanic episodes in people with type 2 BD (AMERICAN PSYCHIATRIC ASSOCIATION, 2022), it is more likely that most participants are in a major depressive episode or euthymic period rather than a hypomanic or manic episode. Thus, in practice, the method we are proposing aims to identify a major depressive episode.

Several articles (CRESCENZO et al., 2017) (WUTHRICH et al., 2022) (NUTT; WILSON; PATERSON, 2008) and DSM-V-TR (AMERICAN PSYCHIATRIC ASSOCIATION, 2022) indicate that total sleep time (TST) is often greater or less in patients with MDD or BD than in healthy individuals. Generally, in depressive episodes or euthymic periods of patients with MDD or BD, total motor activity (TMA) is significantly lower

[1] A better context may be chosen, but choosing the context is a challenging task.

when compared to healthy individuals. Furthermore, considering the total life span of a person with BD, a depressive episode or an euthymic period will be present for longer.

We used the Perception-based Logical Deduction method proposed by Vilém Novák (NOVÁK, 2015)(NOVÁK; PERFILIEVA; DVOŘÁK, 2016) and described in chapter 2 based in the symptoms described in chapter 3, summarized in the previous paragraph. The following two characteristics of a major depressive episode will be the basis for the rules:

1. If the person is in a major depressive episode, the person's TMA tends to be lower than the one of a healthy person.
2. If the person is in a major depressive episode, the person's TST tends to be lower or higher than the one of a healthy person.

Therefore, we formulated rules so that we can conclude the state of an individual's health by evaluating these two symptoms. We wrote fifteen rules based on the combination of all evaluative expressions K_X and K_Y. The implicative rules will be written as

$$\mathscr{R}\textbf{: If TMA is } \mathscr{A} \textbf{ and TST is } \mathscr{B} \textbf{ then the state of health is } \mathscr{C}.$$

As $k_X \times k_Y = 3 \times 5 = 15$, there will be 15 rules in the linguistic description. To facilitate the construction of the rules, we assign a score to each $\mathscr{A} \in K_X$ and then increase or decrease this score depending on the evaluative expression $\mathscr{B} \in K_Y$. Finally, the score obtained defines the $\mathscr{C} \in K_Z$ of each rule $\mathscr{R}$. These rules, and consequently the scores, are based on the following ideas:

1. High TMA indicates that the person is very well, low TMA indicates that they are very unwell, and medium TMA will indicate that they are not well but also not unwell.
2. Lower-than-normal TST can be common. A person cannot sleep more than their physiological needs, but sleeping less than they need can be common depending on their routine. Thus, a TST below normal will have less influence than a TST above normal.
3. If a person's TST is closer to average, there will be less chance of them being in a major depressive episode. The further away from the average, the greater the chances.
4. TMA has a greater weight in the model than TST because actigraphy is not the most effective method of measuring sleep. Furthermore, there are other scores that can be taken into account in addition to the TST to evaluate sleep.

5. If the person's TMA is not "high", then the person will be considered neither "well" nor "very well" because TMA is the main symptom in our model.

Table 4 presents the score of each expression $\mathscr{A}$.

$\mathscr{A}$	Score
Low	5
Medium	3
High	1

Table 4 – Scores of TMA

As $\mathscr{B}$ influences $\mathscr{C}$, then $\mathscr{A}$ will modify the score mentioned above, according to Table 5.

$\mathscr{B}$	Score
Low	+1
Somewhat low	+0
Medium	-1*
Somewhat high	+1
High	+2

Table 5 – Score of TST

The asterisk symbol in Table 5 means that for medium TST there was an exception to this rule because of idea 5. The exception occurs when $\mathscr{A}$ and $\mathscr{B}$ are "medium", then the score of $\mathscr{B}$ is "+0" .

Depending on the final score, we choose the expressions $\mathscr{C}$ according to Table 6.

Score	$\mathscr{C}$
$\leqslant 1$	Very well
2	Well
3	Borderline
4	Unwell
$\geqslant 5$	Very unwell

Table 6 – Relationship between state of health and rule score

Example 40. *Let "TMA is medium and TST is medium then the state of health is $\mathscr{C}$" be a rule.*

Since $\mathscr{A}$ ="medium" and $\mathscr{B}$ ="medium", then score $= 3 - 1 = 2$. However, this case is the exception, thus score$= 3 + 0 = 3$. Therefore $\mathscr{C}$ ="borderline".

Given $\mathscr{A}$ and $\mathscr{B}$, Table 7 informs $\mathscr{C}$, which was chosen to construct the rules[2].

[2] Different rules might achieve a better result. In future work, we hope to improve the rules by choosing

$\mathscr{A}$	$\mathscr{B}$	$\mathscr{C}$
low	low	very unwell
low	somewhat low	very unwell
low	medium	unwell
low	somewhat high	very unwell
low	high	very unwell
medium	low	unwell
medium	somewhat low	borderline
medium	medium	borderline
medium	somewhat high	unwell
medium	high	very unwell
high	low	well
high	somewhat low	very well
high	medium	very well
high	somewhat high	well
high	high	borderline

Table 7 – Implicative Rules

This way,

$$Ext_{<w_X,w_Y,w_Z>}(\mathscr{R}) = (Ext_{w_X}(\mathscr{A}) \otimes Ext_{w_Y}(\mathscr{B})) \Rightarrow Ext_{w_Z}(\mathscr{C}).$$

Thus,

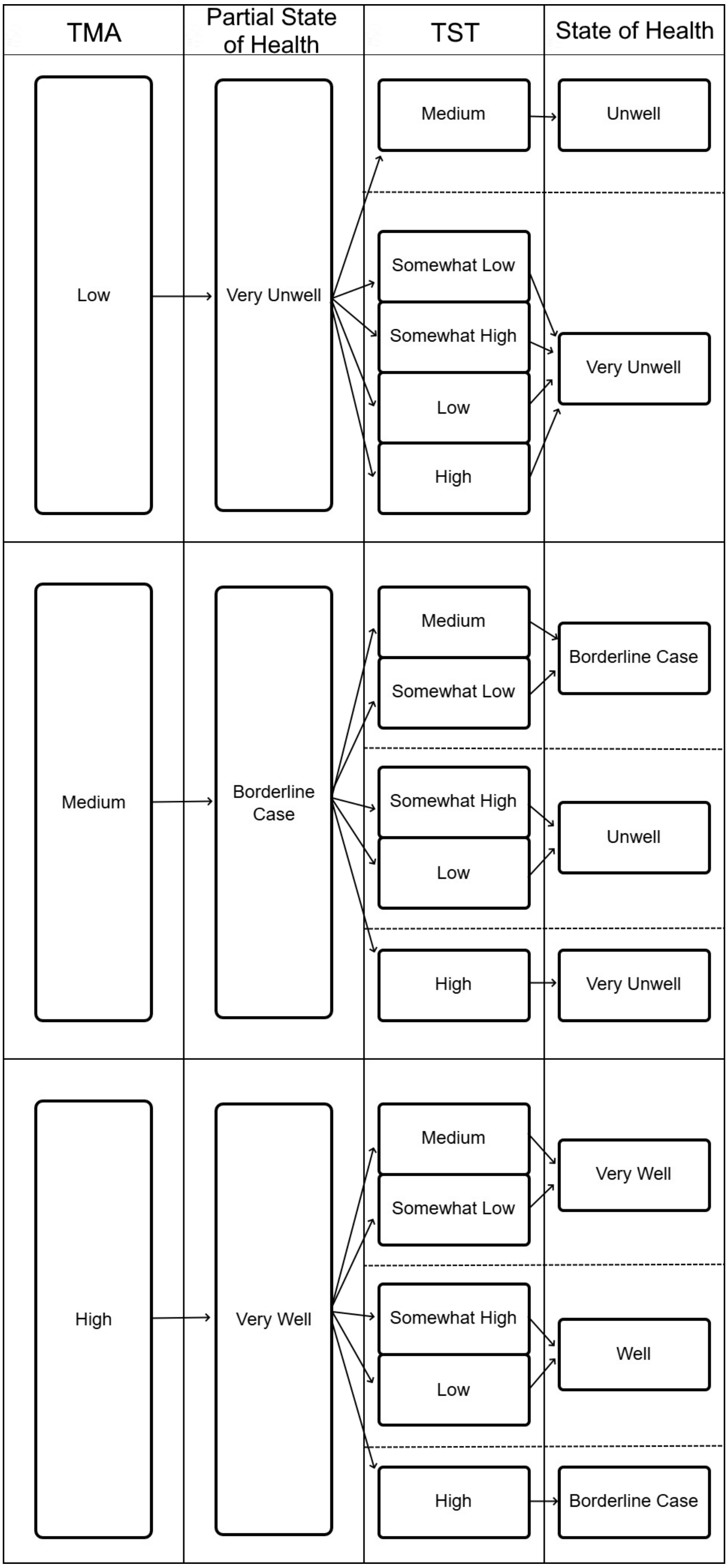
$$Ext_{w_Z}(\mathscr{C}) = Ext_{w_X}(\mathscr{A}) \otimes Ext_{w_Y}(\mathscr{B}).$$

Where w_Z is a context of state of health.

In Table 7, if a person is described as very well, they will also be considered well. The same applies to very unwell. Figure 24 is a schema that may help the visualization of the rules on Table 7. The first and second columns of Figure 24 refer to Table 4 and Table 6. The third and last columns of Figure 24 refer to Table 5 and Table 6.

more appropriate linguistic context, besides other symptoms or sleep scores for evaluation.

Figure 24 – Schema of the rules on Table 7.



Source: Own authorship

4.2.3 Implicative Rules using Intermediate Quantifiers

We resorted to the theory of intermediate quantifiers (NOVÁK, 2008) for two main reasons. First, to decrease the number of control group where participants identified as unwell or very unwell. Second, to reduce the number of rules. Doing so, we generated three implicative rules that evaluate if the patient is unwell or very unwell over a time frame of $5 - 14$ days, depending on data availability:

$\mathscr{R}_1$: If **almost all** daily TMAs are **not high** AND **most** daily TSTs are **somewhat high or low**, then the participant is **very unwell**.

$\mathscr{R}_2$: If **most** daily TMAs are **low**, then the participant is **very unwell**.

$\mathscr{R}_3$: If **almost all** daily TMAs are **not high** OR **most** daily TSTs are **somewhat high or low**, then the participant is **unwell**.

Consider "TMA is not high" as the main symptom to be evaluated and "TST is somewhat high or low" as another symptom. The ideas behind creating the rules were as follows:

1. If both symptoms occur, we consider the person to be very unwell.
2. If the main symptom occurs with a stronger intensity, we consider the person to be very unwell.
3. If only one of the symptoms occurs, we consider the person to be unwell.

Let $A^X_{\neg High}(\mathbf{x})$, $A^X_{Low}(\mathbf{x})$, $A^Y_{Low}(\mathbf{y})$, and $A^Y_{RoHigh}(\mathbf{y})$ be given as follows:

- $A^X_{Low}(\mathbf{x})(i) = Ext_{w_X}(\text{"TMA is low"})(x_i)$;
- $A^X_{\neg High}(\mathbf{x})(i) = Ext_{w_X}(\text{"TMA is not high"})(x_i)$;
- $A^Y_{Low}(\mathbf{y})(i) = Ext_{w_Y}(\text{"TST is low"})(y_i)$;
- $A^Y_{RoHigh}(\mathbf{y})(i) = Ext_{w_Y}(\text{"TST is somewhat high"})(y_i)$.

Where $\mathbf{x} = \{x_1, x_2, .., x_n\}$, $\mathbf{y} = \{y_1, y_2, .., y_n\}$, x_i is the TMA value on the i-th day, y_i is the TST value on the i-th day and n is the number of days under consideration.

Moreover, let $B^X = B^Y = \{1, \ldots, n\}$ be crisp sets with n elements.

4.2.3.1 The Extension of the First Rule

Let Z be a linguistic variable that means “state of health”. The extension of the first rule is

$$Ext_{<w_X,w_Y,w_Z>}(\mathscr{R}_1)(\mathbf{x},\mathbf{y},\mathbf{z}) = [(\mathrm{Q}^{\forall}_{ExBi}\mathbf{x})(B^X, A^X_{\neg High}) \otimes (\mathrm{Q}^{\forall}_{VeBi}\mathbf{y})(B_Y, A^Y_{Low} \oplus A^Y_{RoHigh})]$$
$$\rightarrow Ext_{w_Z}(\text{“Z is Very Unwell”})(\mathbf{z}).$$

Therefore, the truth degree for the a person being very unwell, for $\mathbf{x}^*$ and $\mathbf{y}^*$ given, is

$$Ext_{w_Z}(\text{“Z is Very Unwell”})(\mathbf{z}^*) = (\mathrm{Q}^{\forall}_{ExBi}\mathbf{x})(B^X, A^X_{\neg High}) \otimes (\mathrm{Q}^{\forall}_{VeBi}\mathbf{y})(B_Y, A^Y_{Low} \oplus A^Y_{RoHigh}),$$

where $\mathbf{z}^* = \mathbf{z}(\mathbf{x}^*, \mathbf{y}^*)$ and $\mathbf{z}^*$ may be estimated with an appropriate defuzzification of $Ext_{w_Z}(\text{“Z is Very Unwell”})(\mathbf{z}^*)$.

4.2.3.2 The Extension of the Second Rule

Let Z be a linguistic variable that means “state of health”. The extension of the second rule is

$$Ext_{<w_X,w_Z>}(\mathscr{R}_2)(\mathbf{x},\mathbf{z}) = (\mathrm{Q}^{\forall}_{VeBi}\mathbf{x})(B^X, A^X_{Low}) \rightarrow Ext_{w_Z}(\text{“Z is Very Unwell”})(\mathbf{z}).$$

Therefore, the truth degree of a person being very unwell, for $\mathbf{x}^*$ given, is

$$Ext_{w_Z}(\text{“Z is Very Unwell”})(\mathbf{z}^*) = (\mathrm{Q}^{\forall}_{VeBi}\mathbf{x})(B^X, A^X_{Low}),$$

where $\mathbf{z}^* = \mathbf{z}(\mathbf{x}^*)$ and $\mathbf{z}^*$ may be estimated with an appropriate defuzzification of $Ext_{w_Z}(\text{“Z is Very Unwell”})(\mathbf{z}^*)$.

4.2.3.3 The Extension of the Third Rule

Let Z be a linguistic variable that means “state of health”. The extension of the third rule is

$$Ext_{<w_X,w_Y,w_Z>}(\mathscr{R}_3)(\mathbf{x},\mathbf{y},\mathbf{z}) = [(\mathrm{Q}^{\forall}_{ExBi}\mathbf{x})(B^X, A^X_{\neg High}) \oplus (\mathrm{Q}^{\forall}_{VeBi}\mathbf{y})(B_Y, A^Y_{Low} \oplus A^Y_{RoHigh})]$$
$$\rightarrow Ext_{w_Z}(\text{“Z is Unwell”})(\mathbf{z}).$$

Therefore, the truth degree for a person being unwell, for $\mathbf{x}^*$ and $\mathbf{y}^*$ given, is

$$Ext_{w_Z}(\text{“Z is Unwell”})(\mathbf{z}^*) = (\mathrm{Q}^{\forall}_{ExBi}\mathbf{x})(B^X, A^X_{\neg High}) \oplus (\mathrm{Q}^{\forall}_{VeBi}\mathbf{y})(B_Y, A^Y_{Low} \oplus A^Y_{RoHigh}),$$

where $\mathbf{z}^* = \mathbf{z}(\mathbf{x}^*, \mathbf{y}^*)$ and $\mathbf{z}^*$ may be estimated with an appropriate defuzzification of $Ext_{w_Z}(\text{“Z is Unwell”})(\mathbf{z}^*)$.

4.2.3.4 Firing the Rules

In this method, we fire the three rules and for a number b_0 we choose which expression best describes the state of health of the person, as follows[3]:

1. If we obtain Ext_{w_Z}(“Z is Very unwell”)$(\mathbf{z}^*) > b_0$ from the first rule or second rule, we say that the person is very unwell.

2. If Ext_{w_Z}(“Z is Very unwell”)$(\mathbf{z}^*) > b_0$ from second rule or Ext_{w_Z}(“Z is Unwell”)$(\mathbf{z}^*) > b_0$ from third rule, we say that the person is unwell[4].

3. If the person is not very unwell nor unwell, we say that the state of health is undefined.

[3] b_0 is a value that will help us to present the results, as we can classify the person with an unique expression.

[4] Remember that if a person is very unwell, he/she is unwell.

5 Results and Discussion

5.1 First Method

Table 8 indicates the percentage of people who were classified with each evaluative linguistic expression of state of health according to subsection 4.2.2.

Table 8 – Classification of participants' state of health according to the first method.

Group	Very Unwell	Unwell	Borderline	Well	Very Well
Condition	56.52%	69.56%	26.08%	4.34%	4.34%
Control	6.25%	18.75%	15.62%	65.62%	56.25%

Source: Own authorship.

The method classified 69.56% of participants in the condition group as unwell. Only 4.34% of them were categorized as well. In other words, if only this method was considered, more than 69% of people with a mood disorder would have a correct diagnosis and 4.34% would have an incorrect diagnosis. On the other hand, 18.75% of the control group was misclassified. 20% of all participants (borderline cases) would not be classified as well neither as unwell. The participants in borderline cases are those who presented symptoms but not so strong as to be considered unwell.

5.2 Second Method

Table 9 and Table 10 indicate the percentage of people who were classified as unwell according to subsubsection 4.2.3.4. However, for the former we chose $b_0 = 0$ and for the latter $b_0 = 0.25$.

Table 9 – Classification of groups according to the evaluative expressions of state of health, considering $b_0 = 0$.

Health	Condition	Control
Very Unwell	30.43%	3.12%
Unwell	65.21%	25%
Unclassified	34.79%	75%

Source: Own authorship.

Table 10 – Classification of groups according to the evaluative expressions of state of health, considering $b_0 = 0.25$.

Health	Condition	Control
Very Unwell	26.08%	3.12%
Unwell	60.87%	12.5%
Unclassified	39.13%	87.5%

Source: Own authorship.

Note that all other participants are not classified in this method, i.e., they are associated with the undefined category.

In this method we either classify people as unwell (or very unwell) or we do not classify them at all. In Table 9 we used $b_0 = 0$, thus 65.21% from the condition group were classified correctly and 25% from the control group were classified incorrectly. However, many participants in the control group who were considered unwell in Table 9 had a low membership degree. As a consequence, in Table 10 we used $b_0 = 0.25$ and, as a result, only 12.5% of participants from the control group were incorrectly classified.

5.3 Comparison Between Methods

At first, we could not calculate accuracy because our method is not a binary classification. Nonetheless, after considering condition group's participants who were classified as "unwell" to be true positives and control group's participants who were not classified as "unwell" to be true negatives, we may calculate accuracy. We are also able to calculate sensitivity and specificity. Table 11 compares the accuracy, sensitivity and specificity of both methods.

Table 11 – Accuracy, sensitivity and specificity of first and second method for $b_0 = 0.25$.

	Accuracy	Sensitivity	Specificity
First Method	76.36%	69.56%	81.25%
Second Method	76.36%	60.86%	87.5%

Source: Own authorship.

As seen in Table 11, both methods achieved the same accuracy. For this database, the first method was better in avoiding false negatives (because it achieved higher sensitivity) while the second method was better in avoiding false positives (because it achieved higher specificity). However, note that if in the first method we had chosen the condition group's borderline cases as true positives and the control group's borderline cases as false negatives, we would have achieved an accuracy of 78.18%, a sensitivity of

95.65% and a specificity of 65.62%. Nevertheless, we discarded this classification because we would not be able to calculate the accuracy, sensitivity and specificity of the second method.

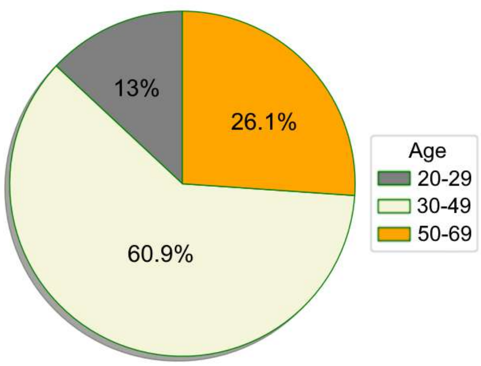
Table 12 compares the results from both methods considering the age of the participants. Figure 25 and Figure 26 show the percentage of people by age in the condition group and control group, respectively.

Table 12 – Percentage of people considered unwell for age and in each group according to Table 8 and Table 10.

	First Method		Second Method	
Age	Condition	Control	Condition	Control
20 – 29	33.33%	20%	0%	10%
30 – 49	57.14%	21.43%	57.14%	7.14%
50 – 69	100%	12.5%	100%	25%

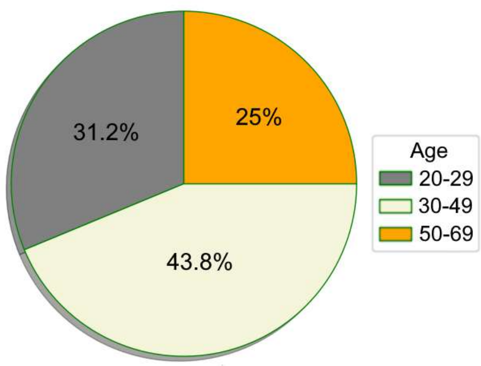
Source: Own authorship.

Figure 25 – Percentage of each age group of condition group.



Source: Own authorship

Figure 26 – Percentage of each age group of control group.



Source: Own authorship

The subgroup aged between 20 and 29 years old had the worst results. In the first method, few were classified correctly and in the second method, none were classified correctly. At the same time, a considerable number of participants were wrongly classified in the condition group. Nevertheless, we must take into account that the database is small and that there are only three participants of this age in the condition group and ten participants in the control group.

The subgroup of participants between 30 and 49 is the largest in the condition and control groups. Consequently, this subgroup is what most influences the group classifications. The second method was more efficient in this case.

In both methods, all participants in the condition group between 50 and 69 years old were correctly classified. 1/8 of the control group was considered unwell in the first method, while 1/4 was considered unwell in the second method. It is important to consider that there are only 6 participants in the condition group and 8 in the control group this age.

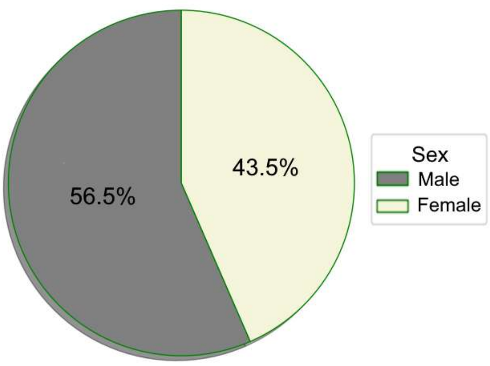
Table 13 compares the results from both methods considering the sex[1] of the participants. Figure 27 and Figure 28 show the percentage of people by sex in the condition group and control group, respectively.

Table 13 – Percentage of people considered unwell for each sex and in each group according to Table 8 and Table 10.

	First Method		Second Method	
Sex	Condition	Control	Condition	Control
Male	69.23%	25%	69.23%	16.66%
Female	70%	15%	50%	10%

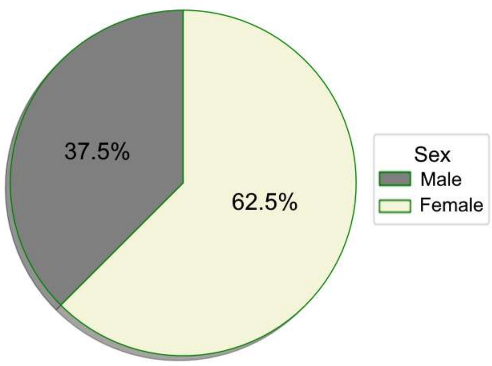
Source: Own authorship.

Figure 27 – Percentage of participants in condition group with a determinate sex.



Source: Own authorship

Figure 28 – Percentage of participants in control group with a determinate sex.



Source: Own authorship

The number of male participants is greater in the condition group, while female participants are the majority in the control group. The first method was more efficient in classifying female participants, while the second method was more efficient in male participants. We cannot say that there are very strong differences in the classification

[1] Although gender roles and social life may be associated with sleep quality and motor activity during the day, physical and physiological issues greatly influence these symptoms. Therefore, we chose the word "sex" and not "gender" to differentiate both groups.

according to the sex of the participants. However, we can notice that classifying female participants varied a lot between methods.

Table 14 – Percentage of people considered unwell considering the mood disorder of condition group according to Table 8 and Table 10.

Mood Disorder	First Method	Second Method	Total[2]
MDD	73.33%	60%	65.21%
BD type 1	100%	100%	4.35%
BD type 2	57.14%	57.14%	30.43%

Source: Own authorship.

The first method was able to classify participants with MDD better than those with BD type 2. The second method was able to classify participants with BD (type 1 and type 2 together) in 62.5% which is better than those with MDD.

Let $p-$MADRS be the product of the first MADRS score and the second MADRS score of each condition gruoup's participant[3].

Table 15 – Percentage of people considered unwell considering the p-MADRS of condition group according to Table 8 and Table 10.

$p-$MADRS	First Method	Second Method	Total[4]
$0 - 324$	66.66%	55.55%	39.13%
$325 - 549$	100%	100%	21.74%
$550 - 674$	40%	0%	21.74%
$\geqslant 675$	75%	100%	17.39%

Source: Own authorship.

If the $p-$MADRS is less than 550, then both MADRS scores were not high. Alternatively, one was high while the other one was low. If the $p-$MADRS is greater than 674 then either both MADRS scores were high or one was very high but the other was not high.

For the dataset used in this research, if $p-$MADRS is between 550 and 674 then the first method was inefficient and the second failed to classify anyone. If $p-$MADRS

2 Refers to the percentage of participants with this mood disorder in relation to the total number of participants in the condition group.

3 $p-$MADRS was chosen to take into account the difference between the two MADRS scores (the smaller the difference, the higher the value tends to be). At the same time, the higher each score, the higher $p-$MADRS is.

4 Refers to the percentage of participants with this $p-$MADRS score in relation to the total number of participants in the condition group.

is less than 325, then the methods were able to classify more than 50%. For the other cases, both methods were very competent in classifying the participants.

Table 16 compares the performances of both methods of this master's thesis with two methods that also used The Depresjon Dataset (GARCIA-CEJA et al., 2018).

Table 16 – Comparative performance with other models using The Depresjon Dataset (GARCIA-CEJA et al., 2018).

Study	Accuracy	Sensitivity	Specificity
Garcia-Ceja et al. (2018)	72.7%	72.9%	72.6%
Kumar et al. (2022)	85.1%	–	–
First Method	76.4%	69.6%	81.2%
Second Method	76.4%	60.9%	87.5%

Source: Own authorship.

While Garcia-Ceja et al. (2018) used the linear kernel support vector machine method, Kumar et al. (2022) used a hybrid of convolutional neural networks and adaptive neuro fuzzy inference system to achieve such results. Both studies were presented in the introduction of this master's thesis. The study of Kumar et al. (2022) did not inform the sensitivity and specificity of their method. The values of accuracy, sensitivity and specificity achieved by the method proposed by Garcia-Ceja et al. (2018) are close to each other. Conversely, our second method achieved the greatest specificity and the lowest sensitivity of all methods.

5.4 Discussion

Both methods presented satisfactory results according to the database we studied. The first method managed to classify participants in the condition group better, but the second method made fewer mistakes in classifying participants in the control group. Furthermore, the second method takes into account more days and by adding intermediate quantifiers the rules become more reliable with the symptoms described in the DSM-V-TR.

Considering the methods as binary classifiers, Table 11 shows that the accuracies achieved were equal but the specificities and sensitivities were different. It was expected that specificity would be greater than sensitivity in both methods, since we considered borderline cases as being classified as negative. As both methods achieved slightly similar results and the database is small, we need future research to define in which situation (screening test or diagnostic test) each method is better recommended, in addition to improving these scores.

The differences between the results of the methods are not large, considering that we have few participants. Results differ the most when we separate participants by age or by $p-$MADRS score. Young participants under 30 years of age had the worst results because the number of participants with a mood disorder not considered unwell was very low in both methods. However, the number of people classified as unwell while being healthy was not low. This indicates that for people this age, we would need different rules that either take into account other symptoms or that take into account more details of sleep or circadian rhythm, besides TST or TMA. The results for this database do not indicate that we can solve these issues by simply changing the contexts of the antecedents of this subgroup.

We only had the results of the MADRS test for the condition group. Consequently, we were unable to add these scores to the rules. However, we can note that if $p-$MADRS is high but not extremely high, the method was very inefficient. We would need more participants to reach a conclusion but we can hypothesize that methods may be inefficient for people suffering from a depressive episode depending on the intensity of the episode. This leads us to consider that including details of other symptoms in the rules may overcome these challenges.

We did not consider that the difference in results for different sexes and different mood disorders was relevant. However, it is worth highlighting that participants with MDD and female participants were those who had the most variation between the two methods. As the second method takes more days into account, we can create rules that take into account the variations that people's bodies experience over several days.

The accuracy of both methods were 76.36%. This value is lower than the accuracy of 85.10% of Kumar et al. (2022), which used a hybrid of convolutional neural network and an adaptive neuro-fuzzy inference system to detect depressive episodes using the same database. However, we consider that our method is easier to understand and to be interpreted by healthcare professionals. Furthermore, our method is not a binary classification, which provides more information for professionals when making decisions.

6 Conclusion

We conclude that the PbLD method without intermediate quantifiers proposed in (NOVÁK, 2015) and the method with intermediate quantifiers (NOVÁK, 2008) have the potential to be great tools for identifying major depressive episodes. The methods need to be improved and tested with a larger database. However, an understanding of each step of the method construction and achieving satisfactory results with such simple rules and little information make us conclude that future work will be relevant. This understanding is possible because the main advantage of FNL is its explainability.

Despite the first method being better at identifying participants in the group condition, we consider the approach based on intermediate quantifiers to be superior. We do so because it leads to an enormous reduction in the number of rules, which are easier to understand and relate to the symptoms of DSM-V-TR (AMERICAN PSYCHIATRIC ASSOCIATION, 2022).

The methodology was explained in details so that professionals in the fields of psychiatry, psychology, and possibly other experts could understand it. Therefore, we may be able to achieve multi-disciplinary research focused on improving the diagnosis of the mood disorders considered in the project. Although future research should be concerned with optimization of the parameters in the FNL approach, a database consisting of a large amount of detailed, relevant, and more comprehensive information would help us to formulate more detailed rules taking into account factors such as age and sex. Finally, the challenges of differential diagnosis of MDD and BD type 2, and differential diagnosis of BD type 2 and borderline personality disorder are some of our interests.

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