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Advanced Trends in Smart Agriculture

Tendências Avançadas em Agricultura Inteligente

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“The people that walked in darkness have seen a great light: they that dwell in the land of the shadow of death, upon them hath the light shined”

(Isaiah 9:2)

Resumo

Compreendendo os desafios associados às mudanças climáticas, as estratégias de manejo agrícola devem avaliar o impacto do aumento das temperaturas e das mudanças nos padrões de precipitação sobre a produtividade das culturas. Agricultura inteligente é a adoção de tecnologias avançadas e operações agrícolas orientadas por dados para otimizar e melhorar a sustentabilidade na produção agrícola. Esta tese explora o tema da agricultura inteligente com uma perspectiva baseada em otimização, abordando o Problema de Rotação de Culturas (PRC), a aplicação estratégica de herbicidas para o controle de ervas daninhas em campos agrícolas, os desafios enfrentados na multiplicação de sementes de milho, questões relacionadas a pragas que afetam a produção de soja e a utilização de previsões de séries temporais no planejamento agrícola. Nosso principal objetivo é auxiliar a comunidade de produtores de grãos oferecendo um conjunto completo de ferramentas de otimização que incorporam diversos conceitos das práticas agrícolas em uma estrutura matemática. Elaborar planos de cultivo de culturas baseados apenas em oportunidades de mercado pode levar a abordagens que não são sustentáveis ao longo do tempo. Portanto, exploramos o PRC como uma solução para auxiliar o manejo agrícola na criação de estratégias de longo prazo que integrem eficiência agrícola e sustentabilidade. Nosso arcabouço de manejo agrícola inclui as dinâmicas entre culturas, solo e água para desenvolver um plano eficaz de rotação de culturas. Também consideramos o papel das culturas de cobertura no balanço hídrico do solo e como indicador de melhorias na estrutura do solo. Discutimos aprimoramentos na estrutura do solo relacionados ao uso recorrente de culturas de cobertura no campo. Nosso método matemático para implementar o PRC introduz diversas inovações na análise de manejo agrícola. O manejo de ervas daninhas é fundamental para sustentar altos rendimentos na fazenda. No âmbito da melhoria das práticas de manejo da soja, construímos um modelo de programação dinâmica para auxiliar no controle de ervas daninhas na soja, dado que a soja possui grande importância na produção mundial de alimentos e em diversas indústrias. A crescente prevalência de ervas daninhas resistentes a herbicidas é atribuída à intensificação agrícola, levando os agricultores a utilizarem diversos herbicidas químicos para um controle eficaz. O uso extensivo de herbicidas na agricultura comercial afeta negativamente o meio ambiente e desequilibra o ecossistema. Adotar uma abordagem pragmática na aplicação de herbicidas promove um uso mais consciente desses produtos químicos. Além disso, introduzimos um modelo de otimização para lidar com o manejo de pragas. Nosso estudo concentra-se nos danos causados por lagartas na soja. Propomos uma estrutura que descreve o equilíbrio entre ação e perdas em um contexto dinâmico. Para além da comunidade agrícola, também propomos um modelo de otimização voltado para a indústria de multiplicação de sementes, que enfrenta limitações nas capacidades de processamento e armazenamento. Ao longo de nossa pesquisa, revisitamos uma variedade de conceitos agrícolas sob a perspectiva da otimização, sugerindo diversas abordagens para enfrentar cada problema agrícola específico.

Palavras-Chave: Rotação de Cultivos; Pragas agrícolas - Controle integrado; Erva daninha - Controle; Otimização matemática; Planejamento agrícola; Agricultura de precisão.

Abstract

Comprehending the challenges associated with climate change, agricultural management strategies ought to evaluate the impact of increased temperatures and shifting precipitation patterns on crop yields. Smart agriculture, also known as intelligent farming, is the adoption of advanced technologies and data-driven agricultural operations to optimize and improve sustainability in agricultural production. This thesis explores the subject of smart agriculture from an optimization perspective, addressing the Crop Rotation Problem (CRP), the strategic application of herbicides for weed control in crop fields, the challenges faced by agribusinesses in corn seed multiplication, issues related to insect pests impacting soybean production, and the use of time series forecasting in agriculture. Our main goal is to assist the grain farm community by offering a complete set of optimization tools that incorporate various concepts from the agricultural practices into the mathematical framework. Designing crop cultivation plans based only on market opportunities may lead to approaches that are not sustainable over time. Therefore, we explore the CRP as an solution to aid farm management in crafting long-term strategies that integrate agricultural efficiency with sustainability. Our farm management framework include the dynamics between crops, soil, and water to develop an effective crop rotation plan. We also consider the role of cover crops within the soil water balance and as an indicator of soil improvements. We discuss enhancements into the soil structure related to the recurrent use of cover crops in the field. Our mathematical method for implementing the CRP introduces various innovations into farm management analysis. Weed management is critical to sustain high yield in the farm. Under the scope of improving soybeans management practices, we have constructed a dynamic programming model to assist weed control in soybeans as soybeans hold major significance in worldwide food production and numerous industries. The increased prevalence of herbicide-resistant weeds is attributed to agricultural intensification, prompting farmers to employ various chemical herbicides for effective weed management. Commercial agriculture's extensive use of herbicides adversely affects the environment and disrupts ecological balance. Adopting a pragmatic approach to herbicide application fosters a more mindful use of these chemicals. Additionally, we introduce an optimization model to tackle pest management. Our study concentrates on the damages caused by caterpillars on soybeans. We propose a framework that outlines the equilibrium between action and losses in a dynamic context. In addition to the agricultural community, we also propose a optimization model designed to tackle the seed multiplication industry, which is constrained by the limits of processing and storage capabilities. Throughout our research, we revisit a variety of agricultural notions from the perspective of optimization, suggesting several approaches to tackle each specific farm issue.

Keywords: Crop Rotation; Agricultural pests - Integrated control; Weed Control; Mathematical Optimization; Agricultural planning; Precision farming.

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1 Introduction

Brazil has been the leading country in net agricultural exports for the last twenty years (VALDES *et al.*, 2020). During the 2020/2021 harvest, Brazilian soybean production reached about 135.9 million tons, setting a record for the highest yield in the country (CONAB, 2021b). Brazil also exported a total of 86.1 million tons of soybeans in the same harvest season. Therefore, Brazilian farms account for about half of the worldwide soybean trade. Soybeans and their derivatives have emerged as the predominant agricultural products traded on global markets (CONAB, 2021a; ARAGAO; CONTINI, 2021). An agency report indicates that Brazil held the position of the second largest corn exporter in 2020 (CONAB, 2021a). These statistics highlight the influence of grain production on the Brazilian economy. Behind the massive agricultural market lies an extensive operation that combines unprecedented grain processing capacity with a global logistics network.

Remarkable figures in the agricultural sector indicate that substantial and numerous opportunities for development are forthcoming. For analysts dedicated to performance maximization, progress requires the skill of representing intricate issues based on real-world parameters while maintaining consistency. There are occasions where achieving what is feasible is enough; at different times, we aim at achieving the very best, the ideal outcome.

Improvement on the horizon is the motivation of our research that merges optimization and sustainability within agribusiness. Overall, we aim to incorporate elements of modularity, task distribution, and a systematic methodology into farm management, as these have shown considerable success in the industrial sector. Even though industrial production sites generally feature a controlled environment and more repetitive tasks, we believe that agricultural practices and agronomic advice can similarly benefit from the practical application of optimization. This approach can help organize their processes and offer fresh perspectives beyond routine practices.

Agricultural tasks are varied, encompassing planting seeds, managing diseases, and overseeing plant development until it is time for harvest. In carrying out these tasks, farmers encounter numerous decisions. Many of these require swift action to avoid a reduction in yield. Introducing an optimization mindset into farm management, we believe, can reduce the need for extreme actions in critical situations by predicting the most effective strategy and making adjustments in advance using data.

In Brazil, favorable weather conditions provide farmers with the opportunity to choose from a diverse selection of commercial crops. Although certain crops dominate due to the necessary infrastructure and market advantages, grain farmers still have the flexibility to explore various unconventional alternatives. Choosing an alternative commercial crop alters particular management methods; however, the existing farm equipment may remain suitable, which is advantageous for agriculture. In contrast, large-scale industrial production requires a complete overhaul to manufacture a different item.

The ability to grow diverse crops on a grain farm prompts an exploration of the Crop Rotation Problem (CRP), a prominent combinatorial problem, which embraces a wide array of viewpoints, leading to diverse applications and consistently yielding novel insights. The fundamental challenge involves planning a cropping sequence tailored to incorporate costs, profitability, environmental indicators, water resources, and other target attributes. Our distinctive CRP strategy for targeting grain farms incorporates the integration of cover crops and their ecoservices, which can lead to various advantages for soil health and enhance the consistency of grain yields. In addition, we include water balance equations in the model to predict the significant effects of cover crops on the overall success of the farming operation.

Weed management approaches such as manual, mechanical, cultural, biological, and chemical techniques each have their own limitations. It is unlikely that any method alone can achieve the required effectiveness in the control of weeds (DAS *et al.*, 2024). To effectively manage weeds in any commercial crop, a well-structured approach known as Integrated Weed Management (IWM) is employed within the agricultural sector.. As outlined by Holt (2013), IWM is an approach to control weeds that integrates knowledge of the biology and ecology of weeds with various control methods. This strategy employs diverse techniques, incorporating non-chemical and preventive measures, to reduce reliance on herbicides.Without a comprehensive understanding of herbicide application throughout the entire crop cycle, farmers are unable to effectively undertake preventative measures. A model-based approach we propose in this research aims to address the decision making about herbicide application in soybean crops.

In Brazil, insect pests are estimated to lead to an average annual production loss of 7.18% (OLIVEIRA *et al.*, 2021), resulting in a significant decrease of millions of tons in the production of biofuels, fiber, and food. Understanding the harm caused by insect pests in agriculture is essential for the development of agronomic policies. The impact of pest damage on commercial crops can have global price implications. Accurately assessing losses caused by pests is essential for allocating additional resources to pest management. This involves a thorough evaluation of control mechanisms that affect both agricultural productivity and the environment. Among the numerous insect pests that concern farmers in Brazil, *Helicoverpa armigera* (Lepidoptera, Noctuidae: Heliothinae) occurrences pose a serious threat to soybean and various other crops (SOSA-GóMEZ *et al.*, 2016). The adaptation strategies of *Helicoverpa armigera* with plants are highly complex and not well understood (SUZANA *et al.*, 2018). The difficulties in controlling *Helicoverpa armigera* infestations in soybean crops drive us to formulate an optimization model. This model aims to represent the density of the caterpillar population, assess the corresponding damage to soybeans, and outline preventative measures to avert outbreaks, ultimately strengthening farm management resilience against *Helicoverpa Armigera.*

Climate influences agricultural production on two different time scales: long-term, historical climate directly affects crop production, affecting land use patterns, while short-term, weather conditions play a crucial role in determining crop failure and productivity reductions (PEREDA; ALVES, 2018). Commodity futures prices are also crucial in determining both the selection of crops and the timing of sowing. Our initiative to tackle climate impacts and fluctuations in agricultural commodity prices involves employing time series forecasting techniques.

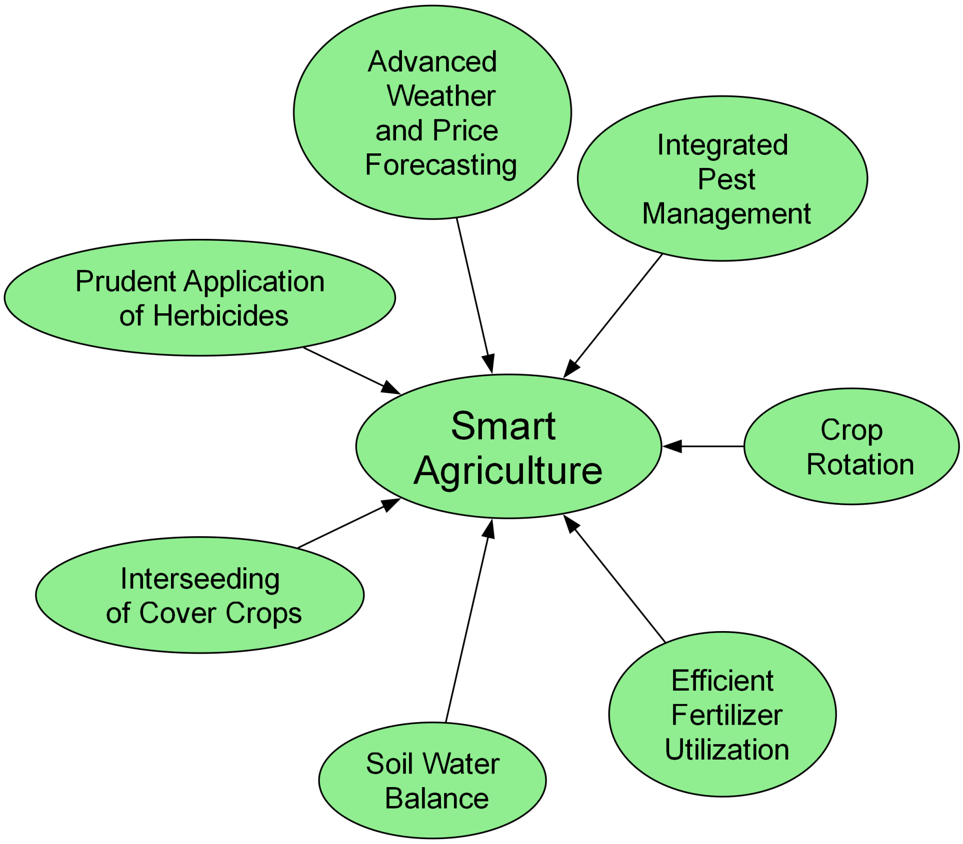

Figure 1.1 – Creating a cohesive framework to facilitate sustainable practices in line with the Smart Agriculture trend.

Figure 1.1 presents an overview of our research framework. In agriculture, the concept of sustainability goes beyond simply addressing environmental issues. Although adopting eco-friendly practices is essential, sustainability ought to be perceived as the holistic evaluation of the entire farm operations. Our research aims to provide algorithm-driven solutions for each subject, continually supporting the sustainability of farming as both a lucrative venture and a protector of the environment: (1) crop rotation, (2) integrated pest management, (3) careful use of herbicides, (4) weather and market forecasting, (5) interseeding with cover crops, (6) managing soil water balance, and (7) efficient fertilizer usage.

Research question and objectives

In this thesis, the main research inquiry that we aim to explore is: *"How can farm management strategies integrate optimization principles to drive sustainability?"*

The aim of our research is to investigate challenges in the grain production chain, mainly in the grain farm context, from an optimization perspective by providing a structured analysis of agronomic approaches and improving strategic planning abilities. To achieve this objective, the following particular targets have been established:

- Create a model for the CRP that integrates soil water balance;
- Design a dynamic programming approach for herbicide application;
- Devising a mixed integer optimization model to address the seed multiplication issue;
- Develop an algorithm for pest management that merges the use of natural predators with artificial control techniques;
- Examine time series forecasting techniques and their applications in predicting weather and agricultural commodity trends;
- Evaluate the effectiveness of the suggested optimization algorithms and decision-support approaches using both actual agricultural data and simulated datasets.

Research motivation

Our methodology emphasizes the use of mathematical models and algorithmic strategies; however, our research is primarily driven by two interdisciplinary themes that are essential to the entire society:

1. **Sustainability and food security**: 17 Sustainable Development Goals (SDGs) were formulated by the United Nations (UN) to tackle the major challenges faced today, protect the planet and improve the quality of life for all. Part of the 2030

Agenda, SDG2, or Sustainable Development Goal 2, aims to "*End hunger, achieve food security and improved nutrition and promote sustainable agriculture*". This objective underscores the interrelated aspects of encouraging sustainable agricultural methods, empowering smallholder farmers, eliminating rural poverty, advocating for healthy living, and combating climate change. Among the targets related to this objective (United Nations, 2023), we can highlight: "*By 2030, double the agricultural productivity and incomes of small-scale food producers, in particular women, indigenous peoples, family farmers, pastoralists and fishers, including through secure and equal access to land, other productive resources and inputs, knowledge, financial services, markets and opportunities for value addition and non-farm employment*"; "*By 2030, ensure sustainable food production systems and implement resilient agricultural practices that increase productivity and production, that help maintain ecosystems, that strengthen capacity for adaptation to climate change, extreme weather, drought, flooding and other disasters and that progressively improve land and soil quality*"; "*Increase investment, including through enhanced international cooperation, in rural infrastructure, agricultural research and extension services, technology development and plant and livestock gene banks in order to enhance agricultural productive capacity in developing countries, in particular least developed countries*".

2. **Transforming farm policies to reduce climate risks**: The increasing frequency and severity of extreme weather events driven by climate change present numerous difficulties: they increase risks and impacts, influence every aspect of food security and nutrition. The social groups most susceptible are disproportionately affected by unstable weather conditions. Weather-related impacts exert pressure on land and water resources, thereby affecting agricultural systems and ecosystems. The United Nations' Food and Agriculture Organization (FAO) argues for urgent measures to address climate risk by improving capabilities in prevention, anticipation, absorption, adaptation, and transformation to inform policies, decisions, and climate measures. This encompasses executing assessments of climate risk, impact, and vulnerability; establishing multi-hazard early warning systems; reinforcing infrastructure against climate risks and ensuring risk transfer methods, including insurance and social protection; alongside anticipatory actions and strategies for emergency preparedness and response to adapt to climate change and enhance resilience in agricultural systems. Referencing (Food and Agriculture Organization (FAO) of the United Nations (UN), 2022), it can be stated that: "*Enabling stakeholders in the agrifood systems to maintain the production, processing, distribution, and consumption of safe and nutritious foods and other valuable goods and services necessitates a series of effective climate resilience and adaptation measures founded on robust*

ecosystems and the sustainable management and preservation of natural resources".

We hold that integrating optimization with farm management strategies is closely linked to the adoption of resilient agricultural practices to improve adaptability to climate change. An effective method for developing farm policies involves integrating them into the design of a mathematical model within the farm decision-making framework. Consequently, both interdisciplinary themes are intrinsically linked to the essence of this research.

Contributions and thesis outline

The introduction to our research is outlined in Chapter 1, and this thesis is further structured into five more chapters. Chapter 2 presents an engaging literature review. Chapter 3 primarily discusses the farm management's land allocation problem. Chapter 4 focuses on a problem related to weed management in soybeans. The study of seed multiplication enterprises is addressed in Chapter 5, where we introduce an optimization solution to assist the seed allocation problem. Chapter 6 centers on pest control, specifically analyzing caterpillar pressure on soybean fields. Our investigation aims to comprehend how caterpillar population density impacts control strategies. We develop an optimization model that integrates insecticide application with the predator-prey dynamics based on the Lotka-Volterra equations. The comprehensive evaluation of our findings is detailed in Chapter 8. The supporting data for our research can be found in Appendix A to Appendix D.

Chapter 2 offers an in-depth analysis of the literature on the application of optimization in agriculture. Not only covers well placement, it also identifies uncharted and promising opportunities within the field. The chosen papers are highly relevant to the context of our study, and as a result, are not limited to a certain publication period, even though we have diligently searched for the most recent ones available. Our review of the literature indicates that, although optimization in agriculture has been studied, there are still numerous emerging research opportunities, as agricultural innovation significantly transforms practices in the field. This thesis has explored several promising topics to address existing gaps.

The renowned land allocation problem, called the Crop Rotation Problem (CRP), is main theme of Chapter 3. In this chapter, we make an important contribution by highlighting the crucial role of cover crops in the crop rotation process, enhancing soil health and promoting water sustainability. Our proposed model capabilities incorporate the equations that define crop evapotranspiration, accounting for the soil water content, the foreseeing precipitation, and the crop water demand to full establishment.

Chapter 4 make a notable impact in the field by introducing our proposed dynamic programming model, which provides a structured method to implement agronomic recommendations regarding herbicide use. Concerned about the excessive application of herbicides, we introduce a novelty approach aimed at optimizing the use of selective herbicides on the preemergent and post-emergent stages of soybeans, focusing on managing the potential yield losses compared with spraying expenses and controlling weeds without over-relying on herbicides.

The key contribution of Chapter 5 is the proposed algorithmic solution designed to tackle the complex challenge of managing corn breeding in large-scale seed multiplication businesses. The algorithm we propose is effective in assessing the maximum storage capacity needed to manage a significant amount of corn seeds and in supporting balanced utilization of the storage capacity during the harvest period.

The contribution of Chapter 6 is the innovative modeling of a pest problem in soybeans combining the Lotka-Volterra prey-predator equations. Given the increased significance of biological control strategies in agriculture, we develop a pest management system that integrates the use of pesticides, alongside either chemical means or artificially introduced predators.

Chapter 7 contributed to the field by exploring the use of recurrent artificial neural networks in time series prediction. We recognize the importance of strengthening the resilience of long-term strategies and seek to achieve this by incorporating time series forecasting.

2 Optimization and Agriculture: a scoping review

Agriculture has long been a risky business, and farmers often navigate uncertainty. The agricultural sector is highly influenced by weather patterns, regional climate instabilities, soil quality, farm management practices, and seasonal market influences. The annual crop yield is substantially impacted by the mean temperature and the sum of accumulated rainfall, yet predicting weather conditions with accuracy over extended crop cycles remains challenging. Farmers rely on estimated crop yields to determine the acreage designated for each commercial crop to satisfy market demands and capture favorable market opportunities. Grasping the entire complexity of farming, it becomes evident that optimization concepts are not only suitable for dealing with this intricate situation, but also crucial to sustain an equilibrated agribusiness with modern farming practices. In this chapter, we review a number of studies within the literature that explore the link between agricultural challenges and the use of certain optimization methods to address them. To select a few research papers from the vast literature, we search for terms like crop yield prediction, farm management, and optimization, optimization methods applied to the agricultural sector.

Our search was mainly conducted on Google Scholar and Science Direct. This chapter aspires to present a comprehensive outlook connecting optimization with farm management, while other chapters contain distinct related works sections that are focused on particular subjects. In this concise scoping review, we are not strictly bound by the latest publications, as our primary focus is to establish a foundation at the crossroads of agriculture and optimization.

Establishing an analytical approach to relate the climate pattern to the yield response would prevent farmers from making extremely risky decisions. Agrarian meteorological models for the forecast of yield have been placed as the right tool to explore the influences of weather variables (BASSO; LIU, 2019; APARECIDO *et al.*, 2021; JUNIOR *et al.*, 2024). The variability in annual yields is predominantly influenced by meteorological factors. Aparecido *et al.* (2021) conducted an analysis on the variability in soybean yields and introduced an agricultural model to predict soybean yield, highlighting the weather factors that significantly impact yield. This research utilized a historical series of 34 climate records and soybean yield data from regions in Mato Grosso do Sul, Brazil, with climate records spanning 20 years (2000-2019). They have used a multiple linear regression approach to model farming and weather data , incorporating information from

36 different locations. They recognized that recurrent moderate droughts in December rank among the most crucial climate factors. The persistent water shortage throughout their study period was recognized for its considerable effect.

The production of Brazilian maize is crucial for global food security and serves as a raw material in numerous supply chains. Given the vital importance of maize, accurately predicting yields on a national scale can alert institutions and stakeholders ahead of time, enabling them to implement precautionary measures to ensure food security before any bias arises. The Joint UK Land Environment Simulator (JULES) is a process-oriented, collaborative model designed to simulate the exchanges of carbon, water, energy, and momentum between terrestrial surfaces and the atmosphere. JULES-crop is a crop parameter module within JULES, designed with the dual purpose of modeling the effects of weather and climate on crop yields and assessing the influence of croplands on weather and climate patterns (Geoscientific Model Development, 2024). From this premise, Junior *et al.* (2024) examined maize yield forecasts by employing JULES-crop outputs under conditions of water constraints and optimal scenarios, along with meteorological indicators. The favorable Brazilian weather allows for several crop cycles annually, and their attention was directed toward forecasting maize yields in the off-season. Their results indicated that, between 2003 and 2016, 60% of the variation from year to year in the off-season maize yields in Brazil was attributed to factors such as rainfall and temperature.

Although more sophisticated strategies for prediction of yields are contemporaneous, the desire to estimate crop yield before harvest has intrigued humans since the dawn of agriculture. Basso and Liu (2019) performed a thorough examination of seasonal crop yield prediction techniques within the scientific literature. The findings of their study indicated that yield predictions are primarily based on field surveys, statistical regressions that correlate historical yields with current seasonal variables with a growing number of remotely detected data, crop simulation models, or an integration of statistical and dynamic process-based crop simulation models. Although few studies use field surveys exclusively for yield forecasting, these remain the primary methods for predicting and estimating yield in many countries.

Multiple linear regression is an established method for predicting yield, but crop yield models offer a compelling alternative. These models can aid decision makers across any agroindustrial supply chain, even in decisions that extend beyond crop production. Data mining techniques, given the properties of mechanisms and yield data, are well suited for modeling purposes. Implementing these methods with feature engineering, feature selection, and optimal tuning can enhance performance beyond simply replacing multiple linear regression. Their relevance is emphasized in Bocca and Rodrigues (2016), which pointed out that contemporaneous yield models that combine weather prediction produce a more meaningful interpretation of the data. Bocca and Rodrigues (2016) asserted that data mining would be a wise choice among several strategies. Their proposed data mining solution is based on a database encompassing 65 characteristics that include soil, crop variety, and management practices.

According to Kaul *et al.* (2005), the mechanistic models in agriculture tend to be quite complex given the biological considerations involved. Empirical models, on the other hand, are generally simpler, but may sacrifice a degree of precision. They developed a prediction model based on artificial neural networks (ANN) to forecast soybean and corn yields. The researchers chose a feed-forward back-propagating architecture because it was more efficient and had lower memory demands, which were significant challenges during their study. Sharing the same data set in which the ANN was developed, they proposed a multiple linear regression model based on the Maryland Agronomic Soil Capability Assessment Program (MASCAP). The regression model has offered a reliable benchmark against the ANN. In summary, following the evaluation and fine-tuning of the ANN, the authors concluded that the ANN was beneficial and appropriate to help predict the yield. They adjusted the ANN by modifying the learning rate, the count of hidden nodes and the training tolerance. Kaul *et al.* (2005) also found that the yield predictions were improved when the geographic area being examined was smaller. As the spatial scale increases, there is more variability in cropping conditions.

Klompenburg *et al.* (2020) investigated the use of Machine Learning in crop yield prediction. Their systematic approach to the literature produced a rich survey. In addition to several important observations, Klompenburg *et al.* (2020) noticed that models with more features did not always provide the best performance, and emphasized that test range ought to be extensive. Among the variety of Machine Learning models, Klompenburg *et al.* (2020) avoided suggesting the best model as the scope of the selected articles differs in data and features. Rather than pointing out the best option, which could be biased, they indicated the most used ones in their survey, which are random forest, neural networks, and gradient-boosting tree. Neural networks have the largest share. From deep learning algorithms, convolutional neural networks (CNN), long-short-term memory (LSTM), and deep neural networks (DNN) are often used. In their research, Klompenburg *et al.* (2020) began with 567 papers collected from the search for specific keywords per database. After filtering, they conducted 50 studies that met their criteria. Most of the papers were located from Springer, Google Scholar, and Scopus.

Alves *et al.* (2018b) stand for the use of artificial neural networks in crop yield prediction due to their ability to effectively handle nonlinear systems, which typically exhibit complex and unclear variable interactions. Moreover, the intricacy involved in solving statistical models for projecting crop yields limits their field application due to the extensive data needed. Alves *et al.* (2018b) conducted an analysis on the effectiveness of ANNs in estimating crop yields. Growth habit, sowing density, and agronomic characteristics provide information. They successfully used a multi-layer perceptron neural network (MLP) to predict soybean yield with considerable certainty. Rather than being concerned with every stage of soybean development, which goes from seeding to harvesting, Alves *et al.* (2018b) focus on collecting agronomic data during or after the stage R6. In soybeans, the R6 stage denotes the full seed stage within the reproductive growth phases. Data collected during these advanced crop stages are usually more precise; however, it is close enough to the harvest period that it diminishes the potential for long-term predictions. According to Alves *et al.* (2018b), the main factors associated with the potential soybean yield include the following list: (1) number of branches per plant, (2) number of pods, (3) number of inter-nodes, (4) insertion of the first pod, (5) stem diameter and (6) plant height.

There is no doubt that chemical fertilizers significantly improve crop production. The application of phosphates and nitrates is on the rise as farmers strive for higher yields, but this leads to nutrient surplus and potential environmental dangers. A significant problem is eutrophication, characterized by a reduction in oxygen levels in the water, which endangers marine organisms. Regarding the extensive use of fertilizers, Cropper and Comerford (2004) devised a genetic algorithm (GA) integrated with a mechanistic nutrient uptake model (SSAND, soil supply and nutrient demand) to determine the minimal phosphorus input required to satisfy the initial four-year nutrient needs of pine plantations. Cropper and Comerford (2004) focused exclusively on the application of phosphorus fertilizers. While phosphorus fertilizers play a crucial role in crop growth, it is important to also consider additional macro and micronutrients. Cropper and Comerford (2004) provides a compelling study on phosphorus requirements, which could be modified to address the demands for other vital nutrients in crops. Their dataset comprises trustworthy field data, featuring records from 10 stands of rapidly growing loblolly pine plantations located in southern Georgia, as well as one plantation in Florida. Their model range includes examining the connection between root length density and nutrient absorption. In addition, the scenario involving grass competition in pine plantations has been investigated. While our study does not specifically address pine plantations, Cropper and Comerford (2004)'s insights into the nutrient absorption of growing pines prove enlightening, as their research traverses several fields of knowledge, thereby broadening our perspective. Cropper and Comerford (2004) highlighted in their research that one motivation for developing genetic algorithm solutions is the complex nature of optimizing fertilizer regimes, mainly due to multiple fertilizer applications. The problem is high-dimensionality suggests that without meticulous attention, deterministic methods would likely face a swift increase in complexity given the vast number of possible combinations inherent in the problem.

In Zheng *et al.* (2009), researchers examined how long-term water shortages reduce productivity. The aim was to gather data and suggest innovative strategies for managing farms, addressing soil traits, and the differences in soybean production. They aimed to identify the main factors that contribute to yield volatility. Their research was carried out in fields at a relatively low elevation of 240 meters. Their research was conducted in Hailun County, located centrally in Northeast China. This region is mostly flat, with an average temperature of about 24.1 ^{circ}C and annual rainfall near 500 mm. The wet season, which coincides with crop growth, runs from May to September and it is recommended to begin planting in early May. Despite advances in modern machinery transforming agriculture, certain resistant areas continue to rely heavily on labor-intensive methods, given that traditional farming practices are culturally important. According to the study by Zheng *et al.* (2009), a significant labor force persists in the rural areas of their study location, leading to fragmentation of the croplands. Households often divide arable land, which is vital for their study as it allows independent management practices and unbiased data collection. Regular interviews were conducted with each household. In our perspective, their extensive door-to-door data collection with multiple farmers was crucial, likely facilitating significant informative exchanges. The discussions covered topics such as soil preparation methods and crop rotations. In their analysis, Zheng *et al.* (2009) employed SYSTAT 12 for statistical processes, which included summary statistics, stepwise linear regression, and Classification and Regression Tree (CART) analysis. They found that phosphorus application, farm manure use, and soil organic carbon levels significantly influenced yield variability in their study. They concluded that it is imperative to revise management approaches to diminish yield fluctuations and boost crop output during drought conditions.

The increasing reliance on chemical fertilizers has prompted the research by Ahmed *et al.* (2021). Repeated applications of chemical fertilizers and other compounds have an impact on soil properties. Ahmed *et al.* (2021) express significant concern about the balance of soil health and crop productivity. In their study, they acknowledge the ongoing shift in agriculture due to the rise of the Internet of Things (IoT), cloud computing, and other technologies within precision agriculture. However, they claim that these advanced technologies alone cannot establish a sustainable production system. Traditional farming practices, which have long been refined by farmers, remain the standard of agriculture. Leveraging this deep-rooted knowledge from farmers can guide environmental understanding and decision making. Ahmed *et al.* (2021) introduced nutrient recommendations through an enhanced genetic algorithm that uses sensor data from time series to recommend diverse crop configurations.

From Cardoso *et al.* (2011), climate forecasts are, foremost, responsible for annual fluctuations in soybean yields. Soybeans have various uses, since they are primarily processed for oil and ground to serve as a rich protein source in animal feed. Additionally, soybeans are incorporated into numerous human foods in smaller amounts, such as soy milk, soy protein, and several consumer food items. Among agricultural products, soybeans stand out as a major contributor to Brazil's farm commodity cash receipts. Cardoso *et al.* (2011) discussed the growth cycles of various soybean cultivars, categorizing the duration of the cycle into different groups: (1) early, lasting 75 to 115 days, (2) semi-early, taking 116 to 125 days, (3) medium, spanning 126 to 137 days, (4) late medium, covering 138 to 150 days, and (5) late, exceeding 150 days. Highlighting the broad variability in soybean cycle length is crucial, as any optimization approach should be able to adjust the optimization period accordingly. Cardoso *et al.* (2011)'s objective was to enhance yields by studying crop forecast models. By linking climate patterns with anticipated crop yields, analyses could be improved, significantly decreasing error margin. Leguminous crops have specific water needs, and moderate to severe droughts can cause flower and pod abortion at the start of the reproductive phase. Crop yields are particularly sensitive during the grain filling stage, where browning of leaves and premature leaf drop are clear indicators of water stress.

Gusso *et al.* (2017) analyzed the Enhanced Vegetation Index (EVI) data for predicting soybean crop yield, deducing from their findings that it reliably estimates production. They addressed the Brazilian production report and their methodology. The Companhia Brasileira de Abastecimento (CONAB) releases grain yield forecasts periodically and, being a government entity, is the main source of production estimates in Brazil. Inaccuracy in these forecasts could potentially lead to food supply crises or disruptions in the market. Gusso *et al.* (2017) noted that although these agency reports provide accurate and high-quality insights, CONAB's methods are still prone to errors related to their survey methodology. Furthermore, fine-resolution data are not widely available. Due to these limitations, the forecasts mainly serve large-scale policies and market regulation. Gusso *et al.* (2017) conducted their study in Mato Grosso, Brazil, a tropical state with typically high humidity; annual precipitation ranges from 1300 to 1700 mm (CARVALHO, 2022). Their research incorporated data from the Landsat satellite along with the Moderate Resolution Imaging Spectroradiometer (MODIS). Landsat is a satellite system that operates to continuously capture images of the global land surface, providing robust data and documenting natural and human-made changes (MASEK, 2022). MODIS, a critical instrument on board the Terra and Aqua satellites, covers the Earth's entire surface every 1 to 2 days, capturing data across 36 spectral bands or wavelength groups (MACCHERONE, 2022).

Oliveira *et al.* (2017)'s intriguing study, targeting not just sugarcane estimates, presented a comprehensive methodology with broad potential applications. Their research goes beyond the calculation of fresh mass yields, aiming to assess the actual sugar content of a harvest right from the field. They highlighted the significant economic impact of precise estimates, noting that setting goals, a critical business activity, is often based on these estimates. Inaccurate growth predictions can exceed processing and storage capacities, resulting in significant logistical problems. Setting ambitious goals creates a risky situation, while overestimating production can lead to multiple breaches of supply contracts. Oliveira *et al.* (2017) mentioned that accurately modeling plant phenology along with its physiological mechanisms often necessitates a comprehensive grasp of their micro-environment alongside laboratory experiments. Empirical models tend to avoid delving into the specific natural events involved, opting instead for a broader perspective. Unsurprisingly, business management typically gravitates towards these empirical approaches. In Oliveira *et al.* (2017), the objective was to assess sugar levels using weather fluctuations and management strategies as input variables. Their findings sought to offer valuable information to the sugar industry, particularly for sugarcane processing facilities. The study explored techniques such as support vector regression, random forests, and regression trees. Although data mining exhibits significant promise, it necessitates adjusting the algorithm's parameters for every new set of data.

Burt (1965) released their study during an era when computers were primarily the tool of select pioneers. In this time frame, as computational power was just beginning to expand, the achievement of practical optimization solutions necessitated the expertise of highly skilled specialists to convert the problem into a format that could be solved using the computational power available at that time. Although trends in academic research had decisively reshaped outlooks over the past decades, some researches from the 1960s managed to grasp the optimistic future of operations research in many fields. We have found in Burt (1965) a confident and precise perspective on Operations Research in farm management. In one of the initial insights presented in Burt (1965), it is observed that viewing Operations Research merely as a collection of tricks or methods limits its ultimate effectiveness in any field. This reflects a modern misunderstanding that without the necessary expertise of optimization experts, even the results derived from certain methodologies may fall short in various ways. Burt (1965) acknowledged the challenge of estimating parameters essential for managing intricate farm management scenarios, although they did not explicitly identify it as a problem. They discussed various farm management applications with optimism, expressing confidence in their feasibility for data collection. A comprehensive farm enterprise budget is crucial for the business's profitability and strategically guides the future use of farm resources. Using linear programming for budgeting in large enterprises is quite commonplace. Even if the entire budget isn't optimized, certain budget components are adjusted to achieve desired maximum or minimum outcomes. Burt (1965) identified linear programming as a commonly used approach to budgeting. They suggested that subsequent development should involve extending from budgeting to the allocation of resources over time, leading to dynamic linear programming. Burt (1965) introduced a creative scenario with respect to farm expansion. The act of purchasing land becomes financially feasible only after some time, following the success in outcomes such as good harvests. To foresee future results, it is essential for mathematical programming to incorporate uncertainties related to land and yield prices. Risk analysis in farm management is another significant challenge, particularly with high-risk crops and risk-averse farming strategies. A dynamic programming model could provide valuable insight for long-term planning. Burt (1965) concluded notably that with the increasing mechanization of farms, the demand for operations research methods would increase. Scheduling would become a crucial aspect of large-scale agriculture. The field of farm management offers limitless opportunities for application.

Neto *et al.* (1998) explored crop modeling by initially presenting a concise semantic overview of *model classification*. They categorized the models into three main types: (1) conceptual, (2) physical, and (3) mathematical. In the realm of mathematics, models can be categorized as either empirical or mechanistic. Neto *et al.* (1998) highlighted the inherent ambiguity of the mechanistic models. Despite these models striving for accurate representation of events and processes, the intricacy of the real world requires omitting certain aspects to maintain practicality and usability. Consequently, all mechanistic models carry limitations and require assumptions. Neto *et al.* (1998) discussed some models in the literature that aim to predict wheat yield using meteorological variables and another yield projection model based on sugarcane leaf and Growing Degree Days (GDD). Their conclusion highlighted the importance of conducting a proper test batch before implementing any model decision.

In order to address the intricate farm management issues present in horse farms, Moghaddam and DePuy (2011) introduced a stochastic optimization model designed to identify the optimal number of acres of hay a farm should cultivate for their horses' needs. The variability in weather conditions is represented within the mathematical model by incorporating random variables into the constraints of the model. In addition, the model helps determine the appropriate amount of hay to purchase and sell with the goal of maximizing the farm's overall profit. This model helps horse farmers make effective decisions about the cultivation, purchasing, and sale of hay. Their model incorporated multiple constraints to manage inventory levels, ensuring that the hay consumption in each period aligns with the estimated demand according to its definition.

Boussios *et al.* (2019) developed a methodology to determine optimal farm management strategies, supported by the use of Dynamic Programming (DP). By utilizing stochastic variables associated with weather patterns and price fluctuations, they developed a novel method aimed at enhancing sustainability and managing risk. Boussios *et al.* (2019) stated that their approach aligns more closely with agricultural practices because it incorporates stochastic weather variables. Their dynamic programming application became efficient by the use of thresholds, which basically replace continuous variables in the model with triggers. To illustrate, let us assume that a certain variable represents the quantity of a highly hazardous chemical compound. If it is continuous and, confined to tank, spans from 0 to 1000 liters, this creates an infinite range of intermediate states. By categorizing the volume into intervals such as empty, half-full, or overflowing, we can significantly reduce this state space, which would suffice for many decision makers. In Boussios *et al.* (2019), a key threshold is identified as the minimum criterion of 100 mm accumulated rainfall necessary to commence planting. This aligns well with the straightforward decision-making process of farmers, who typically employ simple strategies rather than intricate methods in their daily tasks. Agronomic recommendations are based similarly on specified ranges. This threshold model shares similarities with agronomic methodologies and offers significant improvements in computational efficiency through the use of dynamic programming.

Popp *et al.* (2003) reported experiment results in soybean field to establish comparison parameters in the following agricultural practices: (1) no-till versus tilled seed bed preparation, (2) soybean cultivars and their maturity groups and plant seasons, and (3) planting equipment (planter versus grain drill). The experimental site was set up in Keiser, Arkansas, in 1990. A subsequent trial occurred in 1992, this time in Rohwer, Arkansas. These trials generated data that were instrumental in developing an intriguing optimization model. The objective function of the model of Popp *et al.* (2003) is to maximize the area that can be planted with a specified soybean maturity group cultivar. Their constraints accounted for available labor resources and restricted access to equipment and machinery. Their research provided robust information on the probability of timely completion based on the availability of machinery and the planting season. Their study reveals that recommendations that disregard the operational scale implications driven by weather might neglect crucial factors for crop producers.

Manos *et al.* (2013) introduced a multi-criteria programming model designed to optimize agricultural production plans. Their research focused on increasing gross margin while minimizing reliance on fertilizers and reducing labor hours. The authors applied a solution approach inspired by the Weighted Sum Method. Their optimization method was tested in the Thessaly region of Greece and the findings demonstrate that it effectively balances gross margin, fertilizer use, and labor efforts.

In semi-arid countries, achieving agricultural sustainability in the face of climate change poses a significant obstacle. Robert *et al.* (2018) examined the adaptive decision-making processes of farmers in response to climate change. Their study concentrated on long-term investments in bore well irrigation and analyzed short-term decisions about cropping systems and irrigation water application rates. A stochastic dynamic model was employed to evaluate how farmers' decision making would respond to different social and economic aspects under various climate change scenarios. Their study examined multiple policies—including subsidies for rain-fed agriculture, changes to subsidized energy for irrigation, and a water charge adjusted to the ambient groundwater level—and assessed their effects on both farmer profit and groundwater levels.

2.1 Understanding the relevancy of the reviewed material

Tables 2.1 and 2.2 resumes the principal factor from each research paper in this brief literature review. From the work dedicated to crop yield prediction, a variety of techniques were employed, such as multiple linear regression, artificial neural networks, data mining, and evolutionary algorithms. Some methodologies are devised for a multi-crop system, while others are exclusively dedicated to specific crops. While the application of artificial neural networks can be smoothly modified for different crops, several parameters must be customized to develop an efficient algorithm as it is applied to a new crop. This is due to ANNs' high dependence on the specific characteristics of the dataset.

Other investigations, like Burt (1965) and Neto *et al.* (1998), collectively aim to underscore the significance of optimization within the agricultural sector. In conducting our literature review, we opted for two surveys, Klompenburg *et al.* (2020) and Basso and Liu (2019), which offer a comprehensive overview of their application within academic studies. Based on the review by Basso and Liu (2019) that included more than 250 yield prediction studies, remote sensing data was mentioned in slightly more than 50% of the articles. Agrometeorological models were used in approximately one-third of the studies, while process-based models appeared in 15% of the studies. Only a few papers discussed the use of surveys in forecasting crop yields. Although we have discussed only one multicriteria, which is the paper published by Manos *et al.* (2013), incorporating multiple objectives is a prevalent trend in the literature.

Table 2.1 – Evaluating selected articles from the literature, part (a)

Article	**Type of Article**	**Key Data Analysis Techniques**	**Role in Farming**	**Information source**
Ahmed *et al.* (2021)	Original Research	Improved Genetic Algorithm	Cotton / Ground nuts / Maize / Rice / Yield Prediction	Sensor mesh in the area
Alves *et al.* (2018b)	Original Research	Artificial Neural Network	Soybean yield forecast	Evaluation based on agronomic traits of soybeans
Aparecido *et al.* (2021)	Original Research	Multiple linear regression	Soybean yield forecast	Historical series of climate and soybean yield from soybean-producing locations
Basso and Liu (2019)	Survey	Statistical regression	General crop yield prediction	Over 250 papers concerning yield forecasts from the Web of Science database by the beginning of February 2018
Bocca and Rodrigues (2016)	Original Research	Data Mining	Sugarcane yield forecast	Dataset provided by a sugarcane mill with 65 features
Boussios *et al.* (2019)	Original Research	Dynamic programming model	Optimization mathematical models in Farm Management	Crop growth and weather simulators
Burt (1965)	Systematic review	Systematic review of Operations Research in Agriculture	Optimization mathematical models in Farm Management	The author's expertize
Cardoso *et al.* (2011)	Original Research	Statistical analysis using data from COLA-CPTEC Atmospheric Global Circulation Model (AGCM)	Soybean yield forecast	Daily forecast and observed precipitation data for Passo Fundo, Brazil, alongside daily temperature observations, spanning from October 20 to February 21 for the years 2005/2006, 2006/2007, and 2007/2008
Cropper and Comerford (2004)	Original Research	Genetic Algorithm	Minimization of fertilizer consumption in pine plantation	Ten stands of rapidly-growing loblolly pine plantations, ranging from 1 to 4 years old, were selected for a field experiment conducted in the coastal plain area of southern Georgia
Gusso *et al.* (2017)	Original Research	EVI processing, Artificial Neural Network	Soybean yield forecast	Experiment conducted in Mato Grosso, Brazil

Table 2.2 – Evaluating selected articles from the literature, part (b)

Article	**Type of Article**	**Key Data Analysis Techniques**	**Role in Farming**	**Information source**
Junior *et al.* (2024)	Original Research	JULES-crop, multiple linear regression	Maize yield forecast	Meteorological data collected on an hourly basis from 2003 through 2016 and cultivar dataset from several locations
Kaul *et al.* (2005)	Original Research	Artificial Neural Network	Maize / Soybean yield forecast	Historical (1978–1998) Maryland corn and soybean yield data from the Maryland Cooperative Extension (MCE) Hybrid Variety Performance
Klompenburg *et al.* (2020)	Survey	Machine learning	General crop yield prediction	567 studies retrieved from Science Direct, Scopus, Web of Science, and Springer link
Manos *et al.* (2013)	Original Research	Multicriteria programming model	Sustainable Production	Data from the region of Thessaly in Greece
Moghaddam and DePuy (2011)	Original Research	Stochastic optimization model	Optimization of hay land use	Case study located in northern Kentucky
Neto *et al.* (1998)	Survey	The use of optimization models in agriculture	Optimization mathematical models in Farm Management	Literature review and the author's expertize
Oliveira *et al.* (2017)	Original Research	Data Mining	Sugarcane yield forecast	Data collected from a sugarcane mill
Popp *et al.* (2003)	Original Research	Stochastic optimization model	Soybean yield forecast	A field trial in Arkansas
Robert	Original Research	Stochastic dynamic model	The use of groundwater irrigation	A farmer who represents the Berambadi watershed in Karnataka, India
Zheng *et al.* (2009)	Original Research	Statistical Analysis: with general linear model (GLM) and classification and regression trees	Soybean yield forecast	Household interview

3 Designing a crop rotation optimization model

The rising population and increased food consumption are exerting significant pressures on agriculture and natural resources. To address global food security and sustainability challenges, it is essential for farming and food production to expand considerably while also taking environmental stewardship into account. This enhancement in production can be achieved either by agricultural extensification, which involves clearing more land for farming, or intensification, which requires obtaining higher yields by using more production resources, improved agronomic methods, diverse crop varieties, and other innovations (TILMAN *et al.*, 2011). Improving yields on existing farmland, restoring degraded land, and adopting sustainable agricultural practices would alleviate the pressure to clear forests for agricultural production. Halting and reversing land degradation will be essential in reaching the world's growing food needs.

To enhance productivity and optimize waste management, agricultural systems need to evolve. By adopting a comprehensive and integrated approach, farming practices and food systems should strive for sustainability. Elements from traditional farming wisdom, when merged with cutting-edge technologies and scientific discoveries, can lead to more sustainable management strategies. Decision-making should be integrated to effectively address these goals. Agricultural innovations play a crucial role in aiding farmers to boost productivity, minimize environmental harm, and tackle issues arising from variations in soil, climate, and market dynamics (AKKAYA *et al.*, 2021).

According to the United Nations, investment in agriculture is on the decline. The ratio of public spending on agriculture to the agricultural sector's contribution to GDP dropped from 0.50 in 2015 to 0.45 in 2021, in all regions except North America and Europe (United Nations, 2023). Although there has been some advancement, numerous countries need accelerated progress towards sustainable agriculture and rural development. Consequently, farmers need to boost their efficiency to align with public expenditure in the sector.

Continued advancements in agriculture will enable society to achieve results in securing food supply. In the past, policies aimed at modernizing crop production and developing agricultural practices were successful as they helped farmers who were struggling with profitability in the agricultural sector. In the present day, although technological and economic obstacles have diminished somewhat, global advancement in food production has decelerated marginally. Securing food security continues to be a significant challenge, as uncertainties within the worldwide supply chain place strain on the agriculture industry as a whole (SPIELMAN; PANDYA-LORCH, 2009).

Even with the advancements in modern agricultural machinery, anticipating a significant shift solely through technological substitution is unrealistic. The future of farming lies in integrating optimization and decision-making models into agricultural choices. Employing sophisticated planning methodologies to optimize the use of machinery, resources, and the environment can significantly improve the production system. Studies reveal that considerable advancements can be realized by targeting “yield gaps” in underperforming lands, thus enhancing crop production efficiency (FOLEY *et al.*, 2011). Globally, there is a quest for innovative and sustainable projects and pioneering research advancements to enhance the sustainability of food production systems worldwide (United Nations, 2023).

Until recently, most agricultural paradigms focused on improving production, often to the detriment of the environment. Likewise, many environmental conservation strategies have yet to seek to improve food production. However, to achieve global food security and environmental sustainability, agricultural systems must be transformed to address both challenges (FOLEY *et al.*, 2011). Innovations in technology, whether through machinery, chemicals, or biological methods, should enhance farm productivity and boost agricultural profits while minimizing negative environmental effects (PRETTY, 2008).

According to Carravilla and Oliveira (2013), management science plays a crucial role in tackling complex problems related to management by integrating insights from various scientific disciplines. This approach emphasizes the need for problem structuring, modeling, and resolution to enhance decision-making. In the realm of agriculture, the Crop Rotation Problem (CRP) represents the planning process for farmland management. Understanding the depth of agrarian applications, the CRP serves as a key to exploring numerous options and possibilities. With many farmers embracing integrated farming methods, this signifies strides toward sustainability. Implementing more precise and targeted techniques leads to reduced waste and environmental gains, all while maintaining profitability (PRETTY, 2008).

Farmers continually advance and make essential management decisions, yet determining the best crop at the right moment is crucial and surpasses an individual’s ability to evaluate all potential outcomes. This is not merely a choice among the most lucrative crops; their plan takes into account soil type, soil fertility, weather conditions, and previous crops planted in the field. Factors such as machinery, equipment, workforce, and environmental impact also influence this decision. There are considerable opportunities to boost productivity by enhancing the use of current crop varieties through refined management, which can significantly narrow yield gaps (FOLEY *et al.*, 2011).

Based on soil conservation practices, growing cover crops is adequate to keep the land from being degraded. Cover crops provide resilient surface cover between growing seasons in annual crops that can prevent erosion or soil depletion. Cover crop residues still protect the soil after plant termination. Cover crops reduce erosion, and using cover crops as green manure improves soil structure and nutrient availability. The organic matter produced by cover crops provides a sustainable environment for beneficial soil organisms. Hence, using cover crops in the crop rotation problem study is essential for enduring sustainability in intensified cropping systems (BAUMHARDT; BLANCO-CANQUI, 2014).

Implementing the crop rotation system in the agrarian field improves the soil ecosystem services. Crop rotation planning benefits multiple soil functions of water and nutrient cycling. Even so, developing a proper rotation scheme could reduce the need for herbicides and pesticides. Diversification in farmland allocation is also the core component to sustain land-use intensification (PELTONEN-SAINIO *et al.*, 2018) (BALESTRINI *et al.*, 2015). These agricultural benefits strongly rely on the selection of crop species and the cultivation sequence adopted by the producer (VOLSI *et al.*, 2022).

Insufficient information and management abilities significantly obstruct the adoption of sustainable agricultural practices. Agribusiness ranges from small family-run operations to extensive enterprises that manage farms nationwide. Larger businesses require more meticulous planning to ensure smooth operation without significant disruptions. Conversely, small farms often rely heavily on subsidies, either to cover production expenses or to invest in new equipment, and they are frequently more attuned to the regional agricultural landscape, focusing on crops that are readily marketable locally. While both large and small agribusinesses have their distinct characteristics, all farmers face difficulties in understanding the long-term consequences of their choices, as it is challenging to fully grasp the future impact of these decisions.

This chapter aims to fill this void and answer the following questions:

- How does an optimized cropping pattern perform in comparison with other farmland allocations?
- How would the forecasting yield based on meteorological data affect the entire planning horizon?
- How would soil attributes drive decision-making?
- How costly is it to diversify farmland allocation to meet adjacent constraints?

To address these questions, we consider an actual case application from a grain farmer in the state of São Paulo, Brazil. Studies (FOLEY *et al.*, 2011) point out guidelines for developing better agricultural and land use practices through data analysis and implementation of decision support tools to improve environmental management and productivity. By diversifying crop rotations, we can promote the conservation of natural resources and decrease production vulnerabilities. It is essential to highlight that low diversity cropping schemes, even though they might have the best productivity and more significant revenue, are not necessarily the ones with the highest profits in the long run (VOLSI *et al.*, 2022).

In this chapter, our research is structured around a discussion of materials and methods, detailed in Section 3.1. Here, we explore related works and their beneficial influence on existing literature. We examine the features we intend to incorporate into our optimization model and outline the specifics of the proposed model. Section 3.2 discusses the generated crop schemes and how they serve their purpose in agrarian activity. We aim to answer the research quests using comparisons and observations. Section 3.3 states our final observations about this research, pointing our analytical advice into the farmer scenario and the future research possibilities in the agrarian field.

3.1 Materials and Methods

Subsection 3.1.1 surveys the crop planning problem in the literature and discusses our contribution to the agrarian field, which brings novelty from other literature research. Subsection 3.1.2 presents an overview of the case study. We organized in this section the gathered data from the grain producer in Brazil. We extend our survey in the farmer scenario to form a reliable data set. Subsection 3.1.3 delves into the ideas associated with evapotranspiration. The advantages of incorporating cover crops within the crop rotation strategy are discussed in Subsection 3.1.4. Subsection 3.1.5 describes the proposed optimization model. Crop nutrient demand and soil fertility are fundamental in the crop rotation schedule. By creating an innovative mono-objective model of the CRP, we aim to enhance the link between nutrient dynamics on arable lands and the sequence of crops. In this section, we discuss the constraints of the problem and their justification from the agrarian perspective.

3.1.1 Related works

Sustainable practices support economic health in contemporary agribusiness management. Vegetable farms have significantly become sought after since market consumption has been thinking more about a sustainable lifestyle. In the past few decades, as societies seek more responsible and sustainable developments, we have noticed a strong connection between the crop rotation problem and the contemporaneous agricultural transformation. Although farmland allocation studies have been around for many decades, their goals have profoundly changed in recent trends, and farm profitability is not always the primary task. The latest research in the literature takes more ecological criteria in their approaches. Outside the broad field of agricultural sciences, the crop planning problem has also experienced a great deal of attention from operations management and economics (BOYABATL *et al.*, 2019).

Although crop rotation may benefit soybean production, the relationship between the preceding and succeeding crops requires long-term evaluation. Kelley *et al.* (2003) introduced a field study to evaluate the effects of crop rotation on soybean yield and seed weight. The long-term study, which spans from 1979 to 1998, considered three two-year rotations. Their rotation schemes have winter wheat, summer fallow, and grain sorghum aside from the soybean. The comparison also includes the continuous soybean scenario. In their final observations, full-season soybean yields grown in the crop rotation strategy have been 15% higher than monoculture soybean. Crop rotations involving sorghum and wheat also strongly affect the total soil organic carbon and nitrogen concentration levels. Their study gives us a perspective of how hard it is to evaluate crop rotations in the field. They stuck with only a few configurations, one soil type, and one field location, even though it took 20 years to get confident results.

Sehgal *et al.* (2023) and Notaris *et al.* (2023) studied the effects of long-term cropping rotation strategies. A crop rotation based on grain legumes and cereals increased yield stability. These experiments report overall gains in crop yields; however, they also acknowledge that yield responses are deeply dependent on weather conditions and management practices. Even biological N_2 fixation, which is a well-known service of legumes in the field, is subject to fluctuation by year. Sehgal *et al.* (2023) also discussed conservation crop rotation systems, which consist of establishing a combination of high residue producing with low residue producing crops such as corn and soybean rotation. Selecting highly stable crops in the crop rotation scheme can reduce production risk and withstand various weather conditions (NIETHER *et al.*, 2023).

A long-term crop rotation strategy combined with weed management is a reliable alternative for controlling perennial weeds in agrarian fields. Especially, crop rotation and management practices can drastically affect weed seed bank and weed density (OLESEN *et al.*, 2007; MISHRA *et al.*, 2022). In organic farms, crop rotation and residues from the cropping system are essential for weed management (NIETHER *et al.*, 2023).

The work of (VOLSI *et al.*, 2022) indicates that grain production systems that employ crop rotation with species diversification showed greater productivity and profitability than rotation without species diversification. Six-grain production systems were part of the experiments: five rotations with varying levels of species diversification and one corn-soybean rotation without species diversification. The study examined productivity, cropping revenue, production cost, and profit markers. Grain production systems with species diversification showed better efficiency than the well-known corn-soybean system. Economic, sensitivity, and statistical analyses were carried out in addition to sample collections.

Haneveld and Stegeman (2005) provides a valuable insight into crop succession requirements. They understood that planning the future crops to be seeded is fundamental to sustaining fertility. Other options exist than scheduling the same botanical family on the same land in a row. Crop residues that remain in the field after harvest often still contain pests or diseases and can also cause the proliferation of some important agricultural pests in the succeeding crop. Haneveld and Stegeman (2005) also considered in their mathematical framework the necessity of leaving the land lying fallow.

The crop sequence and the selected plant species considerably impact carbon retention. Carbon sequestration in soil organic matter is one of the most efficient climate mitigation strategies. Triberti *et al.* (2016) detailed their experiment, which aims to evaluate the interaction between type of crop rotation, manure, and mineral fertilization on the dynamic of organic matter and nitrogen in the soil. Their final observations include cultivating crops with high carbon-to-nitrogen ratio residues and reintroducing legume-based crop rotations, which would benefit farm management regarding soil fertility and lower the carbon dioxide concentration in the atmosphere.

Dupuis *et al.* (2022) are deeply concerned about food shortage in the future linked to the environmental impact of intensive agriculture systems. They understand that a reliable prediction of the succeeding crop would benefit agronomic decisions and drive them toward more sustainable practices. Optimized fertilizer plans based on the knowledge from future growing seasons can reduce water pollution caused by fertilizer run-off. Their work proposes methods for the prediction and visualization of crop rotations. Based on data from many fields in Quebec, Canada, they combine Markov's principle with process mining to infer the next crop rotation. Their evaluations indicate equivalent performance compared with neural networks, although their methodology is more straightforward to implement.

Aggarwal *et al.* (2022) introduce a crop rotation proposal for the Muzaffarnagar district in India. Sugarcane is the primary crop grown in the region, exceeding 90% of the total cultivable area. Although raindrops are reasonable and soil fertility is high, Aggarwal *et al.* (2022) reported land degradation around the district. Sugarcane monoculture is partially responsible for land degradation and water contamination. Another side-effect of extensive growing of one particular crop is the consumption market; the sugarcane yield in the region exceeds the mill capacity and drives prices down during the harvest season.

Dupuis *et al.* (2022) and Aggarwal *et al.* (2022) handle the crop rotation problem from the regional perspective. Their work observes the whole group of farmers and could support policy decision-making. Aside from their research that explores the community pattern, our study intends to get close to the farmer decision-making framework and the choices that affect the farm performance and profitability in a close assessment of the large environmental impacts.

From the rich set of crop rotation applications, Pahmeyer *et al.* (2021) search for a more friendly interface with the user. Pahmeyer *et al.* (2021) develop a web-based decision support system called *Fruchfolge*, which aims to assist crop management choices in the field. Although they recognize the powerful capacity of mathematical programming, high data requirements are typically assumed to preclude the use of crop rotation models on the farm. These procedures comply with European Union regulations to protect water bodies from large-scale use of nitrogen fertilizers. Behind their user-friendly web solution, the main goal in the optimization model is to reach the farm's maximum total expected contribution margin.

Fenz *et al.* (2023) propose an alternative solution method for the crop rotation problem using reinforcement learning. Fenz *et al.* (2023) combines NDVI (Normalized Difference Vegetation Index) from predecessor crops as an input parameter. From the benefits of this methodology, flexibility and scalability stand out as the most recognized characteristics. Fenz *et al.* (2023) and Pott *et al.* (2023) use satellite-based data to look into the crop rotation problem. Pott *et al.* (2023) focus their evaluation on Southern Brazil and observe that, in drought seasons, the continuous soybean practice in some mesoregions in the state of Rio Grande do Sul showed a yield penalty around 20% in comparison with more diverse fields, which indicates a low climate resilience of current cropping system adopted.

Decisions on the crop rotation and the annually cropping plan can transform agricultural profitability and productivity, with both short and long-term consequences. Planning crop rotations is a complex task in farm management and the decisions must consider favorable and unfavorable factors for reaching the desired outcomes (DURY *et al.*, 2012; KASU *et al.*, 2019).

Some research has already been carried out on optimization models to support sustainable crop planning and farm management. For example, Santos *et al.* (2015) developed an integer programming model to decrease the usage of cultivable area required to produce a regional vegetable demand. Aliano *et al.* (2023) developed a mixed-integer non-linear programming model to schedule planting and harvesting operations for distinct sugarcane varieties. Boyabatl *et al.* (2019) discussed optimal farmland allocation policies under revenue uncertainty and developed a heuristic approach to represent a typical grain farmer from Iowa, USA. Regis Mauri (2019) applied several renowned relaxation methods in the crop rotation problem, such as Lagrangian Relaxation, Lagrangian Relaxation with Clusters, Lagrangian Decomposition, Column Generation based on Lagrangian Relaxation with Clusters, and Column Generation based on Lagrangian Decomposition. Fikry *et al.* (2021) proposed a crop rotation model which maximizes the farm net return based on the estimated yield sales. In their work, limited water resources and the fluctuations resulting from droughts in previous growing seasons are connected during the crop sequence planning . They also presented a compelling study case in Egypt, where the uncertainty in water level forecasting is tackled by robust optimization techniques. A related linear programming model was designed by (KEYSER *et al.*, 2023) to minimize fertilizer cost using bio-based fertilizers in cultivable farms while at the same time complying with the nutrient demand of the considered crop.

Time series forecasting (TSF) is a crucial research issue in diverse areas, such as financial time series, wind power, and traffic forecasting (YADAV; THAKKAR, 2024; LINDEMANN *et al.*, 2021). Over the last decades, non-linear relationships in the TSF have intrigued researches in developing innovative methods and models. Deep neural network (DNN) has been surging in many data-driven applications as an alternative to classical statistical methods since DNNs are able to estimate multi-target time-series (MASINI *et al.*, 2021; REYES; VENTURA, 2019).

Although research with similar intentions has been proposed in the literature, the approach presented in our study here brings novelty to the crop rotation problem. A real case study inspires our proposed model. Our research highlights the grain producers in the state of São Paulo, Brazil. Our objective function and restrictions are fully aligned with the struggles of their managerial activity. The proposed solution considers the nutrient supply based on the farmer's practice experience and the use of carefully selected cover crops adapted to the state's region. This paper is an unmatched combination of farming management expertise that can be easily adapted to grain farming worldwide. We reevaluate the famous agrarian problem under new perspectives and aim to complement the farmer's difficult task of planning crops further ahead.

3.1.2 A case study

We calibrated our proposed optimization model with real data from a farmer. We collected information about types and areas of the crop from a farmer in the city of Tatuí, state of São Paulo. The specific location of the farmer is 23°22'28.7"S 47°52'43.3"W. The farmer is well-skilled and has dedicated many years to raising beans, soybeans, maize, sorghum and wheat. Due to the large sum of investments required to compete, renting fields is common practice as many landowners cannot afford the cost of mechanization.

From seed to harvest, our expert farmer has sufficient technology to perform all the activities with a certain level of mechanization. Regarding the regional aspect, we might classify our farmer as a medium-sized farmer. A minimum labor workforce manages the daily operation. The majority of the arable land that the farmer has is composed of red latosol. A spatial view of the cultivable land is shown in Figure 3.1.

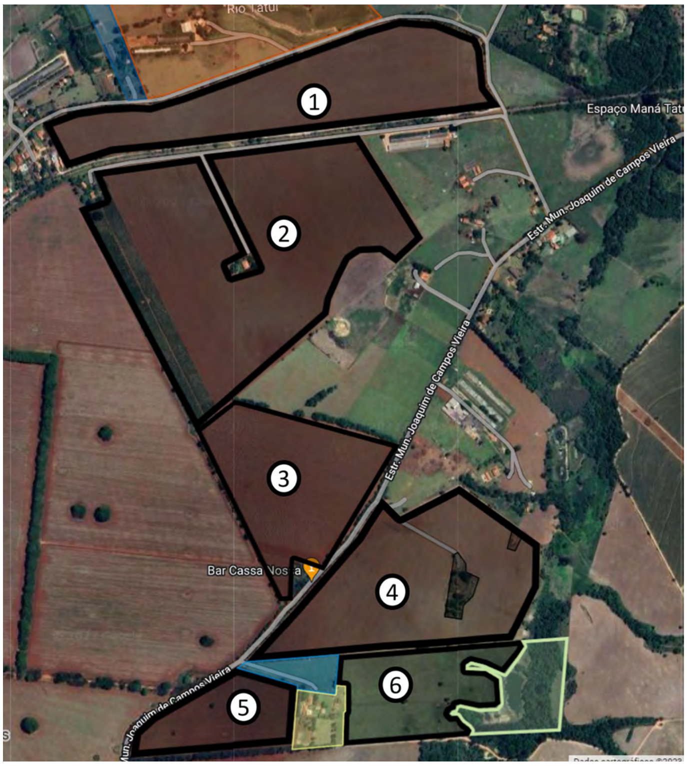

Figure 3.1 – Distinct fields and their respective cultivable area (in *hectares* (ha): (1) Sítio Shalar Gleba A: 18.8 ha (2) Sítio Shalar Gleba B: 34.4 ha (3) Sítio dos Caresias: 13.3 ha (4) Sítio Bom Retiro: 18.9 ha (5) Sítio Boa Esperança Gleba A: 6.1 ha (6) Sítio Boa Esperança Gleba B: 8.1 ha.

Figure 3.2 shows the climate chart that reflects the average climate data for Tatuí, a city characterized by its humid subtropical climate (MOCHIZUKI *et al.*, 2006). This climatological chart is derived from the average data provided by the Instituto Brasileiro de Meteorologia (INMET) (Instituto Nacional de Meteorologia (INMET), 2024).

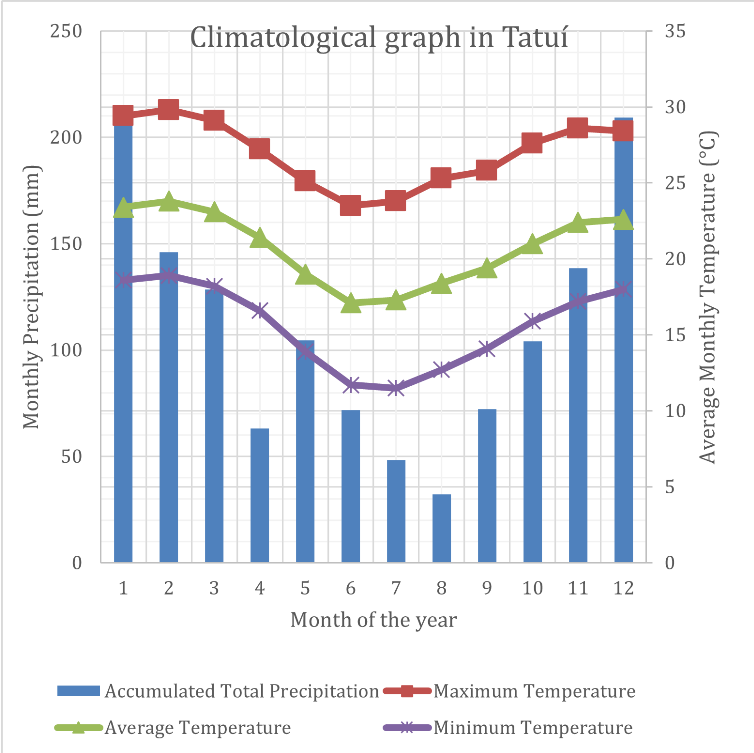

Figure 3.2 – Climate chart from the city of Tatuí, SP.

3.1.3 Comprehending the relationship between evapotranspiration and crop yield

This subsection explores various equations that determine the effect of water on the cultivation of key crops. The constraints related to weather and water incorporated into our proposed solution are grounded in the principles and insights covered in this section of our work.

A sustainable water utilization would rather focus on maximizing the output per unit of water, rather than achieving the greatest yield per unit of cultivable land (FERERES; SORIANO, 2007; ARAYA *et al.*, 2011). Thus, it becomes imperative to establish a water assessment within the CRP. Our methodology bases on the approach proposed in (DOORENBOS; KASSAM, 1979; ALLEN *et al.*, 1998). The calculation procedure follows the following:

I) Determine maximum yield (Y_m) if adopted crop variety, dictated by climate, assuming other growth factors (e. g. water, fertilizer, pests and diseases) are not limiting.

II) Calculate maximum evapotranspiration (ET_m) when crop water requirements are fully met by available water supply.

III) Determine actual crop evapotranspiration (ET_a) based on factors concerned with available water supply to the crop.

IV) Evaluate factors concerned with the interaction between water supply, crop water requirements and actual yield (Y_a); through:

V) Selection of yield response factor (ky) to evaluate relative yield decrease as related to relative evapotranspiration deficit, or $(1 - Y_a/Y_m) = ky \cdot (1 - ET_a/ET_m)$, and obtain actual yield (Y_a).

3.1.3.1 Crop coefficients and the crop evapotranspiration

In this subsection, we present the crop coefficients (K_c), which are vital for estimating crop evapotranspiration (ET_c) under ideal conditions, as referenced in (ALLEN *et al.*, 1998; DOORENBOS; KASSAM, 1979). The parameters do not account for limitations such as soil water stress, salinity impact, crop density, pest and disease impact, weed interference, or nutrient inadequacy. The combined influence of crop transpiration and soil evaporation is represented by a unified crop coefficient. The K_c coefficient reflects the traits of the crop and the mean effects of soil evaporation. The crop coefficient method is used to calculate ET_c, integrating various weather effects into reference evapotranspiration (ET_o) and crop attributes into the K_c coefficient, as outlined in Equation (3.1):

$$ET_c = K_c \cdot ET_o \; [mm \; of \; water/day] \tag{3.1}$$

Table 3.1 illustrates the typical growth cycles for the primary commercial crops we have chosen. The area in which they are cultivated significantly impacts their growth cycle, primarily because of fluctuating weather patterns.

Table 3.1 – Lengths of grain crop development stages for worldwide climatic regions (days)

Crop	**Initial Stage**	**Development Stage**	**Mid-Season Stage**	**Late Season Stage**	**Total**	**Plant Date**	**Region**
Soybeans	15	15	40	15	85	Dec	Tropics
Soybeans	20	30/35	60	25	140	May	Central USA
Soybeans	20	25	75	30	150	June	Japan
Soybeans	20	25	75	30	150	June	Japan
Winter Wheat	20	60	70	30	180	December	Calif., USA
Winter Wheat	30	140	40	30	240	November	Mediterranean
Winter Wheat	160	75	75	25	335	October	Idaho, USA
Maize (grain)	30	50	60	40	180	April	East Africa
Maize (grain)	25	40	45	30	140	Dec/Jan	Arid Climate
Maize (grain)	20	35	40	30	125	June	Nigeria (humid)
Maize (grain)	20	35	40	30	125	October	India (dry, cool)
Maize (grain)	30	40	50	30	150	April	Spain (spr, sum); Calif.
Maize (grain)	30	40	50	30	170	April	Idaho, USA
Sorghum	20	35	40	30	130	May/June	USA, Pakis., Med.
Sorghum	20	35	45	30	140	Mar/April	Arid Region

Source: FAO - Food and Agriculture Organization of the United Nations [1]

The K_c coefficient for any period of the growing season can be derived by considering that during the initial and mid-season stages K_c is constant and equal to the K_c value of the growth stage under consideration. During the crop development and late season stage, K_c varies linearly between the K_c at the end of the previous stage (kc_{prev}) and the K_c at the beginning of the next stage (kc_{next}), which is K_c end in the case of the late season stage. From initial to mid, determine K_c from the slope. Therefore, we could use a interpolation to determine any intermediate K_c. Table 3.2 presents a set of crop coefficients (K_c) and mean maximum plant heights for non-stressed, well-managed crops in sub-humid climates, which has relative humidity over 45% and wind velocity above 2 m/s, for use with the FAO Penman-Monteith ET_o.

Table 3.2 – Empirical crop coefficient (K_c)

Crop	**Initial K_c**	**Mid-stage K_c**	**Late K_c**	**Maximum Crop Height (m)**
Maize, Field (grain) (field corn)	0.3	1.20	0.60	2
Soybeans	0.4	1.15	0.50	0.5 - 1.0
Winter Wheat - non-frozen soils	0.7	1.15	0.25	1
Sorghum -grain	0.3	1 - 1.10	0.55	1-2

Source: FAO - Food and Agriculture Organization of the United Nations [2]

3.1.3.2 Hargreaves method to determine reference evapotranspiration (ETo)

The Hargreaves method (HARGREAVES; SAMANI, 1985) relies on temperature to estimate evapotranspiration using an empirical relationship in which solar radiation and air temperature data were used to model reference evapotranspiration (ETo). The technique can effectively record daily fluctuations in potential evapotranspiration for simulation intervals shorter than 24 hours. The approach has been confirmed for locations globally (HARGREAVES; ALLEN, 2003). The regression utilized a dataset compiled over eight years of precise lysimeter measurements for a reference grass crop in Davis, California. Reference evapotranspiration (ETo) is computed as defined in Equation (3.2):

[1] Available at: <https://www.fao.org/4/X0490E/x0490e0b.htm#TopOfPage>. Access date: October 19th, 2024.

[2] Available at: <https://www.fao.org/4/X0490E/x0490e0b.htm#TopOfPage>. Access date: October 19th, 2024.

$$ETo = 0.0023 \cdot Ra \cdot (Tmax - Tmin)^{HE} \cdot (Tmed + 17.8) \quad [mm\ of\ water\ per\ unit\ of\ time] \tag{3.2}$$

Where:

- ETo: reference evapotranspiration [*mm of water per unit of time*]
- HE: empirical exponent (typically 0.5) [*dimensionless*]
- Ra: extraterrestrial radiation [kJ/m^2]
- $Tmax$: air maximum temperature [$°C$]
- $Tmin$: air minimum temperature [$°C$]
- $Tmed$: mean air temperature [$°C$]

While the FAO Penman-Monteith equation (ALLEN *et al.*, 1998) provides a more comprehensive method for calculating reference evapotranspiration (ET_o), we opted for the Hargreaves method because it requires fewer parameters for computation. In this study, a moderate level of accuracy suffices for assessing long-term crop rotations. The source of these meteorological parameters are the automated weather stations operated by the INMET (Instituto Nacional de Meteorologia (INMET), 2024).

3.1.3.3 Actual Evapotranspiration over monthly periods

Total available soil water (Sa) is defined here as the depth of water in mm/m soil depth between the soil water content at field capacity (Sfc or at soil water tension of 0.1 to 0.2 atmosphere) and the soil water content at wilting point (Sw or at soil water tension of 15 atmosphere). Total available soil water (Sa) can vary widely for soils having a similar texture.

Despite employing an empirical methodology along with recognized parameters to assess a complex dynamic process, it is important to recognize that incorporating localized data on the total available soil water within the root zone is essential. Ultimately, implementing this in the field would demand comprehensive measurements across the cultivable area to enhance both performance and accuracy. We take the reference total available water ($Sa\ mm/m$) as a typical parameter for various soil textures, based on the specifications in (DOORENBOS; KASSAM, 1979), which are outlined in the subsequent list:

- Heavy textured soil: 200 mm/m
- Medium textured soil: 140 mm/m
- Coarse textured soil: 60 mm/m

In our case study, the arable land falls within a spectrum ranging from medium-textured to heavy-textured soil. For reconnaissance and preliminary planning purposes an estimate of mean actual evapotranspiration (ET_a) for a given crop can be obtained using the available soil water index (ASI). The ASI indicates the part of the month when available water is adequate for meeting full crop water requirements ($ET_a = ET_m$). A combination of ASI value, maximum evapotranspiration (ET_m) and remaining available soil water $[(1-p) \cdot S_a \cdot D]$ provides an estimate of the mean monthly ET_a. The following equation is adapted from Doorenbos and Kassam (1979).

$$ASI = \frac{In + Pe + Wb - [(1-p) \cdot Sa \cdot D]}{monthly\ ET_m}\ [dimensionless] \tag{3.3}$$

The symbols in Equation (3.3) are described as follows:

- In: net monthly irrigation application [*mm of water per month*]
- Pe: effective rainfall [*mm of water per month*]
- Wb: actual depth of available soil water at beginning of the month, [*mm of water per root depth in unit of length*] (i. e. available water content in mm water depth per meter soil depth (mm/m) (Wb))
- $[(1-p) \cdot Sa \cdot D]$: depth of remaining available soil water when $ET_a < ET_m$ [*mm of water per root depth in unit of length*]
- ET_m: maximum evapotranspiration [*mm of water per month*]

For the ASI, it is presumed that when the sum of $In + Pe$ is equal to or less than $30 \times ET_m$, it will entirely support evapotranspiration without causing deep percolation or runoff. Additionally, the average monthly ET_a is influenced solely by the aggregate of In, Pe, and Wb, rather than their monthly distribution. The ASI can exceed one or be less than zero. If $ASI \geq 1$, then $ET_a = ET_m$, whereas if $ASI < 0$, then the ratio ET_a/ET_m becomes so diminutive that crop growth is nearly impossible unless ET_m is low and there is a significant amount of remaining soil water $[(1-p) \cdot Sa \cdot D]$ available.

For illustration, let us consider maize, seeded at late September. Potential crop evapotranspiration is 8 mm/day. The cultivable land is a medium textured soil with total available water equals to 140 mm/m ($Sa = 140\ mm/m$) and maize root systems reaches to 1.2 m of depth ($D = 1.2\ m$). Net irrigation (In) was around 120 mm/month. Effective rainfall (Pe) was 30 mm/month. Actual depth of available water at the beginning of the month $Wb = 60 mm/m$. For example, consider the following procedure to determine the actual evapotranspiration:

- $In + Pe + Wb = 120 + 30 + 60 = 210\ mm/month$
- Fraction p from Table 3.3: 0.45
- Available soil water when $ET_a < ET_m$ $(1 - p)Sa \cdot D = 92$ mm
- $ASI = (210 - 92)/(30 \cdot 8) = 0.49$
- According to Figure 3.4, ET_a constitutes approximately 75% of ET_m, which is nearly 6.0 mm/day.

Table 3.3 – An estimative to Soil Water Fraction (p)

Soil Water Depletion Fraction (p)		
	Crop Groups according to Soil Water Depletion	
Evapotranspiration (ET_m) [mm/day]	**Crop Group 3**	**Crop Group 4**
	Wheat	Maize Sorghum Soybean
2	0.80	0.88
3	0.70	0.80
4	0.60	0.70
5	0.50	0.60
6	0.45	0.55
7	0.43	0.50
8	0.38	0.45
9	0.35	0.43
10	0.30	0.40

Source: Partly derived from Doorenbos and Kassam (1979)

We investigate the connection between ASI and ETm to determine ETa, utilizing Figures 3.3 and 3.4. The charts present an approximation of the actual evapotranspiration based on mapping their relationship.

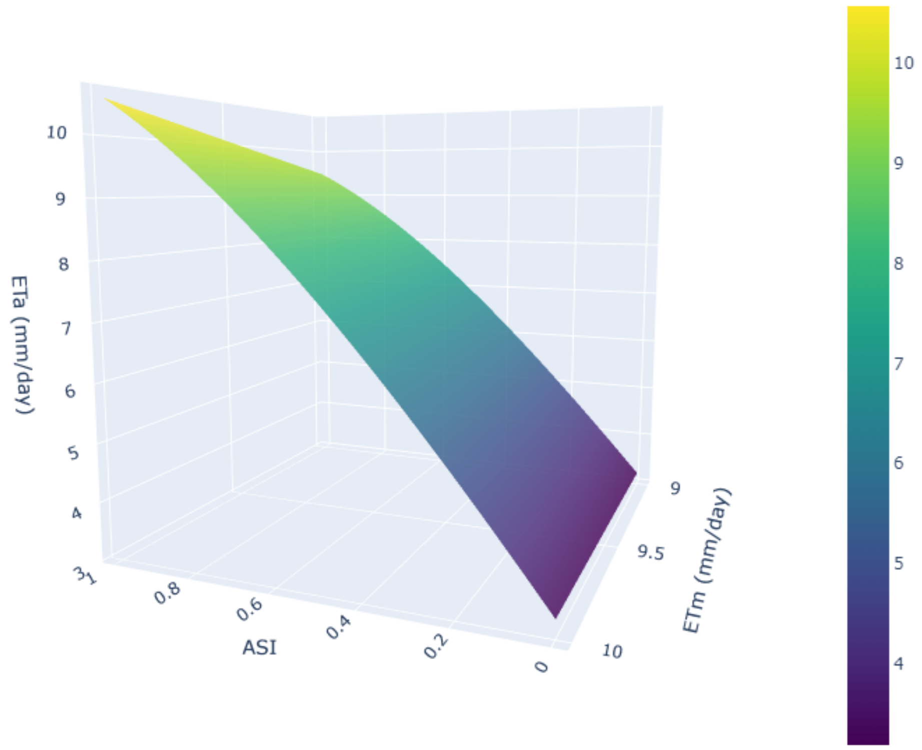

Figure 3.3 – A surface plot of the Actual Evapotranspiration (ET_a)

Actual evapotranspiration (ET_a) expressed as a percentage of ET_m is determined by the residual depth of accessible soil moisture $[(1-p) \cdot S_a \cdot D]$ and the ASI.

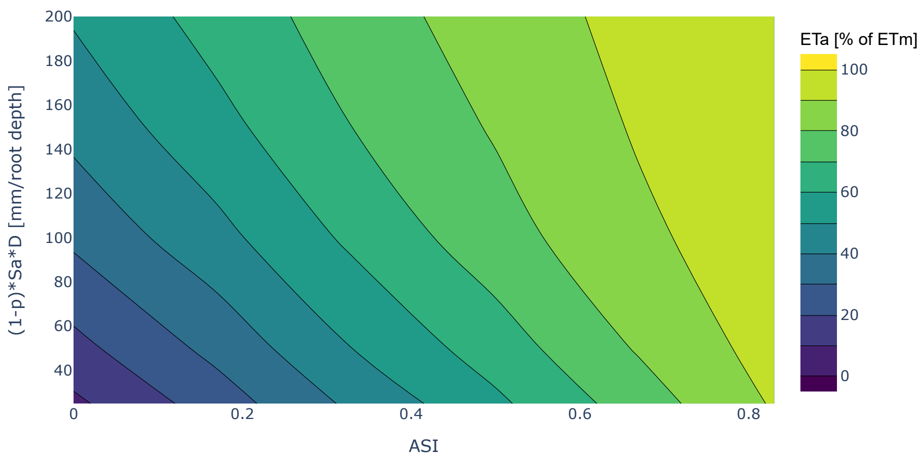

Figure 3.4 – Actual evapotranspiration (ET_a) expressed as a percentage of ET_m.

No matter which soil preparation method is used, the soybean root system generally penetrates to depths ranging from 0.43 to 0.54 m, while corn roots typically extend between 0.40 and 0.46 m. Wheat roots can delve as deep as 1.5 m and are hair-like in structure. For optimal irrigation practices, wheat root depth is generally considered to be 30 to 40 cm.,Moreover, the sorghum root system is extensive and fibrous, with more absorbent hairs. It reaches a depth of up to 1.5 m (with 80% found within the top 30 cm of soil), and laterally it can extend to 2.0 m.,Corn roots, in deep and well-maintained soils, may grow as deep as 2 meters. However, the most branched portion of the corn root system is typically found between 0.4 and 0.6 meters deep, depending on the soil quality, with about 80% of the soil's moisture absorption occurring within this zone.

3.1.3.4 Yield and Water

In order to quantify the effect of water stress it is necessary to derive the relation ship between relative yield decrease and relative evapotranspiration deficit given by the empirically-derived yield response factor (ky). Since the relationship is also affected by factors other than water, such as crop variety, fertilizer, salinity, pests and diseases, and agronomic practices, the relationships presented refer to high producing varieties, well-adapted to the growing environment, growing in large fields where optimum agronomic and irrigation practices, including adequate input supply, except for water, are provided (DOORENBOS; KASSAM, 1979).

With the presented relationships it is possible to plan, design, and operate irrigation supply systems taking into account the effect of different water regimes on crop production. Equation (3.4) defines the ratio of achieved yield to potential yield. The following parameters and equation are adapted from Doorenbos and Kassam (1979).

- Y_a: actual harvested yield [*units of crop yield per area*]
- Y_m: maximum harvested yield [*units of crop yield per area*]
- ky: yield response factor [*dimensionless*]
- ET_a: actual evapotranspiration [*mm of water per day*]
- ET_m: maximum evapotranspiration [*mm of water per day*]

$$(1 - \frac{Y_a}{Y_m}) = ky \cdot (1 - \frac{ET_a}{ET_m}) \Rightarrow \frac{Y_a}{Y_m} = 1 - ky + ky \cdot \frac{ET_a}{ET_m} \tag{3.4}$$

The yield response to water availability is measured by the yield response factor (ky), which connects the proportional yield reduction ($1 - Ya/Ym$) to the ratio of actual evapotranspiration (ETa) to maximum evapotranspiration (ETm). Either consistently throughout the crop's growth period or at distinct stages like establishment, vegetative, flowering, yield formation, or ripening, this relationship can appear. In the former case, the degree of water shortage pertains to the deficit relative to the crop's water needs over the entire growing season, whereas in the latter, it pertains to the water needs of a particular growth stage. The ky values used for many crops assume that the connection between relative yield (Ya/Ym) and relative evapotranspiration (ETa/ETm) is linear, applicable for water shortages reaching about 50 percent or $1 - ETa/ETm = 0.5$. The ky values are derived from an evaluation of field data from experiments conducted under diverse growing conditions. These experimental outcomes pertain to high-yield crop varieties that are well-suited to the growing environment and are cultivated with advanced crop management techniques. The magnitude and duration of water deficit expressed as relative evapotranspiration deficits ($1-ETa/ETm$) are made to correspond closely to the individual crop growth periods. Analysis of the available field experimental data in terms of the more precisely defined stress-day and drought indices proved difficult. Utilizing the yield response factor (ky) in the planning, design, and operation of irrigation projects enables the quantification of water supply and usage with respect to crop yield and total production in the designated area (DOORENBOS; KASSAM, 1979). Table 3.4 displays the yield response factor for primary crops at various stages of crop development.

Table 3.4 – Yield response factor (ky) for major commercial crops.

Crop	**Yield response factor (ky)**				
	Vegetative period (1)	**Flowering period (2)**	**Yield formation (3)**	**Ripening (4)**	**Total growing period**
Maize	0.40	1.50	0.50	0.20	1.25
Sorghum	0.20	0.55	0.45	0.20	0.90
Soybean	0.20	0.80	1.00	1.00	0.85
Winter Wheat	0.20	0.60	0.50	0.5	1.00
Spring Wheat	0.20	0.65	0.55	0.55	1.15

In this research, although these relations are more concerned to the irrigation planning, we focus our attention to the capacity of establishing a yield parameter based on the actual water supply in non-irrigated farms. As reported by the Instituto Brasileiro de Geografia e Estatística (IBGE) (NUNES *et al.*, 2006), data from the 2006 agricultural census indicates that only 6.3% of the country's agricultural lands used irrigation methods, such as flooding, infiltration, sprinkling or similar. The irrigated area comprised 4.45 million hectares or 7.4% of the total area in temporary and permanent crops.

As an example, let us calculate the relative reduction in soybean yield, expressed as $1 - Y_a/Y_m$. According to the seeding calendar for soybeans in São Paulo, planting occurred in early October with an anticipated harvest in February. An estimation for the total growing period is around 115 to 140 days. The total water requirement varies from 400 to 700 mm depending on depending on the climate, growing season length, maturity, planting date, and location. Table 3.5 details the example parameters.

Table 3.5 – Total growing period for soybeans and accumulated precipitation

Growth Period	Duration (days)	Total water requirements (mm)
Establishment and vegetative period	50	90
Flowering period	30	215
Yield formation	30	215
Ripening	20	180
Total	130	700

This analysis will examine the effect on harvest yield, taking into account yield formation and the overall growth duration. The water supply is reduced by 18% (126 mm) from the total requirement of 700 mm, and this deficit is evenly distributed throughout the entire 130-day growing period. The yield response factor (ky) for the total growing period is 0.85. Water supply is 18% less, so actual crop evapotranspiration $ET_a = 595\ mm$ (15% less than $ET_m = 700\ mm$). Using Equation (3.4), we determine that, from the water requirements and supply, soybeans are anticipated to achieve just 0.847 of their full potential, following which a 15.3% reduction in yield is expected when the water drought is evenly distributed.

$$\frac{Y_a}{Y_m} = 1 - 0.85 + 0.85 \cdot \frac{574}{700}$$
$$\frac{Y_a}{Y_m} = 1 - 0.85 + 0.85 \cdot 0.82$$
$$\frac{Y_a}{Y_m} = 1 - 0.85 + 0.697$$
$$\frac{Y_a}{Y_m} = 0.847$$

Now, suppose there was shortage of water during yield formation of the same magnitude of previous example (18% less water supply during yield formation). According to Table 3.4, the yield response factor (ky) for yield formation is 1.0. Thus, the following procedure determines the relative yield response:

$$\frac{Y_a}{Y_m} = 1 - ky + ky \cdot \frac{ET_a}{ET_m}$$
$$\frac{Y_a}{Y_m} = 1 - 1.0 + 1.0 \cdot \frac{176}{215}$$
$$\frac{Y_a}{Y_m} = 1 - 1.0 + 1.0 \cdot 0.82$$
$$\frac{Y_a}{Y_m} = 1 - 1.0 + 0.82$$
$$\frac{Y_a}{Y_m} = 0.82$$

As the yield factor (ky) from yield formation is greater than the total growing period factor, the relative yield response would be more affected. Consequently, with the ratio Y_a/Y_m being merely 0.82 of the total potential in this case, it can be deduced that water stress in yield formation leads to an 18% reduction in potential yield. We could make a similar assessment for vegetative or flowering stages. A solid projection of yield will be based on the minimum result for all the periods.

3.1.4 Diversified crop rotation and beneficial effects

Crop rotation with high diversity and biomass input (plant shoots and roots) under no-tillage (NT) management is one of the pillars of conservation agriculture, positively impact crop yield and agricultural profitability (LI *et al.*, 2019; TELLES *et al.*, 2019; GARBELINI *et al.*, 2022). (GARBELINI *et al.*, 2022) reported several increases to crop yields in a diversified agronomic system. Their study took place in the state of Paraná, Brazil. Soybean yield was 6% higher on average than that in double-cropping systems (wheat-soybean or maize-soybean). The wheat yield was among the most responsive crops in diversified rotations, gathering an increase of 27.3% to 32.4% higher yield in comparison with continuous double cropping system. (GARBELINI *et al.*, 2022) also reported that first maize crop yield in the spring-summer season was from 6.8 % to 9.2% higher than that from maize-maize rotation system. (FRANCHINI *et al.*, 2012a) reported that wheat yield has 6.8 % higher in the crop rotation system than in the crop succession. From their experiment, they noticed that the yield growth coefficient was greater in crop rotation systems that in the crop succession. The study presented by (SMITH *et al.*, 2023) also observed the increasing grain yields with higher species diversity across several fields in Europe and in the United States. The growth pattern in grain yields was consistent over time. Diversity in crop schemes was beneficial to all cereals when was combined with a small nitrogen input. Grain yields increased with diversification in crop rotations are also related to the decreased weed competition, and pest and disease pressures (TELLES *et al.*, 2019; GARBELINI *et al.*, 2022; SMITH *et al.*, 2023; BENNETT *et al.*, 2012). In a sandy soil, the use of cover crops has strong effect on soybean yield. (CORDEIRO *et al.*, 2021) conducted a field experiment in Western São Paulo state, Brazil, to evaluate the increase in soybean production seeding cover crops. A crop succession of soybean and black oats increased soybean yields by 50% in comparison with soybean after fallow. From a six-year field experiment in the North China Plain, (YANG *et al.*, 2024) realized that the large-scale adoption of diversified cropping systems could increase cereal production by 32% when wheat–maize follows alternative crops in rotation and farmer income by 20% while minimizing the environment impact from the agricultural chain. . In conclusion, several studies pointed out significant yield gains from establishing a diversity crop sequence including cover crops. The slope of yield growth is steeper in long-term planned crop rotations. Although soil type and history defines the full yield potential, beneficial effects from crop rotation have been demonstrated all over the world as a profitable and sustainable approach.

3.1.5 Overview of the Optimization Model

The optimization framework we present here aligns to the following representation, incorporating elements from the mixed-integer linear programming models proposed in (MIRANDA *et al.*, 2019b) and (MIRANDA *et al.*, 2021). A cropping set of N crops belong to a N_{fam} number of families. The entire planning horizon is composed of M weeks. We also define a set of K cultivable fields. Cropland area in hectares for each plot k is combined in a vector called $area[k]$. From the sequence of grown crops from each cultivable field, we ensure that a safety interval must be satisfied to seed crops from the same family again, which is β interval in *weeks*.

The main indexes in our proposed optimization model are:

- i: the set of crops that can be seeded $(i = 1, \cdots, N)$
- j: the set of periods to evaluate $(j = 1, \cdots, M)$
- k: the set of cultivable land, divided in K plots $(k = 1, \cdots, K)$
- p: the set of crop's families $(p = 1, \cdots, N_{fam})$
- n: it represents the set of years from the planning horizon $(n = 1, \cdots, \frac{M}{\theta})$, where θ stands for *a year in weeks* $(\theta = 52)$
- h: the crop rotation attributes $(h = 1, \cdots, H)$
- γ_{cov}: impact of cover crops on evapotranspiration

The primary decision variable set, denoted as $u[i][j][k]$, indicates whether Crop i is scheduled to be planted in Plot k during Period j.

$$u[i][j][k] = \begin{cases} 1, & \text{if Crop } i \text{ is to be seeded in Plot } k \text{ during Period } j. \\ 0, & \text{otherwise.} \end{cases}$$

Production cost per area for any Crop i presents in $cprod[i]$ vector, which holds production expenses in terms of *crop yield units*. The notion of how much of the crop yield is compromised by the costs of production is essential in the agrarian field. The projected yield for Crop i seeded in Period j is $YM[i][j]$. Based on the assessment of the water balance, $y_F[i][j][k]$ calculates the proportion of the anticipated yield $YM[i][j]$ that can be achieved. Aside from the commercial crops revenues and costs, establishing cover crops has also a significant cost in the agribusiness. Although cover crops do not require any expensive cultural trait, they occupy the cropland during their cycle and so, farmers could loose an opportunity to seed any particular profitable crop. We present cover crop's expenses as $ccov[i]$, which indicates costs per area (*R$ / hectare*). The crop cycle for each Crop i is $cyc[i]$ and presents in *weeks* of duration. Farmers would have some production goals related to the major crops and we map their position using demand for each Crop i through the $YT[i]$ vector [*unit of crop yield per unit of area*].

Using $Com[i]$, we establish two separate crop sets: commercial crops and cover crops. Each group has its own goal; the first reunites the profitable crops, and the second one, the crops intend to improve sustainable practices, build soil, and cover the soil. We understand that the cover crops ought to be killed or pruned before reaching the mature stages. In general, peak benefits from cover crops are attained while still in a vegetative growth state. Therefore, we replace growing a commercial crop in order to improve soil attributes using cover crops. $Com[i]$ follows the description ahead:

$$Com[i] = \begin{cases} 1, & \text{if Crop } i \text{ is a commercial crop.} \\ 0, & \text{if Crop } i \text{ is a cover crop.} \end{cases}$$

Increasing diversification of crop families in the cropland is a control measure to prevent more widespread crop pests. Establishing rotations in adjacent cultivable fields would limit insect food availability and mobility. We use the $Sadj[k][v]$ adjacent array to characterize the boundaries from each plot. $Sadj[k][v]$ defines a neighborhood crucial to enforce group diversification.

$$Sadj[k][v] = \begin{cases} 1, & \text{if Plot } k \text{ is adjacent to Plot } v \text{ .} \\ 0, & \text{otherwise.} \end{cases}$$

The family array is $F[i][p]$:

$$F[i][p] = \begin{cases} 1, & \text{if Crop } i \text{ is from the Family } p \text{ .} \\ 0, & \text{otherwise.} \end{cases}$$

The remaining parameters are described in the following list:

- $s[i]$: sales price for yield unit [*\$/unit of yield crop*]
- $attr[i][h]$: soil improvements: from 0 (poor) to 4 (excellent) for each crop i [*dimensionless*]
- $attrp[k][h]$: rating soil improvements for each k plot [*dimensionless*]
- $Sp[l][k][n]$: the supply quantity of nutrient l to plot k during the year n [*unit of weight per unit of area*]
- $R[l][i]$: the required quantity of nutrient l for crop i [*unit of weight per unit of area*]
- $y_F[i][j][k]$: yield ratio derived from the assessment of the soil water balance for crop i seeded at period j in plot k [*dimensionless*]
- $ETm_{accum}[i][j][k]$: the accumulated evapotranspiration for crop i seeded at period j in plot k during the whole cycle of crop i [*mm of water per crop cycle*]
- $Pe[j]$: reported precipitation for period j [*mm of water per unit of time*]

The proposed objective function in Equation (3.5) aims to maximize the revenues in the planning horizon. Our proposed model estimates profits based on the sale price of any crop i, the expected yield, and the production costs per area. Aside from the commercial crops, we also introduce the cover crops in the cropping planning study. Cover crops are usually mowed down or killed using herbicide before reaching the reproductive stage. Then, we do not expect direct returns from the cover crops seeding even though they improve soil fertility and structure. The expenditure with cover crops is part of Equation (3.7). The crop assessment, yield, and profit are determined at the seeding period, meaning we account for harvest at the time of seeding rather than at the actual harvesting period as described in Equation (3.6).

$$\textbf{Maximize} \quad comprofits - covcosts \tag{3.5}$$

$$\begin{aligned} comprofits = &\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{K} u[i][j][k] \cdot \Big(area[k] \cdot s[i][j] \cdot \\ &(y_F[i][j][k] \cdot YM[i][j] - cprod[i][j]) \cdot Com[i]\Big) \end{aligned} \tag{3.6}$$

$$covcosts = \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{K} u[i][j][k] \cdot \Big(area[k] \cdot ccov[i][j] \cdot (1 - Com[i])\Big) \tag{3.7}$$

Constraints in Equations (3.8), (3.9) and (3.10) are adapted from Miranda *et al.* (2019b). Equation (3.8) ensures that crops from the same family are not assigned to adjacent plots during the same growing season. We introduce two arrays $F[i][p]$ and $Sadj[k][v]$ in the constraint composition; they are both binary sparse matrices. Using these arrays as a reference to family and adjacency has enhanced the efficiency of the solving process. It represents a new interpretation of crossing only crops from a given set find in Miranda *et al.* (2019b).

$$\begin{gathered} \sum_{i=1}^{N}\sum_{r=0}^{cyc[i]-1}\sum_{v=1}^{K} u[i][j-r][v] \cdot F[i][p] \cdot Sadj[k][v] \leq \\ K \cdot \left(1 - \sum_{i=1}^{N}\sum_{r=0}^{cyc[i]-1} u[i][j-r][k] \cdot F[i][p]\right), \\ p = 1, \cdots, N_{fam}, \quad j = 1, \cdots, M, \quad k = 1, \cdots, K \end{gathered} \tag{3.8}$$

Constraints in Equation (3.9) avoid seeding crops from the same family sequentially without growing another family or assigning a fallow interval. Their objective is to disrupt pest cycles in the cropland.

$$\begin{gathered} \sum_{i=1}^{N}\sum_{r=0}^{cyc[i]+\beta} u[i][j-r][k] \cdot F[i][p] \leq 1, \\ p = 1, \cdots, N_{fam}, \quad j = 1, \cdots, M, \; k = 1, \cdots, K \end{gathered} \tag{3.9}$$

Equation (3.10) represents the spatial and temporal limitation of the problem, which prevents more than one crop from occupying some area at any period in the planning horizon. We should respect the proper seeding interval for each cultivar, which is the period that leads to the maximum crop potential.

$$\sum_{i=1}^{N} \sum_{r=0}^{cyc[i]-1} u[i][j-r][k] \leq 1,\ j = 1, \cdots, M,\ k = 1, \cdots, K \tag{3.10}$$

Constraint in Equation (3.11) defines the strategy to avoid seeding outside the recommended window. An binary array represented by $Seed[i][j]$ holds true when Period j is adequate for seeding Crop i, and false when Period j is outside the recommend seeding window.

$$\begin{aligned} u[i][j][k] \leq Seed[i][j],\ i = 1, \cdots, N,\ j = 1, \cdots, M, \\ k = 1, \cdots, K \end{aligned} \tag{3.11}$$

Constraints in Equation (3.12) establish the minimum yield per Crop i. We take yield demands only to commercial crops.

$$\sum_{j=1}^{M} \sum_{k=1}^{K} area[k] \cdot y_F[i][j][k] \cdot YM[i][j] \cdot u[i][j][k] \geq YT[i] \cdot Com[i],\ i = 1, \cdots, N \tag{3.12}$$

Our nutrient analysis for the crop sequence encompasses both macro and micro nutrient provision throughout the planning period. Nutrient assessment is characterized in Equation (3.13), where $Sp[l][k][n]$ quantifies the amount of nutrient l that needs to be provided in year n to meet the crop's nutritional requirements.

$$\begin{gathered} Sp[l][k][n] = \sum_{i=1}^{N} \sum_{j=1+(n-1)\cdot\theta}^{n\cdot\theta} u[i][j][k] \cdot area[k] \cdot R[l][i], \\ k = 1, \cdots, K, \quad n = 1, \cdots, \frac{M}{\theta}, \quad l = 1, \cdots, L \end{gathered} \tag{3.13}$$

The constraints in Equation (3.14) limit the nutrient supply within specific minimum and maximum values. It should be noted that our approach to the nutrient model reflects the typical practices employed by farmers, although those with precision agriculture equipment may apply fertilizers in a more tailored manner.

$$\begin{gathered} min_{nut}[l] \leq Sp[l][k][n] \leq max_{nut}[l] \\ k = 1, \cdots, K, \quad n = 1, \cdots, \frac{M}{\theta}, \quad l = 1, \cdots, L \end{gathered} \tag{3.14}$$

Cover crops in the crop rotation solution are the subject of Equations (3.15) and (3.16). From analyzing the farmer's context, we selected and classified cover crops according to four desirable effects in the cropland: (i) nitrogen scavenger capacity, (ii) solid builder, (iii) weed fighter, and (iv) lasting residue. Each Plot k must reach a minimum portion in each attribute to satisfy the constraint set in Equation (3.16).

$$attrp[k][h] = \sum_{i=1}^{N} attr[i][h] \cdot u[i][j][k], \quad k = 1, \cdots, K, \quad h = 1, \cdots, H \tag{3.15}$$

$$attrp[k][h] \geq min_{attr}[h], \; k = 1, \cdots, K, \quad h = 1, \cdots, H \tag{3.16}$$

Equation (3.17) defines the domain of the decision variable $u[i][j][k]$.

$$u[i][j][k] \in \{0, 1\}, \quad i = 1, \cdots, N, , \; j = 1, \cdots, M, \quad k = 1, \cdots, K \tag{3.17}$$

Equation (3.18) determines the maximum evapotranspiration (ET_m) when crop water requirements are fully met by available water supply on a weekly basis.

$$ET_m[i][j][k][r] = ETc[i][r] \tag{3.18}$$
$$\forall r = 1, \cdots, cyc[i], \quad i = 1, \cdots, N, \; j = 1, \cdots, M, \; k = 1, \cdots, K$$

Equation (3.19) represents the maximum evapotranspiration (ET_m) when crop water requirements are fully met by available water supply during the whole crop cycle, which is divided on four stages: initial, crop development, mid-season and late season.

$$\begin{aligned} ETm_{accum}[i][j][k] = & \sum_{v=j}^{j+Init[i]} ETm_{ini}[i][j] \\ & + \sum_{v=j}^{j+Dev[i]} \frac{ETm_{ini}[i][j] + ETm_{dev}[i][j]}{2} \\ & + \sum_{v=j}^{j+Mid[i]} ETm_{mid}[i][j] \\ & + \sum_{v=j}^{j+Late[i]} ETm_{late}[i][j]) \end{aligned} \tag{3.19}$$

We mapped idle fields with Equation 3.20. The evapotranspiration from an idle (bare or fallow) field can be similar to the reference evapotranspiration (ETo), although it may not be exactly equal. Mapping idle cropland indicates which is the main source of evapotranspiration during each period j.

$$Empt[j][k] = \sum_{i=1}^{N} \sum_{r=0}^{min\{cyc[i],j\}} u[i][j-r][k\ ], \qquad j = 1, \cdots, M, \quad k = 1, \cdots, K \tag{3.20}$$

Cover crops can reduce evapotranspiration by shading the soil and reducing soil temperature. They can reduce wind speed at the soil surface, further decreasing evaporation by windbreak effect. Equation (3.21) maps the seeding of cover crops into the cultivable land in past seasons.

$$covSeed[j][k] = \sum_{i=1}^{N} \sum_{r=0}^{min\{cyc[i]+\beta,j\}} u[i][j-r][k\ ] \cdot (1 - Com[i]), \qquad j = 1, \cdots, M, \quad k = 1, \cdots, K \tag{3.21}$$

Estimated actual evapotranspiration during period j is defined in Equation (3.22).

$$ETacc[j][k] = \sum_{i=1}^{N} \sum_{r=0}^{min\{Cycle[i],j\}} \left(ET_m[i][j-r][k][r] \cdot u[i][j-r][k] \cdot ASI[i][j-r][k]\right), \qquad j = 1, \cdots, M, \quad k = 1, \cdots, K \tag{3.22}$$

Soil water balance in the proposed solution is part of Equation (3.24). It determines the amount of available water in the soil for crop development. Soil water balance is dynamic and influenced by various biophysical factors. Our modeling of the soil water balance relies on an empirical approximation due to the complexity of capturing detailed physical processes and the availability of field data. The following constraints that model the yield response to water are based on Steduto *et al.* (2012) and Doorenbos and Kassam (1979). The initial soil water content is defined by Equation (3.23). The precipitation amount for each period j is defined in $Pe[j]$.

$$Wb[0] = Wb_{initial}[0] \tag{3.23}$$

$$\begin{aligned} Wb[j][k] \;&=\; Wb[j-1][k] + Pe[j] \\ &- ETo[j][k] \cdot (1 - Empt[j][k]) \\ &+ \gamma_{cov} \cdot (1 - Empt[j,k]) \cdot covSeed[j][k] \\ &- (1 - \gamma_{cov} \cdot covSeed[j][k]) \cdot ETacc[j][k], \\ &\; j = 1, \cdots, M, \quad k = 1, \cdots, K \end{aligned} \tag{3.24}$$

The available soil water index (ASI) for each Crop is defined in Equation (3.25) .

$$ASI[i][j][k] = \frac{(\sum_{v=j}^{j+cyc[i]} Pe[v]) + Wb[j][k] - [(1 - p[i]) \cdot Sa[k] \cdot D]}{ETm_{accum}[i][j][k]}, \tag{3.25}$$
$$i = 1, \cdots, N, \; j = 1, \cdots, M, \; k = 1, \cdots, K$$

The benefits of cover crops are realized when they have optimal development conditions, particularly in terms of water availability. Equation (3.26) ensures that they are not seed under extremely unfavorable conditions.

$$ASI[i][j][k] \geq \gamma_{cov} \cdot (1 - Com[i]), \tag{3.26}$$
$$i = 1, \cdots, N, \; j = 1, \cdots, M, \; k = 1, \cdots, K$$

The actual evapotranspiration ($ET_a[i][j][k]$) is estimated from the index $ASI[i][j][k]$, according to Equation (3.27).

$$ET_a[i][j][k] = u[i][j][k] \cdot cyc[i] \cdot F_{ETa}(ASI[i][j][k]), \tag{3.27}$$
$$i = 1, \cdots, N, \; j = 1, \cdots, M, \; k = 1, \cdots, K$$

The yield assessment estimation is presented in Equation (3.28), which uses the relationship between actual evapotranspiration and maximum evapotranspiration.

$$y_F[i][j][k] = 1 - ky + ky \cdot \frac{ET_a[i][j][k]}{ETm_{accum}[i][j][k]}, \tag{3.28}$$
$$i = 1, \cdots, N, \; j = 1, \cdots, M, \; k = 1, \cdots, K$$

For demonstration purposes, Figure 3.5 showcases a generic solution. Various crop families in this example are indicated by unique colors and hatch patterns. The resultant solution is a comprehensive, meticulously planned seeding schedule. The outcome solution is an entire well-planned seeding calendar. For illustration, we detail a generic solution in Figure 3.5. Crop families from this example are represented using distinct colors and hatch patterns. There is a set of 6 plots available to grow crops, and the farmland allocation problem expands to 104 weeks, which is equivalent to a 2-year-planning evaluation.

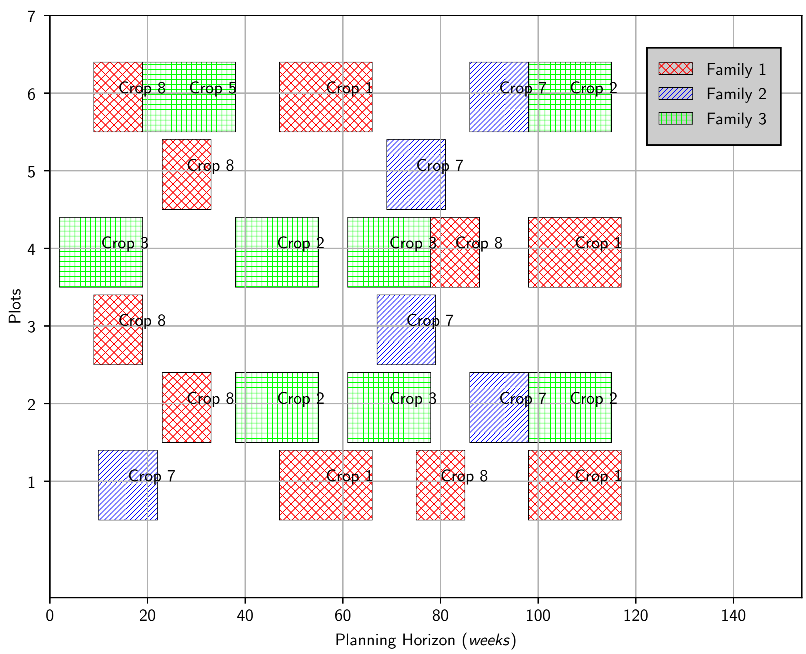

Figure 3.5 – An example of a crop sequence to demonstrate the Crop Rotation Problem in agriculture.

The commercial crops usually grown in our farmer's region are detailed in Table 3.6. As we can notice in this table, not all significant crops are profitable during the data analysis. Severe weather conditions, including longer heatwaves, frost, and droughts, have drastically interfered in the second season yields in the past years. Summer harvests are more stable and usually have higher incomes. Reports on market developments for agricultural commodities could profoundly transform the scenario in Table 3.6.

Many cover crops could be seeded in this subtropical climate. However, we select only the ones that are typically grown in the city of Tatuí and those that our farmer has previous successful experiences. They represent a group of three main cover crops as in Table 3.7. Cover crops generally reach their maximum soil improvement potential before the reproductive stages. Their living cycle should be interrupted before establishing seeds

Table 3.6 – The major commercial crops and average cost and yield from past harvest.

Crop Name	**Family**	**Yield Per Hectare (60-kg bag/ha) in 2023**	**Cost Per Hectare (60-kg bag/ha) in 2023**	**Average Cash Price Received by Farmer Per Unit in 2023**
Soybean	Legume	78.00	34.9	R$ 160.00
Summer Corn	Grass	157.00	67.95	R$ 71.00
Winter Corn	Grass	103.00	67.95	R$ 50.00
Sorghum	Grass	43.38	49.73	R$ 60.00
Wheat	Grass	37.87	26.68	R$ 66.00

by using selective herbicides or with a brush cutter. Consequently, we do not intend to seed cover crops with commercial expectations, but we hope to get better soil performance over the years.

We summarize our qualitative analysis of growing cover crops in Table 3.7. The main attributes we hold are the minimum produced dry matter once the farmer terminates the cover crop and the nitrogen fix from the air that is capacity found in legume crops. We also have an attribute named soil builder, which is the cover crop's ability ratio to produce organic matter and improve soil structure. The nitrogen scavenger ratio is the ability to absorb nitrogen from the surface soil and hold the surplus in the organic matter. Erosion fighter quality rates the capacity to sustain the soil integrity in the face of erosion agents, and it is related to the root system development. Weed-fighter is a measure of competitiveness; it defines how well the cover crops can compete with weeds through their life cycle and after termination. The last residue indicates how long the cover crop would protect the soil surface after termination. A long-lasting mulch even prevents the loss of soil water content over dried seasons. Clark (2012) inspire our cover crop's approach to the crop rotation problem.

Table 3.7 – Cover crop attributes.

Crop name	**Family**	**Cover Crop Costs Per Hectare**	**Minimum Dry Matter (kg /ha /ano)**	**Total N Source (kg/ha)**	**Nitrogen Scav-enger**	**Soil Builder**	**Erosion Fighter**	**Weed Fighter**	**Lasting Residue**
					Rating from 0 (not recommend) to 4 (excellent)				
Black Oats	Grass	R$ 150.00	2200	0	3	2	3	4	2
Fodder Turnip	Mustard	R$142.50	2600	0	2	3	3	3	1
Hairy vetch	Legume	R$36.00	2600	100	0	1	3	2	1

3.2 Computational tests

We implement our model and algorithms in *Python*, using the *Gurobi solver*. The crop assessment, yield, and profit are determined at the seeding period, meaning we account for harvest at the time of seeding rather than at the actual harvesting period. We justify this interpretation as an efficient way to use the index notation; otherwise, we would also need to establish a harvest index variable.

3.2.1 How would the forecasting yield affect the entire planning horizon?

In this subsection, we calibrate our model based on the price curves from CONAB data. Although production in agriculture highly depends on uncertainties, a reliable estimation is irreplaceable in any planning activity. Regional climate conditions and historical yield records are strong references. We introduce evaluations based on deviations from the average yields reported by our collaborative farmer.

The initial yield parameters are presented in Figure 3.6. Our first introduced variation is the small range of soybean yields. From 2023/2024, our farmer reported an average yield of 4680 kg/hectare (78 60-kg bag per ha); we take the range from 40 to 60 bags per hectare.

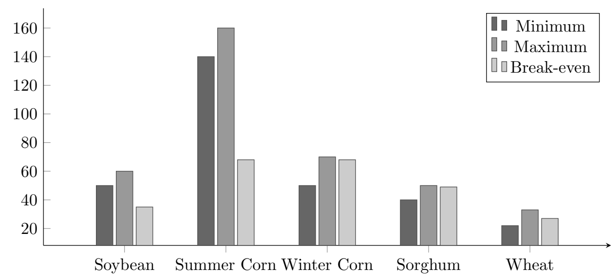

Figure 3.6 – Yield estimation for the entire planning horizon in Test 1.

In this test, soybean became unattractive, and corn took the lead as the primary crop. During the first year (52 weeks), winter corn is profitable; however, in the latter stages, we could not find any good choice for growing in the winter. This pattern matches the reality in the field. The reference farm sits in a subtropical region. Temperatures are chilly most of the year, neither extreme cold nor hot. Considering the pleasant weather in the farm location, irrigation machinery is not essential to reaching good yields, although it should reduce production risks in the winter crop season. Overall, the production forecast does not consider irrigation availability. Climate patterns such as El Nino or La Nina might interfere with the early seeding opportunities for the summer crop by postponing the dried season until mid-September, which delays harvest and stretches the winter crop seeding calendar.

Without accounting for the adjacency constraints or any particular demand for each crop, the most profitable growing pattern is the solution in Figure 3.7. Instead of family rotation, we assign fallow intervals; in other words, we leave the farmland fallow. Under sure soil moisture, seeds from the soil bank would germinate and introduce the required family rotation. In the second year of planning and the third one, fallow intervals are more significant and give plenty of time for soil recovery.

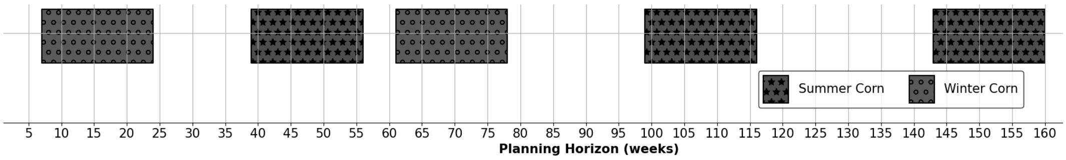

Figure 3.7 – The optimum profitable solution for each plot in Test 1.

Soybean production expectation is higher in Figure 3.8. We are close to the reality in the farm as the interval is set between 70 and 80 bags/hectare.

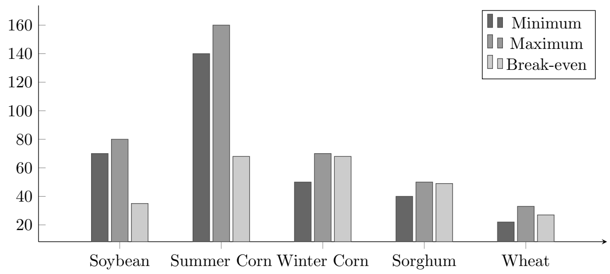

Figure 3.8 – Yield estimation for the entire planning horizon in Test 2.

Under this new trial, soybean is more competitive and secures a spot on the planning horizon. Comfortable yield interval for summer corn also guarantees some places in the optimal mix, as shown in Figure 3.9. Pondering between soybean and summer corn is quite common in the regional scenario; the cash price received by the farmer would make the final decision. From our cash prices charts in the previous subsection, there are some price slows in the most recent year harvests. As our test is based on these curves, soybean sale prices are smaller as we approach the final weeks in the planning horizon.

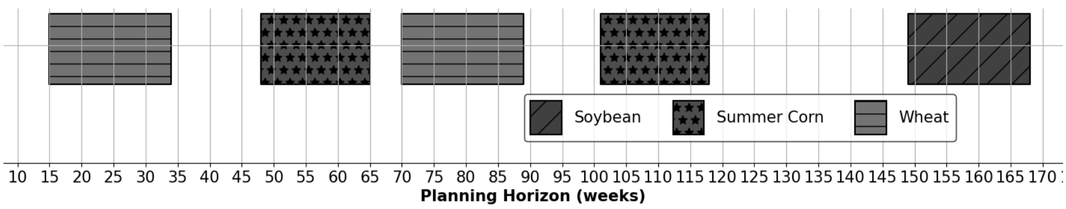

Figure 3.9 – The optimum profitable solution for each plot in Test 2.

Attaining high average corn production on a large scale is usually hard to sustain in the farmer context. His cultivable field is composed of rental fields, which involve distinct soil types, fertility, and tillage. Combining each field with its management history leads to significant variation in the outcome yields. In general, corn is more sensitive to soil characteristics than soybean. In this scenario, we reevaluate the summer corn production parameter. The lower range for summer corn yield is presented in Figure 3.10.

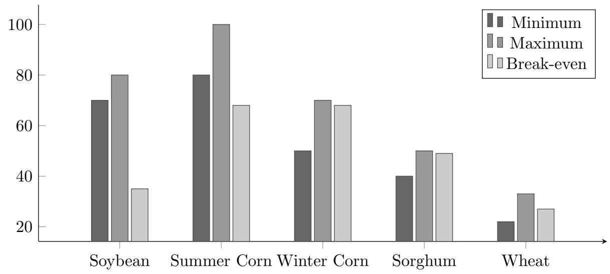

Figure 3.10 – Yield estimation for the entire planning horizon in Test 3.

We find the farmer's preferred crop pattern in Figure 3.11 — the rotation bases on soybean followed by wheat. Over the years, this crop combination provided stable results under several weather patterns, such as short drought and high temperatures. Hence, this crop scheme confirms a well-established cultivable pattern.

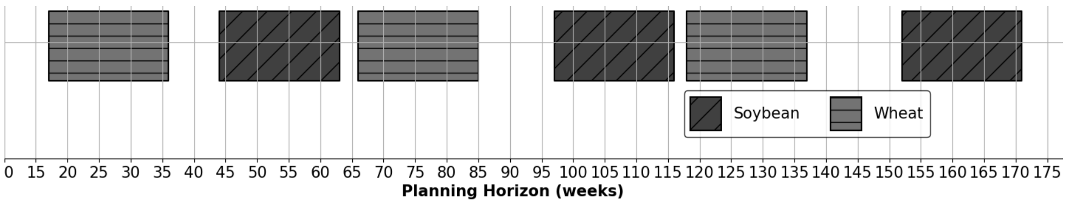

Figure 3.11 – The optimum profitable solution for each plot in Test 3.

We reshape our yield estimation in Figure 3.12. In this scenario, we consider better conditions for winter corn and sorghum.

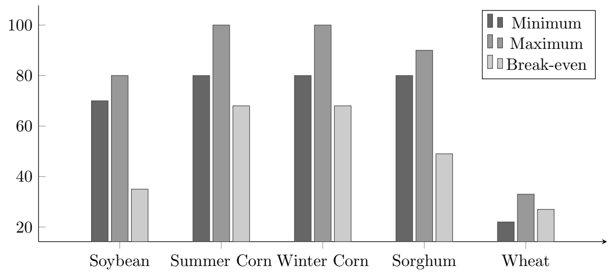

Figure 3.12 – Yield estimation for the entire planning horizon in Test 4.

Boosting the yields for both changes the optimal crop mix, although sorghum remains noncompetitive compared to winter corn and wheat. Higher prices for wheat in the last year of planning still benefit this crop instead of corn and sorghum, and even with poor production performance, wheat remains a good choice. The final combination is presented in Figure 3.13.

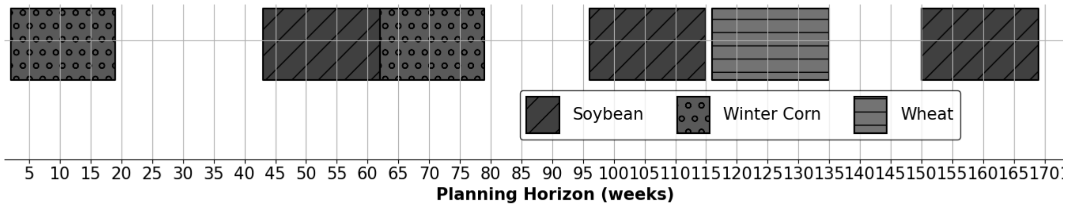

Figure 3.13 – The optimum profitable solution for each plot in Test 4.

In conclusion, based on our analysis of the farmer's experiences, the regional weather characteristics, and the generated cropping schemes in this section, we observe that the scenario favors soybean as a major summer crop. However, once the farmer significantly improves soil fertility and organic matter, producing corn in the summer with higher efficiency and yield will be more reliable. High-yield corn areas are more rewarding than soybeans. Winter crop remains surrounded by uncertainties, and we have indications that wheat is the best option. Without irrigation, high yields from winter corn or sorghum are highly improbable. The cash price farmers receive per unit of wheat is greater than the corn's price; wheat is more profitable, even producing less.

3.2.2 How would soil attributes drive decision-making?

Sustainable practices are in demand. Over the past decade, agriculture has overcome cultural obstacles to embrace environmental initiatives. Soil degradation and water pollution in rural areas become significant concerns in modern society. In the previous section, we discussed the economic approach in agriculture, which combination suits better the farmer's reality. In this current section, we take the crop rotation model as a sustainable initiative from which the farmer could improve soil attributes gradually.

From Test 5 to Test 9, we enable the minimum rating in dry matter, nitrogen source, nitrogen scavenger, soil builder, erosion fighter, weed fighter, and lasting residue. Improvements from each trial are reported in Figure 3.14.

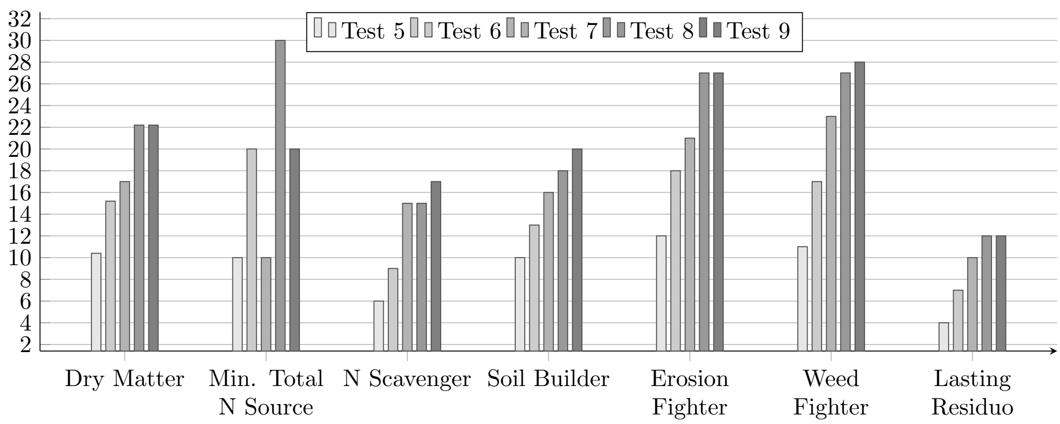

Figure 3.14 – Reported attributes from Test 5 to Test 9.

The solution in Figure 3.15 is the starting point. Even though cover crops are part of the solution, they do not replace the major commercial crops. They fill the gaps between significant crops. From the aggregation of the cover crops in the solution, the soil is more occupied than in Figure 3.7 and in Figure 3.9. Although leaving the field fallow improves the performance of the subsequent crop, there is more space to populate the soil with weeds, which could be challenging to manage.

Hairy vetch is well-known for fixing nitrogen in the soil surface, and corn is one of the crops with the highest nitrogen demand. Crop succession of hairy vetch and maize in the scheme from Figure 3.15 holds a positive impact in the proposed solution.

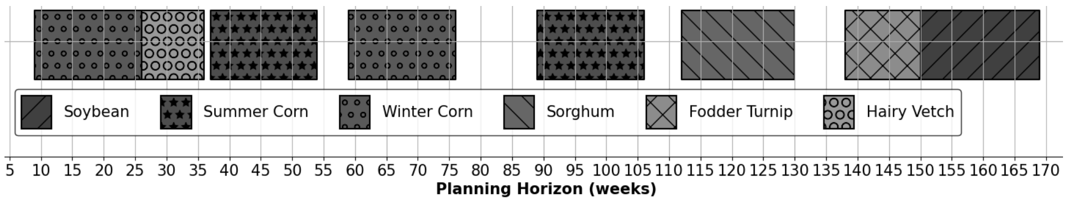

Figure 3.15 – The optimal scheme solution from Test 5.

From Test 6 to Test 9, we notice an increase in the soil occupation ratio. Cover crops usually are killed before reaching maturation, and their cycle could be adjusted to fit in the interval between major commercial crops.

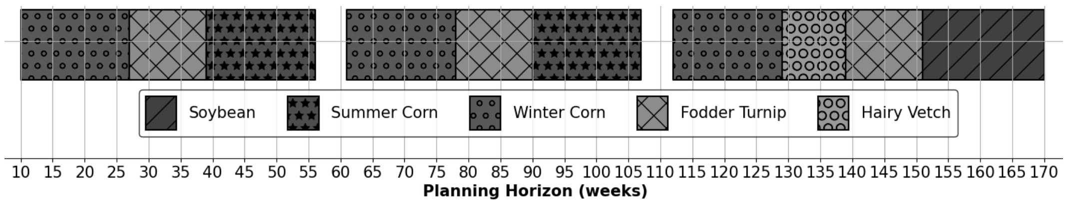

Figure 3.16 – The optimal scheme solution from Test 6.

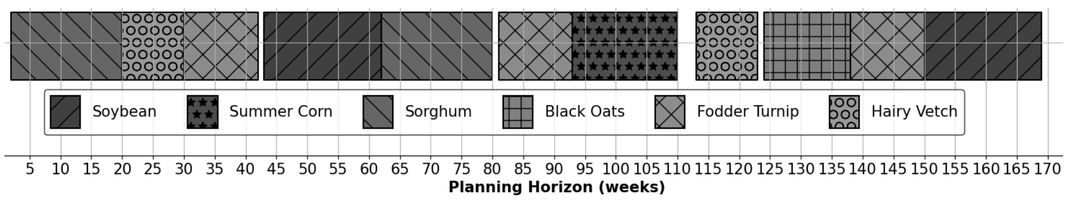

Figure 3.17 – The optimal scheme solution from Test 7.

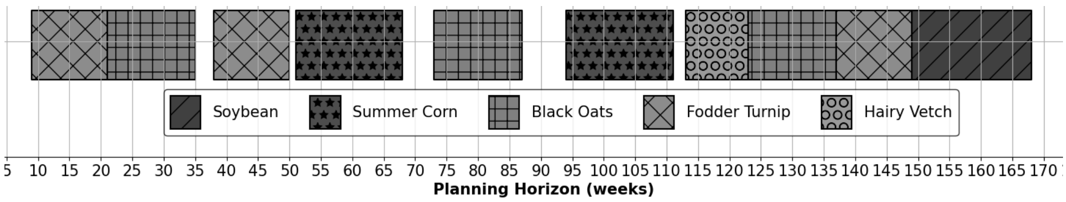

Figure 3.18 – The optimal scheme solution from Test 8.

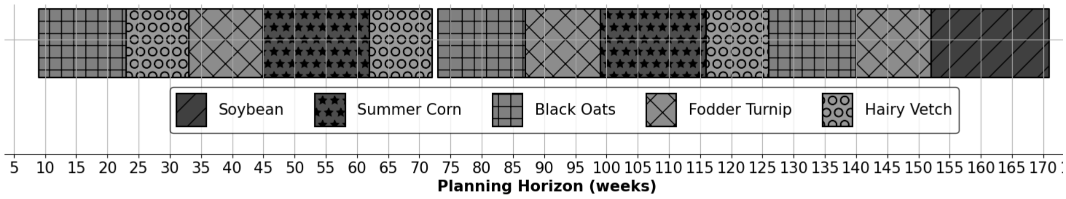

Figure 3.19 – The optimal scheme solution from Test 9.

Test 9 in Figure 3.19 stacked with many crops.

Introducing cover crops in the solution scheme affects the gross profitability of the farm. We should not focus strictly on the amount itself as we do not cover the cover crop's relationship with the following crop and its expected yield improvements in major crops.

One interesting parameter is the gross profitability from Test 3 and Test 6, which are almost equivalent. The crop scheme in Test 3 is composed of soybean and wheat, while the crop pattern in Test 6 replaces wheat with winter corn, fodder turnip, and hairy vetch and introduces summer corn. There are not any initiatives in Test 3 to enhance soil performance over the entire planning horizon. However, Test 6 is composed of many cover crops. Aside from the introduction of cover crops, the other change is the yield expectation from winter corn, which is more significant in Test 6 than in Test 3. Therefore, with a slight increase in winter corn yields, the combination of cover crops in Test 6 could easily outperform the standard soybean-wheat rotation.

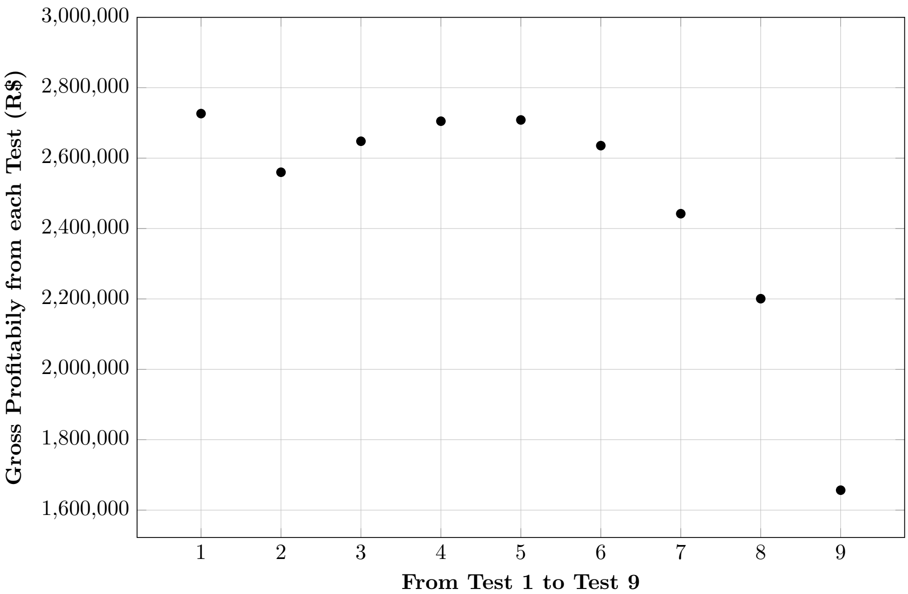

Figure 3.20 – Gross profitability from Test 1 to Test 9.

3.2.3 How costly is it to diversify farmland allocation to meet adjacent constraints?

Cropping family rotation in the field area neighborhood is one of the friendly environmental practices to prevent widespread pest problems. Although crop diversification in the cultivable group areas increases defenses as fortified barriers prevent mobility from one field to another, crop diversification would contribute little to any specific plot area enduring pest attacks.

Enforcing adjacency to all the plots sounds unwise. Figure 3.21 shows the adjacency restriction fully satisfied. A clear negative sign in this solution is the allocation emptiness. The crop set is tiny, as well as the number of crop families in the real database. Working with few crop families leaves short options to attain the constraint of adjacency, and consequently, we find long fallow intervals in the solution. Fulfilling the available spots in the planning horizon would be more accessible when the farmer can grow crops from many families besides legumes and grain.

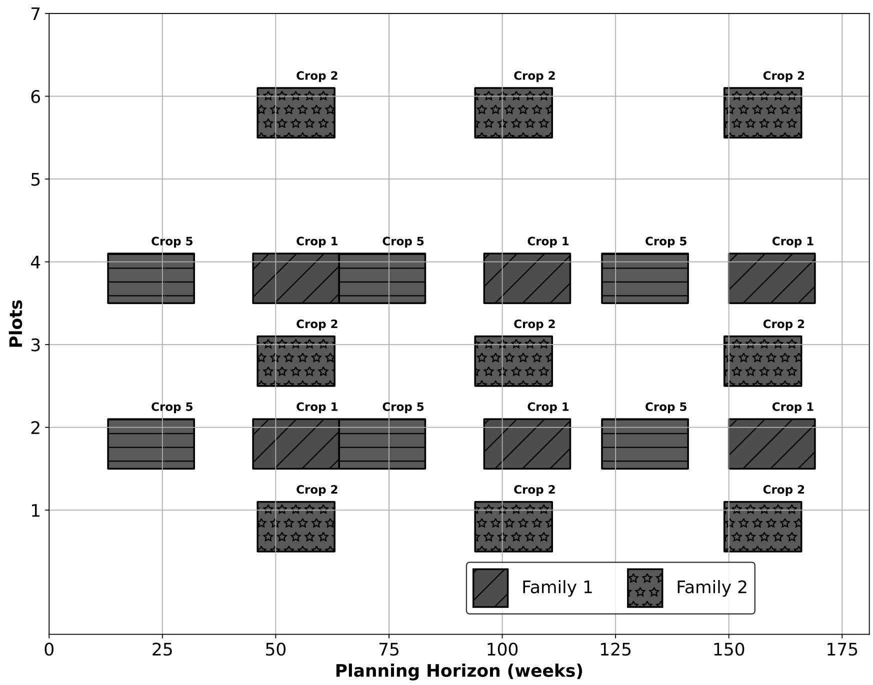

Figure 3.21 – Adjacent plots: fulfilling the family alternation constraint.

The best alternative is to reevaluate the adjacency. Grouping some plots would reduce the neighborhood. Respecting adjacency is critical on the interconnecting plots. Our central field area is Plot 3 in Figure 3.22. Plot 1 and Plot 2 constitute one group, and Plots 4, 5, 6, and 7 form another. The bridge between them is Plot 3.

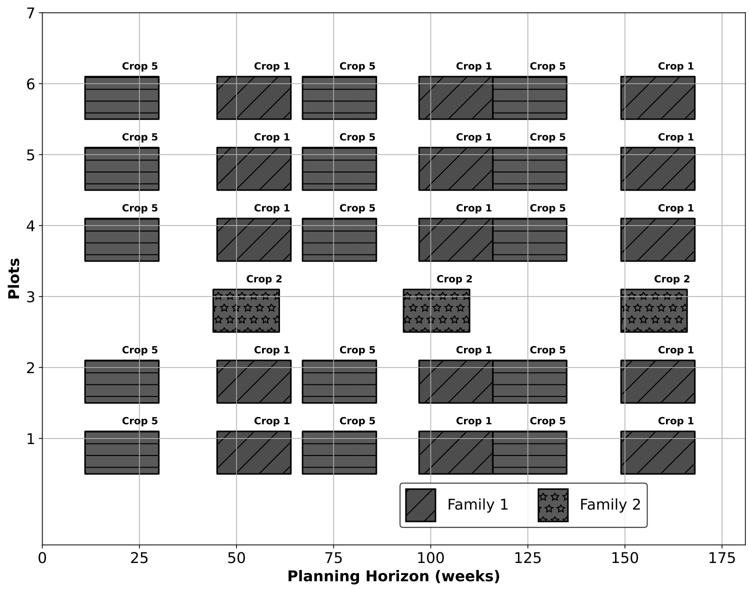

Figure 3.22 – Alternative adjacency plots and crop families.

In Figure 3.23, we observe a seeding schedule including cover crops in the planning horizon. A distinct scheme is presented in the bridge plot 3; others are distributed in two big groups.

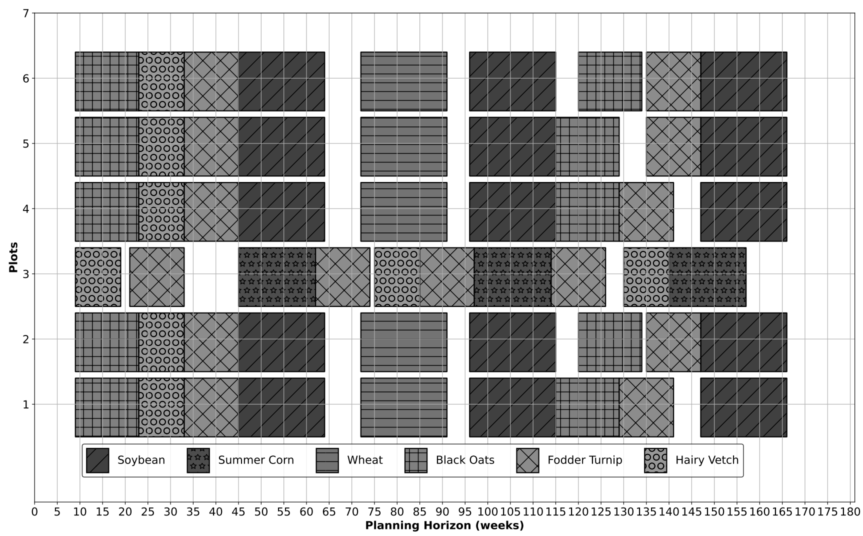

Figure 3.23 – Plots adjacent and crop groups: reduced neighboring crops

Comparing profits, the crop planning in Figure 3.21 generates R$ 1,713,070.00 in gross profits while the second planning in Figure 3.22 results in R$2,361,879.00 in gross profits and the last seeding schedule in Figure 3.23 produces R$ 2,064,957.00.

According to the field area and characteristics from Figure 3.1, the total cultivable area is 99.5 hectares, and the expected return per area is R$17,216.00 in Figure 3.21. From Figure 3.22, the return estimation per area is R$23,737.00. The same parameter for the farmland allocation in Figure 3.23 is R$ 20,753.00. In conclusion, without the relaxation of the adjacency constraint, farmers would endure small gross profits in the planning period.

3.2.4 Discussion on results

This subsection examines how limitations in soil water influence the development of the crop sequence. Yield and price are determined by average reports based on Tables 3.6 and 3.7. We introduce a minor deviation in these parameters to assess the sensibility of the optimization model. Initially, it may seem that cover crops would not be part of the solution, given that they incur expenses rather than generate cash, and the objective function aims to maximize profitability. Cover crops play an essential role in enhancing soil water balance, as illustrated in Equation (3.24), which has a notable impact on the yield of commercial crops. Consequently, integrating cover crops into agricultural practices can be a strategic approach to increasing commercial crops productivity. We have planned crop rotations that extend over a period of five years (10 semesters). This time-span is sufficient to assess the impact of field planning strategies.

We use historical climate data to simulate the next five years. *Relative precipitation* is a scalar parameter that we propose to create testing scenarios in which the precipitation pattern may decrease or increase proportionally. This approach allows us to study how changes in the precipitation pattern influence decision-making over an extended time frame.

For years 1 to 5, we refer to data from a weather station that records temperature, wind speed, humidity, and precipitation. For evaluation purposes, we alter the recorded values to test how variations in precipitation affect the decision-making process. A specific pattern that we look for in these simulations is the increase in cover crops when precipitation decreases. In stressful conditions, agricultural performance is significantly more robust when cover crops are regularly grown. Regular precipitation and overestimated precipitation occur in crop sequences 1 and 2, where the presence of cover crops is relatively lower than in the other sequences. Comparing the estimated profit from crop sequences 2 and 5, we find that the presence of cover crops almost matches the solution without cover crops, especially when considering crop sequences with only 0.9 precipitation over the same period. In the context of the proposed optimization model, we do not attribute increases to commercial crop yields. Instead, we consider their influence on water balance, believing that cover crops will support commercial crops in achieving their maximum evapotranspiration potential. As discussed in the Subsection 3.1.4, various studies have shown gains in crop production with cover crops. Hence, a value of γ_{cov} around 30% serves as a reliable reference for designing crop schedules. The crop sequences in Table 3.8 are generated based on 2010-2014 precipitation data and weather. Leaving uncultivated land is not planned as part of crop rotations; instead, it is a decision made to avoid planting crops that would not yield profit caused by the lack of soil moisture to reach a reasonable production level in any commercial crop available. We did not impose a strict limit on soil water storage capacity because our main concern is insufficient water rather than excessive moisture levels.

In addition to affecting actual evapotranspiration on cultivable land, cover crops offer other benefits that aren't considered in the optimization model as incomes. These benefits include weed suppression, improved nutrient availability for subsequent crops, and erosion control. We rate all crop sequences based on these characteristics, as illustrated in Table 3.9. The larger the attribute index, the more effective the crop sequence.

Although we do not disclose the weather patterns from the simulations, we observe that profitability is lower based on data from 2018. Unfortunately, weather stations recorded smaller volumes of rainfall from 2018 to 2023 compared to the period from 2010 to 2015. Rainfall has been below average in many regions of the world during this decade. Therefore, integrating cover crops may not be beneficial in the short term, but it will be essential for sustaining good average yields in the future.

The crop sequences in Table 3.10 are generated based on changes in relative precipitation. The results from crop sequences are presented in Table 3.11.

Table 3.8 – Pre-generated crop sequences selecting crops in five rotation years (Abbreviations: Bo = black Oats, Ft = fodder turnip, Hv= hairy vetch, Sg=sorghum, Soy = soybeans, Sc = summer corn, Wh=wheat, Wc = winter corn).

Crop Sequence	Relative Precipitation	Semester									
		1	2	3	4	5	6	7	8	9	10
1	0.7	Bo Sg	Bo Sc	Bo	Sc	Hv Wh	Sc	Wh	Sc	Wh	Sc
2	0.8	Bo Sg	Bo Sc		Bo Sc	Wh	Bo Sc		Bo Sc	Wh	Sc
3	0.9	Ft Sg	Sc	Wh	Sc	Wh	Bo Sc	Ft	Sc	Wh	Sc
4	1.0	Ft Sg	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc

Table 3.9 – Outcomes from algorithm-derived crop rotations utilizing meteorological data from 2010 to 2014.

Crop Sequence Number	Estimated Profit per Hectare (ha)	Accumulated Dry Matter (kg/ha)	Total Nitrogen Fixing (kg/ha)	Accumulated Soil Improvements Index				
				Nitrogen Scavenger	Soil Builder	Erosion Fighter	Weed Fighter	Lasting Residue
1	R$ 23,861.41	10,400	100	6	10	12	11	4
2	R$ 30,307.49	13,000	0	10	15	15	15	5
3	R$ 35,283.96	7,000	0	8	7	9	11	5
4	R$ 39,830.23	2,200	0	3	2	3	4	2

Table 3.10 – Algorithm-generated crop rotations over a span of five years without cover crops (Abbreviations: Bo = black Oats, Ft = fodder turnip, Hv= hairy vetch, Sg=sorghum, Soy = soybeans, Sc = summer corn, Wh=wheat, Wc = winter corn).

Crop Sequence	**Relative Precipitation**	**Semester**									
		1	2	3	4	5	6	7	8	9	10
5	0.7	Ft Sg	Sc	Hv Wh	Sc	Wh	Sc	Wh	Soy	Ft	Bo Sc
6	0.8	Bo Sg	Bo Sc		Bo Sc	Wh	Soy	Ft	Bo Sc	Wh	Sc
7	0.9	Bo Sg	Bo Sc		Bo Sc	Sg	Bo Sc	Wh	Sc	Wh	Sc
8	1.0	Hv Sg	Bo Sc	Bo	Bo Sc	Hv	Sc	Sg	Bo Sc	Wh	Sc

Table 3.11 – Outcomes from algorithm-derived crop rotations utilizing meteorological data from 2018 to 2022.

Crop Sequence Number	Estimated Profit per Hectare (ha)	Accumulated Dry Matter (kg/ha)	Total Nitrogen Fixing (kg/ha)	Accumulated Soil Improvements Index				
				Nitrogen Scavenger	Soil Builder	Erosion Fighter	Weed Fighter	Lasting Residue
5	R$ 12,536.32	9,600	100	8	8	12	13	6
6	R$ 18,020.80	12,600	0	11	14	15	16	6
7	R$ 23,821.01	10,400	0	8	12	12	12	4
8	R$ 27,534.17	15,600	200	8	14	18	16	6

If there are no cover crops in the rotation, the yields from commercial crops are likely to suffer. Unprotected soil is highly susceptible to evapotranspiration. Cover crops and their residues are vital to avoid erosion and runoff in wet seasons, as well as to minimize evapotranspiration during important dry periods. Table 3.12 illustrates the crop sequence produced without the inclusion of cover crops within the subset. Despite implementing family rotation by alternating crops between legume and grass families, maize and soybean dominate the summer season, while wheat remains the safest option for winter.

Table 3.12 – Algorithm-generated crop rotations over a span of five years without cover crops (Abbreviations: Bo = black Oats, Ft = fodder turnip, Hv= hairy vetch, Sg=sorghum, Soy = soybeans, Sc = summer corn, Wh=wheat, Wc = winter corn).

Crop Sequence	**Semester**									
	1	2	3	4	5	6	7	8	9	10
9	Sg	Sc	Wh	Soy	Wh	Soy	Wh	Sc	Wh	Sc
10	Sg	Sc	Wh	Soy	Wh	Sc	Wh	Sc		Sc
11	Wh	Soy	Wh	Soy	Wh	Sc		Sc	Wh	Sc
12	Sg	Sc	Wh	Soy	Wh	Soy	Wh	Sc		Sc
13	Wh	Sc	Wh	Soy	Wh	Soy	Wh	Soy	Wh	Sc
14	Wh	Soy	Wh	Sc	Wh	Soy	Wh	Soy	Wh	Sc

Table 3.13 estimates profitability from the crop sequences in Table 3.12. They are quite smaller than previous results in Tables 3.9 and 3.11. When precipitation surpasses historical averages, yield projections for commercial crops achieve their peak potential, and the resulting outputs without cover crops align with estimated profitability

similar to integrating cover crops. However, during dry seasons, we observe that the resilience provided by cover crops becomes evident. The expenses associated with cover crops are outweighed by the benefits observed in the next commercial crop, considering only the effect on evapotranspiration and not accounting for the potential yield increase due to nutrient availability from cover crop residues.

Table 3.13 – A comparative analysis of crop sequences that do not include cover crops.

Crop Sequence Number	**Estimated Profit per Hectare (ha)**	**Relative precipitation**	**Weather historical data**
9	R$ 14,090.05	0.70	Jan/2010 to Dec/2014
10	R$ 30,742.14	1.00	Jan/2010 to Dec/2014
11	R$ 39,952.08	1.50	Jan/2010 to Dec/2014
12	R$ 2,760.78	0.70	Jan/2018 to Dec/2022
13	R$ 17,271.39	1.00	Jan/2018 to Dec/2022
14	R$ 36,455.22	1.50	Jan/2018 to Dec/2022

Under highly adverse conditions, Crop sequence 1 significantly outperforms Crop sequence 9, primarily due to the presence of cover crops. Crop sequence 9 achieves only 59% of the estimated profit generated by Crop sequence 1. The most unfavorable comparison is between Crop sequence 12 and Crop sequence 5. Both are subjected to the same weather conditions, but Crop sequence 5 includes several cover crops, whereas Crop sequence 12 has none. Consequently, Crop sequence 12 reaches only 22% of the output of Crop sequence 5.

When the relative precipitation is adjusted to one or higher —indicating that the precipitation matches or exceeds historical weather data — the disparity between crop sequences with and without cover crops diminishes. For instance, crop sequence 11 generates higher profits compared to crop sequence 4.

3.2.5 Assessing outcomes from various cities involved in grain production across Brazil

In the preceding subsection, we examined crop sequences generated for the Sorocaba region, where our collaborative farmer is situated, and explored various aspects regarding the strategic utilization of our proposed model within this local framework. Now, however, we aim to broaden our analysis by applying it to a larger group of grain farmers, producing crop sequences for numerous locations throughout the country. For different regions of Brazil, optimized cropping plans were developed to understand the agricultural patterns among Brazilian grain producers. The cities listed in Table 3.14 are significant grain production hubs in Brazil.

Table 3.14 – Algorithm-generated crop rotations over a span of five years without cover crops (Abbreviations: Bo = black Oats, Ft = fodder turnip, Hv= hairy vetch, Sg=sorghum, Soy = soybeans, Sc = summer corn, Wh=wheat, Wc = winter corn).

Crop Sequence	**Location**	**Semester**									
		1	2	3	4	5	6	7	8	9	10
15	Barreiras-BA	Ft Sg	Sc	Sg	BoSc	Wh	Sc	Ft	Bo Sc	Wh	Sc
16	Castro-PR	Bo Sg	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc
17	Chapadão do Sul-MS	Ft Sg	Sc	Wh	BoSc	Bo	Sc	Sg	Bo Sc	Wh	Sc
18	Rio Verde-GO	Ft Sg	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc
19	Sorocaba-SP	Bo Sg	Bo Sc	Bo	Sc	Wh	Bo Sc		Bo Sc	Wh	Sc
20	Sorriso-MT	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc
21	Uberaba-MG	Wc	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc
22	Barreiras-BA	Ft Sg	Sc	Hv Wh	Sc	Wh	Soy	Wh	Sc	Wh	Sc
23	Castro-PR	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc
24	Chapadão do Sul-MS	Ft Sg	Sc	Sg	BoSc	Wh	Sc	Wh	Sc	Wh	Sc
25	Rio Verde-GO	Ft Sg	Sc	Wh	Sc	Wh	Sc	Sg	Sc	Wh	Sc
26	Sorocaba-SP	Bo Sg	Bo Sc		Bo Sc	Wh	Sc	Wh	Sc	Wh	Sc
27	Sorriso-MT	Bo Sg	Sc	Wh	Sc	Wh			Sc	Wh	Sc
28	Uberaba-MG	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc	Wh	Sc

Table 3.15 presents the projected profits for each crop rotation. The significant variations in the results are attributed to the weather conditions. The erratic nature of rainfall influences the solver's choice to bypass unprofitable cropping allocations, opting for leaving fields fallow instead of incurring losses.

Cover crops are excluded from the solution only under exceptionally favorable weather conditions, as illustrated in Table 3.14. In all other cases, they are essential for reducing evapotranspiration and assisting commercial crops in achieving maximum yield. Cover crops attributes are displayed in Tables 3.16 and 3.17.

We should consider that the yield estimations are grounded in the analysis provided by a decision maker who utilizes comprehensive weather data. Consequently, the significant decrease in profitability observed in crop sequence 2, compared to other sequences, arises from a water deficit that impairs the allocation of any economically viable crops. Our optimization approach, due to its comprehensive consideration of weather, avoids taking risks. However, farmers may face some risk in their decisions, either aiming for price spikes or hoping for unanticipated shifts in weather conditions.

Table 3.15 – A comparative assessment of projected profits derived from solver solutions across various Brazilian locations.

Crop Sequence Number	**Location**	**Estimated Profit per Hectare(ha) 2010-2014**	**Crop Sequence Number**	**Estimated Profit per Hectare(ha) 2018-2022**
15	Barreiras-BA	R$ 22,783.70	22	R$ 11,995.20
16	Castro-PR	R$ 41,839.46	23	R$ 45,239.73
17	Chapadão do Sul-MS	R$ 34,709.86	24	R$ 38,927.07
18	Rio Verde-GO	R$ 35,821.26	25	R$ 38,268.21
19	Sorocaba-SP	R$ 33,215.84	26	R$ 27,440.63
20	Sorriso-MT	R$ 44,118.75	27	R$ 35,696.93
21	Uberaba-MG	R$ 44,066.03	28	R$ 45,145.96

Table 3.16 – Outcomes from algorithm-derived crop rotations utilizing meteorological data from 2010 to 2014.

Crop Sequence Number	Accumulated Dry Matter (kg/ha)	Total Nitrogen Fixing (kg/ha)	Accumulated Soil Improvements Index				
			Nitrogen Scavenger	Soil Builder	Erosion Fighter	Weed Fighter	Lasting Residue
15	9,600	0	10	10	12	14	6
16	2,600	0	2	3	3	3	1
17	10,000	0	9	11	12	13	5
18	2,200	0	3	2	3	4	2
19	13,000	0	10	15	15	15	5
20	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0

Table 3.17 – Outcomes from algorithm-derived crop rotations utilizing meteorological data from 2018 to 2022.

Crop Sequence Number	Accumulated Dry Matter (kg/ha)	Total Nitrogen Fixing (kg/ha)	Accumulated Soil Improvements Index				
			Nitrogen Scavenger	Soil Builder	Erosion Fighter	Weed Fighter	Lasting Residue
22	4,800	100	3	3	6	6	3
23	0	0	0	0	0	0	0
24	4,800	0	5	5	6	7	3
25	2,200	0	3	2	3	4	2
26	7,800	0	6	9	9	9	3
27	2,600	0	2	3	3	3	1
28	0	0	0	0	0	0	0

3.3 Final remarks and conclusion

Our research in this paper reshapes the famous Crop Rotation Problem. We collect and analyze actual data from a collaborative farmer. The information taken directly from the source and the contributions made by the main stakeholder, our collaborative farmer himself, leads this research in a wealthy pathway that is deeply connected with the agribusiness challenges. With a renewed vision of the agrarian problem, we dedicate our time and effort to proposing an optimization model that embraces the farm management challenge in planning many years from scratch. Our proposed solution follows the standard crop rotation elements and the financial ecosystem surrounding farming activity (revenues, costs, and profits). Aside from the main features of crop rotations, we enhance the model with cover crops and nutrient supply evaluation. In the modeling process, we elaborate an efficient set of constraints, and we can run large instances of the rotation problem. We have fully stretched the boundaries of the proposed optimization methodology and provided a robust analysis of the real problem. Our gathered results attained the current farmer necessity at hand. Our optimization strategy involving cover crops brings environmental benefits, assisting farm management practices dealing with contemporaneous scarcity of mineral resources.

After carefully considering the economic scenario from our actual data application, we observe that corn average yields can sustain significant growth over the following years with increasing investments in farming practices. Therefore, corn could even replace soybean as the main crop due to the greater production. Once farmers reach stability in production averages over the years, we would notice risk reduction in the farm business as corn prices are less volatile in the harvest season than soybeans. Another benefit of corn expansion is that the seeding window is more significant than soybeans.

Our next step is leaning toward the hot topics in Artificial Intelligence. In that sense, exploring artificial neural networks in highly combinatorial models, such as the crop rotation problem, would allow the incorporation of other fundamental ecological patterns that affect farm management decisions. Besides our achievements in agriculture, our modeling approach to the problem mechanism could also reach other research areas as it is a scheduling problem.

4 Developing a dynamic programming model to assist weed management in soybeans

Agriculture technology undertook large evolutionary steps in the past century, reaching unimaginable yield growths along all the cultivable crops (OLIVEIRA *et al.*, 2021). From soybeans, breeding of improved cultivars with higher yield potential combined with no-tillage growing system and herbicide developments transformed a crop grown mostly for forage in the early twentieth century into a worldwide major cultivable crop (WARREN, 1998).

Until the late 1950s, nearly all soybean weed control was done by tillage before and after seeding, managing weeds close to the soybean harvest was only possible when manually removed from the field, weed competition in the harvest causes significant yield losses. Nave and Wax (1971) reported even an average yield could drop 25% by a certain weed infestation and the total machine losses were around 50% in unmanageable weed infestation. The use of pre-emergence and post-emergence herbicides reduced drastically the labor force required to deal with weeds and increased weed efficacy control in the field (OLIVEIRA *et al.*, 2017; GIANESSI; REIGNER, 2007).

Herbicides discoveries have their boom from 1940 to 1980. Aside from new herbicide designs, selectivity (i.e. selectivity refers to the capacity of a particular herbicide to kill a certain group of weeds in a post-emergence state of the grown crop, without affecting yield or grain quality) and the expansion of the application window spanning from many crop stages have profoundly change the agrarian scenario. Although this prosperous era provided almost all the major commercial herbicides, the following decades has been not so much prolific in new herbicide site of action (SOA) (OLIVEIRA *et al.*, 2021; DUKE, 2012).

Diversity is the dominant pattern in the Brazilian agriculture. Brazilian farm production spans from food to fibers and other industrial raw materials (CARBONARI; VELINI, 2021). Farmers have long used pesticides to sustain high yields and efficiency of crops in land use, consequently, Brazil market is the second largest user of pesticides worldwide (FAOSTAT, 2023), Brazilian farms consumes 377 thousands of pesticides tonnes in 2020.

As farms expand, the utilization of pesticides proportionally increases, Carbonari and Velini (2021) defend that the Environmental Impact Quotient (EIQ) continuously drooped as the new market products are more efficient and present lower risks to the environment. They also pointed out that there are many degradation and dissipation processes that reduces pesticide residues when applied in the fields. These set of processes drives the amounts of pesticides present in food products to extremely small fractions and in compliance with regulatory agencies.

Weed management is crucial in any major crop. Uncontrolled pests can lead to production losses over many years and multiple crops. Although chemical control has the greatest efficacy, farmers' reliance on herbicides causes undesirable effects such as the emergence of resistant weeds. Among many samples, populations of horseweed (*Conyza canadensis*, *Conyza bonariensis* and *C. sumatrensis*) resistant to glyphosate are widespread in Brazilian fields. We could see in Figure 4.1 the weed infestation of horseweed competing with soybean. Trezzi *et al.* (2013) conducted an experiment to evaluate the loss of grain yield caused by horseweed competition and the reported average loss was 25%.

Figure 4.1 – Typical horseweed infestation competing with soybeans.[1]

Management of sourgrass (*Digitaria insularis*) is another huge challenge in Brazilian soybean fields. Wide distribution of glyphosate-resistant bio-types sourgrass are one the main reasons to hand sourgrass infestation (GAZZIERO *et al.*, 2012). Figure 4.2 illustrates an infested soybean field with sourgrass and other grass plants.

1 Source: Image from author's ownership

Figure 4.2 – Grass species like sourgrass and others are in competition with soybean plants. [2]

Taking into account the challenges of weed management in soybeans, we develop a dynamic programming model to assist decision-making on the farm. Our special attention to dynamic programming techniques are naturally motivated about the progressing states we observe in the field. Among them are soybean growth pattern, the weed competition impact over the crop development and even the weather influence. There are many state transitions driven by the managerial decisions taken in the agricultural environment. The objective of this chapter is to optimize herbicide use through a dynamic programming solution that balances environmental conditions, agricultural needs, and cost-efficiency.

The research question we would like to answer in this section are:

- How can we translate the weed control problem into an optimization problem?
- Can we reproduce the farmers' expertise in the generated solutions?
- Which are our boundaries with the dynamic programming approach?
- Could we properly assist the farmer with the most recommend course of action?

The influence of weather on optimal herbicide application conditions is addressed in Section 4.1. Moving to a more detailed examination of herbicides, Section 4.2 covers the herbicides commonly used in Brazilian soybean agriculture. Section 4.3 presents a brief discussion about the main weed species in the Brazilian soybean crop. Particularly, sourgrass, as one of the hardest weeds to manage in cultivable areas, is our chosen weed

[2] Source: Image from author's ownership.

to evaluate under the dynamic approach. Our proposed methodology exhibits in Section 4.4. Our current results are part of Section 4.5. Our final evaluation is Section 4.6. In addition, Subsection 4.6.1 holds some remarks about further steps in the research field.

4.1 The impact of weather conditions in Herbicide Spray Tank Applications

Based on the literature (MATZENBACHER *et al.*, 2014; CIESLIK *et al.*, 2013; PENCKOWSKI *et al.*, 2003), the recommended temperature for applying herbicides is between 20°C and 30°C. Relative humidity should exceed 50%, and wind speed must be less than 10 km/h. Rainfall occurring soon after spraying may diminish the absorption and translocation of herbicides, thereby affecting weed control.If precipitation happens before the herbicide reaches rain-fastness, its effectiveness will be diminished. It is not recommended to spray immediately following rainfall. It is essential to allow the leaves to dry before applying the spray to ensure the product's effectiveness. For spraying to be effective and consequently manage weeds, specific weather conditions need to be met. Spraying in poor conditions might result in the need for an additional weed application to achieve desired results. Therefore, the criteria we utilize to assess the timing for herbicide spraying are outlined as follows:

- Temperature should range from 20 $°C$ to 30 $°C$
- Wind speed should be within 3 to 10 m/s
- There should be no rainfall in the last four hours or expected in the next four hours, nor more than 20 mm of rain in the past 24 hours
- Optimal radiation levels for spraying are deemed to exceed 300 kJ/m^2

In crop growth management, applying selective herbicides is essential to minimize the impact of weeds. Failing to properly attend to the appropriate spraying conditions can result in significant signs of phytotoxicity (PENCKOWSKI *et al.*, 2003; MIOLA *et al.*, 2020; KIICHLER *et al.*, 2023). Figure 4.3 illustrates a typical phytotoxicity effect on soybeans, exemplifying inadequate herbicide application under adverse environmental conditions.

Appendix B provides a concise summary of the weather stations from which we collected data to assess spraying viability from 2010 to 2024. Using information from

[3] Source: Image from author's ownership.

Figure 4.3 – Phytotoxicity in soybean crop caused by selective herbicide sprayed under high temperature. [3]

633 weather stations operated by the Instituto Brasileiro de Metereologia (INMET), we examined the number of hours suitable for fieldwork under ideal weather conditions. Our analysis centers on a tractor-operated herbicide sprayer that requires firm soil conditions to function effectively without harming the developing crop. We base our analysis in terms of:

- Total precipitation, hourly [$mm\ of\ water$]
- Air temperature - Dry Bulb, hourly [$°C$]
- Global radiation, hourly [kJ/m^2]
- Wind velocity, hourly [m/s]

For successful herbicide application, it is essential to consider a combination of weather conditions. Unfavorable weather can lead to inadequate weed control, herbicide loss due to wind drift, and other negative consequences. In Table 4.1, we display the mean values across Brazilian states. This table underscores the limited hours during which the optimal conditions for herbicide application are met. The column named "Temp" presents the average monthly hours where temperature is between 20 $°C$ to 30 $°C$, the next column denoted "Temp. and W. Speed" refers to the average monthly hours where temperature criteria is met and also wind speed criteria is satisfied.

Table 4.1 – Soybean stages and the adequate position of herbicides: the proper spraying stage.

Average monthly hours suitable for herbicide application in each Brazilian State						
Federative Unit	**Temp.**	**Temp. and W. Speed**	**Prec.**	**Prec. and Rad.**	**Temp., Prec., W. Speed and Rad.**	**Prec. and Wind.**
PB	303.33	40.07	73.93	44.56	25.43	77.13
RN	288.7	49.75	78.94	40.72	26.65	76.95
PE	291.81	40.99	78.64	46.76	26.23	76.54
DF	227.36	40.48	78.39	69.73	32.82	74.63
AL	313.06	27.98	66.05	36.71	17.25	53.99
BA	288.02	29.99	71.13	44.83	18.63	53.65
SE	319.94	30.43	65.08	37.93	16.45	52.64
CE	277.06	29.73	79.03	36.93	18.24	50.53
RS	160.62	42.71	75.1	61.28	28.71	46.12
PR	204.11	31.31	74.98	58.88	21.03	42.93
GO	255.66	21.47	73.4	52.43	15.74	40.24
ES	264.44	23.97	69.43	47.92	14.98	39.6
MG	227.37	22.97	77.02	60.87	17.23	39.18
RJ	250.64	24.94	73.04	49.54	15.63	39.18
PI	261.47	22.08	80.13	36.66	14.35	37.53
MS	241.95	24.26	73.8	46.75	14.33	34.68
SC	165.77	30.29	71.89	59.81	20.24	33.55
SP	215.9	20.94	74.55	57.25	14.56	31.44
MA	279.72	15.32	71.12	33.46	9.74	27.24
TO	259.37	16.31	67.07	34.61	10.13	26.27
MT	271.64	14.97	68.26	37.04	8.22	22.32
PA	303.51	9.78	64.2	30.58	5.52	16.74
RR	281.05	10.87	58.16	17.56	5.92	16.65
RO	296.71	9.24	62.41	29.8	3.78	11.22
AC	315.59	5.25	65.57	35.45	2.65	8.35
AP	314.48	5.27	46.06	21.75	1.98	6.24
AM	301.8	2.21	52.81	23.75	0.7	2.11

Temp.: Temperature; W. Speed: Wind Speed; Prec.: Precipitation; Rad.: Radiation

We reunite the best stations and the worst stations in terms of mean ideal hours in Table 4.2. In the State of Amazonas, some of the least favorable conditions for applying herbicides include high temperatures, an extensive rainy season, and frequent cloudy days. Despite the fact that much of the region is classified as non-arable and preserved as a natural resource, the average state parameters revealed extremely limited prospects for addressing crop needs because of its tropical nature.

Table 4.2 – Soybean stages and the adequate position of herbicides: the proper spraying stage.

Average monthly hours suitable for herbicide application in each Brazilian State							
Federative Unit	**Weather Stations**	**Temp.**	**Temp. and W.Speed**	**Prec.**	**Prec. and Rad.**	**Prec. Wind. Rad.**	**Prec. Wind.**
RN	NATAL	377.43	74.02	67.23	48.04	41.55	156.83
TO	MATEIROS	279.47	69.76	78.24	60.94	52.88	147.77
MA	FAROL de SANTANA	332.13	70.04	78.73	50.21	44.36	147.32
CE	TIANGUA	350.96	72.88	73.56	42.75	39.6	138.97
BA	REMANSO	315.64	64.13	91.19	54.67	43.97	138.78
PA	DTCEA JACAREA-CANGA	306.04	0.34	92.93	0.0	0.0	0.0
MG	EB PEF BONFIM	310.45	32.39	48.23	0.0	0.0	0.0
AM	CRMN MANAUS	309.17	0.34	82.24	0.0	0.0	0.0
AM	DTCEA TABATINGA	388.82	0.0	7.15	0.0	0.0	0.0
AM	DTCEA TEFE	314.62	0.52	66.55	0.0	0.0	0.0

Figure 4.4 illustrates the annual trend of available hours for two sites, showcasing areas where weather conditions remain optimal for the longest duration within the chosen locations. From May to September, *Mateiros* reported the closest match between the ideal condition and the average hours when temperature is between 20 and 30 $°C$. In *Natal*, the time frame from October to February records the highest averages under ideal conditions. The lines and their distribution depict the average monthly hours during which the appropriate temperature is achieved. The box-plot shows a small interval of hours allocated for herbicide application, during which the conditions of temperature, precipitation, wind speed and radiation are satisfied. For example, although Natal generally experiences over 350 hours with suitable temperatures, it exceeds just 200 hours when it comes to sufficient radiation, wind, and rainfall.

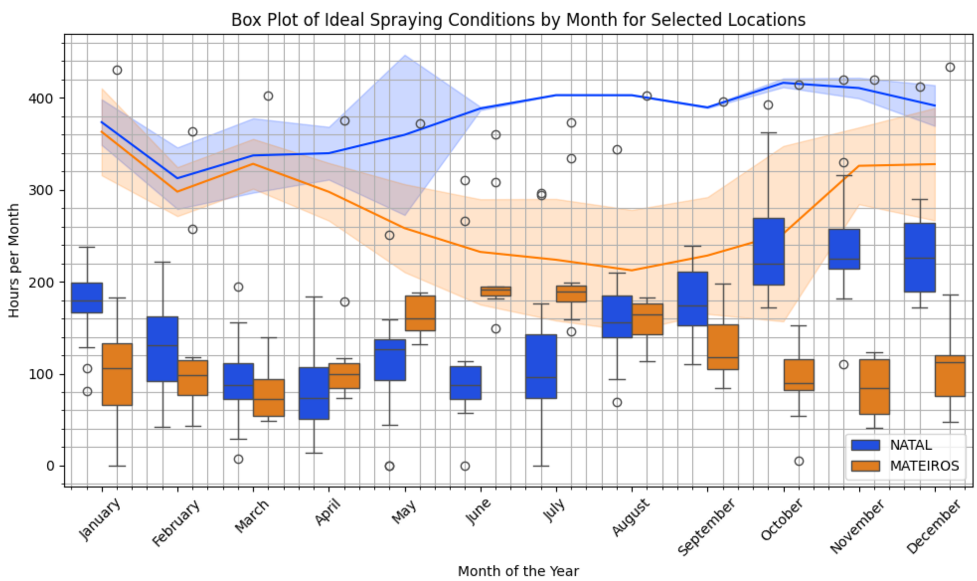

Figure 4.4 – An examination of spraying conditions in Mateiros and Natal.

4.2 A comprehensive approach into typical soybean's herbicides

Based on farmer's expertise and data, we have selected a group of herbicides in another to handle weed infestation in soybean fields. Product's name and their classes are introduced in Table 4.3. Herbicides have their recommended interval based on the soybean growth, we manage to summarize each proper interval from the selected herbicides in Table 4.1.

Table 4.3 – A brief introduction of major herbicides commonly used in a grain farm from the state of São Paulo, Brazil.

Index	Selective Herbicide	Class
1	Cloransulam-methyl	Selective herbicide
2	Chlorimuron-ethyl	Post-emergent, systemic selective herbicide from the sulfonylurea chemical group
3	Saflufenacil	Conditional selective contact herbicide
4	2,4-D	Selective, systemic post-emergence herbicide from the aryloxyalkanoic acid chemical group
5	Glufosinate	Non-selective total action herbicide
6	Glyphosate	Conditional selective, systemic action herbicide
7	Carfentrazone-ethyl + naphtha solvent	Post-emergent, conditional selective herbicide with non-systemic action
8	Sulfentrazone + Diuron + 1,2-ethanediol	Pre-emergent, conditional selective herbicide with systemic action
9	Diclosulam	Selective herbicide
10	Haloxyfop-R-methyl + Ethylene glycol monoethyl ether	Systemic action selective herbicide
11	Cletodim	Systemic herbicide

Table 4.4 – Soybean stages and the adequate position of herbicides: the proper spraying stage.

Index	Selective Herbicide	Soybean Preseed Burn-off	Soybean Pre-emergence	Soybean Post-emergence	Soybean Pre-harvest	Maximum spraying number per crop cycle
1	Cloransulam-methyl	N.R.	N.R.	R.	R.	1
2	Chlorimuron-ethyl	N.R.	N.R.	R.	R.	1
3	Saflufenacil	S (minimum interval R. of 10 days before planting)	N.R.	N.R.	R.	1
4	2,4-D	R (at least 7 days before planting)	N.R.	N.R.	N.R.	1
5	Glufosinate	R (at least 7 days before planting)	R.	N.R.	R.	1
6	Glyphosate	R.	R.	R.	R.	1
7	Carfentrazone-ethyl + naphtha solvent	R.	R.	N.R.	R.	1
8	Sulfentrazone + Diuron + 1,2-ethanediol	R.	R.	N.R.	N.R.	1
9	Diclosulam	R.	R.	N.R.	N.R.	1
10	Haloxyfop-R-methyl + Ethylene glycol monoethyl ether	R.	R.	R.	R.	1
11	Cletodim	R.	R.	R.	R.	1

R.: recommended N.R.: not recommended

4.3 Advanced weed control combined with dynamic programming

Worldwide, weeds have long caused yield losses and increased farmers' production costs. Even though practices to manage weeds have appeared as soon as the first agrarian developments, the lack of species equilibrium increases the pressure as some weed populations have reached high adaptive capacity and distribution (REIS; VIVIAN, 2011). Currently, the greatest challenges of weed control in soybean fields reported by our local farmer are:

- Sourgrass (*Digitaria insularis*)
- Horseweed (*Conyza bonariensis*)
- Morningglory (*Ipomoea grandifolia*)
- Dayflower (*Commelina benghalensis*)
- Alexandergrass (*Brachiaria plantaginea*)

Handling all the main weeds that affect soybean crop at once would be a difficult task to accomplish with dynamic programming techniques. However, we managed to work around the dimensionality problem by considering each weed individually. We take each weed competing with soybean as an independent sub-problem of the weed management challenge. The flow of time does not compromise the feasibility of the dynamic programming approach; in other words, a large number of stages to evaluate is more amenable than a large state space. Although we can manage many stages, there are some main stages related to soybean development that can drastically reduce the span. These stages are strictly associated with the herbicide position in soybeans. Our dynamic programming model takes into account five essential phases:

1) Pre-seed of soybean with safety interval between spraying and seeding
2) Pre-seed of soybean
3) Pre-emergence of soybean
4) Post-emergence of soybean
5) Pre-harvest of soybean

From the whole weed problem in soybeans, we choose **sourgrass** as our initial weed plant to handle in our fields. Based on our independence of states as an assumption, the solution strategy can be reproduce for other pests in the soybean crop.

Our dynamic programming method has the following features:

- The states to manage are the sourgrass density in soybeans.
- The transition function updates the sourgrass density in any given k stage.
- The objective function aims to reduce yield loss combined with expenditures from spraying herbicides.

4.3.1 The relationship between soybean and sourgrass

Sourgrass (*Digitaria insularis*) is one of the critical weed in soybeans. States from Southeastern, Southern and Central-West regions of Brazil have accounted soybean yield loss due to the aggressive establishment of sourgrass in no-till farming (PAULA *et al.*, 2020). Sourgrass, as native South and Central America's specie, and its outstanding adaptability among Brazilian agrarian fields have raised the barrier to suppress weed competition among the crops (KISSMANN, 1995). Gazziero *et al.* (2012) referred to the sourgrass' glyphosate tolerance, even under high dose, as one of the reasons to the continuous growth of this specie.

In the literature, there are several researches trying to assess how substantial are the soybean yield losses due to the sourgrass competition. From following a farmer's experience in the State of Parana, Gazziero *et al.* (2012) reported a reliable assessment in yield loss caused by sourgrass that we could see in Table 4.5.

Reis and Vivian (2011) refer to sourgrass (*Digitaria insularis*) as one of the most present weeds in the soybean crop in the country. Sourgrass resistance to glyphosate has been known since 2008 in Brazilian fields. This specie is extremely competitive and can suppress easily the crop development. The use of graminicide in the early stages of soybean is still effective as long as the sourgrass has not reached 45 days of growth.

Table 4.5 – The assessment of soybean yield loss caused by sourgrass from (GAZZIERO *et al.*, 2012)

Sourgrass plant per square meter	**Yield per hectare ($kg \cdot ha^{-1}$)**	**Losses**
0	3392	0.00
0-3	2595	23.49 %
4-8	1885	44.42 %

Paula *et al.* (2020) reported in their work the damage related to sourgrass density in the initial stage of soybean development. The highest density in their experiment (8 sourgrass plants per square meter) resulted in 16% of germination losses. Sauerwein *et al.* (2019) discussed an experiment that took place in ESALQ/USP department in the

2018/2019 soybean season. In the most severe case of sourgrass density (up to 16 plants per square meter), soybean yield was reduced by 60%.

4.3.2 Understanding the roll of herbicides

From consulting our local farmer, we have selected a group of commercial herbicides. In our proposed model, we considered this selection as our decisions to make. Table4.6 shows this selection of herbicides. Their detailed composition are described in Table 4.7.

Table 4.6 – Current herbicides in use by a grain farmer in São Paulo, Brazil.

Index	**Product**	**Chemical group**
1	Cloransulam-methyl	CLORANSULAM-METHYL: Sulfonanilide triazolopyrimidine
2	Chlorimuron-ethyl	Chlorimuron-ethyl: Sulfonylurea
3	Saflufenacil	(1) Saflufenacil: Pyrimidinedione (uracil) (2) Mixture of sodium methylnaphthalenesulfonate: Naphthalene sulfonate salt
4	2,4-D	2,4-D: Aryloxyalkanoic acid
5	Glufosinate	Substituted homoalanine
6	Glyphosate	Substituted glycine
7	Carfentrazone-ethyl + solvent naphtha	(1) Carfentrazone-ethyl: Triazolone (2) Solvent naphtha (petroleum), light aromatic: Aromatic hydrocarbon
8	Sulfentrazone + Diuron + 1,2-ethanediol	(1) Sulfentrazone: Triazolone (2) Diuron: Urea (3) 1,2-ethanediol: Glycol alcohol
9	Diclosulam	DICLOSULAM: Sulfonanilide triazolopyrimidine
10	Haloxyfop-R-methyl + Diethylene glycol monoethyl ether	(1) Haloxyfop-R-methyl: Aryloxyphenoxypropionic acid (2) Diethylene glycol monoethyl ether: Glycol ethers, polyethers
11	Clethodim	Cyclohexanedione oxime

Table 4.7 – Herbicides and chemical compound description.

Index	**Product**	**Composition**
1	Cloransulam-methyl	CLORANSULAM-METHYL 840 g/kg
2	Chlorimuron-ethyl	CHLORIMURON-ETHYL 250 g/kg
3	Saflufenacil	(1) SAFLUFENACIL 700 g/kg (2) Mixture of sodium methylnaphthalenesulfonate 10 g/kg
4	2,4-D	2,4-D 806 g/L
5	Glufosinate	GLUFOSINATE AMMONIUM SALT 200.00 g/L
6	Glyphosate	Glyphosate 792.5 g/kg
7	Carfentrazone-ethyl + solvent naphtha	(1) CARFENTRAZONE-ETHYL 400.00 g/L (2) Solvent naphtha (petroleum), light aromatic 556.69 g/L
8	Sulfentrazone + Diuron + 1,2-ethanediol	(1) SULFENTRAZONE 175.00 g/L (2) Diuron 350.00 g/L (3) 1,2-ethanediol 65.70 g/L
9	Diclosulam	DICLOSULAM 840 g/kg
10	Haloxyfop-R-methyl + Diethylene glycol monoethyl ether	(1) Haloxyfop-R-methyl 540.0 g/L (2) Diethylene glycol monoethyl ether 531.0 g/L
11	Clethodim	CLETHODIM 120 g/L

In other to pair the dynamic programming evaluation with weed control, we have to make some assumptions about the efficacy of herbicides as we consider them in the problem's universe as **actions**. Although there are many researches about chemical compound response, our study goals are more aligned with the management side of the business and we would rather avoid the complex chemical background surrounding the use of herbicide in grain crops. For our work at hand, the efficiency idea should be enough to grasp a robust track in the research field. Even though many studies tried to investigate pesticides' performance under many conditions, weather related events, physiology, population dynamic, and dispersal of resistant weeds are just a small selection of imbalanced variables.

Both under or over-dose for weed infestation could drastically alter herbicides' performance. Spraying unadvised doses of herbicides leads to poor weed management, phytotoxicity responses of the grown crop, or even, the weed evolution of polygenic resistance from the herbicide site of action. In conclusive thoughts, a combination of herbicide selection, appropriated doses, method of application and weather conditions has considerable influence in the overall performance to manage weeds in the field (SHEKHAWAT *et al.*, 2022).

Our herbicides' efficiency parameters bases on farmer report, which is rather perception built from many years of experience in the field. They are bounded to very local context of place, time and methodology, and, so far, they are rather empirical. Nevertheless our current research does not follow into the chemical field and we have not perform any field experiment to assure any accurate result.

We should have clear in mind that the mechanism of soybean's physiology are far more complex than we could encompass in this research even we decided to neglect the herbicide's peculiarities. Hence, our brief assumption about herbicide's efficacy is in Table 4.8 based **solely** on farmer assistance and experience. From our set of herbicides, some of them are not recommended to control sourgrass and we have not dropped them out due to the potential fitting in other weed management. In the sourgrass scenario, these herbicides without effect in sourgrass population have null efficiency as we can see in Table 4.8 and the outcome solutions will not include them as they represent only costs. *Product 8* and *Product 9* are recommended for controlling weeds at pre-emergent state. Pre-emergent herbicides are designed to hold and kill germinating weed seeds. Their targets are weeds that have not reached the soil surface. Therefore, their action mechanism is distinct in comparison with general spray killers.

Table 4.8 – The author's efficiency estimation of herbicides to handle *Digitaria insularis* in soybeans.

Estimation of herbicide's efficiency in managing sourgrass *(Digitaria insularis)* in soybean crop.										
0.00: Ineffective to 1.00: Full Effective										
1 0.00	2 0.00	3 0.00	4 0.00	5 0.80	6 0.75	7 0	8 0.50	9 0.45	10 0.97	11 0.95

Currently in our research, we have discussed the aspect of herbicides, but we still require a dependable cost relationship. Since purchase prices are closely tied to immediate market fluctuations, the ratio between the unit cost of the herbicide and the unit yield of soybeans is generally more stable. This ratio has been used as the basis for compiling the data in Table 4.9, Table 4.10, and Table 4.11.

Table 4.9 – Understanding the cost of herbicides in soybean crop.

Index	**Chemical Herbicide**	**Last Purchase Price**	**Unit**	**Date**	**Soybean Price per 60kg bag**
1	Cloransulam-methyl	R$ 2.38	g	20/12/2022	R$ 163.10
2	Chlorimuron-ethyl	R$ 0.19	g	05/10/2021	R$ 160.59
3	Saflufenacil	R$ 0.87	g	14/10/2020	R$ 151.98
4	2,4-D	R$ 17.29	L	14/10/2020	R$ 151.98
5	Glufosinato	R$ 55.00	L	15/02/2023	R$ 155.16
6	Glifosato	R$ 28.90	kg	02/08/2023	R$ 124.59
7	Carfentrazona-etílica + solvente naphtha	R$ 643.00	L	09/03/2023	R$ 144.43
8	Sulfentrazona + Diuron + 1.2-ethanediol	R$ 113.90	L	24/10/2022	R$ 165.21
9	Diclosulam	R$ 1.12	g	29/10/2018	R$ 79.48
10	Haloxyfop-R-methyl + Diethylene glycol monoethyl ether	R$ 0.24	L	24/03/2022	R$ 181.69
11	Clethodim	R$ 57.40	L	22/11/2022	R$ 166.69

Table 4.10 – The ratio between soybean yield unit and spraying costs, part (a).

Chemical Herbicide	**Ratio: Soybean 60-kg bag per Unit of Commercial Product**	**Sourgrass (*Digitaria insularis*) Recommended Dose per Hectare** ($Unit/ha$)	**Ratio: Soybean 60-kg bag per Sourgrass Herbicide Spraying**
Clethodim	0.3443462	2	0.6887
Diclosulam	0.0141400	41.7	0.5896
Glufosinate	0.3544781	3	1.0634
Glyphosate	0.2319592	1.5	0.3479
Haloxyfop-R-methyl + Diethylene glycol monoethyl ether	0.0013333	290	0.3867
Sulfentrazona + Diuron + 1.2-ethanediol	0.6894389	1.4	0.9652

Table 4.11 – The ratio between soybean yield unit and spraying costs, part (b).

Chemical Herbicide	**Ratio: Soybean 60-kg bag per Unit of Commercial Product**	**Standard Dosage for Weed Management, excluding Sourgrass** ($Unit/ha$)	**Ratio: Soybean 60-kg bag per Sourgrass Herbicide Spraying**
Cloransulam-methyl	0.0145617	23.8	0.3466
Chlorimuron-ethyl	0.0012037	80	0.0963
Saflufenacil	0.0057311	50	0.2866
2,4-D	0.1137668	1.5	0.1707
Carfentrazone-ethyl + solvent naphtha	4.4520453	0.075	0.3339

4.4 Approach to algorithm design

Based on Bellman (1957), our optimization problem can be represented as a dynamic programming model in the following format:

$$\textbf{Minimize}_{u_0,u_1,\cdots,u_{n-1}} \sum_{k=0}^{n-1}\{e_k(x_k,u_k)+\psi(x_n)\} \tag{4.1}$$

$$\textbf{Subject to } x_{k+1} = f_k(x_k,u_k,w_k) \tag{4.2}$$

$$x_k \in X_k, \quad k = 1,\cdots,K \tag{4.3}$$

$$u_k \in U_k, \quad k = 1,\cdots,K \tag{4.4}$$

$$\textit{Given a } x_0 \tag{4.5}$$

The following list presents the main parameters in the problem's mathematical approach:

- k stages, ranging from the initial stage $k = 1$ to $k = K$;
- x_k states, which represents sourgrass density in soybean crop;
- u_k decisions that are the herbicide selection at some stage, for each k stage, we have a set of possible actions U_k;
- w_k represent exogenous information acknowledged at some k stage;
- $e_k(x_k, u_k)$ is the elementary cost function, which represents the soybean yield loss due to the weed interference;
- $f_k(x_k, u_k, w_k)$ is the transition function that is the sourgrass population density at each k stage;
- $\psi(x_n)$ is the cost function when reaches a x_n state at K (final) stage, which is a soybean yield loss estimation at the crop final stage;
- x_0 is the initial state of the decision system.

For illustration, Figure 4.5 presents the line of though developed in our dynamic programming proposal.

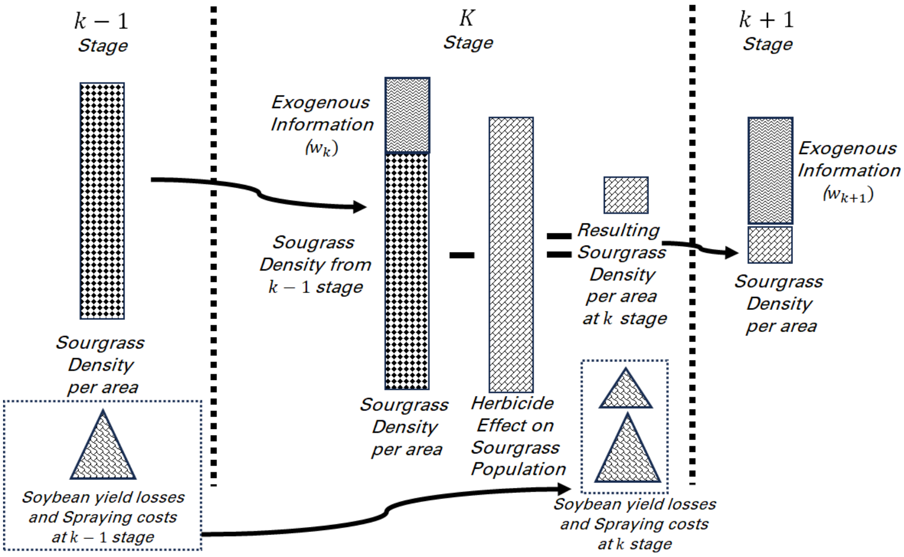

Figure 4.5 – The weed management problem as a dynamic programming model.

The problem's solution would be attained based on the Hamilton–Jacobi–Bellman equation. The cumulative cost function $F(x_k)$ would be:

$$F(x_n) = \psi(x_n) \tag{4.6}$$

$$F(x_k) = \mathbf{Minimize}_{u_k, u_{k+1}, \cdots, u_{n-1}} \{ \sum_{j=k}^{n-1} e_j(x_j, u_j) + \psi(x_n) \}, \ \forall k \neq n \tag{4.7}$$

$$F(x_k) = \mathbf{Minimize}_{u_k, u_{k+1}, \cdots, u_{n-1}} \{ e_k(x_k, u_k) + [\sum_{j=k+1}^{n-1} e_j(x_j, u_j) + \psi(x_n)] \}, \ \forall k \neq n \tag{4.8}$$

By definition:

$$F(x_k) = \mathbf{Minimize}_{u_k} \{ e_k(x_k, u_k) \tag{4.9}$$
$$+ \mathbf{Minimize}_{u_k+1, u_{k+2}, \cdots, u_{n-1}} \{ [\sum_{j=k+1}^{n-1} e_j(x_j, u_j) + \psi(x_n)] \} \}, \ \forall k \neq n$$

$$F(x_{k+1}) = \mathbf{Minimize}_{u_k+1, u_{k+2}, \cdots, u_{n-1}} \{ [\sum_{j=k+1}^{n-1} e_j(x_j, u_j) + \psi(x_n)] \}, \ \forall k \neq n \tag{4.10}$$

At last, we reach the Hamilton-Jacobi-Bellman (HJB) equation in Equation (4.11):

$$F(x_k) = \mathbf{Minimize}_{u_k} \{ e_k(x_k, u_k) + F(x_{k+1}) \}, \ \forall k \neq n \tag{4.11}$$

4.4.1 Pseudo-code of Dynamic Programming

In order to evaluate the weed management in soybean crop, we use Python algorithms based on the pseudo-codes in Algorithm 1 and Algorithm 2. Algorithm 1 presents a recursive method from the optimality assurance in Equation 4.11 . It is a recursive algorithm, which means the iterative process begins at the last stage and moves forward the first stage, tracing all the optimum decision path along the iterations. Algorithm 2 takes the initial conditions (x_0 state) and builds the optimum set of decisions taking the mapped policies from Algorithm 1.

Our choice for coding is Python language due its simplicity. Our algorithms are bounded by a finite number states a priori. States are also discrete, we manage sourgrass density as a state, then it could be one plant per square meter or two plants per square meter for example.

Algorithm 1: Recursive method to get optimum policies.

Input: n, X_k, U_k, $\psi(x_n)$

for $x_n \in X_N$ **do**
    $F(x_n) \leftarrow \psi(x_n)$;
for $k = n-1$ **to** 0 **do**
    for $x_k \in X_K$ **do**
        $F_{aux} \leftarrow \infty$;
        $uo_{aux} \leftarrow \infty$;
        for $u_k \in U_k$ **do**
            $x_{k+1} \leftarrow f_k(x_k, u_k)$;
            $Fxu_{aux} \leftarrow e_k(x_k, u_k) + F(x_{k+1})$;
            if $Fxu_{aux} < F_{aux}$ **then**
                $F_{aux} \leftarrow Fxu_{aux}$;
                $uo_{aux} \leftarrow u_k$;
        $F(x_k) \leftarrow F_{aux}$;
        $\pi(x_k) \leftarrow uo_{aux}$;

Output: $\pi(x_k) = u_k^*$, $\forall x_k \in X_k$, $k = 0, \cdots, n-1$

Algorithm 2: Optimum trajectory recovery algorithm.

Input: x_0, $\pi(x_k) = u_k^*$, $\forall x_k \in X_k$, $k = 0, \cdots, n-1$

$x_0^* \leftarrow x_0$;
for $k = 0$ **to** $n-1$ **do**
    $u_k^* \leftarrow \pi(x_k^*)$;
    $x_{k+1}^* \leftarrow f_k(x_k^*, u_k^*)$;

Output: $\{x_k^*,\ k = 1, \cdots, n\}$, $\{u_k^*,\ k = 0, \cdots, n-1\}$

We tested several parameters using these algorithms and our observations present in the next subsection. Based on the herbicides we have already discussed, the critical stages and the range of sourgrass density in soybean crop, Table 4.12 summarizes the problem's dimension. Dimensionality is a combination of states and action, which could easily place the problem among the intractable under the classic dynamic programming techniques.

Table 4.12 – Problem's dimension.

Number of stages	**Number of actions**	**Number of States**
5	11	16

4.5 Results overview

We tested many combination of input parameters to produce a proper evaluation of the optimization package in the weed management problem. Our methodology aims to support the decision making in the farm, translating a typical agrarian challenge into a fully observable system. From these computational experiments, we would like to observe the problem from an unfamiliar angle, outside standard agronomic approach. The following sub-subsections present the tested parameters and corresponding results. We define **exogenous information** as an indicator of a weed outbreak that occurs beyond the typical developmental pattern.

In Test 1, we examine a case of well-managed sourgrass characterized by minor fluctuations at each soybean stage. Test 2 presents a more challenging starting condition, with an initial density of 5 plants per square meter. Test 3 further amplifies this challenge, beginning with 12 plants per square meter. In Test 4, we incorporate repeated herbicide applications and assess the high initial pressure and during the inter-harvest periods.

4.5.1 Test 1: Initial impressions

The initial state x_0 is 0 sourgrass plants per square meter in soybean field. The exogenous information are $[2, 1, 3, 2, 3]$, which affects the sourgrass density in the field at each k stage. The optimum value of the function, which is the soybean yield loss caused by sourgrass competition plus herbicide costs, is equivalent to 14.43 60-kg soybean bags per hectare. The proposed solution does respect the maximum spraying recommended per crop cycle. The execution was smooth, process time took only 0.140625 seconds and we have around the same time in the other trials.

- Initial stage (x_0): 0 sourgrass plants per square meter;
- Exogenous information at each k stage: $[2, 1, 3, 2, 3]$;

- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed with safety interval	0	0	0	0	0	9	0	9	5	5	5	5	5	5	5	5	5
Pre-seed	0	0	0	0	0	0	0	9	9	9	9	5	5	5	5	5	5
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	14.43	14.43	14.43	14.43	14.8	15.02	15.32	15.54	15.79	15.94	16.09	16.24	16.39	16.54	16.69	16.84	17.36
Pre-seed	13.74	14.43	14.43	14.43	14.43	14.43	14.8	15.17	15.32	15.47	15.99	16.24	16.39	16.54	16.69	16.84	16.99
Pre-emergence	11.84	13.74	14.43	14.43	14.43	14.43	14.43	14.8	15.18	15.55	15.93	16.3	16.68	17.05	17.43	17.8	18.18
Post-emergence	5.1	7.75	10.04	11.84	13.74	15.64	17.54	19.34	21.24	23.14	25.04	26.84	28.74	30.64	32.54	34.34	36.24
Pre-harvest	3	4	5.1	5.85	6.6	7.39	8.14	8.89	9.64	10.39	11.14	11.89	12.64	13.39	14.14	14.89	15.64

- Each x_k state in the final solution: $\{0:0, 1:2, 2:3, 3:4, 4:3, 5:4\}$
- Cumulative yield loss per stage $\{0:0, 1:0, 2:0.69, 3:7.89, 4:1.85, 5:4\}$
- Control actions taken at each k stage: $\{0:0, 1:0, 2:11, 3:10, 4:6\}$
- Optimum function value: 14.43 60-kg bags losses per hectare

Although we begin with a null initial state $x0$ as we did in the test before, the exogenous information has moved to $[0, 0, 2, 1, 3]$. This modification leads to a reduced sourgrass density from pre-emergence stage to harvest. We could observe that the optimum value is much less than the previous test. The optimum value was 9.29 sc/hectare, which the combination of soybean yield losses with expenditures in herbicides.

- Initial stage (x_0): 0 sourgrass density per square meter;
- Exogenous information at each k stage: $[0, 0, 2, 1, 3]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11
Pre-seed	0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	9.29	11.09	11.78	11.78	11.78	11.78	11.78	11.93	12.08	12.23	12.38	12.53	12.68	12.83	12.98	13.13	13.28
Pre-seed	9.29	11.09	11.78	11.78	11.78	11.78	11.78	11.93	12.08	12.23	12.38	12.53	12.68	12.83	12.98	13.13	13.28
Pre-emergence	9.29	11.09	11.78	11.78	11.78	11.78	11.78	12.15	12.53	12.9	13.28	13.65	14.03	14.4	14.78	15.15	15.53
Post-emergence	4	7	9.29	11.09	12.99	14.89	16.79	18.59	20.49	22.39	24.29	26.09	27.99	29.89	31.79	33.59	35.49
Pre-harvest	3	4	5.1	5.85	6.6	7.39	8.14	8.89	9.64	10.39	11.14	11.89	12.64	13.39	14.14	14.89	15.64

- Each x_k state in the final solution: $\{0:0, 1:0, 2:0, 3:2, 4:2, 5:4\}$
- Cumulative yield loss per stage $\{0:0, 1:0, 2:0, 3:4.19, 4:1.1, 5:4\}$
- Control actions taken at each k stage: $\{0:0, 1:0, 2:0, 3:10, 4:6\}$
- Optimum function value: 9.29 60-kg bags losses per hectare

In the following test, we adjust the exogenous input as $[0, 0, 3, 3, 0]$. Here, again, we started without presence of sourgrass before seeding, but some pressure has surfaced after crop's emergence. As the severe losses occur from the post-emergence stage, the best trade-off would be still deploying two weed killers.

- Initial stage (x_0): 0 sourgrass density reported in the soybean field;
- Exogenous information at each k stage: $[0, 0, 3, 3, 0]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11
Pre-seed	0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	9.59	11.49	12.18	12.18	12.18	12.18	12.18	12.33	12.48	12.63	12.78	12.93	13.08	13.23	13.38	13.53	13.68
Pre-seed	9.59	11.49	12.18	12.18	12.18	12.18	12.18	12.33	12.48	12.63	12.78	12.93	13.08	13.23	13.38	13.53	13.68
Pre-emergence	9.59	11.49	12.18	12.18	12.18	12.18	12.18	12.55	12.93	13.3	13.68	14.05	14.43	14.8	15.18	15.55	15.93
Post-emergence	2.85	5.5	7.79	9.59	11.49	13.39	15.29	17.09	18.99	20.89	22.79	24.59	26.49	28.39	30.29	32.09	33.99
Pre-harvest	0	1	2.1	2.85	3.6	4.39	5.14	5.89	6.64	7.39	8.14	8.89	9.64	10.39	11.14	11.89	12.64

- Each x_k state in the final solution: $\{0:0, 1:2, 2:3, 3:4, 4:3, 5:1\}$
- Cumulative yield loss per stage $\{0:0, 1:0, 2:0, 3:5.99, 4:2.6, 5:1\}$
- Control actions taken at each k stage: $\{0:0, 1:0, 2:11, 3:10, 4:6\}$
- Optimum function value: 9.59 60-kg bags losses per hectare

4.5.2 Test 2: Evaluating the exogenous information influence in the final solution

In the following group, we establish the initial state $x0 = 5$ (5 sourgrass plants per square meter). The exogenous information also is increased. The investment level in herbicides completely reshapes as the effort to keep sourgrass under control was much strong. The suggested number of spraying in the final solution is much higher than the previous test.

- Initial stage (x_0): 5 sourgrass plant density;
- Exogenous information at each k stage: $[2, 3, 5, 5, 2]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	9	9	9	9	8	8	0	8	9	9
Pre-seed	0	0	0	5	5	5	6	6	6	9	8	5	5	5	5	5	5
Pre-emergence	0	0	6	6	6	5	6	6	6	5	5	6	6	5	5	5	6
Post-emergence	0	0	6	6	6	11	11	11	11	11	11	11	11	11	11	11	11
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	20.19	20.54	20.54	20.54	20.54	20.69	20.84	21.28	21.58	21.73	22.03	22.56	22.71	24.74	25.24	25.03	25.18
Pre-seed	19.48	19.48	20.19	20.54	20.54	20.54	20.54	20.69	20.84	23.07	23.6	23.09	23.24	23.39	23.54	23.69	24.21
Pre-emergence	17.23	19.13	19.48	19.48	19.48	20.19	21.28	21.66	22.03	23.11	23.49	25.05	25.43	26.51	26.89	27.27	28.83
Post-emergence	6.39	9.04	11.29	13.09	14.99	17.23	19.13	20.93	22.83	24.73	26.63	28.43	30.33	32.23	34.13	35.93	37.83
Pre-harvest	2	3	4.1	4.85	5.6	6.39	7.14	7.89	8.64	9.39	10.14	10.89	11.64	12.39	13.14	13.89	14.64

- Each x_k state in the final solution: $\{0 : 5, 1 : 7, 2 : 5, 3 : 6, 4 : 6, 5 : 3\}$
- Cumulative yield loss per stage $\{0 : 0, 1 : 0.5, 2 : 1.06, 3 : 11.99, 4 : 4.14, 5 : 3\}$
- Control actions taken at each k stage: $\{0 : 0, 1 : 6, 2 : 5, 3 : 11, 4 : 10\}$
- Optimum function value: 20.69 60-kg bags losses per hectare

In the following test, we introduce a more strong sourgrass density during the evolving stages. We notice that the greatest impact in the final solution comes from the post-emergence stage of soybean growth, which is also the stage with the largest exogenous input information.

- Initial stage (x_0): 5 sourgrass plants per square meter;

- Exogenous information at each k stage: $[4, 2, 6, 7, 5]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	0	0	9	9	9	9	9	9	8	8
Pre-seed	0	0	0	9	8	5	5	5	5	6	6	6	6	9	9	8	5
Pre-emergence	0	0	6	6	6	5	6	6	6	5	5	6	6	5	5	5	6
Post-emergence	0	0	6	6	6	11	11	11	11	11	11	11	11	11	11	11	11
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	26.75	26.84	26.84	26.99	27.14	27.29	27.44	27.74	28.04	28.33	28.63	28.93	29.08	29.38	29.53	30.06	30.21
Pre-seed	25.78	25.78	25.78	26.37	26.75	26.84	26.84	26.99	27.14	27.29	27.44	27.59	27.74	31.53	31.68	32.21	30.24
Pre-emergence	23.63	25.43	25.78	25.78	25.78	26.49	27.68	28.05	28.43	29.51	29.89	31.46	31.83	32.91	33.29	33.66	35.23
Post-emergence	10.89	13.54	15.79	17.59	19.49	21.73	23.63	25.43	27.33	29.23	31.13	32.93	34.83	36.73	38.63	40.43	42.33
Pre-harvest	5	6	7.1	7.85	8.6	9.39	10.14	10.89	11.64	12.39	13.14	13.89	14.64	15.39	16.14	16.89	17.64

- Each x_k state in the final solution: $\{0 : 5, 1 : 9, 2 : 5, 3 : 7, 4 : 8, 5 : 6\}$
- Cumulative yield loss per stage $\{0 : 0, 1 : 0.8, 2 : 1.06, 3 : 13.79, 4 : 5.64, 5 : 6\}$
- Control actions taken at each k stage: $\{0 : 0, 1 : 6, 2 : 5, 3 : 11, 4 : 10\}$
- Optimum function value: 27.29 60-kg bags losses per hectare

In this scenario, we smooth the exogenous input at a more tractable level. As the most expressive drop is in the post-emergence stage, the optimum value function also moves in proportion with the introduced changes. In comparison with the previous test, yield drop is much smaller due to the distinct exogenous pattern.

- Initial stage (x_0): 5 sourgrass plants per square meter;
- Exogenous information at each k stage: $[3, 4, 5, 2, 4]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	0	9	9	5	9	9	8	8	9	9
Pre-seed	0	0	0	0	0	9	5	5	5	5	5	5	5	5	5	5	5
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	19.61	19.98	20.2	20.29	20.44	20.59	20.74	21.04	21.48	21.63	21.86	22.23	22.38	22.91	23.06	23.51	23.66
Pre-seed	19.23	19.23	19.23	19.61	19.98	20.2	20.29	20.44	20.59	20.74	20.89	21.41	21.57	21.71	21.87	22.02	22.54
Pre-emergence	16.64	18.54	19.23	19.23	19.23	19.23	19.23	19.61	19.98	20.36	20.73	21.11	21.48	21.86	22.23	22.6	22.98
Post-emergence	6.1	8.75	11.04	12.84	14.74	16.64	18.54	20.34	22.24	24.14	26.04	27.84	29.74	31.64	33.54	35.34	37.24
Pre-harvest	4	5	6.1	6.85	7.6	8.39	9.14	9.89	10.64	11.39	12.14	12.89	13.64	14.39	15.14	15.89	16.64

- Each x_k state in the final solution: $\{0:5, 1:8, 2:6, 3:6, 4:3, 5:5\}$
- Cumulative yield loss per stage $\{0:0, 1:1.36, 2:0.69, 3:11.69, 4:1.85, 5:5\}$
- Control actions taken at each k stage: $\{0:0, 1:5, 2:11, 3:10, 4:6\}$
- Optimum function value: 20.59 60-kg bags losses per hectare

4.5.3 Test 3: Assessing the impact of high density sourgrass at the initial stage

We have two moments of impact in this test. The first one is the initial state $x0$ and the last one is the final stage (pre-harvest stage). This high sourgrass density would require the most efficient herbicides choices.

- Initial stage (x_0): 12 sourgrass plants per square meter;
- Exogenous information at each k stage: $[4, 0, 4, 0, 12]$
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	6	6	6	6	6	6	6	6	6	6	6	6
Pre-seed	0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11
Pre-emergence	0	0	6	6	6	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	23.14	23.48	23.48	23.63	23.78	23.83	23.83	23.98	24.13	24.43	24.58	24.73	24.88	25.18	25.33	25.48	25.63
Pre-seed	20.85	22.79	23.14	23.14	23.14	23.48	23.48	23.63	23.78	23.93	24.08	24.23	24.38	24.53	24.68	24.83	24.98
Pre-emergence	20.85	22.79	23.14	23.14	23.14	23.48	23.48	23.86	24.23	24.61	24.98	25.36	25.73	26.11	26.48	26.85	27.23
Post-emergence	12	14.9	17.15	18.95	20.85	22.79	24.69	26.49	28.39	30.29	32.19	33.99	35.89	37.79	39.69	41.49	43.39
Pre-harvest	12	13	14.1	14.85	15.6	16.39	17.14	17.89	18.64	19.39	20.14	20.89	21.64	22.39	23.14	23.89	24.64

- Each x_k state in the final solution: $\{0:12, 1:7, 2:1, 3:5, 4:1, 5:13\}$
- Cumulative yield loss per stage $\{0:1.25, 1:0.84, 2:0, 3:9.79, 4:0, 5:13\}$

- Control actions taken at each k stage: $\{0:6, 1:11, 2:0, 3:10, 4:0\}$
- Optimum function value: 24.88 60-kg bags losses per hectare

Proceeding with the parameter pattern in previous test, we get the following output:

- Initial stage (x_0): 12 sourgrass plants per square meter;
- Exogenous information at each k stage: $[0, 4, 4, 2, 0]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	5	5	5	5	5	5	5	5	5	5
Pre-seed	0	0	0	0	0	9	5	5	5	5	5	5	5	5	5	5	5
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	13.33	13.33	13.33	13.71	14.08	14.29	14.39	14.54	14.69	14.84	14.99	15.52	15.66	15.82	15.96	16.12	16.64
Pre-seed	13.33	13.33	13.33	13.71	14.08	14.29	14.39	14.54	14.69	14.84	14.99	15.52	15.66	15.82	15.96	16.12	16.64
Pre-emergence	10.74	12.64	13.33	13.33	13.33	13.33	13.33	13.71	14.08	14.46	14.83	15.21	15.58	15.96	16.33	16.7	17.08
Post-emergence	2.1	4.75	7.04	8.84	10.74	12.64	14.54	16.34	18.24	20.14	22.04	23.84	25.74	27.64	29.54	31.34	33.24
Pre-harvest	0	1	2.1	2.85	3.6	4.39	5.14	5.89	6.64	7.39	8.14	8.89	9.64	10.39	11.14	11.89	12.64

- Each x_k state in the final solution: $\{0:12, 1:3, 2:7, 3:5, 4:3, 5:1\}$
- Cumulative yield loss per stage $\{0:1.96, 1:0, 2:1.065, 3:9.79, 4:1.85, 5:1\}$
- Control actions taken at each k stage: $\{0:5, 1:0, 2:11, 3:10, 4:6\}$
- Optimum function value: 15.66 60-kg bags losses per hectare

- Initial stage (x_0): 12 sourgrass plants per square meter;
- Exogenous information at each k stage: $[2, 2, 1, 1, 4]$;
- Optimum policies:
- Optimum function values from previous policy mapping:
- Each x_k state in the final solution: $\{0:12, 1:5, 2:7, 3:2, 4:2, 5:5\}$
- Cumulative yield loss per stage $\{0:1.96, 1:0, 2:1.065, 3:4.15, 4:1.1, 5:5\}$

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	5	5	5	5	5	5	5	5	5	5	5
Pre-seed	0	0	0	0	0	0	9	9	9	9	5	5	5	5	5	5	5
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	10.98	10.98	10.98	11.35	11.57	11.72	12.04	12.19	12.34	12.49	12.64	13.16	13.31	13.46	13.61	13.76	14.13
Pre-seed	10.98	10.98	10.98	10.98	10.98	11.35	11.57	11.72	12.24	12.39	12.64	12.79	12.94	13.09	13.24	13.39	13.54
Pre-emergence	8	10.29	10.98	10.98	10.98	10.98	10.98	11.35	11.73	12.1	12.48	12.85	13.23	13.6	13.98	14.35	14.73
Post-emergence	5	8	10.29	12.09	13.99	15.89	17.79	19.59	21.49	23.39	25.29	27.09	28.99	30.89	32.79	34.59	36.49
Pre-harvest	4	5	6.1	6.85	7.6	8.39	9.14	9.89	10.64	11.39	12.14	12.89	13.64	14.39	15.14	15.89	16.64

- Control actions taken at each k stage: $\{0:5, 1:0, 2:11, 3:10, 4:6\}$
- Optimum function value: 13.315 60-kg bags losses per hectare

4.5.4 Test 4: Relaxing the maximum number of spraying per crop cycle

We have tried a distinct approach in this test. In this scenario, we relax the maximum number of applications just to observe how things change under this perspective. To begin with, let's outline the standard solution for scenarios where the maximum number of applications is adhered to without exception:

- Initial stage: 12 sourgrass plants per square meter;
- Exogenous information at each k stage: $[5, 4, 9, 9, 6]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	9	9	9	9	8	9	9	9	9	9	9
Pre-seed	0	0	6	6	6	9	5	5	5	5	5	5	5	5	5	5	5
Pre-emergence	0	0	6	6	6	5	6	6	6	5	5	6	6	5	5	5	6
Post-emergence	0	0	6	6	6	11	11	11	11	11	11	11	11	11	11	11	11
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	36.75	36.84	36.99	37.14	37.29	37.44	37.88	38.03	38.33	38.48	39.01	39.46	39.61	39.91	40.06	40.36	40.51
Pre-seed	33.98	34.69	35.04	35.04	35.04	36.75	36.84	36.99	37.14	37.29	37.44	37.97	38.12	38.27	38.42	38.57	39.09
Pre-emergence	31.73	33.63	33.98	33.98	33.98	34.69	35.78	36.16	36.53	37.62	37.99	39.55	39.93	41.02	41.39	41.77	43.33
Post-emergence	13.39	16.04	18.29	20.09	21.99	24.23	26.13	27.93	29.83	31.73	33.63	35.43	37.33	39.23	41.13	42.93	44.83
Pre-harvest	6	7	8.1	8.85	9.6	10.39	11.14	11.89	12.64	13.39	14.14	14.89	15.64	16.39	17.14	17.89	18.64

- Each x_k state in the final solution: $\{0:12, 1:12, 2:7, 3:11, 4:10, 5:7\}$

- Cumulative yield loss per stage $\{0 : 1.49, 1 : 1.96, 2 : 0.725, 3 : 21.29, 4 : 7.14, 5 : 7\}$
- Control actions taken at each k stage: $\{0 : 9, 1 : 5, 2 : 6, 3 : 11, 4 : 10\}$
- Optimum function value: 39.60 60-kg bags losses per hectare

The subsequent solution is from the unlimited number of applications, supposing some herbicides can be applied more than once each crop cycle. Even though losses are smaller than the previous test that strictly consider the maximum number of herbicide applications per cycle, soybean yield losses is still exceedingly lofty due to the high sourgrass pressure in the field.

- Initial stage (x_0): 0 sourgrass density per square meter;
- Exogenous information at each k stage: $[5, 4, 9, 9, 6]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	6	10	10	10	10	10	10	10	10	10	10	10	10
Pre-seed	0	0	0	6	6	6	6	6	6	10	10	10	10	10	10	10	10
Pre-emergence	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10
Post-emergence	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	34.07	34.07	34.22	34.37	34.42	34.46	34.46	34.61	34.76	34.91	35.06	35.21	35.36	35.51	35.66	35.81	35.96
Pre-seed	33.68	33.72	33.72	34.07	34.07	34.07	34.07	34.22	34.37	34.56	34.71	34.86	35.01	35.16	35.31	35.46	35.61
Pre-emergence	31.43	33.33	33.68	33.68	33.68	33.72	33.72	34.09	34.47	34.84	35.22	35.59	35.97	36.34	36.72	37.09	37.47
Post-emergence	13.39	16.04	18.29	20.09	21.99	23.93	25.83	27.63	29.53	31.43	33.33	35.13	37.03	38.93	40.83	42.63	44.53
Pre-harvest	6	7	8.1	8.85	9.6	10.39	11.14	11.89	12.64	13.39	14.14	14.89	15.64	16.39	17.14	17.89	18.64

- Each x_k state in the final solution: $\{0 : 12, 1 : 6, 2 : 6, 3 : 10, 4 : 10, 5 : 7\}$
- Cumulative yield loss per stage $\{0 : 1.29, 1 : 0.35, 2 : 0.39, 3 : 19.19, 4 : 7.14, 5 : 7\}$
- Control actions taken at each k stage: $\{0 : 10, 1 : 6, 2 : 10, 3 : 10, 4 : 10\}$
- Optimum function value: 35.36 60-kg bags losses per hectare

In the same course, we consider another parameter set, which characterizes a more amenable competition scenario and the possibility of repeating chemical compounds during the crop cycle. Under these test conditions, the best solution attained by the algorithm is to spray the most efficient herbicide three times.

- Initial stage (x_0): 2 sourgrass plants per square meter;
- Exogenous information at each k stage: $[0, 2, 3, 1, 0]$;
- Optimum policies:

Action mapping																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	0	0	0	0	0	0	0	9	9	9	5	5	5	5	5	5	5
Pre-seed	0	0	0	0	0	0	9	9	9	9	5	5	5	5	5	5	5
Pre-emergence	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
Post-emergence	0	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Pre-harvest	0	0	6	6	6	10	10	10	10	10	10	10	10	10	10	10	10

- Optimum function values from previous policy mapping:

Cumulative yield loss																	
Stage \ State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Pre-seed of soybean with safety interval	10.3	10.3	10.3	10.34	10.34	10.65	10.65	10.8	10.95	11.14	11.29	11.44	11.59	11.74	11.89	12.04	12.19
Pre-seed	10.3	10.3	10.3	10.34	10.34	10.65	10.65	10.8	10.95	11.14	11.29	11.44	11.59	11.74	11.89	12.04	12.19
Pre-emergence	8.05	9.95	10.3	10.3	10.3	10.34	10.34	10.71	11.09	11.46	11.84	12.21	12.59	12.96	13.34	13.71	14.09
Post-emergence	1	4	6.25	8.05	9.95	11.89	13.79	15.59	17.49	19.39	21.29	23.09	24.99	26.89	28.79	30.59	32.49
Pre-harvest	0	1	2.1	2.85	3.6	4.39	5.14	5.89	6.64	7.39	8.14	8.89	9.64	10.39	11.14	11.89	12.64

- Each x_k state in the final solution: $\{0 : 2, 1 : 2, 2 : 4, 3 : 4, 4 : 2, 5 : 1\}$
- Cumulative yield loss per stage $\{0 : 0, 1 : 0, 2 : 0.35, 3 : 7.85, 4 : 1.1, 5 : 1\}$
- Control actions taken at each k stage: $\{0 : 0, 1 : 0, 2 : 6, 3 : 6, 4 : 6\}$
- Optimum function value: 10.68 60-kg bags losses per hectare

4.6 Final observations

The use of herbicides has a profound effect on farm management decisions. Precision and timing are fundamental. Without carefully positioning herbicides, the efficiency of weed control decreases and production risks experience a sharp climb. Spraying many herbicides based only on immediate answers pressures the environmental equilibrium. Therefore, modeling the decision pattern is the right path to enhance strategic thinking in farm management decisions. Our innovative proposal supports weed management using sophisticated optimization strategies, our optimization solution can provide herbicide recommendations that balance soybean yield losses and herbicides costs in the same decision plan. Using our systematic approach, we introduce a more fluid and flexible implementation of break-even and control levels related to herbicides in soybeans. We calibrate our model using data from real grain farming applications. Our algorithm-generated solutions are in line with agronomic expertise.

4.6.1 Further researches and the Forward dynamic programming using the post-decision state variable

Dynamic programming is still appealing and further steps are promising. However, the recursive method of Dynamic programming would not be able to handle large size problem's in the context of farm management. We might find some alternative pathways using *approximate dynamic programming*. Powell (2007) discussed several methods, one of them that ought to be suitable in our case is *forward dynamic programming*.

5 Scheduling corn seed populations: a seed multiplication challenge

The grain farming community is significantly affected by the availability of seeds; thus, our attention is directed towards the seed multiplication industry in this chapter. Commercial corn is converted into numerous food and industrial products, ranking it among the world's most important commodities. Beyond its wide range of applications, corn cultivation significantly influences the agricultural sector. The Brazilian public agency, Companhia Nacional de Abastecimento (CONAB), reported that farms in Brazil are projected to yielded 110.96 million tons of corn during the 2023/2024 season, positioning the country among the world's top three corn producers (CONAB, 2023). Given the extensive supply chain necessary for corn production, there is significant pressure to maintain a steady supply of corn seeds, leading to numerous concerns about potential seed shortages.

Alongside the challenges experienced on the farm, companies engaged in corn seed multiplication encounter obstacles, particularly when introducing new products—a procedure that requires several years of field testing. Recently, groundbreaking technologies have significantly reduced the development time for new corn hybrids, allowing more rapid creation of higher-yielding, better-adapted seed options for farmers. These advanced technologies speed up the creation of commercial corn hybrid parents. Since these hybrids are the result of cross-breeding of two parent plants, decreasing the time required to develop these parents enables scientists to deliver innovative products to farmers more rapidly. Nevertheless, the faster generation of parental lines introduces new obstacles—improving the yield (quantity of ears harvested) may result in complications related to processing and storage capacity. Seed breeding companies would benefit from maintaining a consistent weekly harvest of ears. Irregular weekly harvests, which cause under-use or exceed storage limits, pose logistical and productivity problems. A thoughtfully organized harvest calendar emerges from the strategic seeding of the corn population in the field. Whether to avoid exceeding storage capacity or a shortage of harvesting machinery in the farm, careful planning is essential.

Although our attention is primarily on planning seed schedules, the assignment of tasks and resources to achieve specific objectives is applicable beyond agriculture. Studies on allocation issues can also offer advantages in different fields, like urban planning and enhancements in transportation and logistics (MEMMAH *et al.*, 2015). Within the field of Operations Research, coordinating a timeline for a seed population and assigning tasks are intricately linked in terms of data structure and in problem-solving strategies. Optimization problems in mathematics are categorized according to the nature of the constraint equations, decision variables, and objective functions they encompass. Scheduling problems are an example that focuses on optimizing the allocation of tasks, personnel, and resources in typical production settings. They are regarded critical for decision-making. Using optimization methods to formulate effective schedules increases managerial efficiency, allowing organizations to adhere to deadlines and optimize resource utilization (MIRANDA *et al.*, 2019b).

This chapter aims to introduce an optimization strategy for allocating corn seed populations to achieve a steady weekly harvest, thus preventing storage facility overload while maintaining a regular corn ear count each week. This research presents an optimization strategy employing two mathematical models aimed at allocating seed populations within a designated range, ensuring they do not exceed weekly processing limits. The initial model utilizes Growing Degree Days (GDD) to pinpoint the harvest timeline, whereas the second model constructs the planting plan to enhance efficiency given the restricted processing capacity. We effectively found optimal solutions for a wide spectrum of seed populations, and the time taken to reach these solutions showed substantial efficiency and encouraging results.

In Section 5.1, we explore studies relevant to crop allocation and agricultural land utilization. Models proposed in the literature for crop scheduling among sustainable practices have guided us in formulating the constraints for the seed allocation problem. Section 5.2 provides an overview of the proposed models and their corresponding solution approaches. Section 5.3 presents an examination of the findings. In Section 5.4, concluding observations on the subject matter are offered.

5.1 Related works

There is a scarcity of research papers examining seed allocation from the standpoint of the seed multiplication industry. On the other hand, the allocation of agricultural land use has been extensively studied, boasting many successful implementations, and bears resemblances to the seed allocation issue. The operations research community has developed numerous solutions, since allocation problems typically entail multiple objectives and intricate model constraints.

Land use in agriculture involves complex tasks that typically include various attributes, conflicting goals, and numerous spatial elements. Upon examining agrarian and optimization studies, we identified a significant resemblance between the Crop Rotation Problem (CRP), which involves assigning different crops to areas under specific conditions, and the present issue. Hence, in this subsection, we explore the foundational studies on the scheduling of crop varieties and their connection to the seed multiplication problem.

Exploring the CRP, Aliano *et al.* (2014) introduced a mixed-integer optimization framework alongside hybrid metaheuristic solutions. This study incorporates hybrid algorithms that integrate local search with simulation annealing (SA). To prevent poor initial setup, Aliano *et al.* (2014) designed a heuristic method designed to produce an appropriate initial population. For the sake of clarity in this chapter, we utilize the term **hybrid** in two contexts: firstly, a *hybrid algorithm*, which is an algorithm that integrates multiple strategies, and secondly, *hybrid corn*, which pertains to a type of corn resulting from crossbreeding two or more genetically distinct inbred parent lines to yield a new variety with preferred characteristics.

In their study, Aliano *et al.* (2018) introduced a bi-objective optimization framework for addressing the CRP. They highlighted that diversifying the crop sequence by incorporating different crop families could increase diversity. However, its influence on farm profitability should be assessed, as their rotation scheme assigns equal significance to all crops, including the most profitable. This research explored deterministic strategies to manage the multi-objective challenges, emphasizing the competing objectives within the proposed model.

Santos *et al.* (2011) developed a model for the CRP tailored for organic agriculture, emphasizing the modeling of spatial limitations through their optimization approach. Similarly, Forrester and Rodriguez (2018) devised an integer binary linear optimization model aimed at organizing crop rotations in organic vegetable cultivation. Both studies consider multiple elements such as pest management, irrigation, fallow times, and field capacity. The central aim of their crop rotation strategy is to meet market demands.

Miranda *et al.* (2019b) introduced a mixed integer linear programming model (MILP) that address the CRP on large-scale farms. While the model's objective function focuses on profit maximization, their model incorporates a novel set of constraints that link fertilization parameters to enhance soil nutrient availability and mitigate chemical inputs. This results in a more sustainable and profitable allocation strategy. Additionally, they devised a genetic algorithm employing unique encoding methods, which yielded favorable results.

In their study, Miranda *et al.* (2019a) introduced a multi-objective optimization model for the CRP. The research explored the link between the cultivation of crops and soil fertility, with crop allocation guided by cultural characteristics of the crops. Besides profit maximization, the decision-maker can adjust input parameters to achieve a sequence of crops that controls erosion or offers a solution against weeds.

Zhu *et al.* (2017) presented a MILP along with a heuristic algorithm targeting optimization of seed product packaging planning to minimize total costs. The packaging model is identified as a specialized dynamic lot-sizing problem. Given the NP-hard nature of this problem, solving their MILP model requires significant computational time. Consequently, their solution includes a heuristic algorithm to address this challenge. In this chapter, we aim to propose a mathematical model to aid in the allocation of corn seed populations by framing it as a scheduling problem. Our methodology is guided by the optimization tactics discussed in the CRP literature.

5.2 Methodology

In contrast to typical grain farms that focus on optimizing yield from the available arable land, seed multiplication companies prioritize the quality of seeds. To manage space limitations, these companies can form partnerships, allowing them to adjust the size of their multiplication fields as needed. As seed grain has a higher value compared to ordinary grain, seed companies can afford to pay seed-producing farms more generously and extend agronomic support to collaborating farmers to improve production quality. Therefore, generally speaking, seed multiplication businesses competently manage the spatial limitations of growing areas when it comes to producing seed populations. The primary difficulty they face is not dedicating land for extensive seed cultivation, but guaranteeing the market-ready quality of the seeds, which is their most challenging task. Therefore, seeds must be assessed with standard germination tests, as mandatory, to precisely evaluate their quality prior to marketing. After leaving the farm, essential processes like packaging and storage are crucial to maintaining the seeds' germination potential until they reach the buyers. Post-harvest seed processing is usually limited to a few facilities, typically depending on a single facility operating at full capacity to carry out this critical function.

This research aims to sustain seed quality in the process by introducing a carefully designed plan for corn seed populations. This approach helps prevent storage overflow, which can compromise the quality of corn ears. Planning the optimal planting date for each group with a focus on consistent utilization of storage leads to minimizing the gap between the weekly harvest amount and the maximum weekly processing capacity.

For smaller cases of the issue, the task initially appears manageable. However, the vast scale of the seed market operation results in an exponential increase in the figures. There are numerous variations among parental breeders, indicating that numerous seed populations are expected to emerge each year. Each seed company manages thousands of seed populations, each having distinct seeding schedules and harvesting guidelines.

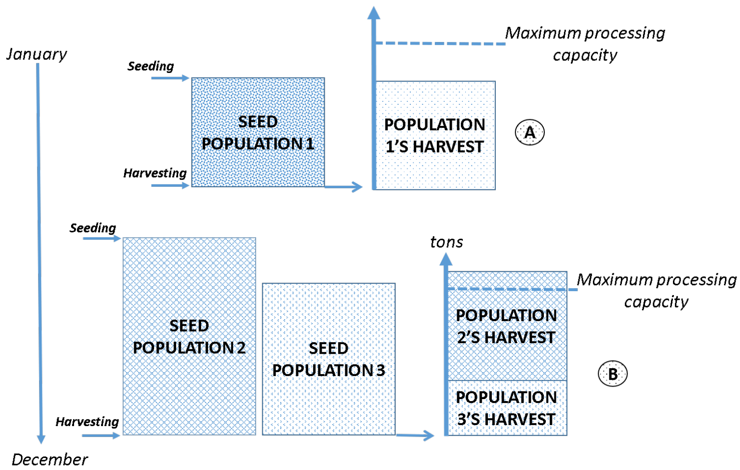

Figure 5.1 – Achieved harvesting weekly quantity from site 0 in Scenario 1.

In the long run, managing a stable weekly harvest quantity becomes difficult due to numerous input parameters. Making predictions about each population's harvest can assist in assessing whether the site's processing capacity will be exceeded or underutilized. Figure 5.1 illustrates the central issue. Here, the production output of population 1 is processed successfully without exceeding the maximum allowable capacity. Upon evaluating the yield from populations 2 and 3, it becomes apparent that it significantly surpasses the site's capacity during that time. Adjusting the planting schedule could prevent excessive production in a specific period, as illustrated in Figure 5.2.

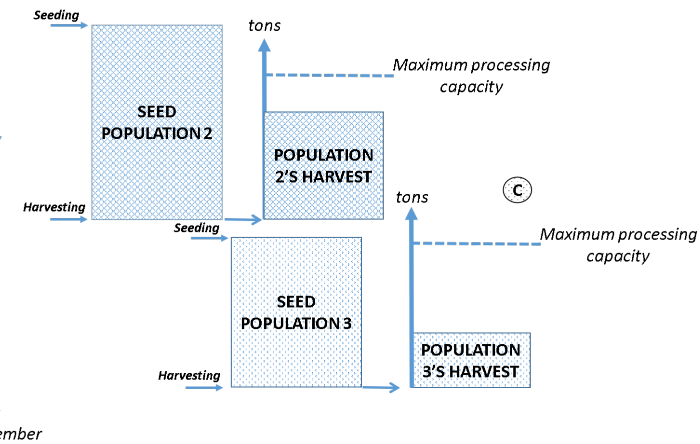

Figure 5.2 – Achieved harvesting weekly quantity from site 0 in Scenario 1.

In the context of harvest planning, our knowledge is based on Growing Degree Days (GDD), a technique for evaluating crop growth and progression during the growing season. The core idea behind GDD is that crops grow when temperatures surpass a defined minimum threshold, called the base temperature (ANANDHI, 2016; NIELSEN *et al.*, 2002). These thresholds are empirically established for each crop. Equation (5.1) provides the formula for calculating the daily GDD using the maximum temperature (T_{max}) [$°C$], the minimum temperature (T_{min}) [$°C$], and the base temperature (T_{base}) [$°C$] specific to each crop.

$$GDD = \frac{T_{max} + T_{min}}{2} - T_{base} \ [°C] \tag{5.1}$$

Upon examining the context and desired outcomes of the problem, we found that it is feasible to separate the GDD information from the planting schedule for each crop type, thereby simplifying the optimization process. Additionally, the method for evaluating GDDs is straightforward, as the outputs tend to be consistent for any given set of inputs. In contrast, scheduling optimization does not easily follow predictable patterns. By evaluating all GDD data first, we reduce the complexity of the task before addressing the core optimization challenge. Our approach involves two optimization models: The first, detailed in Subsection 5.2.1, unites the planting dates with the required GDDs for each seed type to predict harvest periods; the second, described in Subsection 5.3, assesses the scheduling itself.

5.2.1 Model A: finding harvesting periods from each seed population

The growth and development rate of plants is influenced by the ambient temperature. Each species possesses a distinct temperature range, indicated by minimum, maximum, and optimum values (HANEVELD; STEGEMAN, 2005). Temperature affects plant development through various processes such as root growth, water and nutrient uptake, respiration, metabolism, photosynthesis, and the movement of photosynthesis products within the plant (SANS *et al.*, 2002). The Growing Degree Days (GDD) metric is a reliable tool that is used to assess a plant's temperature response. The principle is straightforward: plants grow and develop as long as temperatures exceed a specific baseline (base temperature). The level of growth is directly related to how long the temperature stays above this baseline (STEINMETZ *et al.*, 2009; ALVES *et al.*, 2018a).

For the suggested model, the input indices and parameters are defined as follows:

- N: total count of populations
- J: period for late planting [*the nth day in the sequence*]
- max_{span}: duration for late harvesting [*the nth day in the sequence*]
- $GDD[j]$: vector representing cumulative GDDs
- $H[i]$: the necessary GDDs for crop i to reach the harvest stage

The model includes the following set of ranges:

- $i = 1, \cdots, N$: range of seed populations
- $j = 1, \cdots, J$: range of simulated periods [*in days*]

The set of decision variables is represented by $t[i][j]$. Since every solution corresponds to an index, these values must be integers.

- $t[i][j]$: the harvest time for the seed population i, which was seeded during period j [*the nth day in the sequence*]

The resolution of the described model involves a collection of harvest schedules organized by seed population index i and planting date j. For each specific pairing of seed i and date j, the matrix $t[i][j]$ identifies the corresponding harvest period. Decomposing the seed multiplication issue into two sub-models considerably improves the efficiency of the optimization solution, despite the potential lengthiness of the matrix $t[i][j]$. The objective outlined in Equation (5.2) is to minimize the harvesting period for each combination of i and j.

$$\textbf{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1}^{J} t[i][j] \tag{5.2}$$

Model A produces a dataset that serves as an input parameter for Model B. The constraint described in Equation (5.3) mandates that the harvest period surpasses the requisite growing degree days (GDDs) for each population i. We seek the smallest $t[i][j]$ such that the gap between the GDDs from sowing in period j to harvesting in period $t[i][j]$ is at least $H[i]$.

$$GDD[t[i][j]] - GDD[j] \geq H[i], \quad i = 1, \cdots, N, \quad j = 1, \cdots, J \tag{5.3}$$

Equations (5.4) and (5.5) define the domain for the variables $t[i][j]$. We permit $t[i][j]$ to exceed J due to certain seed populations that are planted one year and harvested in the next, necessitating that the harvest assessment extends beyond J.

$$t[i][j] \geq 0, \quad i = 1, \cdots, N, \quad j = 1, \cdots, J \tag{5.4}$$

$$t[i][j] \leq max_{span}, \quad i = 1, \cdots, N, \quad j = 1, \cdots, J \tag{5.5}$$

Harvesting schedules rely on the local environment and accumulated GDDs, which requires evaluation on a case-by-case basis. Each location will have a unique outcome. After optimizing, we can consolidate the solutions for all sites. In this example, we have two initial populations of seeds ($N = 2$), and the schedule covers six periods (with the late planting period labeled as $J = 3$ and the late harvest period as $max_{span} = 6$). The vector H contains the required GDDs for harvest. The matrix I indicates the planting intervals for seed populations 1 and 2. GDD is a cumulative vector with a length of max_{span}.

- $N = 2$
- $J = 3$ $periods\ of\ time$
- $max_{span} = 6$ $periods\ of\ time$
- $I = [[1, 1, 1], [0, 1, 1]]$
- $H = [20, 35]$
- $GDD = [10 \quad 25 \quad 40 \quad 70 \quad 85 \quad 100]$

The outcome is presented in Table 5.1. For each combination of crop i and planting period j, we have a respective harvesting period j. Each site has its own GDD records; therefore, if there are more than one site, there will be solutions similar to the one in Table 5.1 for each site k. Thinking about many sites, we could gather the results in a single multidimensional matrix $T[i][j][k]$.

Table 5.1 – Outcome solution: harvesting combinations.

$t[i][j]$	$j = 1$	$j = 2$	$j = 3$
$i = 1$	3	4	4
$j = 2$	4	4	5

5.2.2 Model B: scheduling seed populations

In this part, we explain the suggested optimization framework for arranging seed populations. The parameters of the model are as follows.

- N: number of populations
- K: number of sites
- Nw: number of weeks
- B: week size
- M: penalty parameter.

The model includes the following set of ranges:

- $i = 1, \cdots, N$: range of seed populations
- $k = 1, \cdots, K$: range of site locations
- $j = 1, \cdots, J$: range of simulated periods [*in days*]
- $m = 1, \cdots, J$: an alternative set of simulated periods (as we will observe in Equation (5.7), a supplementary range of periods is needed) [*in days*]
- $n = 1, \cdots, Nw$: range of weeks

Additional parameters are outlined as follows:

- $W[k]$: storage capacity limitation of site k [$units\ of\ volume/week$]
- $I[i][j]$: sparse matrix that holds a true value if the interval set by the early and late seeding periods of seed population i includes period j
- $I[i][j]$: sparse matrix indicating whether population i is to be initiated during period j
- $P[i]$: projected yield for seed population i
- $T[i][j][k]$: an array containing the harvest period $j_{harvest}$ for each population i, which is seeded at period j at location k
- $S[i][k]$: a binary array is set to true if population i must be seeded at site k; otherwise, it is set to zero

The proposed model addresses decision variables as detailed in the list below.

- $x[i][j][k]$: a binary decision variable indicating whether population i is seeded in period j at site k
- $p[j][k]$: quantity of harvest during period j at location k
- $a[n][k]$: adjust parameter for storage capacity
- $wp[n][k]$: total harvest during the n week at location k

If storage capacities $W[k]$ were really tight, infeasible solutions would appear often. Taking into account that any solution is better than no solution at all, we introduced the adjust parameter $a[n][k]$.

$$\textbf{Minimize} \quad \sum_{k=1}^{K} \sum_{n=1}^{Nw} |W[k] - wp[n][k]| + M \cdot a[n][k] \tag{5.6}$$

The constraint in Equation (5.7) assesses the production output for each period j. The variable $p[j][k]$ aggregates the harvest for period j at storage site k by summing $P[i]$ whenever $x[i][m][k]$ is true—meaning population i is slated for planting in period m — and the condition for harvesting is also satisfied as per the value in $T[i][m][k]$.

$$p[j][k] = \sum_{i=1}^{N} \sum_{m=1}^{J} x[i][m][k] \cdot P[i] \cdot (T[i][m][k] == j), \quad j = 1, \cdots, J, \quad k = 1, \cdots, K \tag{5.7}$$

The weekly production evaluation, represented by $wp[n][k]$, relies on the total of the accumulated production variables $p[j][k]$, and this sum should span from 1 to B (with B representing 7 days) to determine the fraction of the corresponding week. The constraint is articulated in Equation (5.8).

$$wp[n][k] = \sum_{b=1}^{B} p[(n-1) \cdot B + s][k] =, \quad k = 1, \cdots, K, \quad n = 1, \cdots, Nw \tag{5.8}$$

Equation (5.9) evaluates the weekly production $wp[n][k]$ against the process capacity $W[k]$. This constraint safeguards against exceeding storage process capacity. The adjustment parameter $a[n][k]$ is introduced to maintain an equitable allocation of storage capacity across weeks. Given that B denotes a weekly interval ($B = 7$) and the delayed harvesting phase J spans two years ($J = 730$), the total number of weeks Nw amounts to 104 ($Nw = 104$).

$$wp[n][k] <= W[k] + a[n][k], \quad k = 1, \cdots, K, \quad n = 1, \cdots, Nw \tag{5.9}$$

The parameter $a[n][k]$ must remain non-positive, as dictated by Equation (5.10). It is crucial to impose this restriction to avoid artificially increasing the storage capacity of the warehouse that could occur due to the influence of the adjust parameter in Equation (5.9). In cases where solutions do not affect storage capacity $W[k]$, $a[n][k]$ will be zero, as is ensured by the negative term in the objective function presented in Equation (5.6).

$$a[n][k] \leq 0, \quad k = 1, \cdots, K, \quad n = 1, \cdots, Nw \tag{5.10}$$

Within our proposed model, the constraint outlined in Equation 5.11 guarantees the correct sowing date for each seed variety. Specifically, if $x[i][j][k]$ holds true, indicating crop i is scheduled to be planted in period j at location k, then $I[i][j]$ must also be true, denoting that period j falls within the designated early and late seeding windows for crop i.

$$x[i][j][k] \leq I[i][j], \ i = 1, \ \cdots, \ N, \ j = 1, \ \cdots, \ J, \ k = 1, \ \cdots, \ K \tag{5.11}$$

Equation (5.12) ensures that each seed population, from $i = 1$ to $i = N$, is seeded exactly once and only once.

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x[i][j][k] = 1, \quad i = 1, \ \cdots, \ N \tag{5.12}$$

The constraint detailed in Equation (5.13) guarantees that seed population i is exclusively planted at its designated site. This is achieved by incorporating the product of $x[i][j][k]$ and (1 - $S[i][k]$). Here, $S[i][k]$ is true if crop i is assigned to be planted at location k.

$$\sum_{i=1}^{N}\sum_{j=1}^{J} x[i][j][k] \cdot (1 - S[i][k]) = 0, \quad k = 1, \ \cdots, \ K \tag{5.13}$$

The domain of the main decision variable that sets the allocation of each crop i is defined in Equation (5.14). As a binary set of variables, it can only assume two values.

$$x[i][j][k] \in \{0, 1\} \tag{5.14}$$

5.2.2.1 A minimum example of the proposed model

For illustration purpose only and serve the proposal of this subsection, we have adopted a simplified version of Equation (5.8) that is Equation (5.15). By this modification, a period j accounts by a **whole week**. By that, we also need to introduce a slightly change in the objective function, which is described by Equation (5.16).

$$p[j][k] \leq W[k], \quad j = 1, \ \cdots, \ J, \quad k = 1, \ \cdots, \ K \tag{5.15}$$

$$\textbf{Minimize} \qquad \sum_{k=1}^{K}\sum_{j=1}^{J} |W[k] - p[j][k]| \tag{5.16}$$

Input parameters include two seed populations ($N = 2$), with two sites ($K = 2$). The entire seeding window spans 3 weeks ($J = 3$), although the production evaluation should reach more 3 weeks further the seeding windows ($max_{span} = 6$), and the production vector is $P = [354, 342]$. Each site k has a maximum storage capacity of 400 units per week. When optimizing multiple seed processing sites ($K > 1$), the matrix S will have dimensions of $N \times K$, with each element indicating whether crop i is to be planted at location k, exemplified by the following sample matrix:

$$S = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

The $T[i][j][k]$ array indicates the harvest period for a specific combination of seed population, period, and location. The harvest periods shown in Table 5.2 are derived from the outcomes discussed in Subsection 5.2.1.

Table 5.2 – Harvest period record in the matrix parameter.

$T[i][j][k]$								
$\vdots$								
	$k = 1$				$k = 2$			
	$j = 1$	$j = 2$	$j = 3$	$\cdots$	$j = 1$	$j = 2$	$j = 3$	$\cdots$
$i = 1$	4	5	6		5	6	6	
$i = 2$	3	4	5		4	5	5	
$\vdots$								

Table 5.3 displays the results. It is evident that, for every site k, the outlined weekly production remains within the constraints of storage and processing capacities.

Table 5.3 – Minimum example from scheduling optimization model.

$x[i][j][k]$				$p[j][k]$		
i	j	k	$value$	j	k	p
1	3	1	1	6	1	354
2	2	2	1	5	1	342

5.2.2.2 Transforming the model into into an evaluator for minimal storage capacity

To achieve the minimum processing and storage capacity, modifications in the model are necessary. Specifically, we need to substitute the objective function in Equation (5.6) with that in Equation (5.17). The processing capacity at site k is denoted as $w[k]$:

$$\textbf{Minimize} \quad \sum_{k=1}^{K} w[k] \tag{5.17}$$

Furthermore, it is essential to replace the capacity constraint in Equation (5.9) with Equation (5.18). In our pursuit of finding the minimum storage capacity, the adjustment variables $a[n][k]$ are no longer needed, resulting in the modification shown in Equation (5.18). The maximum storage capacity $W[k]$ continues to appear in the equation, yet only as a boundary constraint.

$$wp[n][k] <= W[k], \quad k = 1, \cdots, K, \quad n = 1, \cdots, Nw \tag{5.18}$$

5.3 Results and observations

Given the extensive nature of the optimization problem tackled in this chapter, we have utilized three different solvers to enhance our comprehension of the difficulties associated with solving the mixed-integer linear programming model. The chosen solvers are:

- *IBM ILOG CPLEX Studio IDE Studio 22.1.1.0*: IBM has created a high-performance optimization solver, known as CPLEX, that is designed to solve challenging mathematical optimization problems. CPLEX efficiently addresses various types of problems, such as linear programming (LP), mixed-integer programming (MIP), quadratic programming (QP), and constraint programming (CP).
- *Gurobi 11.0.3*: A versatile mathematical optimization solver, Gurobi efficiently manages a range of complex optimization tasks, including linear programming (LP), mixed-integer programming (MIP), quadratic programming (QP), among others.
- *PySCIPOpt*: serves as a Python interface for SCIP, a software solution intended to tackle various mathematical optimization challenges. SCIP (Solving Constraint Integer Programs) is distinguished for its proficiency in managing mixed-integer programming (MIP), mixed-integer nonlinear programming (MINLP), and constraint programming (CP).

Each of the three solvers demonstrates adaptability, making them relevant in diverse fields. Their versatility aids operations research, analytics, and decision-making endeavors. They are widely utilized in multiple industries, including finance, logistics, energy, and manufacturing, facilitating data-driven decisions intended to boost efficiency, reduce expenses, or enhance other vital performance indicators (KOCH *et al.*, 2022).

If we do not find an optimal solution within the designated time frame, we use a metric known as the *optimality gap* to evaluate the quality of the solution obtained. The *gap* denotes the difference between the best known solution and the best possible one for an optimization problem. This measure helps determine how close the solver's existing solution is to the real optimal outcome. All three solvers can offer the *optimality gap* as a parameter for analysis. This *gap* represents the disparity between the best integer solution obtained (when relevant) and the best bound provided by the solver to the optimal objective value. Typically, it is expressed as a percentage and is calculated using:

$$gap = \frac{BestIntegerSolution - BestBound}{BestBound} \times 100\ [\%] \tag{5.19}$$

In terms of the test workbench in this study, we created a database derived from actual data and used a probability density function to generate numerous populations. To assess each solver's ability to generate solutions for Model B, we chose several seed population sizes ranging from 100 to 1000 corn seeds. Depending on these population sizes, the number of variables and constraints is illustrated in Table 5.4 to solve the minimum deviation from the weekly maximum capacity and to determine the weekly minimum storage capacity.

For each test, we display the results in Table 5.5. Among the various tests conducted, *CPLEX* outperforms *Gurobi*, while *PySCIPOpt* exhibits relatively weaker performance. It is important to emphasize that our evaluation of performance is restricted to this specific application, and is not meant to undermine the capabilities of any solver, but rather to assess their effectiveness concerning the problem under consideration.

5.3.1 The use of CPLEX into the seed problem

Using CPLEX, we assessed the issue by using a database containing three thousand seed populations ($N = 3000$). The schedule for seeding spans an entire year ($J = 365\ weeks$). Planting at the conclusion of this interval means the harvest will occur in the subsequent year, necessitating an additional number of periods to fully include seed production ($max_{span} = 730\ weeks$). Initially, one must create all possible pairings of seed population i and period j that result in the harvesting period $t[i][j]$, based on Model

Table 5.4 – Describing the size of the optimization problem.

	Minimum Variation			
	The total number of seed populations			
	100	**200**	**500**	**1000**
Integer	73000	146000	365000	730000
Binary	73000	146000	365000	730000
Continuous	1876	1876	1876	1876
Constraints	75394	148494	367794	733294

	Minimum Capacity			
	The total number of seed populations			
	100	**200**	**500**	**1000**
Integer	73002	146002	365002	730002
Binary	73000	146000	365000	730000
Continuous	1668	1668	1668	1668
Constraints	74978	148078	367378	732878

Table 5.5 – A comparative study of linear solvers used for seed population distribution.

Number of seed populations	**Sites**	**Allocation window**	**Solver**	**Solving Time (seconds)**	**Gap**	**Objective**	**Objective function**
100	2	365	Gurobi	0.28	0.00%	M.C.	[771, 741]
100	2	365	pyscipopt	3	0.00%	M.C.	[771, 741]
100	2	365	CPLEX	0.67	0.00%	M.C.	[771, 741]
100	2	365	Gurobi	0.18	0.00%	M.V.	164942
100	2	365	pyscipopt	16	0.00%	M.V.	164942
100	2	365	CPLEX	0.23	0.00%	M.V.	164942
200	2	365	Gurobi	1500.18	0.10%	M.C.	[848, 1063]
200	2	365	pyscipopt	1500	1.68%	M.C.	[858, 1063]
200	2	365	CPLEX	513.44	0.10%	M.C.	[848, 1063]
200	2	365	Gurobi	0.52	0.00%	M.V.	160876
200	2	365	pyscipopt	105	0.00%	M.V.	160876
200	2	365	CPLEX	0.53	0.00%	M.V.	160876
500	2	365	Gurobi	1503	3.44%	M.C.	[1878, 2194]
500	2	365	pyscipopt	3	8.76%	M.C.	[2034, 2236]
500	2	365	CPLEX	0.67	1.57%	M.C.	[1840, 2162]
500	2	365	Gurobi	1.22	0.00%	M.V.	309612
500	2	365	pyscipopt	1500	-	M.V.	-
500	2	365	CPLEX	1.09	0.00%	M.V.	309612
1000	2	365	Gurobi	1501.72	1.5887	M.C.	[3782, 4338]
1000	2	365	pyscipopt	1500	4.318%	M.C.	[3895, 4440]
1000	2	365	CPLEX	1500	1.380%	M.C.	[3831, 4324]
1000	2	365	Gurobi	4.94	0.00%	M.V.	608817
1000	2	365	pyscipopt	1500	-	M.V.	-
1000	2	365	CPLEX	3.94	0.00%	M.V.	608817

Note: M.C. stands for Minimum Capacity, and M.V. stands for Minimum Variation.

A's outcomes. Given the vast amount of input data, we divide the seed population into groups of 500 each. Since there are 3000 seed populations, this procedure is performed six times to cover all necessary harvesting combinations. If there are no modifications in the

GDD data, it is unnecessary to re-solve Model A. The following section provides a short overview of the parameters. The list ahead provides a fair description of the parameters required to evaluate Model B:

- $N = 3000$
- $J = 365\ weeks$
- $max_{span} = 730\ weeks$
- $P = [354, 342, 300, \cdots, 346, 268]$ (the dimension of P vector is N)
- $K = 1$
- $W = 23500\ units\ of\ volume/week$
- $T = [[109 \quad 110 \quad 111 \quad \cdots; \quad \vdots \quad ; \cdots \quad 472 \quad 473 \quad 473] \cdots]$ (the dimension of T is $N \times J \times K$)

Solving the problem and generating a seed schedule solution took 500 seconds. Table 5.6 shows other solver parameters related to the optimization of Model B.

Table 5.6 – Model B: solver's parameters.

IBM ILOG CPLEX: Statistics		
Solver	Cplex	
Constraints	7146	
Variables	1096250	
	Binary	1095000
	Integer	938
	Other	312
Non-zero coefficients	2995779	
Iterations	3130	

From the optimal solution found, Figure 5.3 exhibits weekly harvesting quantities of Site 1. Processing capacity in this test is $W = 23500\ units\ of\ volume/week$. We got the minimum deviation from site processing capacity, increasing the efficiency of the whole seed processing facility.

Looking for minimum capacity, we found the results in Figure 5.4. Is also praised because there is a consistency among every weekly harvest amount. The minimum capacity value was 22047 *units of volume/week*.

Model B tackles infeasible issues by employing the *adjust* variables, which diminish the processing capability. In scenarios where this capacity is insufficient, the resulting scheduling solution would generate spikes, as illustrated in Figure 5.5.

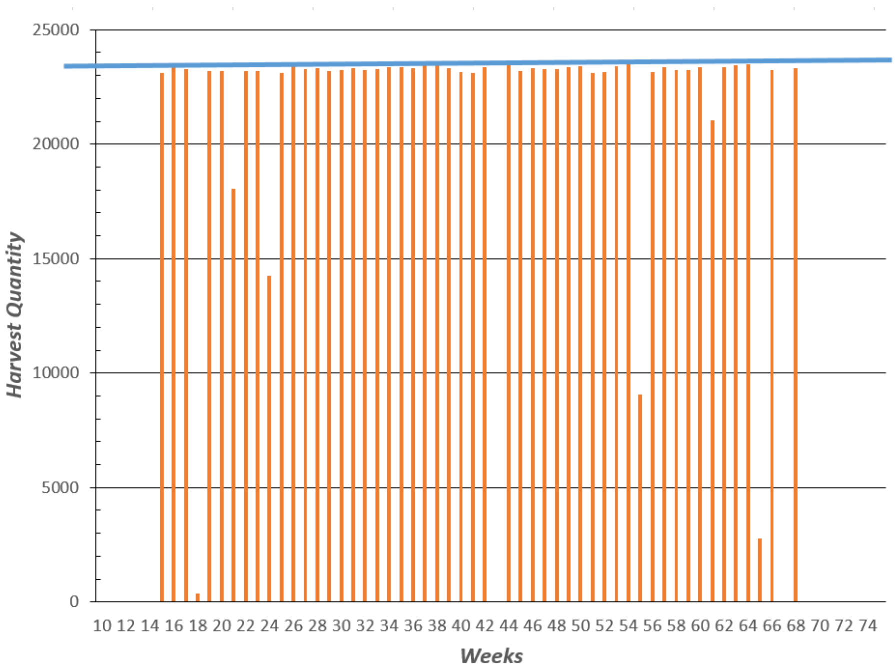

Figure 5.3 – Efficient seed distribution: minimum deviation.

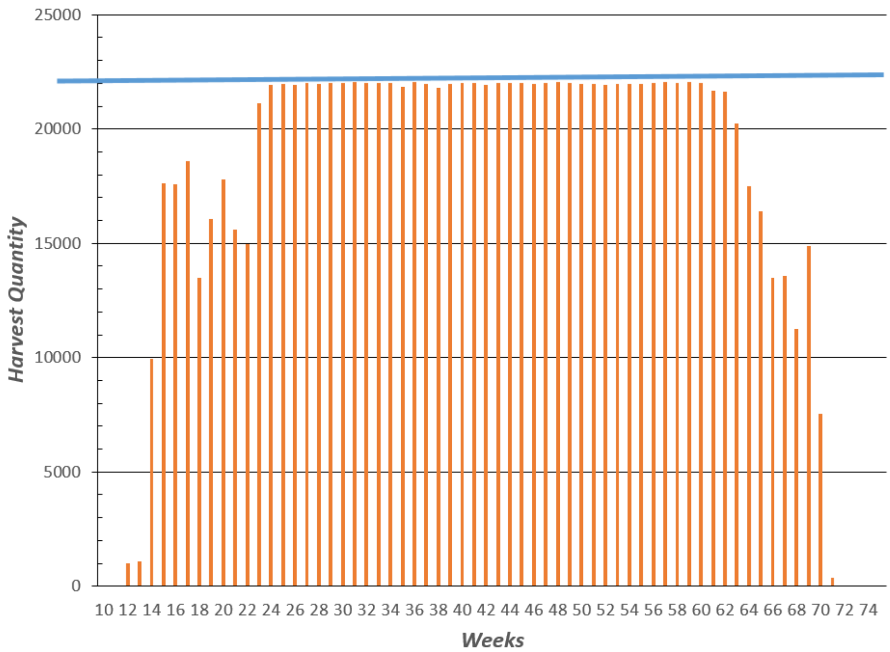

Figure 5.4 – Minimum processing capacity.

5.3.2 A comparison about goals shaping the seed harvesting schedule

For this subsection, we produce another 3000 seed population dataset to compare the objectives in Equations (5.6) and (5.17). Figure 5.6 and Figure 5.7 illustrate two contrasting situations. The former focuses on identifying the minimum storage requirements based on the crop harvest timetable, while the latter seeks to optimize storage usage by minimizing the discrepancy between a fixed storage capacity and the harvest schedule. By examining Figure 5.6, we can see that the facility's processing and storage

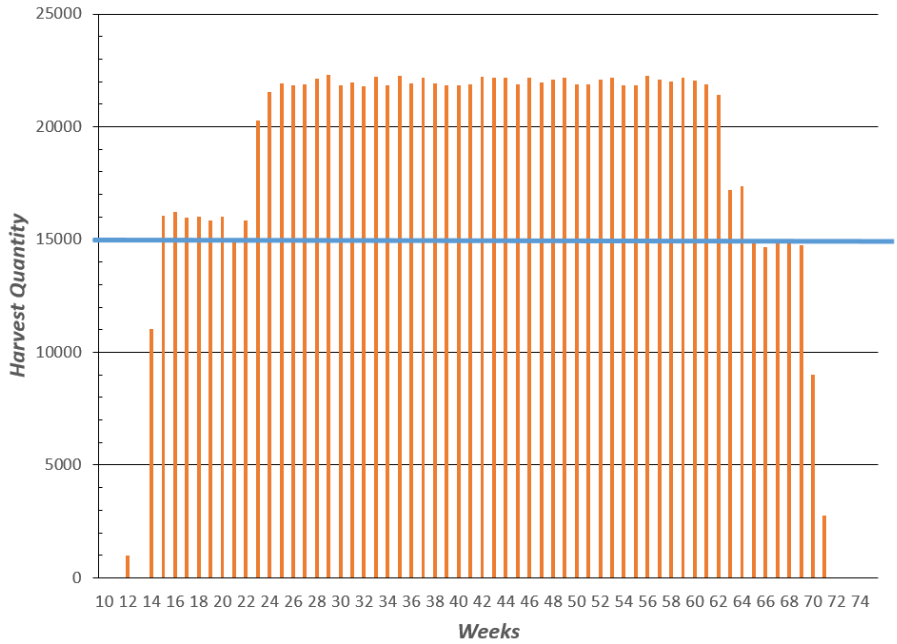

Figure 5.5 – Undesirable solution: harvest overflows processing capacity.

capabilities are over-sized. The site experiences peak activity from the 10th to the 20th week, followed by a period of minimal capacity usage between the 20th and 40th weeks.

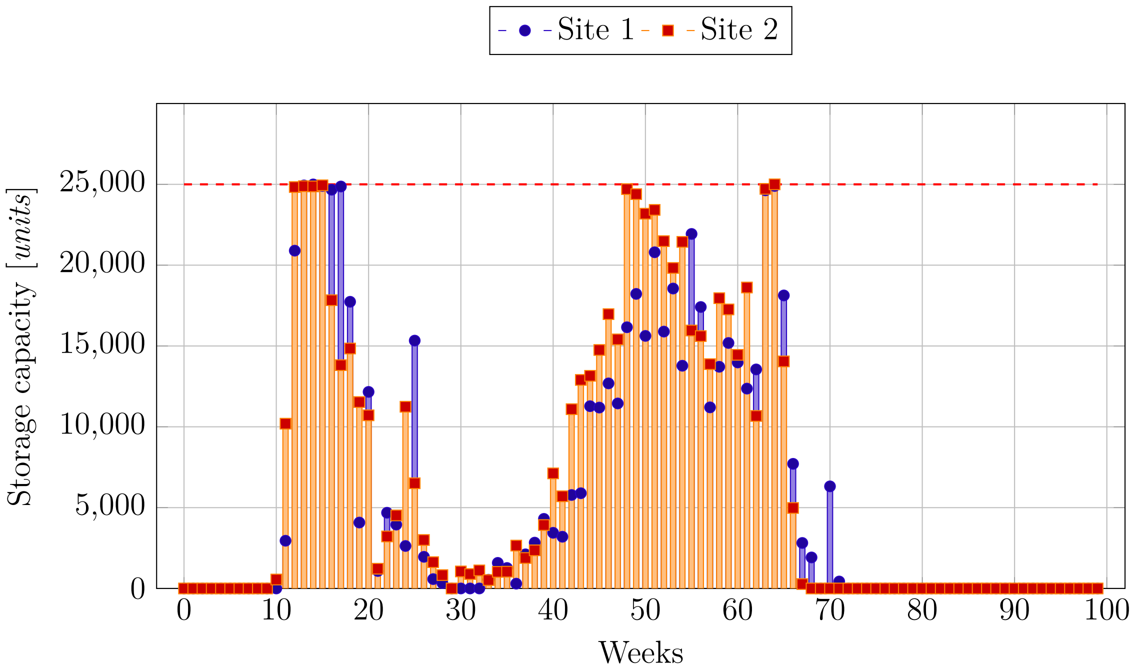

Figure 5.6 – Exploring a balanced utilization of storage capacity, with a maximum limit of 25000 units weekly.

The results presented in Figure 5.6 and Figure 5.7 are derived from the same database, however reflect different goals. Figure 5.7 illustrates the pursuit of minimizing storage capacity. Rather than establishing a maximum capacity and aiming to achieve that target, we adjust seed populations to attain the smallest possible overall capacity.

Attempting to accommodate all weekly productions within a limit also resulted in a well-organized seed schedule. Unusual harvests that deviate from the typical data pattern can be further adjusted to align with storage capacity.

Minimum capacity can be achieved by employing both early and late seeding of the seed population. However, if the gap between these seeding times is too narrow, it results in uneven capacity distribution and often leads to capacity overflow during critical stages.

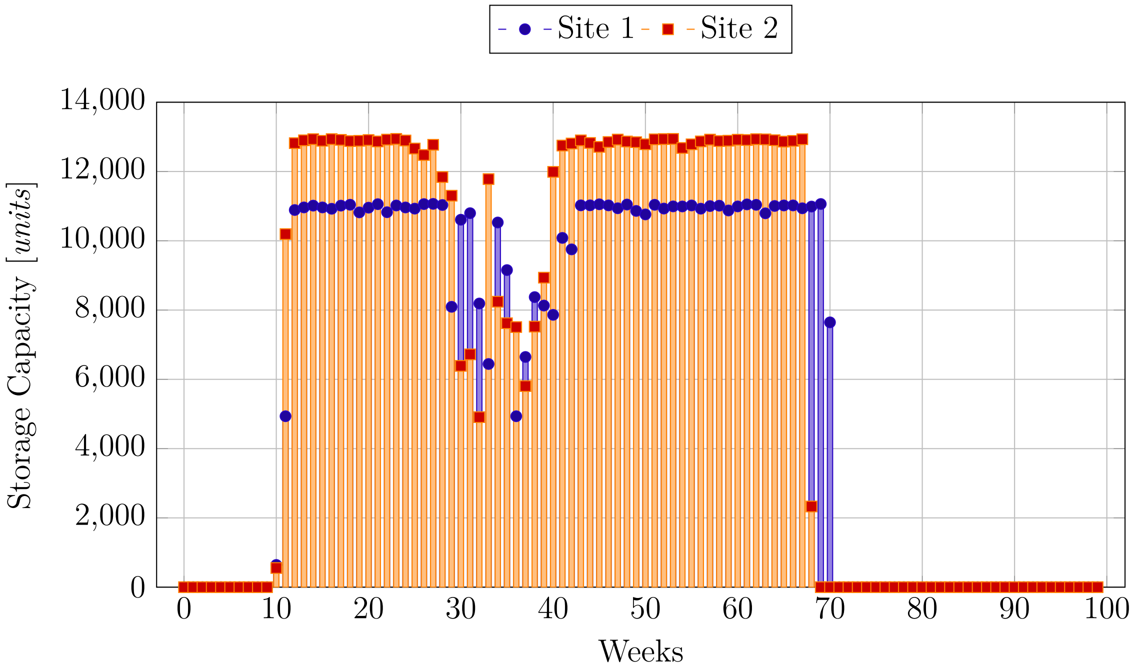

Figure 5.7 – Searching for minimum storage capacity for a two-site seed allocation problem.

5.4 Conclusion

Conducting large-scale operations without utilizing optimization can be challenging and inefficient. In this chapter, we introduce a streamlined approach to handle the distribution of seed populations under processing capacity constraints. We develop mathematical models and assess the performance of the selected solvers using a testing environment with several seed populations based on real data. By applying deterministic techniques to assess our proposed models, we obtain outstanding solutions. We advise starting by determining the minimum required storage capacity, and then implementing the model variant that optimally balances production distribution across weeks. Although addressing agricultural challenges requires abstraction to manage the complexities of living organisms, connecting Agriculture with Operations Research is an intriguing pursuit.

Developing strategies to solve difficulties in the agricultural sector is a continuous process as we improve our understanding of the problem area.

6 Pest management in soybeans

Warm temperatures, sunny days, mild winter and regular rainy seasons provide opportunities for growing crops all year round. In Brazilian southeast, many farmers have well-established the two cropping system, which defines an annual crop sequence which combines soybean in the summer and corn, sorghum or wheat in the winter (FRANCHINI *et al.*, 2012b). Crop intensification increases farm profitability and double cropping allows agricultural producers to improve soil fertility during all year. Although some Brazilian regions have for long developed the double cropping system, reducing fallow interval combined with warm temperatures along the year transform pest outbreak in a ongoing challenge as pest cycles remains unshaken due to the stable food supply and mild weather. Figure 6.1 depicts the occurrence of caterpillars in soybean fields and their effect on the total area of leaves consumed.

Figure 6.1 – Caterpillars consuming soybean foliage. [1]

One of the toughest challenges in Brazilian agriculture is to manage Helicoverpa armigera (*Lepidoptera Noctuidae*). Crop damages caused by *Helicoverpa armigera* (*H. armigera*) are noticed in several large commercial crops. In soybean fields, typical prejudices are the artificial defoliation in the vegetative stages of plant growth and pod-consumption in the reproductive stages. The damages caused by *H. armigera* can even reduce soybean's growth and prevent the crop compensatory ability (STACKE *et al.*, 2018). In 2020/2021 year-crop, Companhia Nacional de Abastecimento (CONAB) reported Brazilian soybean harvest around 135.9 million tons, which is the largest soybean

[1] Source: Image from author's ownership.

yield in world (CONAB, 2021b). Brazilian soybean exports from the same period were estimated at 86.1 million tons (CONAB, 2021a). Understanding how big is the Brazilian soybean supply chain, any pest outbreak spreading across the country drives farmers concerned about enormous yield losses.

Stacke *et al.* (2018) investigated the impact of *H. armigera* on soybean crops. The study examined the damage during two reproductive stages of the soybean plant. They conducted field experiments to verify the actual damage to soybeans caused by *H. armigera*. Inside cage units, they analyzed the effects of distinct population densities. In their conclusive remarks, they observed smaller losses in the early reproductive stages compared to the final stages. This is related to the plant's compensatory ability slowing down in the late reproductive stages, as the time to recover is short at the end of the plant cycle. From their experiments, they also reported that seed size does not vary in proportion to the damage caused by H. armigera. The most significant production loss was observed during the pod-filling reproductive stages.

The study conducted by (SUZANA *et al.*, 2018) examined the soybean consumption by *H. armigera*. Distinct vegetative and reproductive organs from the soybean plant are qualified as the source of food at different stages of development. The economic threshold, ranging from 2 to 3.5 *H. armigera* larvae per row-meter, is recommended during soybean reproductive stages (HAILE *et al.*, 2021; ADAMS *et al.*, 2016).

Maintaining soybean crops in excellent conditions is always one of the farmer's goals. There are several pests that can damage soybean crops, scouting fields is routine and leads to quick responses. The best scenario in pest management is to keep all the damaging agents under strict control. However, most of the time, completely eradicating the entire insect population is impractical , if no impossible. Even if farmers have the means to complete eradicate certain pest agents, their actions might disrupt the crop ecosystem. Agribusiness is complex and dynamic, with decisions balancing technical, financial and sustainable implications. Hence, once a reasonable level is reached, appropriate actions should be taken. In the literature, it is referred as *economic threshold*, introduced in (STERN *et al.*, 1959).

Stern *et al.* (1959) were pioneers in the economic control applied to pest management. The terminology introduced by Stern *et al.* (1959) have become widely accepted, such as the terms economic injury level and economic control. The economic injury level refers to the lowest pest population density that will cause significant economic damage. Another fundamental concept is the economic threshold which is a break-point in the curve of growing pest density. Some action at this point should be taken to avoid reaching the economic injury-level. At the economic threshold, enough time must be left to perform the economic control actions; otherwise, the economic injury level would be reached anyway. Another branch of their research is biological control, which attempts to enhance the resistance of the surround environment over a particular pest. Introducing selected natural enemies is a biological solution to reduce pest densities, and, hopefully, these potential predators will keep the pest population below the economic threshold.

In the field, farmers have plenty of tools to deal with pests. Spraying pesticides is the conventional way to keep pests under control, but biological control of caterpillars has become a lot more appealing lately. Hence, our proposed model should accommodate either chemical or biological solutions to handle the caterpillar pest problem in soybeans. Farmers do more than just keep pests in check; they also work hard to minimize the overseen damage using techniques such as foliar feeding and amino acid application, then there are some space in the proposed model for actions that would minimize defoliation over the crop cycle or help to reduce pod-damage.

This chapter aims to answer the following questions:

- How to evaluate conventional pesticides, biological pesticides and the action of natural agent predators under the same pest management perspective?
- Can an empirical assessment of complex population dynamics produce a trustworthy optimization model to the integrate pest management practices on soybeans?
- Would our algorithm generated solutions produce insights to improve the decision-making in the agrarian field?

Section 6.1 discuss the proposal of the mathematical model. Our proposal is to introduce an empirical model that combines the growth population pattern of *H. armigera* with the control mechanisms in the field, including the caterpillar's natural predators and the use of chemical or biological pesticides. Results from this study are described in Section 6.2. Conclusion and our future perspective about this ongoing research are addressed in Section 6.3.

6.1 Methodology

Our optimization proposal evaluates the damage caused by *H. armigera* in soybeans based on the three observable parameters: (1) the population density of caterpillars; (2) the soybean artificial defoliation; and (3) the soybean pod-damage related to *H. armigera* presence in the field. By introducing new policies to assist pest management decisions, we aim to minimize defoliation and pod-damage during the whole crop cycle. We understand that pod-damage is the most prejudicial side-effect because it represents irreversible yield losses. Given the growth pattern of caterpillar populations and the interplay between pesticide application and natural predators, we suggest a dynamic programming model for effective pest management.

We define a set of control decisions with Na actions and each action i has attributes $attr1[i]$, $attr2[i]$ and $attr3[i]$. The first one ($attr1[i]$) represents the killing efficiency of the caterpillar population; the next one ($attr2[i]$) is a secondary effect to the soybean defoliation; and the third one ($attr3[i]$) represents the effect from action i on reducing the soybean pod-damage. Typical control actions involve either spraying chemical pesticides or using biological pesticides. However, other mechanisms of action are also possible, as we focus on the effect of the action rather than the method by which it is carried out. Another fundamental characteristic of our modeling approach is that the effects of a control action i can last for several weeks. Although contact pesticides which exterminate pests directly upon contact, yield immediate results, there are also plenty of other options with distinct patterns. For example, pesticides that act by ingestion as insect growth regulator have effects on caterpillar population that last for several days after spraying. Our proposed model does not restrict simultaneous actions during any stage t.

Fundamental parameters in the proposed model are described ahead:

- $X1$: pest population density at the initial stage $t = 0$ [*number of individuals/area*]
- $X2$, $X3$: recorded defoliation and pod-damage at the initial stage $t = 0$, respectively, which are usually null values when the optimization covers all the soybean growth stages [*injuries in percentage*]
- Z: predator population density at the initial stage [*number of individuals per unit of area*]
- Na: the total number of available pesticides, chemical pesticides or bio-pesticides
- T: the total number of stages
- B: the lasting effect of pesticides [*in weeks*]
- I: the beginning of soybean reproductive stages [*the nth week in the sequence*]
- $W1$, $W2$, $W3$: weighted sum factors [*dimensionless*]
- α: Lotka-Volterra parameter: prey population growth rate
- δ: Lotka-Volterra parameter: prey population death rate
- β: Lotka-Volterra parameter: predator population death rate

- γ: Lotka-Volterra parameter: predator population growth rate

In the model, the main indexes are:

- $i = 1, 2, \cdots, Na$
- $t = 1, 2, \cdots, T$
- $j = 1, 2, \cdots, B$

Other parameters in the model are:

- $ac[i]$: the cost of taking action i [*$ per unit of area*]
- bc: the cost of increasing predator density in the field [*$ per unit of predator*]
- $s[t]$: a binary vector that represents suitable weather to perform actions in field
- $e[t]$: a binary vector that defines the equipment availability to execute any action
- $\Omega[t]$: exogenous information that represent unpredictable increases or decreases in caterpillar population density [*number of individuals per unit of area*]
- $attr1[i][j]$, $attr2[i][j]$, $attr3[i][j]$: attributes of action i in percentage of efficiency: (1) reduce the number of caterpillars; (2) defoliation impact and (3) reduces soybean pod damage. j index holds the effect over time from applying to the current evaluation.
- $ca[t]$: the cumulative costs of taking actions at stage t [*$ per unit of area*]
- $c1[t]$, $c2[t]$, $c3[t]$: the cumulative effect of control actions at stage t [$\%$]
- $y1[t]$, $y2[t]$, $y3[t]$: the cumulative effect of control actions at stage t expressed in terms of $x1[t]$, $x2[t]$, and $x3[t]$ states
- $x1[t]$: the caterpillar density in the field at stage t [*number of individuals per unit of area*]
- $x2[t]$: the percentage of artificial defoliation caused by caterpillars at stage t [*number of individuals per unit of area*]
- $x3[t]$: the percentage of soybean pod-damage caused by caterpillars at stage t [*number of individuals per unit of area*]
- $zp[t]$: the artificial introduction of predators in the soybean field at stage t [*number of individuals per unit of area*]

Decision variables are:

- $u[t][i]$: the decision set of actions, represented as a binary variable:

$$u[t][i] = \begin{cases} 1, & \text{action } i \text{ is taken at stage } t \\ 0, & \text{otherwise} \end{cases}$$

In general, decisions on the farm have a gradual effect extending over multiple stages. Therefore, the actual state depends on the cumulative effects of actions taken during previous stages. Equations (6.1), (6.2) and (6.3) compute the cumulative effects of actions taken from stage t to $t-B$, in which recursive assessment of the actions undertaken in prior stages. For each given period t, these equations consider what happened during the past B previous stages. Action attributes in Equation (6.1), (6.2) and (6.3) represent the percentage of efficiency based on the magnitude of the problem, Thus, $c1[t]$, $c2[t]$, and $c3[t]$ represent the cumulative percentages derived from the attributes of each executed action. The attributes of an action set are presented in Tables 6.2, 6.3, and 6.4.

$$c1[t] = \sum_{i=1}^{Na}\sum_{j=0}^{\psi} u[t-j][i] \cdot attr1[i][j], \quad \forall t \in \{0, 1, 2, \cdots, T-1\}, \tag{6.1}$$

$$\psi = \begin{cases} t, & \text{if } t - B < 0 \\ B, & \text{otherwise} \end{cases}$$

$$c2[t] = \sum_{i=1}^{Na}\sum_{j=0}^{\psi} u[t-j][i] \cdot attr2[i][j], \quad \forall t \in \{0, 1, 2, \cdots, T-1\}, \tag{6.2}$$

$$\psi = \begin{cases} t, & \text{if } t - B < 0 \\ B, & \text{otherwise} \end{cases}$$

$$c3[t] = \sum_{i=1}^{Na}\sum_{j=0}^{\psi} u[t-j][i] \cdot attr3[i][j], \quad \forall t \in \{0, 1, 2, \cdots, T-1\}, \tag{6.3}$$

$$\psi = \begin{cases} t, & \text{if } t - B < 0 \\ B, & \text{otherwise} \end{cases}$$

Equation (6.4) states the initial condition of the system. The caterpillar population density at stage $t = 0$ should be informed as well as the predator population density. The starting population density of caterpillars is denoted by $x1[0]$, while the initial defoliation level is indicated by $x2[0]$, and the initial damage to pods is represented by $x3[0]$. The initial density of predators is represented by $z[0]$.

$$x1[0] = X1,\ x2[0] = X2,\ x3[0] = X3,\ z[0] = Z \tag{6.4}$$

Equation (6.5) models the dynamic of the predator population at each stage t. We use the Lotka-Volterra equations in the problem for describing the relationship between the caterpillar predators and pest control management (LOTKA, 1956; VOLTERRA, 1926; DAS; GUPTA, 2011). We assume that the predator population, $z[t]$, represents a specific group of caterpillar predators. Additionally, we assume that the pesticides and all the control decisions affect the pest population, but not the predator population dynamics. Natural mortality and limited food supply (prey abundance) keep the predator population in check. Another assumption is about the mutualistic relationship between these predators and the soybean crop: we consider that these predators cause no damage whatsoever. The decision variable $zp[t]$ represents the capacity for artificially increasing predators in soybeans.

$$z[t] = (1-\beta)\cdot z[t-1] + \gamma \cdot z[t-1]\cdot x[t-1] + zp[t-1],\ \forall t \in \{1,2,\cdots,T\} \tag{6.5}$$

As $c1[t]$, $c2[t]$ and $c3[t]$ represent percentage amounts, we need Equations (6.6), (6.7) and (6.8) to get the action size properly at each stage t based on the previous states.

$$y1[t] = c1[t-1]\cdot x1[t-1] + \delta \cdot z[t-1]\cdot x1[t-1],\ \forall t \in \{1,2,\cdots,T\} \tag{6.6}$$

$$y2[t] = c2[t-1]\cdot x2[t-1],\ \forall t \in \{1,2,\cdots,T\} \tag{6.7}$$

$$y3[t] = c3[t-1]\cdot x3[t-1],\ \forall t \in \{1,2,\cdots,T\} \tag{6.8}$$

Equations (6.9), (6.10) and (6.11) are the transition functions and they update the states of $x1[t]$, $x2[t]$ and $x3[t]$. Stink bugs or any other pathogen that could cause soybean damages are overlooked. Artificial defoliation and pod-damage considered in the model are only caused by caterpillar populations.

$$x1[t] = (1+\alpha)\cdot x1[t-1] + \Omega[t] - y1[t],\ \forall t \in \{1,2,\cdots,T\} \tag{6.9}$$

$$x2[t] = \lambda \cdot x1[t-1] - y2[t],\ \forall t \in \{1,2,\cdots,T\} \tag{6.10}$$

$$x3[t] = \begin{cases} 0,\ \forall t \in \{0,1,2\cdots,I-1 \mid I<T\} \\ \mu \cdot x1[t-1] - y3[t],\ \forall t \in \{I,I+1,\cdots,T\} \end{cases} \tag{6.11}$$

Field activities in the farm are dependent on good weather. Hence, we have $s[t]$, a vector that holds an estimate of good weather stages during the crop cycle. Equipment availability must be accounted for in decision-making and it is represented by $e[t]$. Equation (6.12) prevents taking actions in bad weather and Equation (6.13) avoids setting decisions when the required equipment or personnel are busy with other activities.

$$u[t][i] - s[t] \leq 0, \ \forall t \in \{0, 1, 2, \cdots, T-1\}, \ \forall i \in \{0, 1, 2, \cdots, Na\} \tag{6.12}$$

$$u[t][i] - e[t] \leq 0, \ \forall t \in \{0, 1, 2, \cdots, T-1\}, \ \forall i \in \{0, 1, 2, \cdots, Na\} \tag{6.13}$$

The cumulative cost of actions, $ca[t]$, is defined in Equation (6.14). It includes the cost of general pesticides, either biological or chemical, represented by $u[t][i]$, and the cost of introducing natural agent predators.

$$ca[t] = \sum_{i}^{Na} ac[i] \cdot u[t][i] + zp[t] \cdot bc, \ \forall t \in \{0, 1, 2, \cdots, T\} \tag{6.14}$$

The objective function in Equation (6.15) is a weighted sum. We aim to minimize both the maximum damage caused by caterpillar and the cost of taking actions. This combination is essential to produce balanced pest management strategies. Without considering the cost of actions, we could use all the available pesticides to achieve minimal damage, but this would clearly be impractical and unsustainable. We do not employ a multi-objective method to assess costs and damages to soybeans, as our primary interest lies in analyzing the damages themselves. Since we utilize an artificial predator, we are not entirely focused on achieving precision in cost assessment.

$$\textbf{Minimize} \quad \sum_{t=0}^{T} W1[t] \cdot x2[t] + W2[t] \cdot x3[t] + W3[t] \cdot ca[t] \tag{6.15}$$

Equation (6.16) defines the binary variable $u[t][i]$ in the model. As the states are non-negative, we enforce that with Equation (6.17). The artificial introduction of predators at stage t is $zp[t]$. Equation (6.18) ensures that $zp[t]$ is non-negative.

$$u[t][i] = \{0, \ 1\}, \forall t \in \{0, 1, 2, \cdots, T-1\}, \ \forall i \in \{0, 1, 2, ..., Na\} \tag{6.16}$$

$$x1[t] \geq 0, \ x2[t] \geq 0, \ x3[t] \geq 0, \ \forall t \in \{0, 1, 2, \cdots, T\} \tag{6.17}$$

$$zp[t] \geq 0, \ \forall t \in \{0, 1, 2, \cdots, T-1\} \tag{6.18}$$

6.2 Computational tests and results

Our proposed optimization model is built in *Python* language and we use *PySCIPOpt*, which is well-known open source software suite for optimization and a popular choice in the community. For our computational tests, we use real-data inspired dataset and parameters. Although we did not refer to the actual pesticide compound, the set of actions is a solid virtualization of pesticide effect mechanisms. The damage caused by *H. armigera* is also defined from real-data reports. The essential parameters outlined in the following list are drawn from the research conducted by Suzana *et al.* (2018). By constructing the predator model based on theoretical assumptions, we presume that certain capabilities have yet to be fully realized in practice.:

- The caterpillar population doubles every two weeks ($\alpha = ln(2)/2 = 0.3466$)
- Each predator consumes 5 caterpillars per week ($\delta = 5$)
- Half of predator population dies every four weeks ($\beta = ln(2)/4 = 0.1733$)
- Each predation event results in 0.1 new predators ($\gamma = 0.1$)

Crop cycle is divided by weeks and becomes the set of stages $t = 1, \cdots, T$. Table 6.1 presents the set of soybean growth stages ad each corresponding week.

Table 6.1 – Soybean growth stages reorganized in days and weeks.

Soybean growth stages	**Description**	**Average days interval after sowing**	**Soybean growth stages duration**	**From sowing to full maturity (days)**	**From sowing to full maturity (weeks)**
Vegetative Stages					
VE	Emergence	4-7		7	1
VC	Cotyledon	3-10	10	17	2
V1	First node	5-6	6	23	3
V2	Second node	5-6	6	29	4
V3	Third node	5-6	6	35	5
V4	Fourth node	5-6	6	41	6
V5	Fifth node	3-4	4	45	6
V(n)	Nth node	3-4	4	49	7
Reproductive Stages					
R1	Beginning bloom	2-7	7	56	8
R2	Full bloom	2-7	7	63	9
R3	Beginning pod	2-7	7	70	10
R4	Full pod	2-7	7	77	11
R5	Beginning seed	2-7	7	84	12
R6	Full seed	2-7	7	91	13
R7	Beginning maturity	5-10	10	101	14
R8	Full maturity	5-10	10	111	16

There are several model input parameters that we could performance a sensibility analyzes, but we focus our evaluation in a comparison between general pesticides (chemical or biological) and the natural control agent (the group of caterpillar's predators). We define **exogenous information** as an indicator of a pest outbreak that occurs beyond the typical developmental pattern.

Our proposed set of control actions are not fully effective, which is in line with the application in the practice. Several pesticides are recommended to handle caterpillars, we propose a set of general pesticides with distinct efficiency in managing caterpillars and a residual effect that lasts at most three weeks after spraying. Hence, Tables 6.2, 6.3 and 6.4 presents some estimations in days after spraying.

Table 6.2 – Efficiency of decision control actions to handle caterpillar population density on soybeans.

The effect of control action in caterpillar population density				
Action ID	**Percentage of efficiency in reducing caterpillar population density**			
	Weeks After Spraying 0: immediate effect after application to 3: residual effect after three weeks			
	0	1	2	3
1	0	0	0	0
2	60	20	10	0
3	0	40	30	10
4	75	0	0	0
5	0	20	40	40
6	20	30	35	0

Table 6.3 – Efficiency of decision control actions to manage soybean defoliation caused by caterpillar population.

The effect of control action in soybean defoliation				
Action ID	**Percentage of efficiency in reducing defoliation**			
	Weeks After Spraying 0: immediate effect after application to 3: residual effect after three weeks			
	0	1	2	3
1	0	0	0	0
2	20	40	80	0
3	30	50	70	10
4	80	15	0	0
5	0	40	45	0
6	40	30	10	0

Table 6.4 – Efficiency of decision control actions to manage soybean pod-damage caused by caterpillar population.

The effect of control action in soybean pod-damage				
Action ID	**Percentage of efficiency in reducing pod-damage**			
	Weeks After Spraying 0: immediate effect after application to 3: residual effect after three weeks			
	0	1	2	3
1	0	0	0	0
2	30	70	10	0
3	0	40	30	10
4	20	75	0	0
5	0	10	40	45
6	50	70	10	5

From the computational tests we present ahead, the initial state are $x1[0] = 5$, $x2[0] = 0$ and $x3[0] = 0$. Predator population density is 0 ($z1[0] = 0$). The beginning of the soybean reproductive is at stage $t = 8$ ($I = 8$). The relationship between the caterpillar population density and defoliation damage is $\lambda = 0.05$ and for soybean pod-damage is $\mu = 0.03$. From the objective function in Equation 6.15, the model requires the weights $W1$, $W2$, and $W3$. This weight group defines the break-even between damage and action costs. We set the weights to be $W1 = 10$, $W2 = 10$ and $W3 = 1$, representing that controlling defoliation and pod-damage is more valuable than the decision costs from handling the pest outbreak. Thus, selecting the weight is subjective and accentuates the damages over the costs,although the solution will be designed to minimize costs.

The reported test in Figure 6.2 presents a pest problem solution based only on introducing natural agent predators in the cropping area. The exogenous information vector $\Omega[t]$ disturbs the pest management equilibrium and makes it possible to emulate pest outbreak throughout the entire crop cycle.

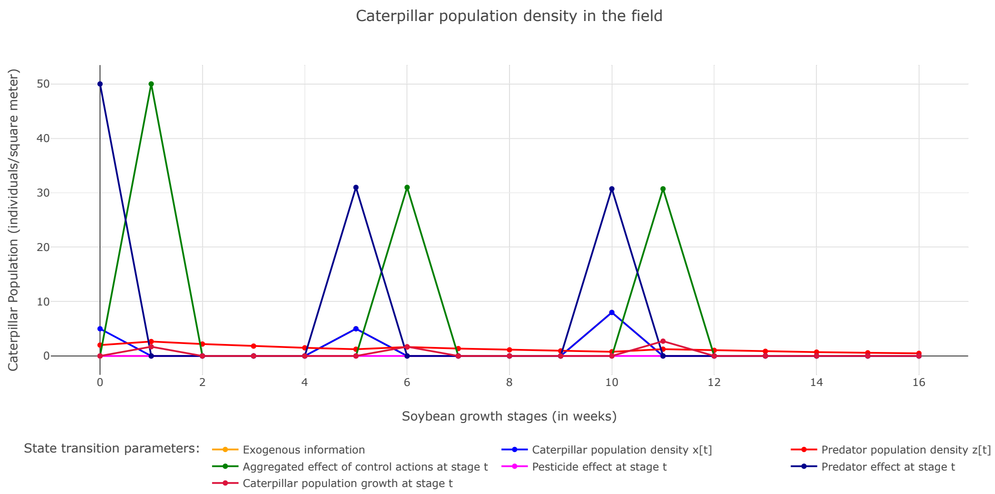

Figure 6.2 – Caterpillar population density from Test A.

Using a typical pesticide spraying as value reference, controlling caterpillar population by increasing predator population density becomes viable until its cost per unit reaches 4.03 times the cost of the typical spraying. Above this threshold, the algorithm provide solutions based on the set of available pesticides.

Figure 6.3 presents a combined action set involving pesticides and natural agent predators to manage the pest population outbreak. Although a relative small percentage of defoliation does not severely affect productivity, we observe some decisions taking by caterpillar density control acting in defoliation levels. Soybean pod-damage is the most undesirable effect caused by caterpillar. Figure 6.4 indicates the percentage damage caused by the caterpillar population on soybeans. Although the perception of high pressure from pest population is immediate and control actions are taken as soon as possible, a certain level of caterpillar in the field would cause inevitable damage until the control mechanism reaches full performance. Our methodology provides a strong response and brings down any pod-damage to prevent significant impacts during all the soybean growth stages.

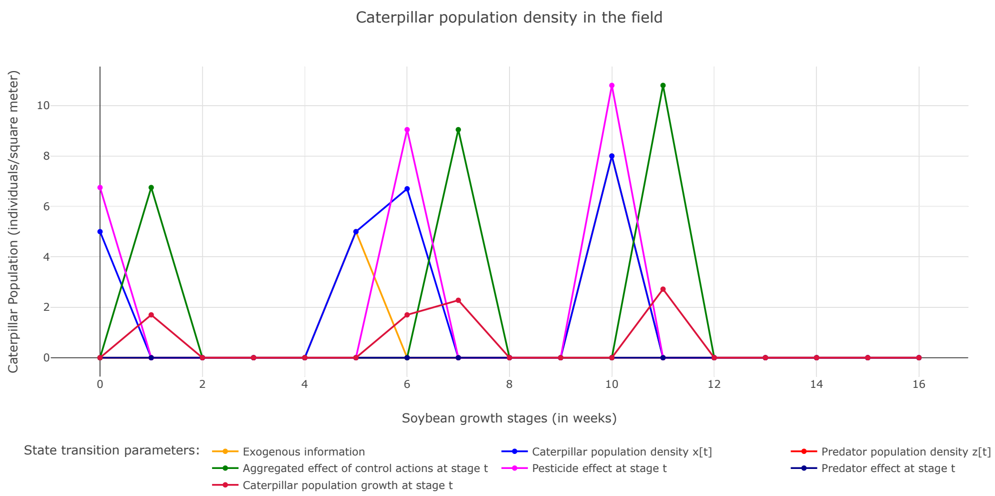

Figure 6.3 – Caterpillar population density on soybeans from Test B.

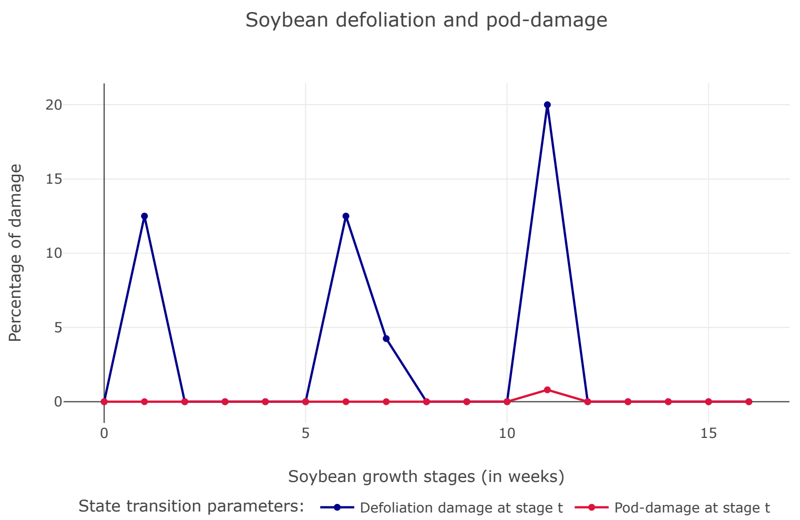

Figure 6.4 – Soybean defoliation and pod-damage from second Test B.

6.3 Conclusion and further steps

Integrated pest management guides farm decisions to prevent the spread of plant pests, including diseases, weeds, and insects. We combine the available mechanisms to handle one particular pest problem on soybeans under a new mathematical perspective. Our optimization model allows farmers to evaluate what is the best solution to reduce damages caused by *H. armigera*. Designing a tailored approach for each scenario of infestation brings more sustainable practices compared to spraying a fixed number of preventive pesticides without carefully considering a broad perspective. From the same standpoint, we provide a mathematical representation for divergent study branches that are hard to join: spraying chemical and biological pesticides and introducing dynamic prey-predator relationships in the farm management policies. Although this work addresses one particular pest on soybeans, further researches in field could easily adapt the same modeling structure to handle other critical pest agents. Our proposed strategy should to assist the farm management decision-making due to its capacity to assess the impact of *H. armigera* on soybeans and faithfully reproduce the pest challenge in a simulated environment, advising the best decisions. Once properly calibrated, our proposed model could reduce the use of pesticides and increase farm efficiency.

7 Time series forecasting in Agriculture

The agricultural sector relies heavily on climate factors and is often regarded as the most vulnerable to changes in weather trends (SANTOS *et al.*, 2021). The impact of climate change on agriculture is influenced by both the pace and intensity of these changes, as well as how well farmers can adjust their crop management practices. Strategies such as crop rotation and integrated pest management are among the techniques that can help farmers adapt. A natural outcome of this scenario is that agricultural production is consistently exposed to risks and uncertainties, with numerous factors having more negative impacts than positive ones, such as price volatility, disease outbreaks, and prolonged droughts (ÖZDEN; BULUT, 2023).

The Food and Agriculture Organization stated that for those involved in agrifood systems to continue producing, processing, marketing, and consuming safe and nutritious foods, as well as other goods and services, it is vital to employ numerous effective strategies for climate resilience and adaptation (Food and Agriculture Organization (FAO) of the United Nations (UN), 2022). Improving climate change preparedness is closely linked to initiatives in the agricultural sector, and transitioning towards more sustainable practices will demand significant long-term investments. Thus, it is virtually impractical to outline long-term investments in climate change mitigation strategies without taking into account any form of price or weather forecasting over an extended period. Anticipating trends in agricultural commodity time series plays a crucial role in sustaining agricultural revenues, thereby supporting a robust financial enterprise. Accurate forecasting of agricultural products can support decision-making and help farmers globally in achieving consistent outcomes in the years ahead.

The Box-Jenkins method (JENKINS; BOX, 1976) is a key traditional technique for forecasting time series. It relies on a linear combination of prior values with assigned weights and explicitly incorporates seasonality to account for recurring fluctuations in seasonal cycles. Combining machine learning alongside the Box-Jenkins approach, the application of artificial neural networks (ANNs) has been demonstrated as an effective method for addressing the non-linear aspects of time series forecasting (KHASHEI; BIJARI, 2011; FARUK, 2010; KURUMATANI, 2020). Another important ability of machine learning solutions is to uncover concealed relationships within target systems and accelerate the modeling of non-linear behaviors through the use of ingenious mechanisms like the gate memory in long short-term memory (LSTM) (HOCHREITER, 1997).

Predicting prices in any market is challenging, and agricultural commodities are no exception. Price forecasts involve numerous non-linear elements and unpredictable fluctuations. Among machine learning tools, Prophet stands out as a robust prediction algorithm. Its most accurate outcomes are achieved when predicting time series with pronounced seasonal patterns, such as those related series to climate (KANINDE *et al.*, 2022; GARLAPATI *et al.*, 2021) .

In this chapter, we discuss the use of artificial neural networks (ANN) for time series forecasting. Section 7.1 discusses the methodology we have utilized to predict time series relevant to the agricultural sector. Section 7.2 presents our forecasting approach into the major grain commodities. Applying time series forecasting methods to weather prediction is the topic of Section 7.3. We present our concluding reflections on the methodology for time series forecasting in Section 7.4.

7.1 The chosen techniques for forecasting time series

We have tailored our approach to time series forecasting by employing Long Short-Term Memory (LSTM) and the Prophet. While ARMA (AutoRegressive Moving Average) models are typically used for these tasks, the intricate non-linear characteristics of the time series we aim to address make standard ARMA models inadequate for reliable predictions. Thus, we utilize artificial neural networks with the aim of effectively identifying the nonlinear component of the time series. A comprehensive overview of LSTM is presented in Subsection 7.1.1. An overview of the Prophet presented in Subsection 7.1.2. The metrics chosen for performance evaluation found in Subsection 7.1.3.

7.1.1 Describing the structure of Long short-term memory (LSTM)

Recurrent neural networks (RNN) are artificial neural networks focused on sequential data or time series data. Although many deep learning architectures have been developed to deal with intrinsic dataset properties, gathered information from previous iterations are discarded and each step processes only their corresponding input. The lack of memory in several deep learning methods turn out to be ineffective models for time series forecasting. (BHANDARI *et al.*, 2022).

The vanishing gradient problem is a phenomenon that takes place in the train step of deep neural networks, where the gradients that are used to update the network's mechanism become extremely small as they are back-propagated from the output layers to the preceding layers. Even though it presents one of the major difficulties for conventional RNNs, Long short-term memory (LSTM), which is a variant RNN, has successfully dealt with the vanishing gradient problem. Innovations in LSTM memory cell overpass the challenge by incorporating a selective mechanism that hold or discard information, downsizing the gradient problem and allowing learning long-term dependencies in sequential data (SYED; AHMED, 2023). The use of LSTM for non-linear time series forecasting has established its efficacy in several studies, exceeding classical approaches in complex task as stock prediction (YADAV; THAKKAR, 2024; MOGHAR; HAMICHE, 2020; BHANDARI *et al.*, 2022; LINDEMANN *et al.*, 2021).

Figure 7.1 details the information flow inside a LSTM cell. Input features are a combination of the output from previous cell (c_{t-1}) and the hidden state (h_{t-1}). Three gates compose a LSTM cell: a forget gate, an input gate, and an output gate. Discarding information is very carefully weighed in the forget gate. Input gate updates the cell state after filtering process in the forget gate. A sigmoid layer and a tahn layer compose the input gate, the first one decides which segment to update and the last one creates a vector of new candidate values that could be added to the state. The next hidden state is provided by the output gate. The flow of information is fully refined within these three gates, leaving behind irrelevant information or redundant data.

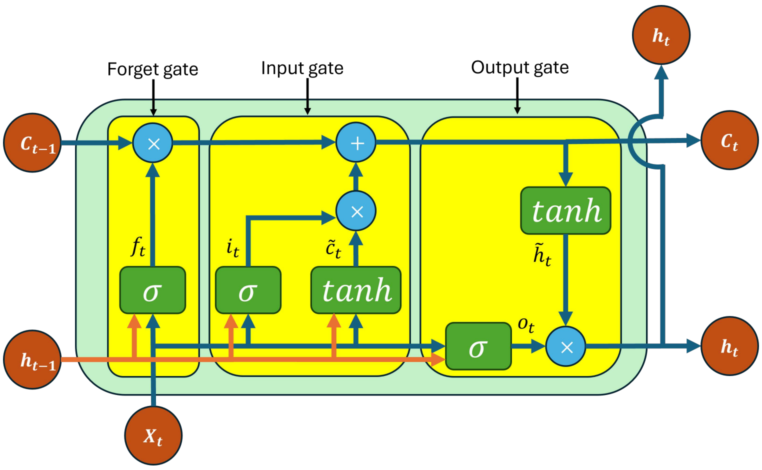

Figure 7.1 – Long short-term memory cell architecture: sh

An input time series can be represented as $X = (x_1, x_2, \cdots, x)$ and the hidden state of the memory cell as $H = (h_1, h_2, \cdots, h_t)$. Equation (7.1) defines the critical task of assessing which data fragments to retain or discard in the forget gate. The preceding hidden state h_{t-1} and the actual input x_t are the input features and the generated output is the forget vector f_t. Therefore, we have σ representing a sigmoid activation function, which has the property that it map the entire number line into a small range, such as between 0 and 1. Another features related to the forget gate are the weight matrix W_f and the bias vector b_f, both are learnable parameters acquired from the training process of the neural network (GREFF *et al.*, 2017; SYED; AHMED, 2023; BHANDARI *et al.*, 2022).

$$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f) \tag{7.1}$$

The input gate controls the inflow of fresh data into the memory cell. There are two key components in the input gate: the input activation gate and the candidate memory cell gate. The input features are the previous hidden state h_{t-1} and the actual node x_t. The degree to which the new input is integrated into the memory cell is controlled by the input activation gate, which produces an input vector i_t. The candidate memory cell gate determines the portion of new data that is kept in the memory cell illustrated in a candidate memory cell vector $\tilde{c}_t$. Equation (7.2) describes the operation of the input activation gate, whereas Equation (7.3) represents the use of the hyperbolic tangent activation function ($tanh$) to determine the candidate memory cell $\tilde{c}_t$. The input activation gate has a weight matrix Wi and a bias vector b_i , whereas a weight matrix W_c and a bias vector b_c holds a crucial role in the candidate memory cell gate. Equation (7.4) denotes the combination of the input vector i_t and the candidate memory cell vector $\tilde{c}_t$ to update the output from preceding cell c_{t-1} using element-wise multiplication.

$$i_t = \sigma(W_i[h_{t-1}, x_t] + b_i) \tag{7.2}$$

$$\tilde{c} = tanh(W_c[h_{t-1}, x_t] + b_c) \tag{7.3}$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \tag{7.4}$$

The output gate defines which data fragments from the current memory cell ought to reach the current hidden state. The output vector o_t, produced in the output gate, is produced in Equation (7.5) . The weight matrix W_o and the bias vector b_o are associated with the output gate. The final step in the memory cell operation is to set the current hidden state h_t, which is is achieved through Equations (7.6) and (7.7).

$$o_t = \sigma(W_o[h_{t-1}, x_t, c_t] + b_o) \tag{7.5}$$

$$\tilde{h}_t = tahn(c_t) \tag{7.6}$$

$$h_t = o_t \odot \tilde{h}_t \tag{7.7}$$

To enhance LSTM performance, it is necessary to perform hyperparameter tuning on the algorithm in order to discover the optimal configuration of various parameters that influence the learning process. Below, we identify the key parameters:

- Batch size: the quantity of samples analyzed prior to the adjustment of the model's internal parameters;

- Epochs: the number of times the model goes through the training data;
- Optimizer: common optimizers for LSTMs include Adam, RMSprop, and SGD;
- Number of layers: adding additional layers enhances complexity but can also render the model harder to train and susceptible to over-fitting;
- Hidden Units: the number of units in each LSTM layer;
- Learning rate: the step size in optimization; setting it too high may result in sub-optimal convergence, whereas a too low value may cause the learning process to be sluggish.

7.1.2 An overview of Prophet for time series prediction

Prophet, offered as an open-source utility by Facebook Inc., is accessible in both Python and R programming languages. It is crafted to include user-friendly parameters that can be modified without the need to understand the specifics of the underlying model (MENCULINI *et al.*, 2021). As detailed in the author's research in Taylor and Letham (2018), Prophet employs a decomposable time series model that is composed of three primary elements: (1) trend, (2) seasonality, and (3) holidays, if applicable. These elements are incorporated into Equation (7.8). The trend function $g(t)$ captures non-periodic shifts in the time series, while $s(t)$ models periodic fluctuations such as weekly and yearly cycles. The function $h(t)$ accounts for the holiday effects that can occur on irregular schedules that span one or more days. The error term ϵ_t accounts for any unique variations not covered by the model. This approach resembles a generalized additive model (GAM) (HASTIE, 2017), a category of regression models that can apply non-linear smooth transformations to predictors. The GAM framework facilitates decomposition and adaptability, allowing the integration of new components, such as a newly detected seasonal influence.

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t. \tag{7.8}$$

7.1.3 Model performance metrics

Any model creation necessitates a form of validation; however, there lacks an established or theoretical framework uniquely tailored for validating ANN models. Typically, the practice involves assessing model validation using a particular performance metric applied to data excluded from the model's construction, known as a *test set*. Commonly cited performance metrics include Mean Absolute Error (MAE) and Root Mean

Squared Error (RMSE) (TWOMEY; SMITH, 1995). In the subsequent sections, we will examine various time series forecasting models, assessing the precision and reliability of the models by calculating six different performance metrics: MSE, RMSE, MAE, MAPE, SMAPE, and R^2. Below are the mathematical equations for these metrics. We begin by describing the symbols:

- $\hat{y}_i$: predicted variable
- y_i: actual value
- $\bar{y}_i$: mean actual value
- N: number of observations

1. Mean Squared Error (MSE):

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{7.9}$$

2. Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\text{MSE}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{7.10}$$

3. Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \tag{7.11}$$

4. Mean Absolute Percentage Error (MAPE):

$$\text{MAPE} = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \tag{7.12}$$

5. Symmetric Mean Absolute Percentage Error (SMAPE):

$$\text{SMAPE} = \frac{100}{n}\sum_{i=1}^{n}\frac{|y_i - \hat{y}_i|}{\frac{|y_i|+|\hat{y}_i|}{2}} \tag{7.13}$$

6. Coefficient of Determination (R^2):

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{7.14}$$

7.2 Historical time series for major agricultural commodities

This research utilizes a collection of agricultural commodities for model forecasting. The feature selection process entails pinpointing the crucial elements that influence the pricing structure of commercial grains within Brazil's market. We focus on four primary grain commodities in Brazil: (1) corn, (2) sorghum, (3) soybeans, and (4) wheat. The grain prices reported by farmers in São Paulo are derived from weekly data collected by the Companhia Nacional de Abastecimento (CONAB) (CONAB, 2023), a state-owned company focused on insights into the agricultural sector.

From the average cash prices paid to farmers in the state of São Paulo shown in the appendix A, we compiled the price chart in Figure 7.2. Our historical dataset spans the period from 2014-01-06 to 2024-02-05. Anticipating commodity trends is a complex endeavor, as market globalization has significantly altered agricultural production. In recent decades, prices have become considerably more influenced by global trends rather than localized opportunities, resulting in a diminished seasonal impact from harvest times in specific regions. The principle of supply and demand remains valid, yet their impact should be evaluated on a global scale. Without correlating various financial indices, it would be difficult to predict major price variations, like those observed in the latter half of 2020.

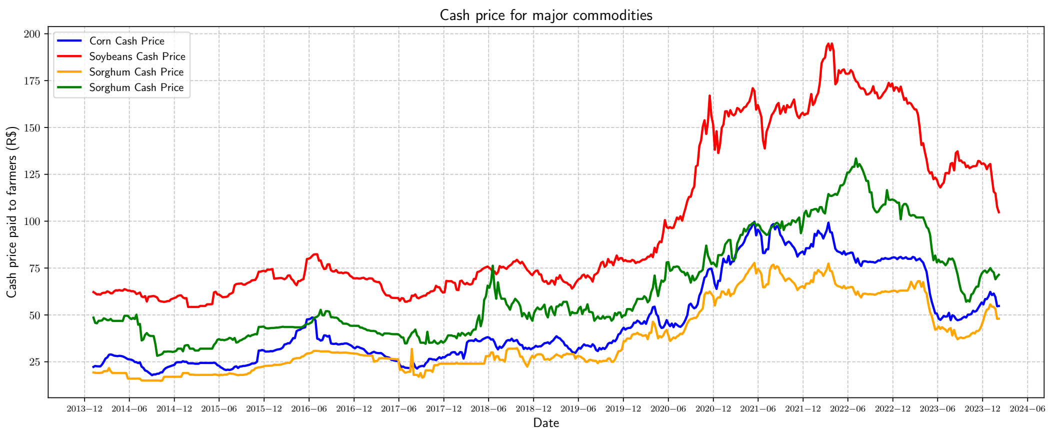

Figure 7.2 – The average cash price received by farmers for soybeans in São Paulo.

As illustrated in the price chart in Figure 7.2, we note a decline in prices for the 2022/2023 harvest. The record yield coupled with constrained storage capacity significantly affects prices. Indeed, throughout the harvest season, both domestic and international markets exert downward pressure on the cash prices producers receive, with increases being rare during this time. The factors contributing to the growth observed in 2021/2022 and 2022/2023 are more aligned with the international supply crisis.

Corn cash prices received by producers tend to remain more stable during the summer and winter harvest seasons. In the state of São Paulo, this price steadiness is attributed to strong domestic demand and its advantageous geographic position. According to historical price data, 2023 experienced significant price declines. The reported cash prices for major grains in 2023 brought an end to the growth seen in previous harvest seasons. Even excluding yield unpredictability, low grain prices significantly impact farm profitability. Thus, engaging in long-term planning initiatives would enhance business stability and better prepare farmers for the upcoming growth cycle. To predict the weekly prices of agricultural commodities, we employ both univariate and multivariate LSTM models, which will be detailed in the upcoming subsections.

7.2.1 Predicting price commodities using univariate LSTM network

We combine the prediction for corn cash price from the Long Short-Term Memory in Figure 7.3. For a clearer presentation within the graphic, we omitted a substantial segment of the training from the plot, concentrating instead on the test set and projections. At times, a sole focus on metrics evaluation can mislead us. By utilizing plots, we can more effectively determine whether the model truly possesses any predictive ability.The primary distinction between test evaluation and future projections hinges on batch information. Testing utilizes historical data contained within the batch; in contrast, predictions for corn futures extending beyond the batch size are achieved by incorporating these future forecasts into the batch and fully substituting the real data from the input parameters. This method results in error accumulation because predictions rely on other predictions.

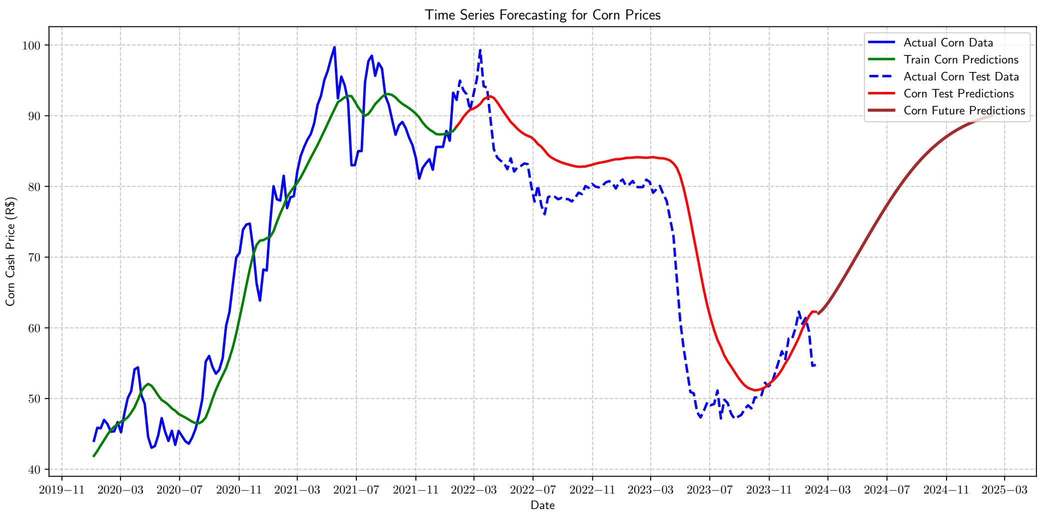

Figure 7.3 – Predicting corn cash price using a univariate LSTM model.

From sorghum, we present the forecast in Figure 7.4. We define *future pre-*

dictions as those made beyond the available historical data. In the training and testing phases, the LSTM generates what we simply refer to as *predictions*.

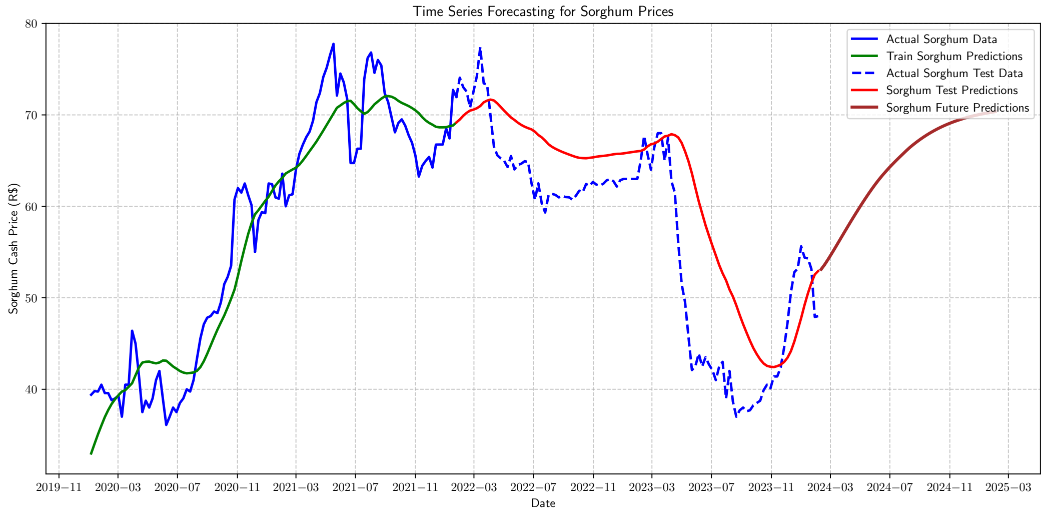

Figure 7.4 – Predicting sorghum cash price using a univariate LSTM model.

Figure 7.5 shows prediction for cash price paid to farmers for soybeans. By integrating the data from this chart with the metrics presented in Table 7.1, we observe that the correlation of the model with soybean data had the least accurate approximation to the actual historical data in the test set. The performance metrics for predicting soybean cash prices with the univariate LSTM model were inferior to those of other agricultural commodity models.

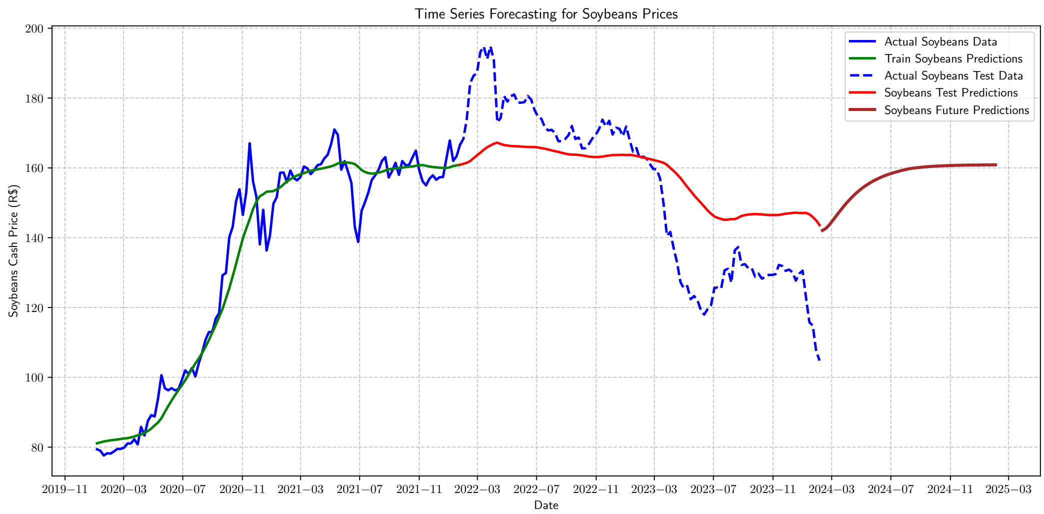

Figure 7.5 – Predicting soybean cash price using a univariate LSTM model.

While the model's performance is not yet optimal, it is important to consider that volatility in agricultural commodities has surged significantly since 2020, with market fluctuations becoming so frequent that even market analysts are struggling to deliver reliable forecasts. Despite the LSTM making attempts to manage certain non-linear behaviors, the evaluation period corresponds to a highly volatile phase in commodity prices, so the suboptimal performance observed in Table 7.1 should not be unexpected.

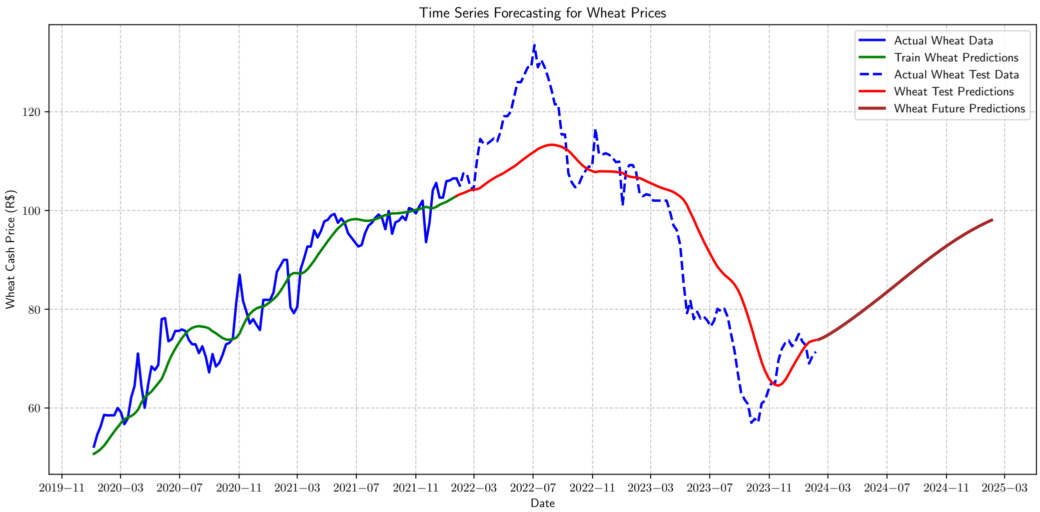

Figure 7.6 – Predicting wheat cash price using a univariate LSTM model.

By evaluating these measurements, we note that predicting sorghum cash achieved the highest performance in terms of MSE, RMSE, and MAE. In contrast, wheat predictions scored top in R2, whereas corn predictions excelled in SMAPE. While LSTMs offer an extensive array of parameters to fine-tune their performance, we present solely the optimal results obtained from numerous tests, aiming for conciseness in this thesis.

Table 7.1 – Evaluating the univariate LSTM performance for forecasting agricultural commodity futures prices.

Time Series	MSE	RMSE	MAE	MAPE	SMAPE	R2
Corn	62.82	7.93	5.91	9.39	8.57	0.73
Sorghum	53.85	7.34	5.74	11.56	10.54	0.57
Soybeans	299.2	17.3	14.51	10.34	9.8	0.48
Wheat	102.6	10.13	8.11	9.1	8.75	0.78

7.2.2 Using a multivariate LSTM network for forecasting agricultural commodities

In this subsection, we explore the application of a multivariate LSTM for predicting agricultural commodities. The input variables consist of four commodities: corn, sorghum, soybean, and wheat. The output is a unified forecast for all four commodities. The predictions for the train set are omitted as it is required to combine the four series into a single entity to properly represent the training context of multivariate LSTMs.

Figure 7.7 displays the cash prices for corn paid to farmers in São Paulo. After completing the training and testing phases, we predict prices for the subsequent year based on historical data. Although our training incorporates data starting from early 2013, for clarity, we only show the final segment of training data from 2019 on-wards in the following figures.

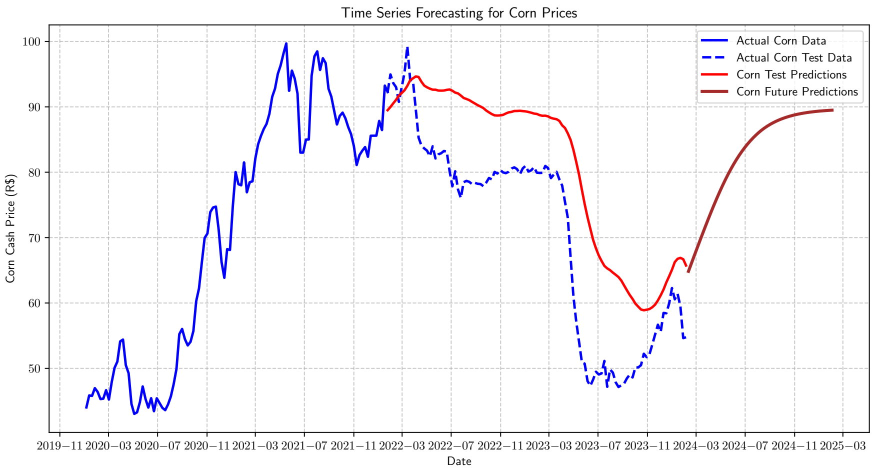

Figure 7.7 – Predicting corn cash price using a multivariate LSTM model.

For sorghum, we have price prediction in Figure 7.8. Though corn prices exceed those of sorghum, the pricing trends for both display a remarkably similar pattern.

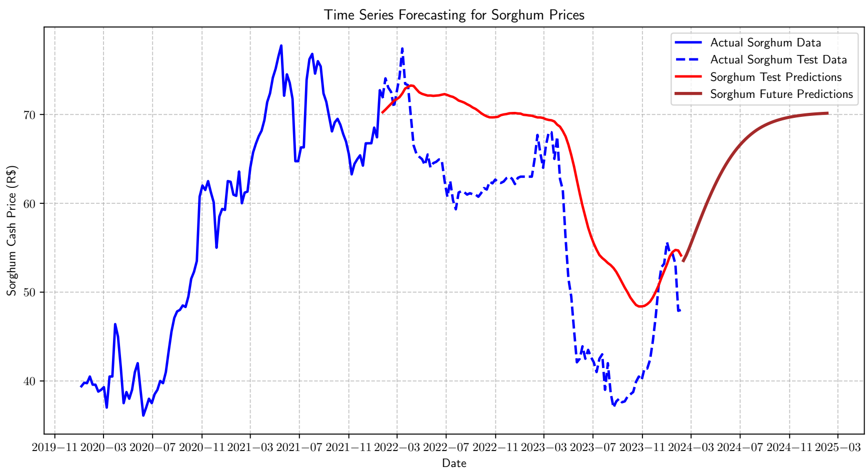

Figure 7.8 – Predicting sorghum cash price using a multivariate LSTM model.

Soybean forecasting in presented in Figure 7.9. The discrepancy between the algorithm's predictions and soybean prices in the last quarter of 2023 should not be viewed as a flaw in the algorithm since economic volatility was exceptionally high during that period.

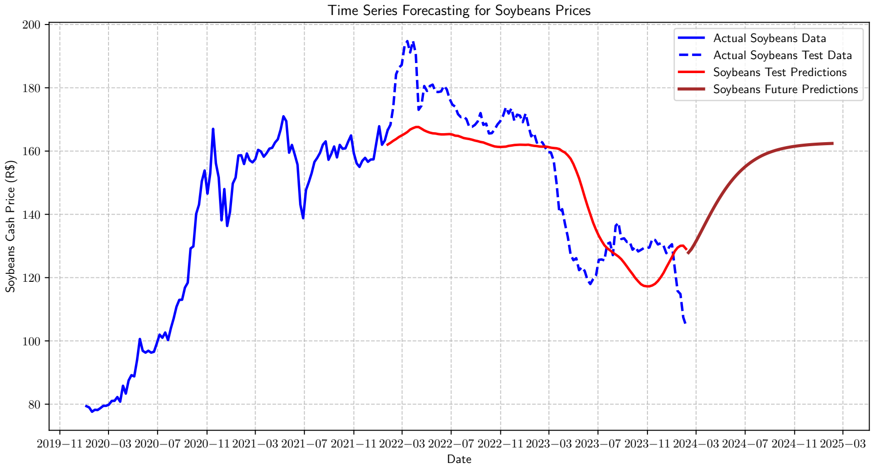

Figure 7.9 – Predicting soybean cash price using a multivariate LSTM model.

Wheat forecasting is part of Figure 7.10. By employing the multivariate variant of LSTM, we aim to correlate additional time series with wheat predictions, and the converse is equally applicable. The separation between the historical wheat series and the test predictions might seem disconcerting; however, it reflects the algorithm's inference derived from the other price series.

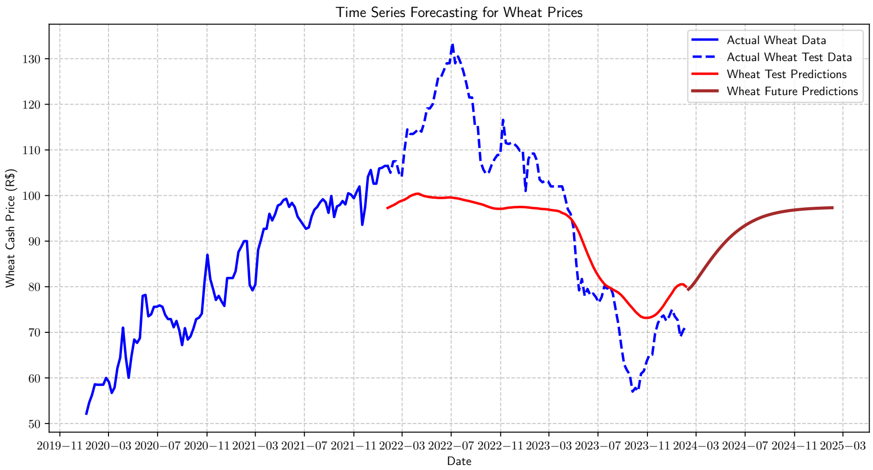

Figure 7.10 – Predicting wheat cash price using a multivariate LSTM model.

A consolidated view of the metric errors from the multivariate LSTM to predict commodity prices is depicted in Table 7.2.

Table 7.2 – Time series forecast evaluation using a multivariate LSTM.

Column	MSE	RMSE	MAE	MAPE	SMAPE	R2
Soybeans	193.45	13.91	11.55	7.88	7.79	0.67
Sorghum	83.26	9.12	7.89	15.66	14.02	0.34
Wheat	197.79	14.06	11.68	11.84	12.12	0.57
Corn	143.16	11.97	10.58	16.8	14.98	0.39

7.3 Historical weather data and forecasting

In combination with price history, the weather history plays a crucial role in price fluctuations. We collected weather history from the Instituto Nacional de Metereologia (INMET). We conducted multiple forecasts using Prophet, a univariate LSTM, and a multivariate LSTM, which are detailed in the subsections below. To be concise and improve clarity, we have chosen the following list for display in our chart analysis:

- Weekly average atmospheric pressure at station level (mB)
- Weekly average total precipitation (mm)
- Weekly average global radiation (KJ/m^2)
- Weekly average air temperature ($°C$)
- Weekly relative air humidity (%)
- Weekly average wind speed (m/s)

In order to concisely assess the algorithms chosen for predicting weather variables, we employ data from a weather station in Sorocaba, São Paulo, to conduct various tests and facilitate the ensuing discussions. In Subsection 7.3.1, the application of the Prophet for predicting weather-related time series is discussed. Subsection 7.3.2 focuses on employing the univariate LSTM, while Subsection 7.3.3 showcases outcomes from the multivariate LSTM model. It is anticipated that any latent patterns within these combination of time series data would emerge in the multivariate model if they exist.

7.3.1 Using Prophet to forecast weather time series

For time series that exhibit significant seasonal dependence, a model using Prophet can effectively align with the data.

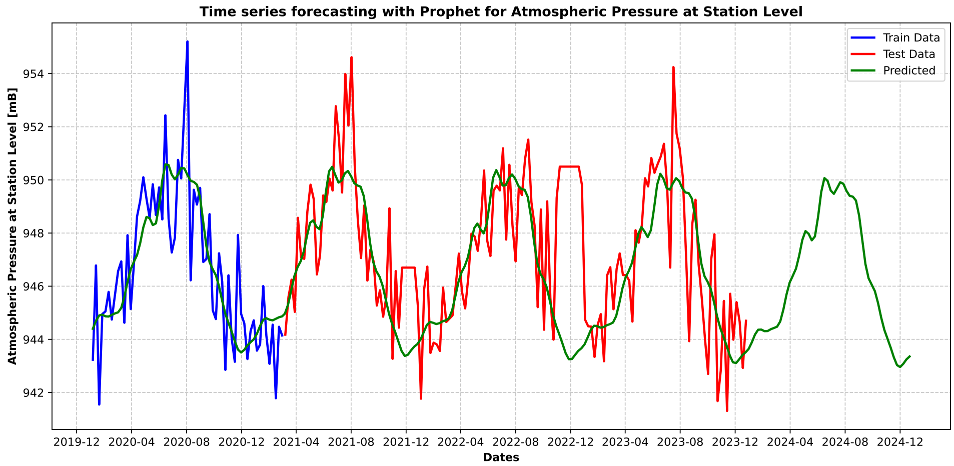

Figure 7.11 – Predicting Atmospheric Pressure at Station Level using Prophet.

Clearly, it is quite challenging to integrate extreme outliers observed in April 2021 into the time series forecasts. The weekly cumulative precipitation depicted in Figure 7.12 shows a distinct pattern with pronounced annual seasonality, featuring peaks during spring and summer and notable lows from autumn to winter.

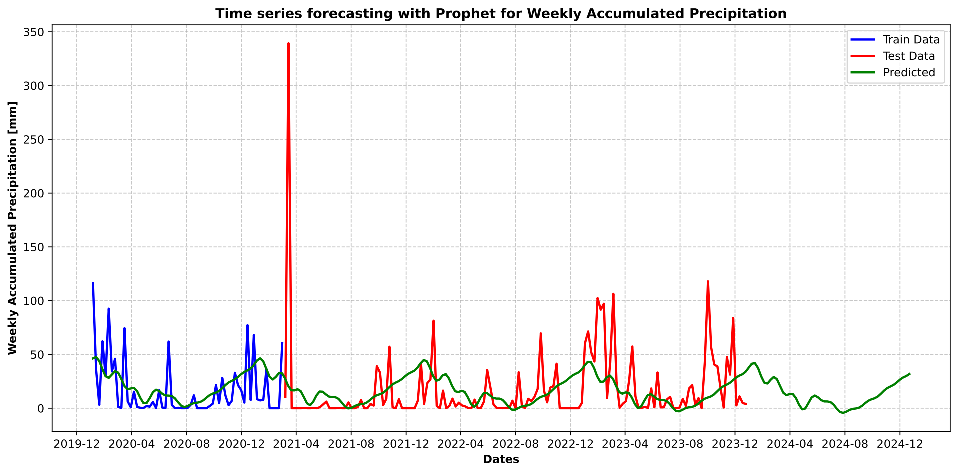

Figure 7.12 – Predicting Precipitation using Prophet.

Total weekly accumulated radiation depends on the ratio of cloudy and sunny days. This weather variable is highly difficult to grasp as we can see in Figure 7.13.

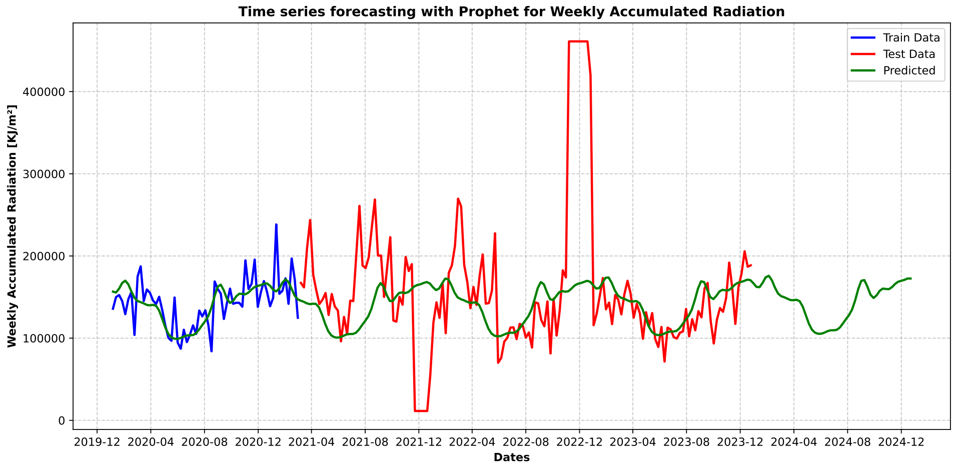

Figure 7.13 – Prophet Model for Accumulated Radiation Prediction.

Extended periods of drought typically occurring during winter show a notable correlation with declines in the average relative humidity depicted in Figure 7.14.

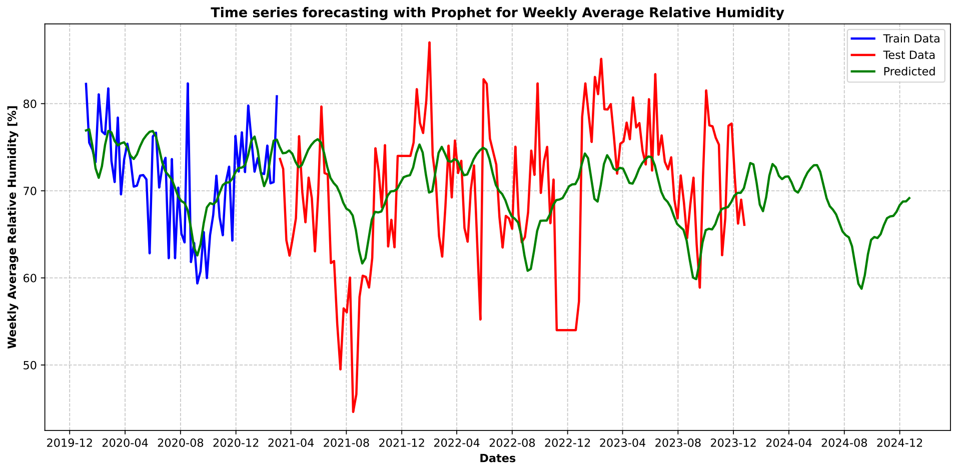

Figure 7.14 – Utilizing Prophet to forecast relative humidity.

In the graph shown in Figure 7.15, one can observe a modest year-to-year rise in the temperature forecast by Prophet. Even algorithmic predictions are starting to reveal the effects of global warming.

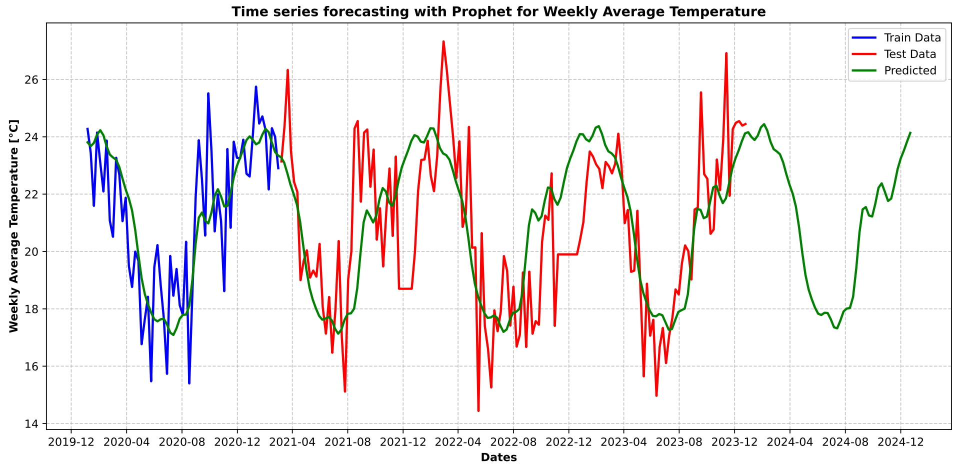

Figure 7.15 – Forecasting mean temperature with the Prophet model.

In analyzing the data illustrated in Figure 7.16, we observed a regular trend in wind velocity spanning 2019 to 2022. However, a significant decline in the weekly average wind velocity is evident in 2023. Prophet successfully identified this outlier trend and accurately captured the overall annual behavior.

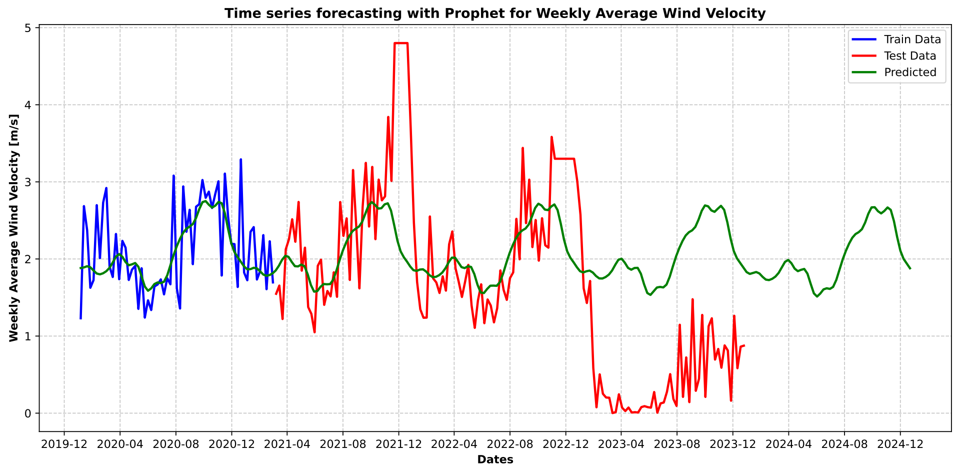

Figure 7.16 – Utilizing Prophet to forecast mean wind speed.

Table 7.3 summarizes the evaluation of Prophet metrics. We have included various graphics throughout this section to forecast variables, aiming to prevent relying solely on the table's metrics for model assessment. Despite not achieving outstanding results from these metrics, it is evident that the algorithm demonstrates some predictive capability. The lower performance can be attributed to the numerous outliers in the weather-related parameters.

Table 7.3 – Evaluating Prophet performance in predicting weather parameters.

Forecasting variable	MSE	RMSE	MAE	MAPE	SMAPE	R2
Weekly Accumulated Precipitation	1270.25	35.64	18.81	inf	141.51	0.04
Weekly Accumulated Radiation	7025772647.92	83819.88	49032.93	69.54	29.95	0.04
Atmospheric Pressure at Station Level	5.76	2.4	1.72	0.18	0.18	0.21
Weekly Maximum Atmospheric Pressure in the Previous Hour	5.76	2.4	1.71	0.18	0.18	0.21
Weekly Minimum Atmospheric Pressure in the Previous Hour	5.95	2.44	1.73	0.18	0.18	0.21
Weekly Average Temperature	4.67	2.16	1.67	8.24	8.06	0.39
Weekly Average Temperature - Dew Point	5.4	2.32	1.7	13.85	12.55	0.5
Weekly Maximum Temperature	15.27	3.91	2.53	9.7	8.76	0.01
Weekly Minimum Temperature	6.46	2.54	1.94	32.82	17.82	0.67
Weekly Maximum Dew Point Temperature in the Previous Hour	8.44	2.91	1.96	12.27	11.07	0.22
Weekly Minimum Dew Point Temperature in the Previous Hour	10.33	3.21	2.58	161.11	38.9	0.5
Weekly Maximum Relative Humidity	74.63	8.64	3.74	5.21	4.46	-0.02
Weekly Minimum Relative Humidity	116.49	10.79	8.05	25.66	23.91	0.14
Weekly Average Relative Humidity	68.98	8.31	6.66	9.96	9.76	0.01
Weekly Average Wind Direction	745.53	27.3	22.0	13.12	13.12	-0.18
Weekly Maximum Wind Gust	4.54	2.13	1.63	102.6	43.76	0.13
Weekly Average Wind Velocity	1.33	1.15	0.9	2007.41	62.09	0.03

7.3.2 Univariate LSTM

Using a multivariate LSTM, we predict atmospheric pressure at stations level depicted in Figure 7.17.

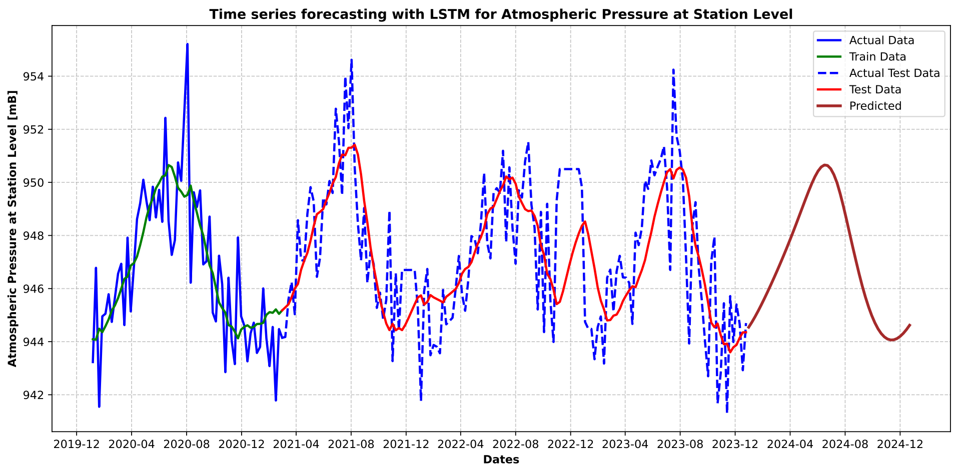

Figure 7.17 – Using LSTM univariate to predict atmospheric pressure at station level.

A prediction for weekly precipitation is part of Figure 7.18.Forecasting precipitation for the upcoming years undoubtedly stands as the foremost task in time series forecasting in relation to the topics we have investigated in this study. Accurate rainfall predictions for the upcoming season can significantly influence the decisions on crop sequences that we explored in Chapter 3. Being aware of a forthcoming dry season can influence farmers' choices to enhance resilience and prevent crop failures.

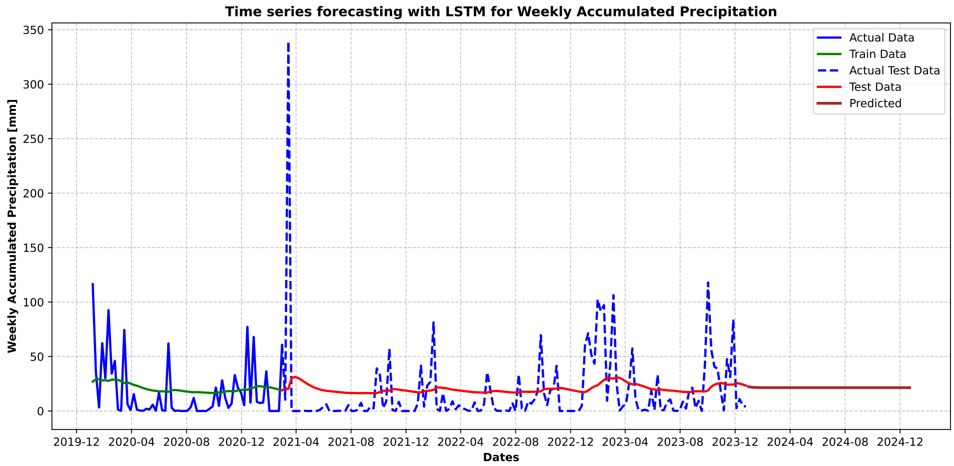

Figure 7.18 – Weekly total precipitation prediction with univariate LSTM.

Weekly accumulated radiation using univariate LSTM is part of Figure 7.19. Radiation is as crucial as water for crop growth and can significantly influence yield.

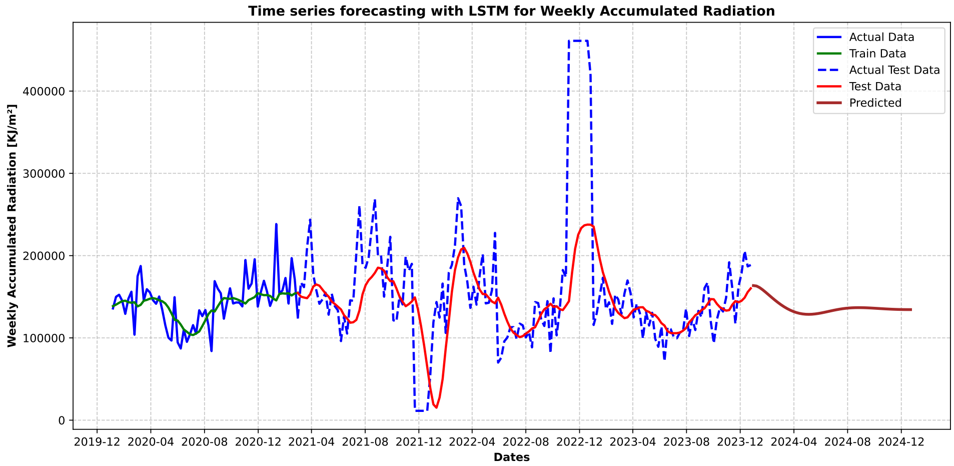

Figure 7.19 – Weekly accumulated radiation estimation from a univariate LSTM.

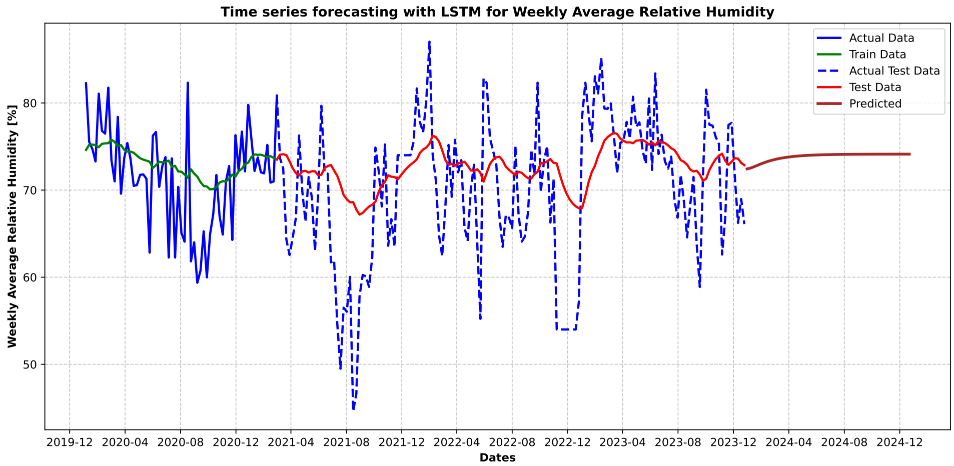

Figure 7.20 – Mean relative humidity using a univariate LSTM.

Predicting mean temperature using univariate LSTM is presented in Figure 7.21.

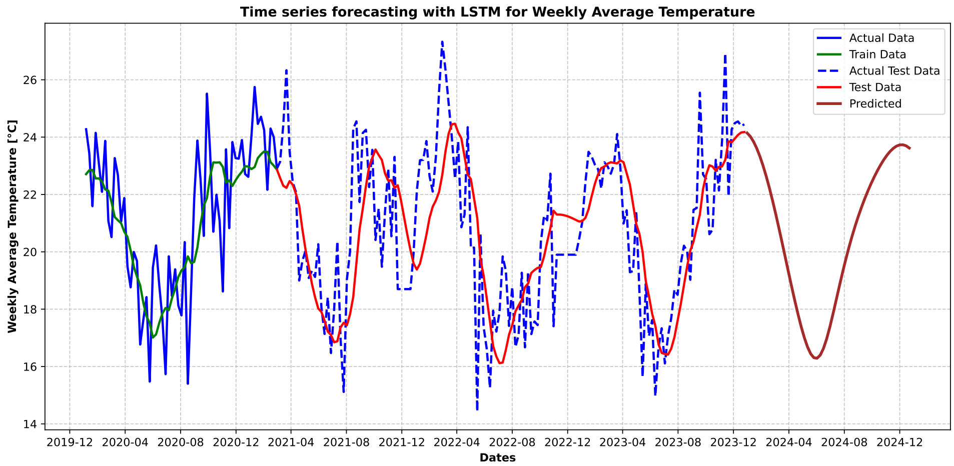

Figure 7.21 – Mean temperature forecasting using univariate LSTM.

Wind velocity is predicted in Figure 7.22.

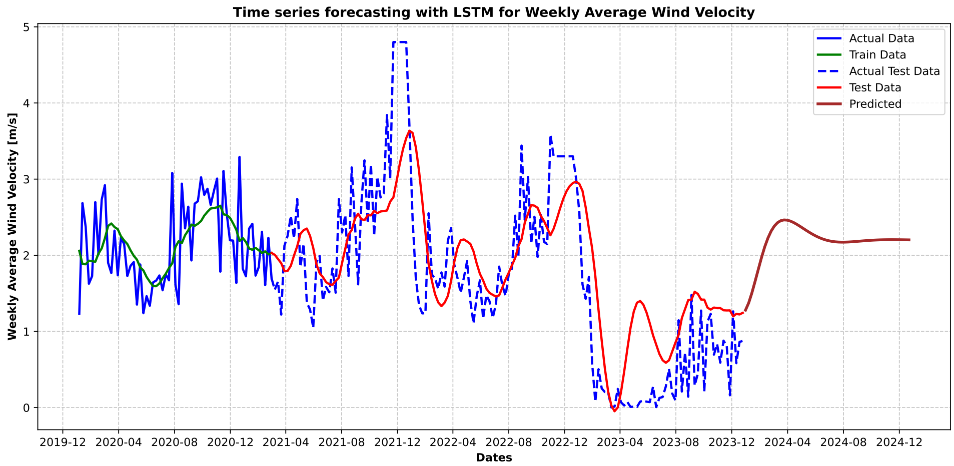

Figure 7.22 – Wind velocity prediction based on univariate LSTM network.

Errors from the selected metrics are displayed in Table 7.4. Errors associated with accumulated radiation are not surprising, given their distinction from other types of errors because of the large quantities that characterize accumulated radiation.Other meteorological variables are expressed in smaller units.

Table 7.4 – Assessing the effectiveness of LSTM models for forecasting weather patterns.

Time series	MSE	RMSE	MAE	R2
Weekly Accumulated Precipitation	1304.51	36.12	22.04	0.01
Weekly Accumulated Radiation	5170616000.0	71906.99	43702.19	0.29
Atmospheric Pressure at Station Level	4.05	2.01	1.62	0.45
Weekly Maximum Atmospheric Pressure in the Previous Hour	4.4	2.1	1.64	0.4
Weekly Minimum Atmospheric Pressure in the Previous Hour	4.39	2.09	1.68	0.42
Weekly Average Temperature	3.99	2.0	1.55	0.48
Weekly Average Temperature - Dew Point	5.57	2.36	1.83	0.49
Weekly Maximum Temperature	9.88	3.14	2.32	0.36
Weekly Minimum Temperature	6.12	2.47	1.97	0.68
Weekly Maximum Dew Point Temperature in the Previous Hour	7.55	2.75	1.99	0.3
Weekly Minimum Dew Point Temperature in the Previous Hour	10.33	3.21	2.58	0.5
Weekly Maximum Relative Humidity	66.64	8.16	3.41	0.08
Weekly Minimum Relative Humidity	101.07	10.05	7.51	0.25
Weekly Average Relative Humidity	62.18	7.89	6.15	0.11
Weekly Average Wind Direction	432.83	20.8	16.69	0.32
Weekly Maximum Wind Gust	1.63	1.28	1.0	0.69
Weekly Average Wind Velocity	0.56	0.75	0.58	0.59

7.3.3 Using LSTM multivariate version to predict weather forecast

In the same manner we did with price series forecasting, we use a multivariate LSTM predict input several weather parameters and predicting all of them together to see if there are interrelated.

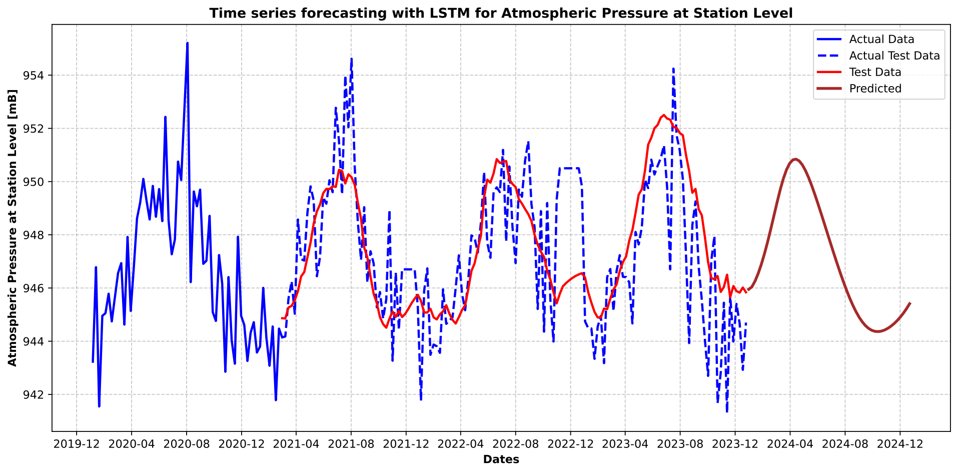

Figure 7.23 – Predicting atmospheric pressure with a multivariate LSTM network.

Weekly accumulated precipitation and radiation are presented in Figures 7.24 and 7.25. We note the peak occurring around April 2021 and acknowledge the difficulty in devising a precise prediction strategy for long-term evaluation.

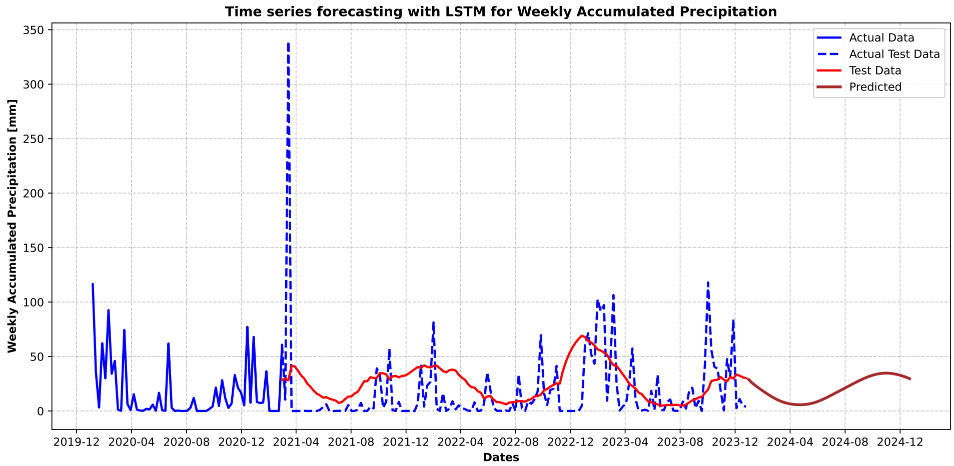

Figure 7.24 – Weekly accumulated precipitation using a multivariate LSTM network.

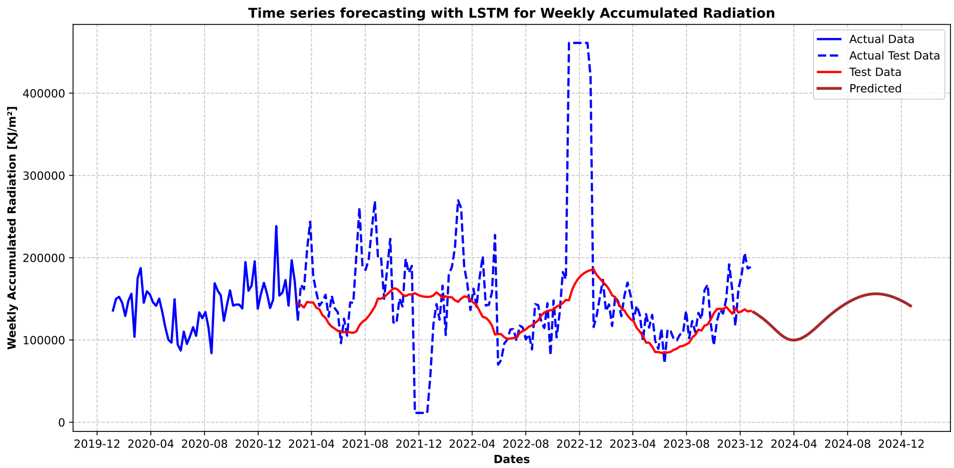

Figure 7.25 – Weekly accumulated radiation using a multivariate LSTM network.

Air moisture prediction is the subject of Figure 7.26. The low levels of air moisture observed in December 2022 stand out as outliers. Forecasts conducted beyond the actual test data indicate a predictable trait, which is a drop in air humidity throughout the winter months, with a particularly notable low occurring in July 2024.

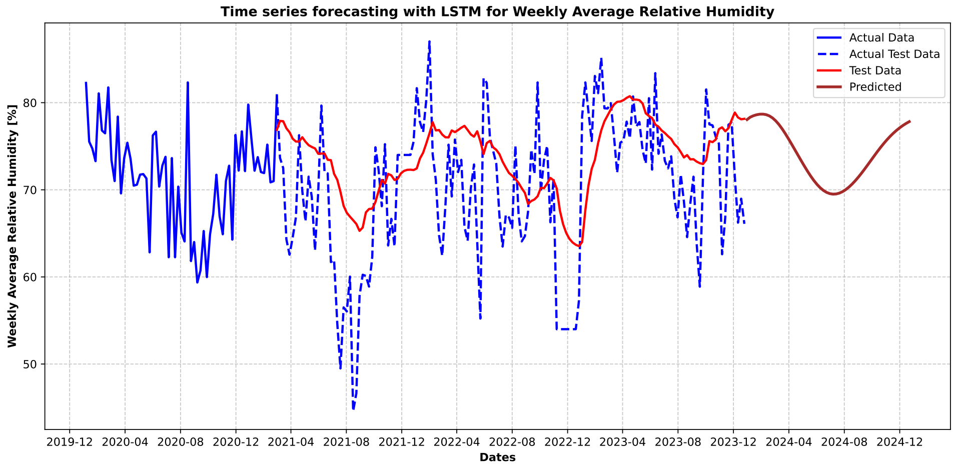

Figure 7.26 – Weekly average relative humidity using a multivariate LSTM network.

Predicting mean temperature using univariate LSTM is presented in Figure 7.27. We observed a modest rise in the annual trend shown in the forecast beyond the test dataset, aligning with the global warming trends for the upcoming years.

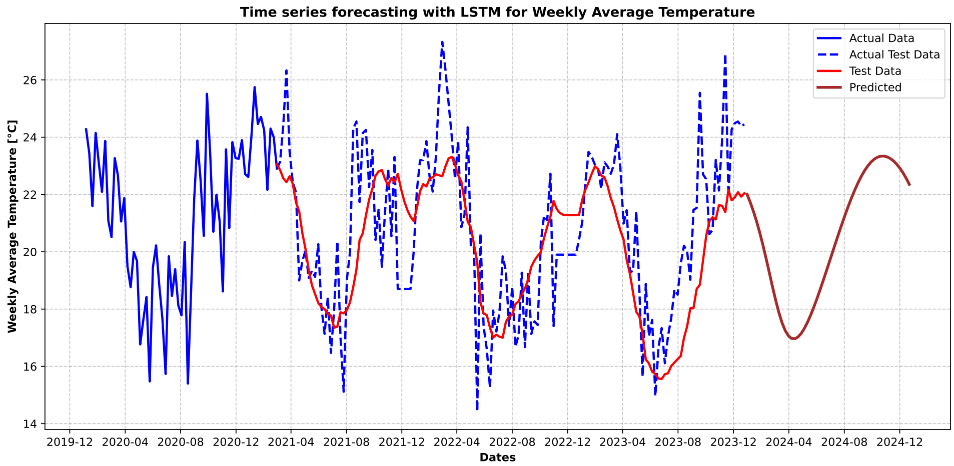

Figure 7.27 – Predicting average temperature using a multivariate LSTM network.

Wind velocity is predicted in Figure 7.28. The climatic factors discussed in this chapter are closely linked to the various themes explored in prior chapters of this thesis. For instance, the effectiveness of herbicide spraying conducted by tractors is impacted by wind velocity. The evapotranspiration rate in crops is also influenced by wind conditions.

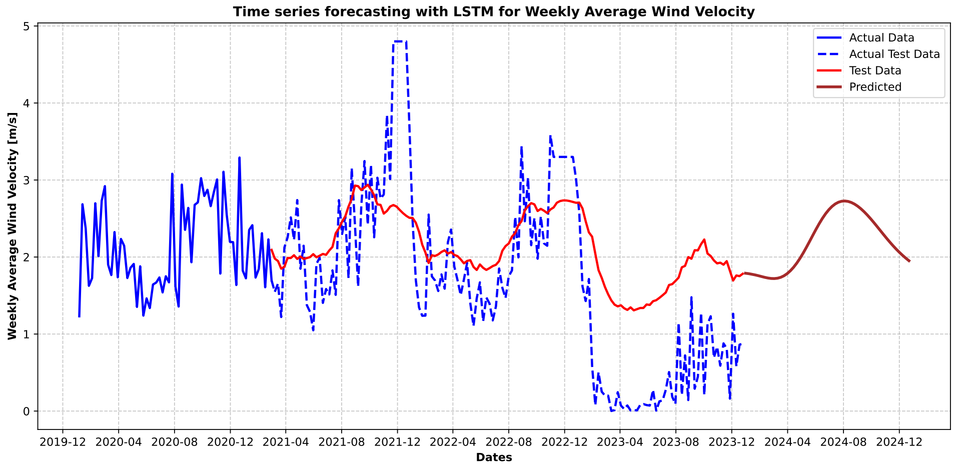

Figure 7.28 – Weekly average wind velocity from a multivariate LSTM network.

Computed errors for the multivariate LSTM is the theme of Table 7.5. Upon comparison with the results in Tables 7.4 and 7.3, it is evident that none of the three methods align perfectly with the historical weather data. It is important to note that the significant weekly fluctuation in weather data results in metric inaccuracies. Nevertheless, charts illustrate that these methods effectively capture the seasonal component.

Table 7.5 – Evaluating LSTM multivariate performance.

Time series	MSE	RMSE	MAE	R2
Weekly Accumulated Precipitation	1379.77	37.15	23.31	-0.04
Weekly Accumulated Radiation	6686341600.0	81770.06	49024.26	0.08
Atmospheric Pressure at Station Level	4.54	2.13	1.66	0.38
Weekly Maximum Atmospheric Pressure in the Previous Hour	4.5	2.12	1.6	0.39
Weekly Minimum Atmospheric Pressure in the Previous Hour	4.32	2.08	1.6	0.43
Weekly Average Temperature	4.37	2.09	1.64	0.42
Weekly Average Temperature - Dew Point	4.52	2.13	1.63	0.58
Weekly Maximum Temperature	16.65	4.08	2.87	-0.08
Weekly Minimum Temperature	8.62	2.94	2.31	0.56
Weekly Maximum Dew Point Temperature in the Previous Hour	6.63	2.58	1.81	0.38
Weekly Minimum Dew Point Temperature in the Previous Hour	11.14	3.34	2.68	0.46
Weekly Maximum Relative Humidity	55.27	7.43	3.74	0.24
Weekly Minimum Relative Humidity	115.13	10.73	8.0	0.14
Weekly Average Relative Humidity	63.98	8.0	6.5	0.08
Weekly Average Wind Direction	625.21	25.0	19.27	0.01
Weekly Maximum Wind Gust	3.08	1.76	1.37	0.4
Weekly Average Wind Velocity	0.93	0.96	0.8	0.32

7.4 Final remarks about time series forecasting in the agrarian field

Forecasting time series in agriculture is not merely a promising strategy; it is critical to maintain consistent agricultural output amid the challenges posed by climate change, which affects both weather patterns and commodity prices. The increasing expenses that farmers incur for crop cultivation should be carefully evaluated against potential revenue before proceeding. For a precise assessment of future earnings, it is essential to establish accurate predictions for agricultural commodities and substantiate the necessity of enhancing foreseeability within the agricultural industry.

Our findings suggest that predicting time series in agricultural settings is feasible and merits additional investigation. Although making accurate long-term forecasts, spanning several years, may remain challenging in the immediate future, we cannot overlook the potential transformation they may bring to agricultural planning as they become more dependable. Additional research is necessary to determine the reliability of forecasts over multiple steps based on historical data. Instead of attempting to forecast time series events, we should reinterpret predictions as the likelihood of events occurring. For instance, predicting the probability of receiving sufficient rainfall in the upcoming cropping season. Potential modifications may enhance the accuracy of forecasts when compared to straightforward predictions of weekly rainfall several months in advance.

8 Conclusion

This thesis's findings have demonstrated their worth in addressing the agricultural challenges we explored through the perspective of optimization. We offer an engaging methodology, enhancing the efficiency of various planning activities associated with farm management. Through our proposal of optimization models, we have provided several services that encompass the daily operations of numerous grain farmers both in Brazil and around the world.

We address real-world challenges in agriculture by modeling their effects on the agribusiness sector, and developed practical solutions to help overcome obstacles in farming activities. Our findings indicate that agriculture can gain substantially by merging planning with a systematic methodology, especially in daily farm decisions. In our quest for sustainability, we prioritize evaluating the environmental impacts of agricultural management choices on ecological balance, considering both resource consumption and pesticide application. Our approach is meticulously crafted to reduce environmental impact through a deliberate management strategy.

We have presented an extensive review of the agricultural sector along with a broad perspective on grain farm practices, encompassing the application of herbicides, pest management, the utilization of cover crops, and the rotation of different crop families. Our study sets itself apart in the literature by addressing farm management decisions in a practical manner, enabling the incorporation of typically complex elements into farm production forecasts. This includes factors like water demand and availability, nutrient absorption, herbicide selection, and pest control components. We do not aim to replace agronomic expertise; instead, we seek to offer a framework for evaluating strategies that extends beyond cost analysis in order to assess decisions in farm management.

Addressing the research question

The primary research inquiry driving this thesis was as follows: "*How can farm management strategies integrate optimization principles to drive sustainability?*". This study demonstrates that integrating an algorithm-based approach into agriculture should be approached incrementally, as we have done in this thesis. Initially, the review of the literature in Chapter 2 indicates that farmers face multiple problems in their daily routine. Addressing the entire scope of farm management within a single framework would be excessively complex and likely impractical.

Upon evaluating our efforts in this thesis, we found that sustainability on the farm is achieved through the appropriate blend of profitability and environmentally friendly practices. In the preceding chapters, the application of optimization methods has consistently focused on balancing these two occasionally conflicting goals. While we did not directly address multi-criteria optimization, by ensuring compliance with environment-related constraints, we inherently integrate the utilization of ecosystem services, thereby benefiting the environment and subsequently shifting our focus to achieve profitability.

Therefore, employing optimization in agriculture is vital for converting agronomic principles into measurable attributes, even enabling us to compare more preferable outcomes, such as fortifying soil structure to prevent erosion versus combating weeds.

Summary of contributions

This thesis have embraced several ecosystem services in the agricultural sector. The novel method we describe in Chapter 3 involves incorporating cover crops into the crop rotation sequence in such a manner that allows for measurement, comparison, and analysis, even during the planning stage. While cover crops are frequently utilized in agriculture, this thesis thoroughly examines the essential tools needed to assess their on-site performance, offering a substantial contribution to the field. Our empirical methodology linking soil moisture levels with crop requirements further elevates the significance of the Crop Rotation Problem.

The themes we have examined delve into the concept of sustainable agriculture. In this thesis, we advocate for a strategic approach to using resources in agriculture. Our efforts focus on raising awareness about the application of herbicides, the strategy for managing pests, and the adaptation of farm management practices to mitigate the effects of climate change while enhancing farm profitability. We explore the application of optimization techniques to improve farm management practices. Our study distinguishes itself from existing literature as it addresses the challenges farmers face in managing and evaluating the vast amount of information necessary for making efficient decisions.Once we have devised mathematical methods to evaluate the impacts of climate, provide agronomic recommendations, and assess the utilization of insecticides and herbicides in crop planning, the optimal approach to decision making will naturally arise from data analysis.

Contributions to the conscious use of herbicides are part of Chapter 4. In this chapter, we introduce an optimization model that outlines a comprehensive weed management approach to maintain weed levels within control throughout the entire crop cycle. Within the optimization framework, we explore a wide range of control actions, not limited to modeling solely the effects of chemical herbicides. Future agricultural strategies employing alternative weed control mechanisms can also be incorporated into the model assessment. In addition to the straightforward use of herbicides, Chapter 3 explores the concept of utilizing crop rotation as a means to manage weeds.

Ecological equilibrium is the key component to sustain farm profitability on the long-therm. Although most chemical pesticides have a immediate impact, we aim to reduce the use of pesticides by sustain a stable predator population in the field. Chapter 6 plays a crucial role in facilitating a comparison between chemical pesticide use and the impact of an artificially managed predator population. As practical advances enable the maintenance of a predator population with desired characteristics for crop symbiosis, the equations we proposed within the pest optimization model will effectively guide the decision between chemical-based and more environmentally friendly solutions.

Related works

Throughout our research, we published two conference papers and submitted an article to a journal, which is currently under review. The paper titled "OPTIMIZATION MODEL TO HANDLE INTEGRATED PEST MANAGEMENT IN SOYBEANS" has been chosen as one of the top five contenders for the *Roberto Diéguez Galvão* award, recognizing the best English presentation and publication at the conference.

MIRANDA, B. S.; YAMAKAMI, A.; RAMPAZZO, P. C. B. AN OPTIMAL STRATEGY FOR SCHEDULING SEED POPULATIONS. In: ANAIS DO SIMPóSIO BRASILEIRO DE PESQUISA OPERACIONAL, 2021, João Pessoa. Anais eletrônicos... Campinas, Galoá, 2021. Disponível em: <https://proceedings.science/sbpo/sbpo-2021/trabalhos/an-optimal-strategy-for-scheduling-seed-populations?lang=pt-br>

MIRANDA, B. S.; YAMAKAMI, A.; RAMPAZZO, P. C. B. DESIGNING AN OPTIMIZATION MODEL TO HANDLE INTEGRATED PEST MANAGEMENT IN SOYBEANS. In: ANAIS DO SIMPóSIO BRASILEIRO DE PESQUISA OPERACIONAL, 2024, Fortaleza/CE. Anais eletrônicos... Campinas, Galoá, 2024.

Remaining questions and future work

Moving beyond the contributions of this thesis, open questions remain, and additional research could greatly enhance understanding in this field. We are progressing with our study based on these topics:

Ensemble-based optimization: adapting the developed algorithms to handle ensemble-based optimization, where several scenarios are constructed, each with its own probability of occurrence, would enhance the algorithms' capacity to address problems in uncertain environments. This, in turn, would necessitate exploring methods to reduce the population size without substantially compromising the algorithm's performance, making it a critical focus for future research.

- Uncertainty modeling: adapting our algorithm-driven strategy to optimize problems under uncertainty by utilizing variations in problem variables with methods such as robust optimization, stochastic programming, and techniques based on Bayesian networks;
- Multi-criteria decision making: improving the algorithm's design by transforming ecosystem concepts, originally modeled as constraints, into a functional objective, and identifying the optimal alternative by evaluating multiple criteria during the selection process.
- Transitioning from a static to a dynamic framework: adapting the methods to consider objective function values that evolve over time, reassessing the developed plans to address economic functions that change with time.
- Heuristics evaluation: exploring heuristic methods and evaluating their effectiveness against the mathematical optimization solvers that we have thoroughly examined in our study.

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APPENDIX A – Appendix A

Tables presented below are derived from the average weekly price reports paid to farmers in São Paulo State, Brazil. These assessments are carried out regularly by the **Companhia Nacional de Abastecimento (CONAB)** and can be accessed at <https://sisdep.conab.gov.br/precosiagroweb/>.

Table A.1 – Cash commodities prices paid to farmers in the State of São Paulo, part (a)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2014-01-06	62.21	22.27	48.59	19.3
2014-01-13	61.62	22.78	45.77	19.1
2014-01-20	60.92	22.59	45.52	19.07
2014-01-27	61.06	22.56	46.89	19.03
2014-02-03	60.81	22.54	46.89	19.1
2014-02-10	61.62	23.54	47.03	19.1
2014-02-17	61.82	25.01	48.21	20.0
2014-02-24	61.82	26.74	47.83	20.0
2014-03-03	62.59	27.27	47.2	20.0
2014-03-10	62.23	28.74	47.2	21.68
2014-03-17	61.51	28.93	47.88	20.0
2014-03-24	61.58	28.67	46.86	19.0
2014-03-31	62.55	28.26	46.8	19.0
2014-04-07	63.18	28.13	46.83	19.0
2014-04-14	63.01	28.01	46.83	19.0
2014-04-21	63.28	28.18	46.83	19.0
2014-04-28	63.02	28.09	46.77	19.0
2014-05-05	62.93	27.87	46.77	19.0
2014-05-12	63.75	27.46	49.54	19.0
2014-05-19	63.43	26.93	49.54	19.0
2014-05-26	62.96	26.4	49.54	16.0
2014-06-02	63.05	26.22	48.47	16.0
2014-06-09	62.6	26.1	47.76	16.0
2014-06-16	62.69	25.7	48.44	16.0
2014-06-23	62.32	24.88	47.8	16.0
2014-06-30	61.14	24.8	50.16	16.0
2014-07-07	61.49	24.28	44.75	16.0
2014-07-14	61.95	24.74	45.21	16.0
2014-07-21	59.82	22.36	36.36	15.0
2014-07-28	59.43	21.77	36.91	15.0
2014-08-04	60.06	20.33	39.36	15.0
2014-08-11	57.13	20.05	40.35	15.0
2014-08-18	59.71	19.35	39.21	15.0
2014-08-25	59.85	18.61	38.79	15.0
2014-09-01	60.01	17.84	38.74	15.0
2014-09-08	60.11	18.28	38.74	15.0
2014-09-15	59.76	18.25	34.35	15.0
2014-09-22	59.31	18.62	28.18	15.0
2014-09-29	57.63	18.66	28.5	15.0
2014-10-06	57.27	19.32	28.76	14.83
2014-10-13	57.13	19.38	29.22	15.0
2014-10-20	56.95	20.8	30.46	17.0
2014-10-27	57.07	21.62	30.35	17.0
2014-11-03	57.4	22.09	30.5	17.0
2014-11-10	57.37	22.44	30.44	17.0
2014-11-17	58.34	22.8	30.44	17.0
2014-11-24	58.66	23.08	30.23	17.0
2014-12-01	58.99	23.36	30.46	17.0
2014-12-08	59.38	24.62	30.77	17.0
2014-12-15	60.12	24.79	32.0	17.0
2014-12-22	60.42	24.76	32.0	17.0
2014-12-29	60.42	24.76	32.0	17.0

Table A.2 – Cash commodities prices paid to farmers in the State of São Paulo, part (b)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2015-01-05	59.3	25.31	30.56	19.0
2015-01-12	58.84	24.96	33.57	19.0
2015-01-19	58.97	25.04	33.86	19.0
2015-01-26	54.25	24.19	33.86	19.0
2015-02-02	54.25	24.19	32.0	18.14
2015-02-09	54.24	24.19	32.0	18.14
2015-02-16	54.24	24.19	32.0	18.14
2015-02-23	54.24	23.96	31.0	17.97
2015-03-02	54.24	23.96	31.0	18.0
2015-03-09	54.24	23.96	31.0	18.0
2015-03-16	54.79	24.4	32.0	18.0
2015-03-23	54.79	24.4	32.0	18.0
2015-03-30	54.79	24.4	32.0	18.0
2015-04-06	54.79	24.4	32.0	18.0
2015-04-13	55.67	24.44	32.0	18.0
2015-04-20	55.67	24.44	32.0	18.0
2015-04-27	55.67	24.44	32.0	18.0
2015-05-04	55.77	24.36	32.0	18.0
2015-05-11	62.0	24.57	34.0	18.2
2015-05-18	62.37	24.19	35.0	18.2
2015-05-25	60.92	23.2	36.74	18.0
2015-06-01	60.85	22.8	37.09	17.7
2015-06-08	60.21	22.02	36.63	18.17
2015-06-15	59.43	21.52	36.75	17.87
2015-06-22	59.54	20.89	36.48	18.13
2015-06-29	59.63	20.58	36.75	18.15
2015-07-06	59.86	20.8	37.07	18.32
2015-07-13	61.11	20.63	37.78	18.9
2015-07-20	61.42	20.85	37.49	18.67
2015-07-27	61.95	21.28	36.69	18.6
2015-08-03	65.85	22.32	37.72	18.33
2015-08-10	66.49	22.57	35.24	18.0
2015-08-17	66.62	21.66	35.5	17.83
2015-08-24	66.67	22.45	36.3	18.0
2015-08-31	66.8	22.53	36.61	18.0
2015-09-07	66.18	22.78	37.2	18.0
2015-09-14	66.12	22.79	37.55	18.2
2015-09-21	66.79	23.02	38.4	18.3
2015-09-28	67.24	23.31	39.1	18.87
2015-10-05	67.59	23.52	38.6	19.03
2015-10-12	68.45	24.11	39.18	20.35
2015-10-19	68.55	24.09	39.44	20.5
2015-10-26	68.64	24.32	39.79	21.5
2015-11-02	70.24	25.03	39.62	21.8
2015-11-09	73.06	31.0	43.6	22.07
2015-11-16	73.06	31.0	43.6	22.07
2015-11-23	73.37	31.28	43.67	22.3
2015-11-30	73.52	31.02	43.79	22.63
2015-12-07	73.17	30.4	42.98	22.83
2015-12-14	74.17	30.48	42.85	22.88
2015-12-21	74.13	30.48	42.85	22.88
2015-12-28	74.27	30.69	43.02	23.03

Table A.3 – Cash commodities prices paid to farmers in the State of São Paulo, part (c)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2016-01-04	74.42	30.59	43.06	22.97
2016-01-11	69.31	30.96	43.06	23.5
2016-01-18	69.48	31.29	43.22	23.33
2016-01-25	69.56	31.58	43.22	23.33
2016-02-01	69.61	31.7	43.35	23.43
2016-02-08	69.73	31.97	43.52	23.52
2016-02-15	69.43	32.64	43.69	23.83
2016-02-22	69.0	34.24	43.55	24.13
2016-02-29	69.66	34.44	43.55	24.27
2016-03-07	71.03	34.91	43.55	24.5
2016-03-14	71.09	34.85	43.55	24.37
2016-03-21	69.26	36.15	43.55	25.27
2016-03-28	68.45	37.64	43.55	26.5
2016-04-04	66.88	37.57	43.38	26.73
2016-04-11	65.82	41.36	43.38	26.93
2016-04-18	68.05	41.87	43.38	27.0
2016-04-25	68.08	42.1	43.38	26.97
2016-05-02	68.75	42.8	43.38	27.73
2016-05-09	69.08	43.14	43.95	27.9
2016-05-16	73.81	43.4	44.7	28.75
2016-05-23	79.85	47.59	45.12	29.43
2016-05-30	79.98	47.7	45.18	29.5
2016-06-06	80.41	47.92	45.65	29.6
2016-06-13	81.4	48.66	46.25	30.0
2016-06-20	82.25	48.42	48.33	30.82
2016-06-27	82.35	47.01	48.73	30.82
2016-07-04	82.38	37.32	49.08	30.75
2016-07-11	78.93	37.04	50.07	30.73
2016-07-18	79.09	36.29	52.75	30.52
2016-07-25	76.18	36.61	49.82	30.55
2016-08-01	74.46	39.1	49.79	30.55
2016-08-08	74.65	39.03	52.0	30.58
2016-08-15	76.97	38.41	52.0	30.58
2016-08-22	77.05	38.91	51.75	30.5
2016-08-29	73.8	34.79	48.78	30.0
2016-09-05	73.22	34.56	48.78	29.9
2016-09-12	73.2	34.54	48.55	29.9
2016-09-19	74.01	34.74	48.58	29.94
2016-09-26	72.68	34.25	48.15	30.0
2016-10-03	72.43	34.31	46.88	29.8
2016-10-10	72.63	34.53	46.55	29.87
2016-10-17	72.19	34.52	45.33	29.88
2016-10-24	72.58	34.81	45.33	29.9
2016-10-31	72.58	34.81	45.32	29.88
2016-11-07	72.22	34.47	45.0	29.73
2016-11-14	70.9	33.96	44.3	29.6
2016-11-21	70.38	33.51	44.2	29.48
2016-11-28	69.52	32.62	44.2	29.48
2016-12-05	69.5	32.2	44.23	29.35
2016-12-12	69.61	33.03	44.23	29.35
2016-12-19	69.41	32.89	44.19	29.0
2016-12-26	69.41	32.41	43.23	28.65

Table A.4 – Cash commodities prices paid to farmers in the State of São Paulo, part (d)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2017-01-02	69.71	32.12	43.23	28.6
2017-01-09	69.84	31.72	42.73	28.4
2017-01-16	69.19	30.78	42.48	28.4
2017-01-23	68.98	29.76	41.83	28.3
2017-01-30	69.5	29.08	41.83	28.3
2017-02-06	69.5	29.41	41.48	28.33
2017-02-13	69.39	29.98	41.46	28.32
2017-02-20	68.12	30.34	41.44	25.0
2017-02-27	67.78	29.86	41.23	25.37
2017-03-06	67.52	29.65	40.15	25.42
2017-03-13	67.55	29.59	40.25	26.87
2017-03-20	66.18	29.69	39.63	26.9
2017-03-27	63.31	29.09	39.13	26.9
2017-04-03	61.74	28.21	40.15	26.83
2017-04-10	60.5	27.56	40.15	26.9
2017-04-17	60.42	27.43	40.1	26.77
2017-04-24	60.3	27.11	40.05	26.61
2017-05-01	59.22	26.05	39.81	26.55
2017-05-08	59.09	25.85	39.74	26.52
2017-05-15	59.04	25.63	39.7	26.43
2017-05-22	59.02	25.45	39.74	26.33
2017-05-29	59.02	25.41	39.69	26.38
2017-06-05	57.51	23.23	39.0	20.67
2017-06-12	58.95	22.27	37.97	21.17
2017-06-19	58.83	22.44	38.0	20.17
2017-06-26	57.04	21.85	34.78	19.17
2017-07-03	57.04	21.79	34.83	19.38
2017-07-10	57.47	21.7	37.41	19.67
2017-07-17	57.58	21.52	38.69	19.67
2017-07-24	60.35	24.25	40.46	31.89
2017-07-31	60.11	24.16	41.03	18.0
2017-08-07	60.97	22.16	42.21	18.0
2017-08-14	59.21	21.37	39.08	18.0
2017-08-21	58.87	22.27	38.1	17.0
2017-08-28	59.47	22.61	35.17	19.0
2017-09-04	59.87	22.82	35.17	16.55
2017-09-11	60.28	22.58	35.17	17.0
2017-09-18	60.28	24.16	34.58	20.4
2017-09-25	61.55	24.98	41.03	20.4
2017-10-02	61.7	26.55	35.17	20.4
2017-10-09	62.59	26.56	35.17	20.41
2017-10-16	62.63	26.25	35.25	19.56
2017-10-23	63.08	26.23	36.34	23.2
2017-10-30	65.55	25.84	35.17	23.2
2017-11-06	64.79	26.61	34.59	23.2
2017-11-13	64.76	27.46	35.17	24.0
2017-11-20	62.92	26.72	36.14	24.0
2017-11-27	61.92	27.5	35.17	24.0
2017-12-04	62.08	27.35	35.76	24.0
2017-12-11	68.16	28.2	37.12	24.0
2017-12-18	68.01	28.77	39.08	24.0
2017-12-25	67.98	28.77	39.08	24.0

Table A.5 – Caption

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2018-01-01	68.02	29.89	39.08	24.0
2018-01-08	67.26	30.15	38.68	24.0
2018-01-15	67.09	30.12	38.68	24.0
2018-01-22	63.41	28.89	37.52	24.4
2018-01-29	63.22	28.95	40.11	24.0
2018-02-05	68.03	30.18	38.88	24.0
2018-02-12	68.29	30.06	40.44	24.0
2018-02-19	66.9	26.67	39.9	24.0
2018-02-26	66.33	30.02	39.0	24.0
2018-03-05	67.13	35.32	39.86	24.0
2018-03-12	66.36	36.19	40.44	24.0
2018-03-19	66.26	36.25	39.27	24.0
2018-03-26	66.6	36.35	41.2	24.0
2018-04-02	69.21	36.03	42.2	24.0
2018-04-09	69.11	37.12	42.2	24.0
2018-04-16	72.82	36.2	46.89	24.0
2018-04-23	73.14	35.96	49.82	24.0
2018-04-30	73.58	36.08	43.96	24.0
2018-05-07	74.49	36.92	47.82	24.0
2018-05-14	75.04	37.41	58.03	24.0
2018-05-21	75.45	37.85	59.88	28.0
2018-05-28	75.84	38.14	65.65	28.0
2018-06-04	75.07	37.82	64.48	28.0
2018-06-11	74.44	36.84	67.9	30.0
2018-06-18	72.74	37.04	76.28	30.0
2018-06-25	73.22	35.48	64.48	26.0
2018-07-02	71.7	33.06	58.62	28.0
2018-07-09	72.75	31.59	67.41	28.0
2018-07-16	76.07	33.12	61.55	26.0
2018-07-23	75.26	32.41	58.62	26.0
2018-07-30	75.72	33.3	58.62	26.0
2018-08-06	75.06	34.76	58.62	25.0
2018-08-13	74.55	36.18	55.68	31.0
2018-08-20	75.77	36.61	54.51	32.0
2018-08-27	76.4	36.32	52.75	32.0
2018-09-03	77.06	37.36	55.68	32.0
2018-09-10	78.45	36.01	56.86	32.0
2018-09-17	78.55	36.25	58.62	32.0
2018-09-24	79.42	36.63	57.62	32.3
2018-10-01	78.35	34.91	56.56	30.68
2018-10-08	77.44	33.95	54.51	27.67
2018-10-15	78.34	33.93	49.24	27.75
2018-10-22	76.93	31.82	51.0	25.6
2018-10-29	75.38	32.54	49.82	23.9
2018-11-05	73.96	31.88	47.87	22.4
2018-11-12	73.1	32.43	51.25	24.55
2018-11-19	71.94	31.65	48.5	26.88
2018-11-26	71.17	32.2	46.89	28.24
2018-12-03	72.72	32.69	51.0	27.13
2018-12-10	73.54	33.54	52.8	28.4
2018-12-17	73.7	33.0	54.0	29.0
2018-12-24	71.63	33.25	53.0	29.0
2018-12-31	71.63	33.25	53.0	29.0

Table A.6 – Cash commodities prices paid to farmers in the State of São Paulo, part (e)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2019-01-07	68.34	34.1	54.18	28.4
2019-01-14	67.6	34.9	55.5	26.17
2019-01-21	68.13	35.05	50.58	26.29
2019-01-28	66.56	35.39	48.65	26.54
2019-02-04	66.76	34.61	53.5	25.96
2019-02-11	68.39	35.69	54.51	26.77
2019-02-18	68.13	36.32	51.58	27.24
2019-02-25	66.12	37.58	49.82	28.18
2019-03-04	69.94	38.63	54.51	28.97
2019-03-11	69.23	37.85	54.51	28.39
2019-03-18	71.23	37.48	55.68	28.11
2019-03-25	69.17	36.4	55.1	27.3
2019-04-01	67.88	35.59	57.0	28.7
2019-04-08	67.38	33.78	57.0	28.7
2019-04-15	67.21	33.58	57.0	29.4
2019-04-22	66.2	32.13	53.65	29.1
2019-04-29	65.92	31.61	49.82	29.1
2019-05-06	64.03	30.36	51.58	29.1
2019-05-13	65.02	29.89	50.99	29.1
2019-05-20	66.84	29.75	50.99	26.7
2019-05-27	68.12	31.17	50.99	28.0
2019-06-03	69.18	32.5	53.4	28.0
2019-06-10	69.49	31.96	54.35	27.7
2019-06-17	71.11	33.26	50.41	27.0
2019-06-24	71.52	34.31	52.75	26.5
2019-07-01	69.85	33.65	52.75	25.0
2019-07-08	71.11	33.75	55.68	25.0
2019-07-15	70.26	33.36	51.58	26.0
2019-07-22	69.44	33.02	51.58	26.0
2019-07-29	67.75	34.37	51.25	25.7
2019-08-05	71.34	33.46	46.89	25.5
2019-08-12	71.84	31.3	48.0	25.5
2019-08-19	72.59	30.7	49.2	24.2
2019-08-26	75.17	32.35	48.06	25.5
2019-09-02	77.85	31.58	48.0	25.0
2019-09-09	78.39	32.42	49.35	25.5
2019-09-16	75.61	31.81	48.06	25.5
2019-09-23	76.58	32.88	48.94	25.5
2019-09-30	79.87	34.64	48.94	26.75
2019-10-07	78.01	35.17	48.94	26.75
2019-10-14	78.41	34.95	46.9	25.0
2019-10-21	76.51	36.31	47.9	25.0
2019-10-28	79.26	35.92	51.0	26.25
2019-11-04	81.74	37.23	49.82	28.5
2019-11-11	79.31	39.61	51.3	30.67
2019-11-18	80.3	40.07	51.3	30.67
2019-11-25	80.31	41.96	48.65	31.6
2019-12-02	78.7	43.19	49.82	35.93
2019-12-09	79.27	42.13	49.82	35.97
2019-12-16	78.64	42.81	53.1	37.4
2019-12-23	78.64	43.05	53.1	37.4
2019-12-30	78.5	43.1	53.1	37.4

Table A.7 – Cash commodities prices paid to farmers in the State of São Paulo, part (f)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2020-01-06	79.38	44.03	52.17	39.4
2020-01-13	78.97	45.87	54.6	39.8
2020-01-20	77.61	45.79	56.27	39.75
2020-01-27	78.24	46.98	58.6	40.5
2020-02-03	78.16	46.4	58.5	39.58
2020-02-10	78.78	45.31	58.51	39.58
2020-02-17	79.49	45.36	58.51	38.8
2020-02-24	79.49	46.67	60.0	39.0
2020-03-02	79.87	45.23	59.1	39.3
2020-03-09	81.04	47.91	56.71	37.0
2020-03-16	81.06	50.08	57.88	40.5
2020-03-23	82.27	51.04	62.11	40.5
2020-03-30	80.79	54.13	64.45	46.4
2020-04-06	85.83	54.41	71.03	45.0
2020-04-13	83.34	50.5	64.55	41.5
2020-04-20	87.51	49.25	60.03	37.5
2020-04-27	89.18	44.58	64.71	38.75
2020-05-04	88.78	43.04	68.42	38.0
2020-05-11	93.94	43.28	67.68	39.0
2020-05-18	100.62	44.82	68.71	41.0
2020-05-25	96.87	47.24	78.0	42.0
2020-06-01	96.29	45.33	78.21	39.0
2020-06-08	96.9	44.0	73.5	36.1
2020-06-15	96.29	45.44	73.93	37.0
2020-06-22	96.57	43.45	75.6	38.0
2020-06-29	99.19	45.43	75.6	37.5
2020-07-06	101.99	44.69	75.9	38.5
2020-07-13	100.99	43.96	75.6	39.0
2020-07-20	102.66	43.61	73.8	40.0
2020-07-27	100.25	44.49	72.9	39.75
2020-08-03	103.99	45.71	72.9	41.0
2020-08-10	107.1	47.59	71.1	43.35
2020-08-17	110.79	49.93	72.5	45.56
2020-08-24	112.98	55.23	70.41	47.1
2020-08-31	113.01	56.03	67.2	47.83
2020-09-07	116.84	54.47	70.92	48.0
2020-09-14	118.4	53.51	68.4	48.5
2020-09-21	129.23	54.07	69.12	48.33
2020-09-28	129.87	55.71	70.8	49.5
2020-10-05	140.17	60.27	72.9	51.5
2020-10-12	143.07	62.24	73.2	52.29
2020-10-19	150.45	66.18	74.1	53.5
2020-10-26	153.87	69.95	81.3	60.75
2020-11-02	146.54	70.63	87.0	62.0
2020-11-09	153.03	73.89	81.7	61.5
2020-11-16	167.03	74.61	79.5	62.5
2020-11-23	156.13	74.74	77.1	61.23
2020-11-30	151.67	71.16	78.0	60.11
2020-12-07	138.1	66.29	76.8	55.0
2020-12-14	148.0	63.85	75.78	58.5
2020-12-21	136.33	68.24	81.9	59.37
2020-12-28	140.7	68.11	81.9	59.26

Table A.8 – Cash commodities prices paid to farmers in the State of São Paulo, part (g)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2021-01-04	149.74	74.75	81.9	62.5
2021-01-11	151.62	80.03	83.4	62.42
2021-01-18	158.61	78.17	87.6	60.97
2021-01-25	158.69	78.0	88.76	60.84
2021-02-01	155.9	81.51	90.0	63.58
2021-02-08	159.25	76.93	90.0	60.01
2021-02-15	157.09	78.47	80.4	61.21
2021-02-22	156.43	78.61	79.2	61.32
2021-03-01	157.43	82.08	80.44	64.02
2021-03-08	160.39	84.28	88.08	65.74
2021-03-15	159.88	85.55	90.24	66.73
2021-03-22	158.21	86.61	92.7	67.56
2021-03-29	159.31	87.4	92.7	68.17
2021-04-05	160.77	88.93	96.0	69.37
2021-04-12	161.03	91.53	94.53	71.39
2021-04-19	162.69	92.79	95.85	72.38
2021-04-26	163.73	95.05	97.8	74.14
2021-05-03	166.79	96.35	98.1	75.15
2021-05-10	170.99	98.14	99.0	76.55
2021-05-17	169.45	99.7	99.3	77.77
2021-05-24	159.48	92.45	97.5	72.11
2021-05-31	161.94	95.54	98.4	74.52
2021-06-07	158.92	94.31	97.5	73.56
2021-06-14	155.7	91.99	95.4	71.75
2021-06-21	143.07	83.0	94.5	64.74
2021-06-28	138.77	83.0	93.6	64.74
2021-07-05	147.68	85.0	92.7	66.3
2021-07-12	150.18	85.0	93.0	66.3
2021-07-19	152.99	94.74	95.4	73.9
2021-07-26	156.55	97.72	96.9	76.22
2021-08-02	157.86	98.49	97.5	76.82
2021-08-09	159.46	95.64	98.5	74.6
2021-08-16	162.02	97.44	99.19	76.0
2021-08-23	163.07	96.68	98.52	75.41
2021-08-30	157.24	92.78	96.2	72.37
2021-09-06	159.11	91.56	99.9	71.42
2021-09-13	161.46	89.42	95.28	69.75
2021-09-20	157.99	87.3	97.62	68.09
2021-09-27	161.96	88.61	97.92	69.12
2021-10-04	160.73	89.12	98.8	69.51
2021-10-11	160.85	88.25	98.05	68.84
2021-10-18	163.0	86.91	100.5	67.79
2021-10-25	164.94	85.84	100.2	66.96
2021-11-01	159.29	84.02	99.4	65.54
2021-11-08	156.12	81.1	100.8	63.26
2021-11-15	154.99	82.61	102.0	64.44
2021-11-22	156.9	83.29	93.57	64.97
2021-11-29	157.87	83.85	97.43	65.4
2021-12-06	156.6	82.36	104.1	64.24
2021-12-13	157.33	85.58	105.6	66.75
2021-12-20	157.38	85.59	102.6	66.76
2021-12-27	162.63	85.59	102.6	66.76

Table A.9 – Cash commodities prices paid to farmers in the State of São Paulo, part (h)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2022-01-03	167.85	87.85	105.93	68.52
2022-01-10	161.99	86.45	106.08	67.43
2022-01-17	163.34	93.25	106.5	72.74
2022-01-24	166.57	92.26	106.5	71.96
2022-01-31	168.29	94.96	105.0	74.07
2022-02-07	173.93	93.63	107.5	73.03
2022-02-14	184.25	93.01	107.5	72.55
2022-02-21	186.37	90.76	105.0	70.79
2022-02-28	187.21	93.0	104.1	72.54
2022-03-07	193.17	95.19	110.0	74.25
2022-03-14	194.76	99.27	114.5	77.43
2022-03-21	191.16	94.17	113.5	73.45
2022-03-28	194.82	93.98	113.5	73.3
2022-04-04	190.74	89.61	114.0	69.9
2022-04-11	173.04	85.29	114.6	66.53
2022-04-18	174.32	84.08	114.0	65.58
2022-04-25	180.56	83.64	116.0	65.24
2022-05-02	178.99	83.3	119.1	64.97
2022-05-09	180.57	82.44	119.1	64.3
2022-05-16	181.0	83.97	120.0	65.5
2022-05-23	178.73	82.1	123.0	64.04
2022-05-30	178.67	82.73	126.0	64.53
2022-06-06	178.85	82.9	126.0	64.66
2022-06-13	180.61	83.25	127.5	64.94
2022-06-20	179.51	83.15	129.0	64.86
2022-06-27	176.55	80.19	129.0	62.55
2022-07-04	174.66	77.85	133.5	60.72
2022-07-11	174.06	80.15	129.0	62.52
2022-07-18	171.68	77.36	130.5	60.34
2022-07-25	170.73	76.06	129.0	59.33
2022-08-01	170.91	78.48	127.0	61.21
2022-08-08	170.07	78.68	124.5	61.37
2022-08-15	167.64	78.53	121.5	61.25
2022-08-22	167.68	78.18	121.5	60.98
2022-08-29	168.27	78.38	115.4	61.14
2022-09-05	169.36	78.23	115.4	61.02
2022-09-12	172.01	78.19	107.5	60.99
2022-09-19	168.28	77.87	105.7	60.74
2022-09-26	168.68	78.4	104.7	61.15
2022-10-03	165.57	79.12	105.15	61.71
2022-10-10	165.66	78.91	106.8	61.55
2022-10-17	166.81	80.05	108.0	62.44
2022-10-24	168.39	79.8	108.9	62.24
2022-10-31	169.45	80.35	108.9	62.67
2022-11-07	171.13	79.97	116.6	62.38
2022-11-14	173.82	79.86	111.5	62.29
2022-11-21	171.87	80.07	111.3	62.45
2022-11-28	173.49	80.56	111.6	62.84
2022-12-05	169.55	80.75	111.3	62.98
2022-12-12	171.51	80.46	110.7	62.76
2022-12-19	171.21	79.7	109.8	62.17
2022-12-26	169.12	80.59	109.9	62.86

Table A.10 – Cash commodities prices paid to farmers in the State of São Paulo, part (i)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2023-01-02	171.84	80.98	101.0	63.0
2023-01-09	168.08	80.1	108.0	63.0
2023-01-16	164.66	80.29	109.2	63.0
2023-01-23	165.68	80.78	109.2	63.0
2023-01-30	162.66	79.93	107.7	63.0
2023-02-06	163.25	79.89	103.5	65.0
2023-02-13	162.68	79.91	102.9	67.7
2023-02-20	161.08	80.96	103.3	65.7
2023-02-27	159.68	80.63	103.1	64.0
2023-03-06	159.56	79.1	102.0	66.5
2023-03-13	156.91	79.62	102.0	68.0
2023-03-20	149.85	80.07	102.0	68.0
2023-03-27	140.6	78.9	102.0	65.0
2023-04-03	141.64	77.96	102.0	67.5
2023-04-10	136.98	75.46	99.7	62.75
2023-04-17	132.87	73.03	97.0	61.5
2023-04-24	127.26	66.89	96.0	56.0
2023-05-01	125.5	60.99	93.0	51.5
2023-05-08	126.14	57.14	85.2	49.5
2023-05-15	122.37	54.06	79.2	45.7
2023-05-22	123.29	50.93	81.7	42.1
2023-05-29	122.11	50.72	78.0	42.5
2023-06-05	119.26	48.07	79.5	43.9
2023-06-12	117.97	47.33	78.1	42.5
2023-06-19	119.56	48.3	78.5	43.5
2023-06-26	120.62	49.49	77.7	42.7
2023-07-03	125.64	49.06	76.5	42.1
2023-07-10	125.74	49.23	77.7	41.0
2023-07-17	125.46	51.14	80.1	42.5
2023-07-24	130.65	47.18	79.59	43.0
2023-07-31	131.1	49.84	80.1	39.0
2023-08-07	127.11	49.36	78.47	42.0
2023-08-14	136.38	47.88	75.0	38.67
2023-08-21	137.31	47.15	71.64	37.0
2023-08-28	132.21	47.41	67.0	37.7
2023-09-04	132.42	47.68	63.0	38.0
2023-09-11	131.27	48.48	61.7	37.6
2023-09-18	131.3	49.05	60.75	37.7
2023-09-25	128.82	48.61	57.0	38.25
2023-10-02	129.68	50.16	57.8	38.5
2023-10-09	128.27	50.14	57.1	38.75
2023-10-16	128.78	50.49	60.9	39.9
2023-10-23	129.34	52.24	61.5	40.5
2023-10-30	129.34	51.72	63.6	40.1
2023-11-06	129.54	52.15	65.1	41.4
2023-11-13	132.19	53.63	65.1	41.4
2023-11-20	131.9	55.17	69.7	42.4
2023-11-27	130.49	56.68	72.0	44.53
2023-12-04	130.88	55.58	73.2	47.25
2023-12-11	130.11	58.48	73.7	50.6
2023-12-18	127.72	58.43	72.5	52.8
2023-12-25	129.7	59.94	73.5	53.2

Table A.11 – Cash commodities prices paid to farmers in the State of São Paulo, part (j)

Price paid to farmers in São Paulo (R$)				
Week	Soybeans	Corn	Wheat	Sorghum
2024-01-01	130.55	62.29	75.0	55.63
2024-01-08	122.88	60.56	73.5	54.4
2024-01-15	115.77	61.4	72.7	54.3
2024-01-22	114.84	59.47	69.0	53.15
2024-01-29	107.48	54.63	70.45	47.9
2024-02-05	104.79	54.76	71.4	48.0

APPENDIX B – Appendix B

Based on Instituto Nacional de Meteorologia (INMET) (2024), we collected information from weather stations across Brazil. A total of 633 automated stations are involved in data collection. To keep it concise, we find it pertinent to provide the station's location along with certain annual average parameters. Our observations span from 2010 to 2024, with the average values for compiled in the tables below. Apart from the station's specific location, we present the **mean annual values** for the following meteorological variables:

- Total Precipitation (mm)
- Global Radiation (KJ/m^2)
- Atmospheric Pressure at Station Level (mB)
- Air Temperature - Dry Bulb ($°C$)

Table B.1 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (a)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
ABROLHOS	BA	-17.96	487	4299861	1013.07	25.65
ACARAU	CE	-3.12	597	7066392	1004.2	26.9
AFONSO CLAUDIO	ES	-20.1	839	5934722	957.87	22.16
AGUA BOA	MT	-14.02	2418	5648212	962.23	26.24
AGUA CLARA	MS	-20.44	1139	6714083	975.9	24.66
AGUAS EMENDADAS	DF	-15.6	1338	7237078	900.65	21.37
AGUAS VERMELHAS	MG	-15.75	771	6969732	930.79	21.31
AIMORES	MG	-19.53	787	7154414	983.28	24.53
ALEGRE	ES	-20.75	1282	7025661	1000.68	24.15
ALEGRETE	RS	-29.71	1757	6612988	1000.42	19.68
ALFREDO CHAVES	ES	-20.64	1333	6575169	1012.94	24.45
ALMAS	TO	-11.28	1083	8159029	954.73	26.46
ALMENARA	MG	-16.17	743	7605682	994.14	25.59
ALTA FLORESTA	MT	-9.85	1127	7045464	978.29	26.02
ALTAMIRA	PA	-3.27	954	2807084	989.64	26.44
ALTO ARAGUAIA	MT	-17.34	1343	7327771	929.1	23.52
ALTO PARAISO DE GOIAS	GO	-14.13	1100	7061056	877.14	20.67
ALTO PARNAIBA	MA	-9.11	1015	8658871	979.53	26.82
ALTO TAQUARI	MT	-17.82	945	7387514	917.56	22.68
ALVORADA DO GURGUEIA	PI	-8.44	441	8703248	982.29	27.83
AMAMBAI	MS	-23.0	1180	6933317	963.38	22.5
AMARGOSA	BA	-13.01	956	6763176	971.12	22.94
ANGELICA	MS	-22.15	1060	6692422	973.73	24.12
ANGICAL DO PIAUI	PI	-6.09	237	5056298	989.79	27.73
ANGRA DOS REIS	RJ	-22.98	1704	4580289	1015.46	23.04
APIACAS	MT	-9.56	1390	9427542	986.78	27.19
APODI	RN	-5.63	609	8774395	997.71	28.14
APUI	AM	-7.22	4104	6924831	992.8	26.57
AQUIDAUANA	MS	-20.48	1042	6264962	995.19	24.94
ARACAJU	SE	-10.95	1168	7311668	1014.26	26.78
ARACUAI	MG	-16.85	660	6588810	979.86	24.76
ARAGARCAS	GO	-15.91	1311	7258153	975.39	26.51
ARAGUACU	TO	-12.59	2103	6677792	985.8	26.84
ARAGUAINA	TO	-7.1	1417	6677023	985.32	25.79
ARAGUATINS	TO	-5.64	1250	6363503	996.14	27.25
ARAL MOREIRA	MS	-22.96	1270	6349244	944.74	22.24
ARAPIRACA	AL	-9.8	468	6564504	987.88	24.89
ARARANGUA	SC	-28.93	1480	5763866	1015.74	19.92
ARAXA	MG	-19.61	1337	9783718	902.48	21.76
ARCO VERDE	PE	-8.42	456	8801352	937.72	24.11
AREIA	PB	-6.97	1141	4828642	948.75	22.52
ARIQUEMES	RO	-9.95	1372	6468987	995.68	26.77
ARIRANHA	SP	-21.13	740	8000874	954.43	23.41
ARQ.SAO PEDRO E SAO PAULO	RN	0.9	649	1365406	1011.64	27.36
ARRAIAL DO CABO	RJ	-22.98	895	6828885	1015.8	23.47
AUTAZES	AM	-2.06	2190	6671753	1007.71	27.47
AVARE	SP	-23.1	1790	6393069	928.18	21.2
BACABAL	MA	-4.24	1278	6012355	1008.48	28.2
BAGE	RS	-31.35	1395	6457437	989.06	17.83
BAIXA GRANDE DO RIBEIRO	PI	-8.34	1052	8607888	953.41	26.64
BALIZA	RR	0.95	12139	0	994.5	25.24
BALSAS	MA	-7.46	764	6646617	980.98	27.52
BAMBUI	MG	-20.03	1177	6425138	937.02	21.58
BANDEIRANTES	MS	-19.95	189	2102559	943.04	23.33
BARBACENA	MG	-21.23	1460	5753176	889.0	18.66
BARBALHA	CE	-7.32	571	7777457	966.52	26.09
BARCELOS	AM	-0.96	2688	6716727	1008.8	27.03
BARRA	BA	-11.08	576	8826844	966.69	27.72
BARRA BONITA	SP	-22.37	935	5767972	954.16	22.76
BARRA DO CORDA	MA	-5.51	814	7586510	993.52	28.19
BARRA DO TURVO	SP	-24.96	1662	4495540	941.99	17.93
BARREIRAS	BA	-12.15	843	7841145	959.2	25.78
BARRETOS	SP	-20.56	1232	6107217	953.46	23.26
BARUERI	SP	-23.52	1120	4672608	928.59	19.86
BATAGUASSU	MS	-21.75	1066	7013992	968.8	24.18
BAURU	SP	-22.36	1211	5946751	943.31	22.11
BEBDOURO	SP	-20.95	1004	5285194	945.35	23.53
BEBEDOURO	SP	-20.95	617	3896822	950.34	23.61
BELA VISTA	MS	-22.1	920	5725090	988.96	22.43
BELEM	PA	-1.41	3406	5583913	1008.86	26.92

Table B.2 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (b)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
BELMONTE	BA	-16.09	1002	7216988	1005.01	23.88
BELO HORIZONTE (PAMPULHA)	MG	-19.88	1323	6103780	919.39	22.02
BELO HORIZONTE - CERCADINHO	MG	-19.98	1359	6127116	883.02	19.98
BENTO GONCALVES	RS	-29.17	1695	5996772	944.33	17.63
BERTIOGA	SP	-23.84	1927	7086333	1015.14	23.34
BOA VISTA	RR	2.82	1250	9107670	1001.61	28.22
BOCA DO ACRE	AM	-8.78	1676	6979776	998.27	26.91
BOM JARDIM DA SERRA - MORRO DA IGREJA	SC	-28.13	2605	6178200	821.51	11.36
BOM JESUS DA LAPA	BA	-13.42	1752	8655839	962.24	28.01
BOM JESUS DO PIAUI	PI	-9.08	555	8390839	978.27	27.79
BONITO	MS	-21.25	928	7634540	974.89	24.32
BRAGANCA	PA	-1.05	1847	6658441	1007.46	27.14
BRAGANCA PAULISTA	SP	-23.95	1498	5908149	916.41	20.82
BRASILANDIA	MS	-21.3	285	2542063	974.93	22.71
BRASILIA	DF	-15.79	1358	7179598	887.43	21.51
BRASNORTE (MUNDO NOVO)	MT	-12.87	1233	8695059	963.14	25.23
BRASNORTE (NOVO MUNDO)	MT	-12.52	912	11244198	963.17	27.03
BRAZLANDIA	DF	-15.6	1496	6612352	888.65	21.98
BREJO GRANDE	SE	-10.48	1655	8787984	1014.59	26.54
BREVES	PA	-1.68	842	5972789	1009.95	28.16
BRUMADO	BA	-14.18	528	7291644	961.57	25.14
BURITICUPU	MA	-4.32	3354	7335551	990.87	27.13
BURITIRAMA	BA	-10.72	394	8103179	955.85	26.96
BURITIS	MG	-15.52	1483	8194913	912.01	22.83
CAARAPO	MS	-22.66	1231	709921	962.37	23.25
CABACEIRAS	PB	-7.48	833	7174207	968.61	25.73
CABROBO	PE	-8.5	376	8150842	974.47	27.34
CACADOR	SC	-26.82	1402	4804219	909.86	16.64
CACAPAVA DO SUL	RS	-30.55	1700	6322936	966.82	17.44
CACERES	MT	-16.07	824	7413885	998.09	26.58
CACHOEIRA PAULISTA	SP	-22.69	1254	5347194	949.28	22.08
CACOAL	RO	-11.45	1724	6985518	989.73	27.06
CAIAPONIA	GO	-16.97	1200	7297469	930.91	24.03
CAICO	RN	-6.46	329	9943390	993.01	29.0
CALCANHAR	RN	-5.16	2837	4649912	1011.5	26.72
CALDAS	MG	-21.92	1298	6435834	896.8	18.07
CAMAPUA	MS	-19.59	408	3852117	952.0	24.82
CAMAQUA	RS	-30.81	1473	5671681	1005.01	18.51
CAMARATUBA	PB	-6.56	833	7237939	997.68	25.78
CAMBARA DO SUL	RS	-29.05	1473	5383756	902.14	15.68
CAMBUCI	RJ	-21.57	1058	6343846	1009.67	23.89
CAMETA	PA	-2.25	2906	5884008	1010.53	27.76
CAMPINA DA LAGOA	PR	-24.57	1245	8282277	946.68	23.39
CAMPINA GRANDE	PB	-7.24	636	7519154	952.36	23.98
CAMPINA VERDE	MG	-19.54	1304	7377143	950.72	24.05
CAMPO BOM	RS	-29.67	1662	4912722	1012.54	20.61
CAMPO GRANDE	MS	-20.45	1330	6601301	953.01	24.0
CAMPO MAIOR	PI	-4.86	359	7563030	997.34	28.76
CAMPO NOVO DOS PARECIS	MT	-13.78	922	6794045	952.07	25.24
CAMPO VERDE	MT	-15.53	1600	6974996	930.19	23.82
CAMPOS	RJ	-21.72	846	6950288	1013.61	23.88
CAMPOS DO JORDAO	SP	-22.75	1483	5621362	836.87	15.13
CAMPOS DOS GOYTACAZES	RJ	-21.71	1042	5992062	1013.51	23.94
CAMPOS DOS GOYTACAZES - SAO TOME	RJ	-22.04	685	7472678	1015.46	24.38
CAMPOS LINDOS	TO	-8.15	681	6216559	963.73	26.42
CAMPOS NOVOS	SC	-27.39	1825	5878143	908.61	17.12
CAMPOS SALES	CE	-7.08	301	6048105	948.31	25.3
CANELA	RS	-29.37	2094	6014436	921.46	15.95
CANGUCU	RS	-31.41	1423	5539559	963.21	16.3
CANTO DO BURITI	PI	-8.12	610	7752156	976.4	27.53
CAPAO DO LEAO (PELOTAS)	RS	-31.8	1243	5302249	1014.63	18.21
CAPELINHA	MG	-17.71	961	7214923	911.97	20.25
CAPITAO POCO	PA	-1.73	1894	6862390	1002.6	26.24
CARACOL	PI	-9.29	384	6073791	954.41	26.03
CARATINGA	MG	-19.74	2625	6466880	947.19	23.81
CARAVELAS	BA	-17.73	1303	6037894	1016.03	24.51
CARIRA	SE	-10.4	1807	9072495	981.29	24.89
CARLINDA	MT	-10.01	2064	8340641	978.0	25.57
CARMO	RJ	-21.94	1085	5327365	982.31	22.88
CAROLINA	MA	-7.34	2608	7424749	990.41	27.86
CARUARU	PE	-8.24	501	6098915	937.16	22.45
CASA BRANCA	SP	-21.78	869	5373604	933.37	22.34

Table B.3 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (c)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
CASSILANDIA	MS	-19.12	1274	9152082	957.21	24.91
CASTANHAL	PA	-1.3	1862	6619402	1006.17	26.87
CASTELO DO PIAUI	PI	-5.35	620	7577418	981.11	28.11
CASTRO	PR	-24.79	1087	5487840	905.25	17.9
CATALAO	GO	-18.16	1419	6997040	914.79	23.22
CAXIAS	MA	-4.82	1289	6878670	1001.54	27.05
CHAPADA GAUCHA	MG	-15.31	936	8162124	917.42	23.36
CHAPADAO DO SUL	MS	-18.8	1033	7634263	922.46	23.28
CHAPADINHA	MA	-3.74	1452	6308423	999.28	27.88
CHAPECO	SC	-27.96	1761	6026748	937.38	19.71
CIDADE GAUCHA	PR	-23.38	1373	6858387	971.42	23.77
CLEVELANDIA	PR	-26.42	1949	5782704	906.84	17.7
COARI	AM	-4.1	1767	8771077	1006.61	27.37
COLINAS	MA	-6.03	819	8379759	990.56	27.34
COLINAS DO TOCANTINS	TO	-8.09	1415	7056812	986.31	26.35
COLOMBO	PR	-25.32	1397	5341486	910.7	17.17
COMODORO	MT	-13.71	1648	5340583	947.62	23.0
CONCEICAO DAS ALAGOAS	MG	-19.99	1200	6703397	949.31	23.41
CONCEICAO DO ARAGUAIA	PA	-8.26	1373	7822952	991.59	26.67
CONDE	BA	-11.8	1847	7505288	1011.22	25.95
CORONEL PACHECO	MG	-21.55	1213	5625354	967.35	23.33
CORRENTE	PI	-10.43	722	7232465	960.69	26.91
CORRENTINA	BA	-13.33	1322	8101784	951.11	24.69
CORUMBA	MS	-19.0	829	7879268	998.46	26.61
CORURIPE	AL	-10.15	4631	6737342	1005.12	26.18
COSTA RICA	MS	-18.82	962	6645517	931.89	23.47
COTRIGUACU	MT	-9.91	1758	8283783	981.46	26.31
COXIM	MS	-18.51	816	7284617	983.76	25.27
CRATEUS	CE	-5.19	587	6909615	978.4	28.06
CRIOSFERA	SP	-84.0	0	3497640	836.5	N/A
CRISTALINA	GO	-16.79	1483	7313187	882.42	21.11
CRISTALINA (FAZENDA SANTA MONICA)	GO	-16.4	1228	7323630	909.23	21.78
CRMN MANAUS	AM	-3.02	624	0	1001.15	27.16
CRUZ ALTA	RS	-28.6	1770	6332472	965.61	18.59
CRUZ DAS ALMAS	BA	-12.65	1070	7168661	989.63	24.36
CRUZEIRO DO SUL	AC	-7.61	1522	4819901	986.74	26.71
CUIABA	MT	-15.56	1369	4955115	987.64	27.19
CURACA	BA	-9.0	290	5616351	972.13	27.46
CURITIBA	PR	-25.43	1468	5732453	913.01	18.13
CURITIBANOS	SC	-27.29	1569	6660125	906.16	17.06
CURVELO	MG	-18.76	1026	8179914	939.85	23.29
DELFINO	BA	-10.46	300	5547648	941.58	24.36
DIAMANTE DO NORTE	PR	-22.63	712	7742249	971.22	23.85
DIAMANTINA	MG	-18.23	1278	6934530	868.42	18.61
DIANOPOLIS	TO	-11.59	1194	7889482	931.66	25.02
DIONISIO CERQUEIRA	SC	-26.29	2152	6874182	923.37	19.62
DIVINOPOLIS	MG	-20.17	1202	6122874	926.59	22.14
DOIS VIZINHOS	PR	-25.69	1564	7426217	951.91	20.93
DOM ELISEU	PA	-4.29	2724	4828723	982.56	26.26
DOM PEDRITO	RS	-31.0	1257	7092745	997.18	18.6
DORES DO INDAIA	MG	-19.48	1248	7319646	933.51	22.63
DOURADOS	MS	-22.19	771	8161253	960.84	23.32
DRACENA	SP	-21.46	1003	6534297	969.67	24.95
DTCEA GUAJARA-MIRIM	RO	-10.79	615	0	995.17	26.79
DTCEA JACAREACANGA	PA	-6.24	266	0	998.77	26.24
DTCEA TABATINGA	AM	-4.25	19280	0	1002.84	24.46
DTCEA TEFE	AM	-3.38	1229	0	1005.1	26.1
DTCEA VILHENA	RO	-12.75	189	0	943.11	25.36
DUQUE DE CAXIAS - XEREM	RJ	-22.59	1894	6723778	1013.37	23.15
EB_PEF_BONFIM	MG	3.36	760	0	1001.53	27.97
ECOLOGIA AGRICOLA	RJ	-22.8	1548	5924804	1011.75	23.91
ECOPORANGA	ES	-18.29	777	6492583	989.9	24.84
EDEIA	GO	-17.34	1023	6987434	944.7	25.22
EIRUNEPE	AM	-6.65	1022	6174819	998.04	26.89
ENCRUZILHADA DO SUL	RS	-30.54	1414	5759236	967.38	17.95
EPITACIOLANDIA	AC	-11.02	938	7174166	988.33	25.51
ERECHIM	RS	-27.66	1657	7047521	927.79	18.04
ESPERANTINA	PI	-3.9	1143	8724186	1000.92	27.74
ESPINOSA	MG	-14.91	342	7489649	950.63	25.67
ESTREITO	MA	-6.65	583	8065944	990.38	26.96
EUCLIDES DA CUNHA	BA	-10.54	579	8502944	962.32	25.14
FAROL de SANTANA	MA	-2.27	635	5553847	1010.53	28.05
FATIMA DO SUL	MS	-22.31	1170	10910376	973.42	25.26
FEIJO	AC	-8.24	1879	6573696	992.83	25.61
FEIRA DE SANTANA	BA	-12.2	1200	7159940	988.5	25.05
FLORESTA	PE	-8.59	621	8016586	976.29	27.37
FLORESTAL	MG	-19.89	1431	6798541	930.98	20.69
FLORIANO	PI	-6.77	538	6464551	996.94	29.22
FLORIANOPOLIS	SC	-27.6	1739	5707605	1015.91	21.28
FORMIGA	MG	-20.45	1098	6965012	916.98	21.67
FORMOSA DO RIO PRETO	BA	-11.05	700	6995724	958.38	24.23
FORMOSO DO ARAGUAIA	TO	-11.99	798	8086529	987.13	27.05
FORMOSO DO RIO PRETO	BA	-11.05	891	8081789	957.93	25.48

Table B.4 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (d)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
FORTALEZA	CE	-3.83	1248	6560304	1009.39	27.48
FORTE DE COPACABANA	RJ	-22.99	995	6530493	1011.51	23.51
FORTE PRINCIPE	RO	-12.43	61	0	996.8	28.33
FOZ DO IGUACU	PR	-25.6	1206	7173706	985.89	22.79
FRANCA	SP	-20.58	1368	6753776	904.29	21.87
FREDERICO WESTPHALEN	RS	-27.4	1995	6139454	958.27	19.58
GAMA (PONTE ALTA)	DF	-15.94	1238	6689182	905.09	22.33
GARANHUNS	PE	-8.91	1641	6992655	922.93	21.34
GAUCHA DO NORTE	MT	-13.18	1100	8845080	969.1	27.06
GENERAL CARNEIRO	PR	-26.4	1659	5658666	903.5	16.56
GILBUES	PI	-9.87	930	8990032	963.96	27.93
GOIANESIA	GO	-15.22	1092	7716500	938.5	25.03
GOIANIA	GO	-16.64	1324	6723841	932.4	23.8
GOIAS	GO	-15.94	1339	7388722	954.75	25.85
GOIOERE	PR	-24.16	4256	12222845	960.26	23.34
GOVERNADOR VALADARES	MG	-18.83	906	6620787	990.73	24.38
GRAJAU	MA	-5.82	1144	7356270	984.87	27.88
GUANAMBI	BA	-14.21	955	9180898	951.51	26.23
GUANHAES	MG	-18.79	966	6551303	920.33	20.8
GUARAMIRANGA	CE	-4.26	1058	5767443	917.81	21.44
GUARANTA DO NORTE	MT	-9.95	977	4918064	979.13	26.39
GUARDA-MOR	MG	-17.56	950	7446449	905.07	21.75
GUIRATINGA	MT	-16.34	1750	7216482	953.85	26.16
GURUPI	TO	-11.75	1344	7607397	980.18	25.77
HUMAITA	AM	-7.92	1891	6663442	1004.35	27.37
IBIMIRIM	PE	-8.51	423	7811042	964.27	25.86
IBIRITE (ROLA MOCA)	MG	-20.03	1547	7081379	884.23	20.06
IBIRUBA	RS	-28.65	1846	5795395	962.41	19.26
IBITINGA	SP	-21.86	931	7931212	958.32	22.75
IBOTIRAMA	BA	-12.19	639	11117707	964.53	27.66
ICARAIMA	PR	-23.39	1511	5984816	963.13	23.98
IGUAPE	SP	-24.72	2306	6042859	1015.2	21.96
IGUATEMI	MS	-23.64	831	8340143	975.88	23.73
IGUATU	CE	-6.4	812	5802021	986.86	27.77
ILHA DO MEL	PR	-25.49	3501	6393303	1014.66	22.11
ILHEUS	BA	-14.66	1477	5754223	1005.79	24.05
IMPERATRIZ	MA	-5.56	1141	6870137	997.83	27.52
INACIO MARTINS	PR	-25.57	1831	6079988	882.09	16.28
INDAIAL	SC	-26.91	1622	6020009	1007.62	21.03
IPANGUACU	RN	-5.53	673	7988725	1009.47	28.62
IPERO	SP	-23.43	681	4453496	947.73	21.33
IPIAU	BA	-14.17	815	6005942	1000.59	24.05
IPORA	GO	-16.42	1256	7453908	944.87	25.09
IRECE	BA	-11.33	455	8299549	928.41	24.08
ITABAIANA	SE	-10.67	368	2361646	990.24	24.24
ITABAIANINHA	SE	-11.27	966	7210931	990.72	24.92
ITABERABA	BA	-12.52	315	13905026	986.14	25.6
ITACOATIARA	AM	-3.13	1127	9047012	1005.69	27.37
ITAITUBA	PA	-4.28	1712	6880070	1007.71	27.71
ITAJAI	SC	-26.95	1534	4926640	1015.65	20.02
ITAMARAJU	BA	-17.01	1036	5288789	1004.24	24.33
ITAOBIM	MG	-16.58	602	7583708	984.28	25.76
ITAPACI	GO	-14.98	1049	6745066	951.04	24.07
ITAPETINGA	BA	-15.24	741	7330827	983.81	24.62
ITAPEVA	SP	-23.98	1522	6580136	931.94	19.96
ITAPIPOCA	CE	-3.48	609	6365456	999.96	28.11
ITAPIRA	SP	-22.42	1222	7199778	943.61	22.11
ITAPOA	SC	-26.08	1953	5658727	1015.65	20.82
ITAPORA	MS	-22.09	1206	7208554	971.42	24.12
ITAPORANGA	PB	-7.32	1185	6983121	979.27	28.14
ITAQUIRAI	MS	-23.45	1356	6887574	974.45	23.01
ITATIAIA	RJ	-22.37	1836	2105191	762.44	10.58
ITAUBAL	AP	0.57	1574	4589280	1008.14	28.2
ITIQUIRA	MT	-17.3	661	6449926	947.08	24.85
ITIRUCU	BA	-13.53	767	6655840	931.11	21.19
ITUIUTABA	MG	-18.95	1050	7579981	952.64	24.3
ITUMBIARA	GO	-18.41	1123	7421166	957.88	24.41
ITUPORANGA	SC	-27.42	1547	6232876	960.8	18.41
ITUVERAVA	SP	-20.36	1095	7378667	945.09	22.97
IVAI	PR	-25.01	1402	6247796	925.23	18.97
IVINHEMA	MS	-22.31	1046	7456470	969.83	24.12
JACOBINA	BA	-11.2	532	6229253	964.58	24.16
JAGUARAO	RS	-32.54	1354	6043048	1012.33	17.85
JAGUARIBE	CE	-5.91	288	7232774	994.1	29.11
JAGUARUANA	CE	-4.85	219	8341197	1009.4	28.11
JALES	SP	-20.16	1222	7566325	961.77	24.42
JANAUBA	MG	-15.8	735	7622370	953.65	25.89
JANUARIA	MG	-15.45	744	7481594	959.5	25.19
JAPIRA	PR	-23.77	1693	8179765	936.48	20.93
JARDIM	MS	-21.48	995	6784726	983.83	24.92
JATAI	GO	-17.92	1382	7808268	938.73	23.15
JEREMOABO	BA	-10.08	463	7957884	984.2	26.34

Table B.5 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (e)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
JOACABA	SC	-27.17	1907	6235486	927.91	18.04
JOAO PESSOA	PB	-7.17	1681	8384130	1009.24	26.88
JOAO PINHEIRO	MG	-17.78	1050	7796181	917.39	22.73
JOAQUIM TAVORA	PR	-23.51	1111	7126081	956.17	21.86
JOSE BONIFACIO	SP	-21.09	2624	7314456	966.97	24.08
JUARA	MT	-11.28	1327	6523695	976.15	26.32
JUINA	MT	-11.37	1396	7584961	970.3	25.3
JUIZ DE FORA	MG	-21.77	1561	5996016	911.94	19.36
JUTI	MS	-22.86	1142	5615099	970.34	23.2
LABREA	AM	-7.44	2521	7165708	1003.75	26.82
LAGES	SC	-27.8	1424	5831567	908.68	16.62
LAGOA DA CONFUSAO	TO	-10.81	1273	7105087	990.61	26.51
LAGOA VERMELHA	RS	-28.22	2855	6353796	921.05	17.2
LAGUNA CARAPA	MS	-22.58	1208	6792502	956.27	22.45
LARANJEIRAS DO SUL	PR	-25.37	1730	6452746	921.0	19.09
LENCOIS	BA	-12.56	955	6946511	964.63	24.66
LINHARES	ES	-19.36	1035	6694276	1011.96	24.35
LINS	SP	-21.67	935	6896552	962.53	23.55
LUIZ EDUARDO MAGALHAES	BA	-12.15	1055	8676713	928.51	24.22
LUZIANIA	GO	-16.27	1361	7293901	903.89	22.6
Laguna - Farol de Santa Marta	SC	-28.6	1279	6135773	1011.12	19.76
MACAE	RJ	-22.38	1156	6797605	1012.92	23.59
MACAJUBA	BA	-12.12	716	13481988	975.14	24.95
MACAPA	AP	0.04	2707	7356420	1009.81	27.86
MACAU	RN	-5.15	217	8458536	1010.23	28.14
MACAUBAS	BA	-13.04	486	8530599	949.14	25.91
MACEIO	AL	-9.55	1566	7457492	1005.09	25.67
MACHADO	MG	-21.68	1268	6229142	907.05	20.68
MAJOR VIEIRA	SC	-26.39	1040	5430741	925.5	17.27
MAL. CANDIDO RONDON	PR	-24.54	1505	6932407	968.45	22.07
MANACAPURU	AM	-3.29	2531	6628297	1006.01	27.4
MANAUS	AM	-3.1	2211	5905646	1004.62	27.82
MANHUACU	MG	-20.26	953	7226653	923.79	20.34
MANICORE	AM	-5.81	4885	4951128	1005.8	26.81
MANTENA	MG	-18.78	965	6636119	986.08	24.11
MARABA	PA	-5.17	1251	6076678	997.98	26.88
MARACAJU	MS	-21.61	832	7467102	967.96	24.47
MARAMBAIA	RJ	-23.05	1075	6855987	1014.08	23.28
MARAU	BA	-13.91	2756	7198214	1014.78	25.33
MARECHAL THAUMATURGO	AC	-8.95	2446	7123426	983.71	26.36
MARIA DA FE	MG	-22.31	1449	6167952	876.42	16.8
MARIANOPOLIS DO TO	TO	-9.58	1355	7833849	989.95	27.28
MARILANDIA	ES	-19.41	919	6210814	1004.35	24.65
MARILIA	SP	-22.24	1231	6605808	939.66	22.73
MARINGA	PR	-23.41	1464	6599675	952.36	22.91
MATEIROS	TO	-10.43	1113	7856743	924.92	24.62
MAUES	AM	-3.4	1714	6790194	1007.92	26.55
MEDICILANDIA	PA	-3.51	3249	6400254	981.97	26.16
MINA DO PALITO	PA	-6.32	1020	5131064	980.42	26.03
MINEIROS	GO	-17.45	1292	7340020	926.42	23.22
MIRANDA	MS	-20.38	2369	7651165	997.27	24.63
MOCAMBINHO	MG	-15.09	677	8074917	962.32	25.06
MONTALVANIA	MG	-15.09	1435	8388614	954.98	24.59
MONTE ALEGRE	PA	-2.0	1091	7418744	999.45	27.75
MONTE ALEGRE DE GOIAS	GO	-13.25	963	7883424	950.9	25.77
MONTE VERDE	MG	-22.86	1603	6239537	849.11	15.35
MONTEIRO	PB	-7.89	1089	8481863	945.58	25.09
MONTES CLAROS	MG	-16.72	761	7336683	942.02	24.06
MORADA NOVA	CE	-5.14	617	7051115	1008.05	28.03
MORRETES	PR	-25.51	2035	5671694	1009.0	21.13
MORRINHOS	GO	-17.72	1191	7610862	930.16	23.11
MOSSORO	RN	-4.9	17554	8443113	1009.93	27.82
MOSTARDAS	RS	-31.25	2165	6068537	1011.25	20.01
MURIAE	MG	-21.1	1636	6095380	982.45	23.19
NATAL	RN	-5.9	1181	21471439	1007.3	26.84
NHUMIRIM	MS	-18.99	816	7825509	1000.18	26.93
NITEROI	RJ	-22.87	986	5489141	1015.35	24.72
NOSSA SENHORA DA GLORIA	SE	-10.21	461	7953363	983.38	25.3
NOVA ALVORADA DO SUL	MS	-21.45	1084	7753849	968.74	24.71
NOVA ANDRADINA	MS	-22.08	385	10112584	971.48	25.07
NOVA FATIMA	PR	-23.41	2150	6986453	939.34	21.44
NOVA FRIBURGO	RJ	-22.33	1614	6167742	898.54	17.16
NOVA FRIBURGO - SALINAS	RJ	-22.33	1595	6238409	899.3	17.17
NOVA MARINGA	MT	-13.07	1394	6539221	973.74	25.92
NOVA PORTEIRINHA (JANAUBA)	MG	-15.8	672	7215943	954.07	25.48
NOVA TEBAS	PR	-24.44	1277	7120614	940.58	21.56
NOVA UBIRATA	MT	-13.69	1225	7042547	959.6	25.91
NOVA VENECIA	ES	-18.7	902	6865768	998.29	24.0

Table B.6 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (f)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
NOVO ARIPUANA	AM	-8.09	1340	8639767	1006.58	27.97
NOVO HORIZONTE	SC	-26.41	2302	7132183	908.09	18.21
NOVO REPARTIMENTO	PA	-4.23	1593	5991670	999.43	26.95
OBIDOS	PA	-5.37	2129	5803672	1000.9	26.59
OEIRAS	PI	-6.97	707	6783948	993.98	27.78
OIAPOQUE	AP	3.81	2463	5157560	1010.3	26.32
OLIVEIRA	MG	-20.71	1200	6337792	902.72	21.09
OURICURI	PE	-7.88	336	9256056	961.09	26.9
OURINHOS	SP	-22.95	1104	6278556	966.4	22.06
OURO BRANCO	MG	-20.55	1333	5962468	899.64	20.1
PACAJA	PA	-3.84	845	5849505	1000.93	27.67
PACARAIMA	RR	4.48	3610	0	915.48	18.0
PALMARES	PE	-8.67	938	4625137	995.13	24.88
PALMAS	TO	-10.18	3750	6630609	979.21	27.53
PALMEIRA DAS MISSOES	RS	-27.92	1420	6350090	944.44	19.02
PALMEIRA DOS INDIOS	AL	-9.42	495	7810246	982.64	25.22
PAMPULHA	MG	-19.88	1529	7077853	919.4	22.01
PAO DE ACUCAR	AL	-9.77	392	7432506	1011.88	27.59
PARACATU	MG	-17.24	750	6920727	934.21	23.89
PARAGOMINAS	PA	-3.01	2123	8162327	999.53	27.18
PARANA	TO	-12.62	890	7803980	979.64	26.5
PARANAIBA	MS	-19.7	962	6935407	967.58	24.89
PARANAPOEMA	PR	-22.49	1879	8235601	978.22	23.72
PARANATINGA	MT	-14.42	1209	7637067	955.95	25.76
PARANOA (COOPA-DF)	DF	-16.01	1312	7127080	899.61	21.75
PARATI	RJ	-23.22	2005	6475395	1015.29	22.99
PARATY	RJ	-23.22	1613	5168887	1015.38	22.88
PARAUNA	GO	-16.95	1052	7432631	937.51	24.69
PARINTINS	AM	-2.64	2043	7190628	1008.14	27.76
PARNAIBA	PI	-3.09	2525	6905358	1002.95	27.54
PARQUE ESTADUAL CHANDLESS	AC	-9.36	1479	7221131	993.18	25.17
PASSA QUATRO	MG	-22.4	1085	6089990	901.54	19.16
PASSO FUNDO	RS	-28.23	1857	6187557	937.44	17.97
PASSOS	MG	-20.74	1196	7418744	927.37	21.6
PATOS	PB	-7.08	439	10184411	982.89	28.39
PATOS DE MINAS	MG	-18.52	1141	6800863	909.64	22.16
PATROCINIO	MG	-19.0	1286	7018952	906.84	20.76
PAULISTANA	PI	-8.08	496	9776971	969.72	28.04
PEDRO AFONSO	TO	-8.97	1390	7918190	990.06	27.39
PEDRO GOMES	MS	-18.07	1308	6910090	984.53	24.54
PEF ESTIRAO DO EQUADOR	AM	-4.53	715	0	1000.79	27.22
PEF IPIRANGA	AM	-2.93	187	0	1002.95	30.6
PEF YAUARETE	AM	0.6	487	0	998.41	26.14
PEIXE	TO	-12.02	1197	8297555	983.11	26.97
PETROLINA	PE	-9.39	269	5915072	971.19	27.28
PIATA	BA	-13.16	787	7143868	874.32	19.61
PICO DO COUTO	RJ	-22.48	2033	5715893	828.83	14.41
PICOS	PI	-7.07	349	7186591	985.44	28.76
PILAO ARCADO	BA	-10.0	524	7684304	967.78	28.12
PIRACICABA	SP	-22.7	1062	7230465	951.16	22.09
PIRANHAS	AL	-9.62	506	7256021	992.75	26.95
PIRAPORA	MG	-17.26	968	8075656	957.35	24.13
PIRES DO RIO	GO	-17.3	1638	7425554	930.04	23.35
PIRIPIRI	PI	-4.26	1265	7277643	992.68	28.2
PIUM	TO	-10.48	1523	6684459	990.4	26.35
PLACAS	PA	-3.88	2726	5563411	999.3	26.59
PLANALTO	PR	-25.72	1517	8065500	967.97	21.67
POCO VERDE	SE	-10.74	632	7451197	972.73	24.43
POMPEU	MG	-19.23	802	9084505	938.36	23.57
PONTA PORA	MS	-22.55	1615	5899611	938.15	21.93
PONTES E LACERDA	MT	-15.25	928	6490547	979.98	25.44
PORANGATU	GO	-13.51	998	7214047	970.18	27.13
PORTO ALEGRE	RS	-30.05	1513	5982066	1009.8	20.0
PORTO ALEGRE - JARDIM BOTANICO	RS	-30.05	1576	5012476	1009.69	20.0
PORTO ALEGRE- BELEM NOVO	RS	-30.19	1114	3225675	1013.54	20.96
PORTO DE MOZ	PA	-1.82	1931	4967055	1009.55	27.07
PORTO ESTRELA	MT	-15.55	539	7161731	994.85	26.43
PORTO GRANDE	AP	0.69	3706	7375588	1002.44	26.6
PORTO MURTINHO	MS	-21.71	745	5876571	1002.39	25.39
PORTO SEGURO	BA	-16.39	1287	6093869	1006.18	23.74
PORTO VELHO	RO	-8.76	1845	5832050	995.87	26.46
PORTO WALTER	AC	-8.27	1327	6566752	987.49	26.25
POSSE	GO	-14.09	866	8142999	921.77	24.82
PRADOPOLIS	SP	-21.34	911	5577805	953.24	22.48
PREGUICAS	MA	-2.59	1348	6684649	1010.77	27.49
PRES. KENNEDY	ES	-21.1	981	7212223	1006.9	24.01
PRESIDENTE FIGUEIREDO	AM	-2.08	2242	5721036	1003.44	27.03
PRESIDENTE KENNEDY	ES	-21.1	814	6673927	1006.52	24.82
PRESIDENTE PRUDENTE	SP	-22.12	1173	5462301	964.35	23.92

Table B.7 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (g)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
PRIMAVERA DO LESTE	MT	-15.58	1418	7893095	936.82	25.56
PATY DO ALFERES - AVELAR	RJ	-22.35	636	3997627	955.13	21.2
QUARAI	RS	-30.37	1353	6502826	1000.67	19.0
QUEIMADAS	BA	-10.98	546	7729012	978.59	26.27
QUERENCIA	MT	-12.6	1092	7757898	972.46	26.02
QUIXADA	CE	-4.98	1045	4908574	991.14	27.43
QUIXERAMOBIM	CE	-5.17	898	8466626	986.89	27.99
RANCHARIA	SP	-22.37	1969	7481489	968.66	22.11
RANCHO QUEIMADO	SC	-27.68	1645	5359321	917.78	16.05
RECIFE	PE	-8.06	1755	6693808	1012.8	25.83
REDENCAO	PA	-8.04	1401	6484167	988.08	27.56
REGISTRO	SP	-24.53	1072	7628179	1012.16	22.92
REMANSO	BA	-9.6	324	9842972	967.5	27.3
RESENDE	RJ	-22.45	1354	6009665	965.5	21.88
RIBAS DO RIO PARDO	MS	-20.47	1144	6656393	962.89	24.77
RIBEIRA DO AMPARO	BA	-11.06	544	6148236	992.79	25.87
RIO BRANCO	AC	-9.96	2068	7005299	988.26	26.67
RIO BRILHANTE	MS	-21.77	1125	6815417	975.78	23.34
RIO CLARO	RJ	-22.65	1236	5771526	958.35	20.86
RIO DE JANEIRO - FORTE DE COPACABANA	RJ	-22.99	1202	5659702	1011.58	23.73
RIO DE JANEIRO - JACAREPAGUA	RJ	-22.94	1334	5422536	1013.78	23.09
RIO DE JANEIRO - VILA MILITAR	RJ	-22.86	1185	5761186	1012.46	24.09
RIO DE JANEIRO-MARAMBAIA	RJ	-23.05	1061	6088850	1014.11	23.55
RIO DO CAMPO	SC	-26.94	2321	6789035	947.54	18.68
RIO GRANDE	RS	-32.03	1171	5700248	1015.17	18.73
RIO NEGRINHO	SC	-26.25	1525	5433670	919.58	17.06
RIO PARDO	RS	-29.87	1480	5991588	1002.64	19.49
RIO PARDO DE MINAS	MG	-15.72	747	7145309	920.8	21.42
RIO SONO	TO	-9.16	1217	7913850	976.05	26.62
RIO URUBU	AM	-2.72	2545	6599514	1005.6	26.24
RIO VERDE	GO	-17.79	1286	8527756	927.3	23.46
RONDON DO PARA	PA	-4.83	2532	6885484	985.67	25.93
RONDONOPOLIS	MT	-16.45	899	6717027	979.3	25.9
ROSARIO OESTE	MT	-14.83	804	6542436	988.38	27.42
S. G. DA CACHOEIRA	AM	-0.12	2932	6276961	1001.02	26.33
S.J. DO RIO CLARO	MT	-13.45	1044	7127755	973.44	25.75
SACRAMENTO	MG	-19.88	1371	7369404	913.22	22.17
SALGUEIRO	PE	-8.06	353	9330353	962.39	27.42
SALINAS	MG	-16.16	830	7487427	959.71	24.26
SALINOPOLIS	PA	-0.62	2158	6873189	1009.05	27.5
SALTO DO CEU	MT	-15.14	947	7163196	977.81	25.5
SALVADOR	BA	-13.02	1700	6712889	1008.91	25.77
SALVADOR (RADIO FAROL)	BA	-12.81	1540	5725201	1010.13	25.35
SANTA CRUZ	RN	-6.23	248	7255035	987.2	26.88
SANTA FE DO ARAGUAIA	TO	-7.12	1549	7384503	988.49	26.58
SANTA ISABEL DO RIO NEGRO	AM	-0.41	245	0	1007.65	28.09
SANTA MARIA	RS	-29.71	1747	5969349	1003.11	19.39
SANTA MARIA DAS BARREIRAS	PA	-8.73	1119	8551996	990.0	27.74
SANTA MARIA MADALENA	RJ	-21.95	1055	5325175	951.81	20.66
SANTA RITA DE CASSIA	BA	-11.02	609	9178753	961.55	26.84
SANTA RITA DO PARDO	MS	-21.31	900	7366504	970.3	23.57
SANTA ROSA	RS	-27.89	2426	6575920	982.64	20.52
SANTA ROSA DO TOCANTINS	TO	-11.05	1008	7463865	977.68	26.64
SANTA TERESA	ES	-19.99	1433	6054571	907.3	18.42
SANTANA DO ARAGUAIA	PA	-9.34	1334	6929246	990.6	27.29
SANTANA DO LIVRAMENTO	RS	-30.84	1220	4889772	987.83	17.98
SANTAREM	PA	-2.5	3082	7494576	994.53	27.04
SANTIAGO	RS	-29.19	1795	6642577	969.3	19.1
SANTO ANTONIO DO LESTE	MT	-14.93	579	9482899	938.06	24.93
SANTO AUGUSTO	RS	-27.85	1852	6394332	958.16	19.72
SAO BENTO	MA	-2.7	4155	0	1008.51	29.6
SAO BORJA	RS	-28.65	1230	7015559	1004.23	21.14
SAO CARLOS	SP	-21.98	1346	6708090	920.04	21.2
SAO FELIX DO ARAGUAIA	MT	-11.62	1192	6847731	988.41	28.2
SAO FELIX DO XINGU	PA	-6.64	1915	7389780	988.38	26.33
SAO GABRIEL	RS	-30.34	1446	6027613	1001.57	19.73
SAO GABRIEL DO OESTE	MS	-19.42	1097	7915551	940.2	23.78
SAO GONCALO	PB	-6.76	769	8424049	985.68	27.45
SAO JOAO DEL REI	MG	-21.23	1402	6793947	910.77	19.88
SAO JOAO DO PIAUI	PI	-8.36	334	8078410	985.31	28.74
SAO JOAQUIM	SC	-28.28	1793	6347487	862.27	14.0
SAO JOSE DO XINGU	MT	-10.48	897	6714703	977.98	27.04
SAO JOSE DOS AUSENTES	RS	-28.75	2292	6174521	879.06	13.89
SAO LUIS	MA	-2.53	1956	7245124	1005.56	26.82

Table B.8 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (h)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
SAO LUIS DO PARAITINGA	SP	-23.23	1135	7697032	918.58	19.14
SAO LUIS DO QUITUNDE	AL	-9.29	1318	7110908	1012.88	25.57
SAO LUIZ DO PARAITINGA	SP	-23.23	549	5696742	918.92	19.31
SAO LUIZ GONZAGA	RS	-28.42	1685	6806692	985.66	20.88
SAO MATEUS	ES	-18.71	1205	6901068	1011.93	24.05
SAO MATEUS DO SUL	PR	-25.83	1325	4666775	928.15	17.31
SAO MIGUEL ARCANJO	SP	-23.85	1041	6737471	939.68	19.83
SAO MIGUEL DO ARAGUAIA	GO	-12.82	1036	8917619	987.19	28.07
SAO MIGUEL DO OESTE	SC	-26.78	1669	6588369	940.31	20.4
SAO PAULO - INTERLAGOS	SP	-23.72	1320	5383160	928.55	19.55
SAO PAULO - MIRANTE	SP	-23.48	1548	6358827	927.18	20.53
SAO PEDRO DO PIAUI	PI	-5.91	694	8194333	978.1	27.39
SAO RAIMUNDO NONATO	PI	-9.03	277	8079526	969.03	27.5
SAO ROMAO	MG	-16.36	1253	8300864	958.55	24.99
SAO SEBASTIAO	SP	-23.81	992	8322203	1014.04	25.34
SAO SEBASTIAO DO PARAISO	MG	-20.91	1623	8965256	920.29	21.84
SAO SIMAO	GO	-18.97	2403	9985072	955.24	24.45
SAO TOME	RJ	-22.04	700	7907735	1015.54	24.06
SAO VICENTE DO SUL	RS	-29.7	1558	6528927	999.33	19.34
SAPEZAL	MT	-10.17	1205	6642101	950.18	25.11
SAQUAREMA	RJ	-22.87	1010	5617816	1013.63	23.87
SAQUAREMA - SAMPAIO CORREIA	RJ	-22.87	1662	6066720	1013.76	23.49
SELVIRIA	MS	-20.35	614	3715336	971.46	23.81
SENHOR DO BONFIM	BA	-10.44	640	4314587	954.14	24.18
SERAFINA CORREA	RS	-28.7	1875	5873535	958.97	17.88
SEROPEDICA-ECOLOGIA AGRICOLA	RJ	-22.76	1183	6221740	1011.89	24.06
SERRA DOS AIMORES	MG	-17.8	1034	6388096	991.9	23.64
SERRA DOS CARAJAS	PA	-6.08	2543	6240659	931.84	24.76
SERRA NOVA DOURADA	MT	-11.99	931	8229976	961.05	27.71
SERRA TALHADA	PE	-7.95	550	7618740	956.97	26.18
SERRINHA	BA	-11.66	485	5612236	976.28	24.49
SETE LAGOAS	MG	-19.45	1167	6414267	933.35	21.95
SETE QUEDAS	MS	-23.97	1418	8665154	967.46	22.94
SIDROLANDIA	MS	-20.98	821	6974565	959.4	23.79
SILVA JARDIM	RJ	-22.65	1452	5319743	1013.74	23.34
SILVANIA	GO	-16.68	1077	5677463	908.83	22.19
SINOP	MT	-11.98	1113	6130993	970.28	25.79
SOBRAL	CE	-3.73	541	7268669	1000.42	27.77
SOLEDADE	RS	-28.85	1737	6685936	939.71	17.79
SONORA	MS	-17.9	2116	5834250	956.74	24.86
SOROCABA	SP	-23.35	1119	7242863	946.98	20.71
SORRISO	MT	-12.55	1643	6628601	968.88	26.57
SOURE	PA	-0.81	2172	5804393	1009.93	27.4
SURUBIM	PE	-7.84	450	8210346	966.31	24.62
Santa Vitoria do Palmar - Barra do Chui	RS	-33.74	1029	7296151	1014.8	17.39
TANGARA DA SERRA	MT	-14.65	1440	14471995	962.32	25.05
TAUA	CE	-6.02	380	6673967	965.86	27.63
TAUBATE	SP	-23.04	2514	5779055	949.82	21.15
TEOFILO OTONI	MG	-17.9	3011	6743478	962.8	23.2
TERESINA	PI	-5.07	1183	6586561	1002.8	27.98
TERESOPOLIS	RJ	-22.45	2754	5586595	907.75	18.13
TERESOPOLIS-PARQUE NACIONAL	RJ	-22.45	2735	5048234	907.96	18.24
TEUTONIA	RS	-29.45	1505	5659185	1006.05	20.23
TIANGUA	CE	-3.73	1104	6528028	929.06	23.36
TIMOTEO	MG	-19.57	1307	5731999	960.4	22.62
TOME ACU	PA	-2.6	1546	7087964	1006.2	26.92
TORRES	RS	-29.35	1556	6941464	1015.17	19.65
TRAMANDAI	RS	-30.01	1370	6247772	1015.12	19.73
TRES LAGOAS	MS	-20.79	977	7474636	976.27	25.87
TRES MARIAS	MG	-18.2	1109	7404252	912.16	22.16
TRES RIOS	RJ	-22.1	1099	5105744	981.37	22.35
TUCUMA	PA	-6.74	1772	4178883	975.42	26.16
TUCURUI	PA	-3.82	1443	6277285	994.7	27.09
TUPA	SP	-21.93	1246	6824508	957.55	23.83
TUPANCIRETA	RS	-29.89	1695	5884831	962.35	18.74
TURIACU	MA	-1.66	1764	6789729	1006.17	26.96
UAUA	BA	-9.83	367	7268971	962.63	25.0
UBERABA	MG	-19.71	1491	6433433	927.46	22.6
UBERLANDIA	MG	-18.92	1395	7040143	917.34	23.46
ULIANOPOLIS	PA	-3.8	0	0	N/A	N/A

Table B.9 – Average values for precipitation, radiation, pressure and air temperature across the collection of weather stations, part (i)

Station	Federation Unit	Latitude	Total Precipitation (mm)	Global Radiation (KJ/m^2)	Atmospheric Pressure at Station Level (mB)	Air Temperature - Dry Bulb ($°C$)
UNA	BA	-15.28	1306	6719631	1007.12	23.94
UNAI	MG	-16.55	1184	7903895	942.0	23.89
URUCARA	AM	-2.9	2061	6103958	1008.18	27.33
URUCUI	PI	-7.47	842	7728643	967.4	26.57
URUGUAIANA	RS	-29.84	1302	6622765	1005.63	20.03
URUSSANGA	SC	-28.53	1938	5675922	1011.48	20.22
VACARIA	RS	-28.51	1794	6327272	906.76	15.68
VALENCA	BA	-13.34	2390	13573660	989.34	23.05
VALENCA DO PIAUI	PI	-6.4	1974	8119401	974.3	27.65
VALPARAISO	SP	-21.32	1408	7182408	970.13	24.03
VARGINHA	MG	-21.57	1276	6754094	909.96	20.36
VENDA NOVA DO IMIGRANTE	ES	-20.25	1283	6262500	935.62	19.97
VENTANIA	PR	-24.24	1172	5130270	894.79	18.23
VICOSA	MG	-20.76	1320	5882549	937.26	20.03
VILA BELA DA SANTISSIMA TRINDADE	MT	-15.06	1084	5761425	987.29	25.85
VILA MILITAR	RJ	-22.86	1099	6338465	1012.55	24.12
VILA VELHA	ES	-20.47	1080	6714676	1012.19	23.91
VILHENA	RO	-12.73	1736	6833653	946.56	24.9
VITORIA	ES	-20.27	1427	6686121	1016.16	24.59
VITORIA DA CONQUISTA	BA	-14.89	1196	7822505	918.07	20.61
VOTUPORANGA	SP	-20.4	1059	6701148	959.49	24.93
XANXERE	SC	-26.94	2687	6404418	915.97	18.73
XEREM	RJ	-22.59	1907	5910708	1012.96	22.85
XINGUARA	PA	-7.11	1728	6568366	983.93	26.48
ZE DOCA	MA	-3.27	1621	5217725	1005.77	27.82

APPENDIX C – Appendix C

From the *Instituto Nacional de Meteorologia* (Instituto Nacional de Meteorologia (INMET), 2024), we gathered data from various weather stations throughout Brazil. The network consists of 633 automated stations engaged in data gathering. Our data covers the period from 2010 to 2024, with annual average values summarized in the tables below. We provide the **mean annual values** for the following meteorological parameters:

- Temperature - Dew Point ($^\circ C$)
- Relative Humidity (%)
- Gust Wind (m/s)
- Wind Velocity (m/s)

Table C.1 – Mean values for weather stations across the entire data collection period, part (a)

Station	Temperature - Dew Point ($^{\circ}C$)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
ABROLHOS	19.15	68.27	8.36	5.35
ACARAU	21.75	75.87	6.34	3.11
AFONSO CLAUDIO	16.51	72.65	4.22	1.88
AGUA BOA	17.51	63.77	4.95	2.31
AGUA CLARA	17.18	69.78	3.94	1.33
AGUAS EMENDADAS	14.01	67.51	4.98	1.98
AGUAS VERMELHAS	15.24	72.18	4.94	2.01
AIMORES	17.7	68.23	5.38	2.47
ALEGRE	17.98	71.72	3.74	1.38
ALEGRETE	14.11	73.25	5.44	2.06
ALFREDO CHAVES	18.25	70.65	4.82	1.93
ALMAS	18.81	67.3	4.71	1.77
ALMENARA	18.14	66.64	4.26	1.52
ALTA FLORESTA	20.46	75.45	4.22	1.75
ALTAMIRA	22.8	81.92	2.77	0.86
ALTO ARAGUAIA	15.8	66.3	5.08	2.42
ALTO PARAISO DE GOIAS	12.56	60.6	4.84	1.95
ALTO PARNAIBA	17.95	63.65	3.42	1.03
ALTO TAQUARI	15.37	67.73	4.96	2.23
ALVORADA DO GURGUEIA	16.28	55.22	4.65	1.8
AMAMBAI	15.79	70.41	4.97	1.7
AMARGOSA	17.64	73.56	4.8	1.83
ANGELICA	16.44	65.96	5.18	2.25
ANGICAL DO PIAUI	19.12	63.6	3.61	1.19
ANGRA DOS REIS	19.65	82.64	3.13	0.62
APIACAS	21.4	73.88	2.98	0.97
APODI	18.93	60.74	6.22	2.68
APUI	21.54	76.57	3.23	0.91
AQUIDAUANA	17.6	68.03	3.47	0.94
ARACAJU	19.64	65.99	5.92	2.49
ARACUAI	15.09	58.46	4.28	1.33
ARAGARCAS	17.09	61.13	3.67	1.23
ARAGUACU	19.51	69.76	4.33	1.9
ARAGUAINA	20.8	77.46	2.52	0.6
ARAGUATINS	20.65	70.3	3.78	1.57
ARAL MOREIRA	15.2	67.35	6.58	3.11
ARAPIRACA	19.33	73.73	5.12	1.87
ARARANGUA	16.97	82.65	4.84	2.27
ARAXA	13.68	63.32	5.41	2.45
ARCO VERDE	16.56	66.38	7.39	3.13
AREIA	19.31	83.75	7.46	3.56
ARIQUEMES	21.79	77.22	4.89	1.44
ARIRANHA	15.64	66.57	4.46	1.75
ARQ.SAO PEDRO E SAO PAULO	26.53	88.39	9.69	4.58
ARRAIAL DO CABO	20.16	80.09	8.51	4.73
AUTAZES	21.63	75.75	1.43	0.39
AVARE	14.67	69.84	5.9	2.34
BACABAL	21.6	70.42	4.06	1.18
BAGE	12.83	75.36	6.67	3.35
BAIXA GRANDE DO RIBEIRO	17.95	63.05	5.74	2.0
BALIZA	21.98	75.3	2.03	0.63
BALSAS	18.59	63.25	3.96	1.35
BAMBUI	16.63	77.36	3.56	1.34
BANDEIRANTES	16.91	70.66	5.95	3.12
BARBACENA	14.33	74.23	4.3	1.66
BARBALHA	17.37	62.44	4.78	1.7

Table C.2 – Mean values for weather stations across the entire data collection period, part (b)

Station	Temperature - Dew Point ($^{\circ}C$)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
BARRA DO TURVO	15.73	88.6	2.9	0.62
BARREIRAS	17.06	63.4	3.51	1.2
BARRETOS	15.26	65.53	3.69	1.21
BARUERI	14.69	75.14	4.39	1.35
BATAGUASSU	16.29	64.66	5.86	3.06
BAURU	15.87	70.83	4.82	1.4
BEBDOURO	16.17	66.46	5.13	2.12
BEBEDOURO	15.62	65.38	4.64	1.68
BELA VISTA	16.12	69.45	4.57	1.89
BELEM	23.2	81.38	3.64	0.82
BELMONTE	20.28	82.08	4.44	1.73
BELO HORIZONTE (PAMPULHA)	14.45	64.98	4.66	1.51
BELO HORIZONTE - CERCADINHO	13.62	69.11	8.86	5.24
BENTO GONCALVES	13.2	77.14	6.42	2.81
BERTIOGA	20.57	86.31	4.44	1.59
BOA VISTA	20.6	65.61	5.2	1.97
BOCA DO ACRE	21.38	74.95	3.58	1.24
BOM JARDIM DA SERRA - MORRO DA IGREJA	8.53	75.94	12.35	7.75
BOM JESUS DA LAPA	16.64	54.94	4.71	1.48
BOM JESUS DO PIAUI	16.76	57.56	3.95	1.09
BONITO	18.1	71.59	5.6	2.89
BRAGANCA	22.99	79.43	4.77	1.39
BRAGANCA PAULISTA	15.14	73.37	5.37	1.84
BRASILANDIA	15.83	67.58	3.61	1.36
BRASILIA	13.67	65.1	5.14	2.28
BRASNORTE (MUNDO NOVO)	19.4	73.64	5.07	1.93
BRASNORTE (NOVO MUNDO)	19.37	67.19	3.84	0.33
BRAZLANDIA	13.28	62.12	5.64	2.25
BREJO GRANDE	21.34	73.98	4.87	1.62
BREVES	22.76	74.25	3.98	1.1
BRUMADO	16.2	61.23	4.85	1.52
BURITICUPU	20.43	69.68	4.95	2.29
BURITIRAMA	15.95	55.38	4.81	1.59
BURITIS	13.94	61.61	5.52	2.58
CAARAPO	16.26	67.96	5.91	2.78
CABACEIRAS	18.09	66.31	6.53	2.49
CABROBO	16.49	56.92	6.89	2.36
CACADOR	12.01	76.64	4.01	1.19
CACAPAVA DO SUL	13.34	79.06	7.54	4.06
CACERES	20.15	71.17	3.9	1.38
CACHOEIRA PAULISTA	16.52	74.06	4.0	1.22
CACOAL	19.84	68.15	4.0	1.46
CAIAPONIA	15.34	63.19	4.02	1.38
CAICO	17.58	53.97	6.8	2.85
CALCANHAR	18.96	65.16	10.77	7.22
CALDAS	13.19	77.66	3.01	0.86
CAMAPUA	17.66	70.95	5.18	2.83
CAMAQUA	15.57	83.94	3.81	0.87
CAMARATUBA	19.49	67.46	5.67	2.14
CAMBARA DO SUL	12.47	83.26	6.46	2.96
CAMBUCI	18.53	74.67	3.02	1.11
CAMETA	22.48	74.22	4.76	1.7
CAMPINA DA LAGOA	16.25	67.41	5.98	3.12
CAMPINA GRANDE	19.25	77.16	7.24	3.25
CAMPINA VERDE	15.71	64.26	4.39	1.71
CAMPO BOM	14.79	68.94	3.75	1.23

Table C.3 – Mean values for weather stations across the entire data collection period, part (c)

Station	Temperature - Dew Point (°C)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
CAMPOS	19.27	77.62	6.35	3.3
CAMPOS DO JORDAO	11.74	81.92	4.17	1.44
CAMPOS DOS GOYTACAZES	19.19	76.74	5.74	2.73
CAMPOS DOS GOYTACAZES - SAO TOME	20.7	79.26	7.21	3.59
CAMPOS LINDOS	17.78	65.4	5.15	2.22
CAMPOS NOVOS	12.68	77.68	6.66	3.43
CAMPOS SALES	17.37	65.31	5.35	2.01
CANELA	12.59	81.87	6.61	2.82
CANGUCU	12.92	82.04	8.55	4.56
CANTO DO BURITI	16.41	55.68	5.65	2.67
CAPAO DO LEAO (PELOTAS)	14.12	83.49	6.4	3.21
CAPELINHA	15.01	74.78	4.88	2.0
CAPITAO POCO	22.28	80.2	4.4	1.81
CARACOL	17.42	63.89	5.29	2.17
CARATINGA	18.03	72.67	4.21	1.44
CARAVELAS	20.72	80.72	5.86	2.64
CARIRA	20.15	77.19	5.59	2.54
CARLINDA	19.67	73.1	3.67	1.26
CARMO	17.84	75.91	2.79	0.92
CAROLINA	19.88	65.9	3.55	1.15
CARUARU	18.0	77.92	7.34	3.02
CASA BRANCA	14.24	64.15	4.41	1.46
CASSILANDIA	16.47	63.86	3.55	1.2
CASTANHAL	23.02	80.26	3.94	1.06
CASTELO DO PIAUI	17.68	57.66	5.1	1.98
CASTRO	14.18	81.37	4.56	1.46
CATALAO	14.13	60.87	4.83	1.85
CAXIAS	20.88	72.47	3.67	1.28
CHAPADA GAUCHA	13.99	59.93	5.66	2.73
CHAPADAO DO SUL	15.52	65.69	5.39	2.19
CHAPADINHA	21.22	70.87	4.96	1.91
CHAPECO	13.8	72.83	6.5	3.44
CIDADE GAUCHA	16.15	65.96	4.74	1.61
CLEVELANDIA	12.74	72.76	6.88	3.44
COARI	22.97	78.35	3.72	1.23
COLINAS	18.99	64.37	3.62	1.02
COLINAS DO TOCANTINS	21.75	76.8	3.87	1.59
COLOMBO	14.18	84.69	4.28	1.38
COMODORO	18.34	77.65	3.46	1.16
CONCEICAO DAS ALAGOAS	15.82	66.94	4.68	2.0
CONCEICAO DO ARAGUAIA	20.17	71.55	3.58	1.38
CONDE	21.91	79.25	5.86	2.69
CORONEL PACHECO	18.2	75.74	3.51	1.41
CORRENTE	16.59	58.95	4.16	1.44
CORRENTINA	17.7	69.61	4.07	1.52
CORUMBA	18.44	63.75	4.71	1.67
CORURIPE	21.62	77.15	5.61	2.29
COSTA RICA	15.91	67.33	6.05	3.29
COTRIGUACU	21.3	76.9	3.36	1.17
COXIM	18.17	69.24	3.33	0.9
CRATEUS	17.98	58.47	5.26	2.07
CRIOSFERA	nan	61.18	12.2	10.32
CRISTALINA	13.21	64.36	4.61	2.11
CRISTALINA (FAZENDA SANTA MONICA)	15.18	70.46	4.91	1.92
CRMN MANAUS	22.33	76.74	2.31	0.73
CRUZ ALTA	13.42	75.13	5.96	2.69

Table C.4 – Mean values for weather stations across the entire data collection period, part (d)

Station	Temperature - Dew Point ($°C$)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
CURITIBA	13.43	73.98	5.1	1.88
CURITIBANOS	12.76	78.19	5.75	2.62
CURVELO	14.93	63.62	4.19	1.69
DELFINO	14.42	58.38	6.08	2.57
DIAMANTE DO NORTE	17.11	69.38	5.17	2.02
DIAMANTINA	13.28	73.8	5.73	2.94
DIANOPOLIS	16.03	61.79	5.6	2.63
DIONISIO CERQUEIRA	13.65	71.01	6.42	3.02
DIVINOPOLIS	16.15	72.68	4.57	2.14
DOIS VIZINHOS	15.15	72.24	4.42	1.82
DOM ELISEU	21.37	75.36	4.13	1.74
DOM PEDRITO	13.07	72.88	6.75	3.67
DORES DO INDAIA	15.05	66.0	4.31	1.8
DOURADOS	16.14	67.7	5.76	2.32
DRACENA	17.57	70.4	6.76	3.04
DTCEA GUAJARA-MIRIM	21.32	74.87	4.11	1.39
DTCEA JACAREACANGA	22.35	81.41	1.39	0.34
DTCEA TABATINGA	20.69	80.93	1.04	0.14
DTCEA TEFE	24.37	90.9	1.83	0.48
DTCEA VILHENA	20.25	76.55	6.4	3.07
DUQUE DE CAXIAS - XEREM	18.98	78.06	2.81	0.63
EB_PEF_BONFIM	21.0	67.84	4.42	2.54
ECOLOGIA AGRICOLA	19.28	77.93	4.63	2.19
ECOPORANGA	19.16	73.66	4.75	1.91
EDEIA	16.27	62.85	4.76	1.81
EIRUNEPE	18.99	67.36	3.25	1.01
ENCRUZILHADA DO SUL	13.02	77.44	6.37	2.63
EPITACIOLANDIA	21.42	80.41	3.1	1.02
ERECHIM	13.25	76.23	4.99	1.83
ESPERANTINA	20.78	69.75	4.17	1.3
ESPINOSA	13.75	51.42	6.31	2.77
ESTREITO	20.52	73.24	3.35	1.04
EUCLIDES DA CUNHA	18.25	68.91	6.69	3.07
FAROL de SANTANA	23.22	75.46	8.57	3.57
FATIMA DO SUL	18.91	57.24	6.17	3.03
FEIJO	22.31	81.44	3.18	1.37
FEIRA DE SANTANA	19.59	74.14	5.65	2.48
FLORESTA	16.6	55.38	7.1	3.18
FLORESTAL	14.89	73.68	2.86	0.77
FLORIANO	18.96	59.04	3.98	1.25
FLORIANOPOLIS	16.89	77.17	4.91	1.58
FORMIGA	14.71	68.01	4.38	1.63
FORMOSA DO RIO PRETO	17.76	72.8	3.88	1.29
FORMOSO DO ARAGUAIA	18.67	65.1	4.45	1.69
FORMOSO DO RIO PRETO	17.34	66.29	3.91	1.36
FORTALEZA	21.49	71.02	6.52	2.52
FORTE DE COPACABANA	19.39	77.93	5.72	2.7
FORTE PRINCIPE	16.61	51.4	2.68	1.52
FOZ DO IGUACU	16.61	72.07	5.36	2.43
FRANCA	13.41	62.64	4.97	1.88
FREDERICO WESTPHALEN	14.53	75.24	4.16	1.43
GAMA (PONTE ALTA)	13.93	63.75	5.17	2.29
GARANHUNS	18.08	82.5	5.78	2.11
GAUCHA DO NORTE	17.9	61.96	3.94	1.29
GENERAL CARNEIRO	13.41	84.34	3.53	0.95
GILBUES	16.67	55.13	5.48	2.37

Table C.5 – Mean values for weather stations across the entire data collection period, part (e)

Station	Temperature - Dew Point ($^{\circ}C$)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
GOVERNADOR VALADARES	17.78	69.81	4.16	1.63
GRAJAU	18.99	61.34	4.2	1.68
GUANAMBI	14.54	51.99	7.83	3.75
GUANHAES	15.57	74.77	4.26	1.63
GUARAMIRANGA	18.58	84.92	7.43	2.84
GUARANTA DO NORTE	20.91	75.56	3.01	0.79
GUARDA-MOR	14.53	67.06	4.56	1.71
GUIRATINGA	17.65	64.29	3.33	0.9
GURUPI	18.83	70.5	3.43	1.09
HUMAITA	22.28	76.22	3.35	1.24
IBIMIRIM	16.37	61.15	6.48	2.89
IBIRITE (ROLA MOCA)	13.44	68.05	7.76	3.52
IBIRUBA	14.51	76.88	5.68	2.47
IBITINGA	15.63	68.44	4.46	1.64
IBOTIRAMA	15.61	53.0	5.04	1.86
ICARAIMA	16.47	66.16	5.46	2.5
IGUAPE	18.69	83.39	4.17	1.32
IGUATEMI	15.91	67.57	6.38	3.15
IGUATU	18.29	60.23	5.77	2.51
ILHA DO MEL	19.57	83.52	5.03	2.46
ILHEUS	20.96	83.65	4.93	2.22
IMPERATRIZ	20.48	68.69	2.84	0.73
INACIO MARTINS	12.78	81.6	4.98	2.12
INDAIAL	17.7	82.84	3.23	0.99
IPANGUACU	20.79	65.67	5.8	2.4
IPERO	15.23	71.62	1.16	0.1
IPIAU	19.32	77.51	3.63	1.46
IPORA	15.99	62.08	4.77	1.86
IRECE	15.53	62.39	6.91	2.73
ITABAIANA	21.38	85.76	4.67	2.33
ITABAIANINHA	19.95	76.21	5.99	2.63
ITABERABA	20.14	74.28	5.08	2.05
ITACOATIARA	23.22	79.35	3.1	0.97
ITAITUBA	22.92	76.11	2.79	0.81
ITAJAI	16.93	83.16	3.84	1.52
ITAMARAJU	20.85	82.47	5.36	2.63
ITAOBIM	16.57	60.44	5.07	1.66
ITAPACI	17.09	70.05	3.16	1.09
ITAPETINGA	18.31	71.23	4.79	2.12
ITAPEVA	14.91	75.24	5.25	2.14
ITAPIPOCA	20.35	65.57	8.27	3.95
ITAPIRA	15.0	68.26	3.56	0.97
ITAPOA	18.24	86.14	3.02	1.01
ITAPORA	16.52	66.35	5.07	2.13
ITAPORANGA	19.08	62.07	5.5	2.3
ITAQUIRAI	16.85	71.39	4.88	1.81
ITATIAIA	7.78	82.01	6.46	2.77
ITAUBAL	22.02	71.45	6.7	2.91
ITIQUIRA	17.5	68.11	4.22	1.52
ITIRUCU	17.09	79.66	5.28	2.28
ITUIUTABA	16.33	65.76	4.41	1.5
ITUMBIARA	16.5	65.79	3.83	1.42
ITUPORANGA	14.99	82.53	2.83	0.82
ITUVERAVA	15.23	66.72	4.24	1.68
IVAI	14.04	75.75	4.24	1.42
IVINHEMA	16.36	65.46	5.52	2.46

Table C.6 – Mean values for weather stations across the entire data collection period, part (f)

Station	Temperature - Dew Point (°C)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
JALES	15.7	63.1	5.2	2.25
JANAUBA	16.36	59.58	6.1	2.47
JANUARIA	16.67	64.32	4.6	1.93
JAPIRA	15.0	71.49	4.99	1.91
JARDIM	17.34	66.45	4.22	1.58
JATAI	16.04	69.64	4.19	1.55
JEREMOABO	18.38	65.26	6.11	2.7
JOACABA	13.8	78.98	5.07	2.27
JOAO PESSOA	21.64	73.9	5.93	1.98
JOAO PINHEIRO	13.75	60.59	4.38	1.72
JOAQUIM TAVORA	15.8	71.13	5.47	2.11
JOSE BONIFACIO	16.04	64.8	5.22	2.18
JUARA	20.26	72.99	3.48	1.05
JUINA	19.97	74.94	3.6	1.1
JUIZ DE FORA	15.32	79.49	5.8	2.59
JUTI	16.75	70.84	4.68	1.49
LABREA	22.34	78.45	3.14	0.91
LAGES	12.99	80.75	6.21	3.05
LAGOA DA CONFUSAO	21.42	77.57	3.5	1.28
LAGOA VERMELHA	12.64	77.19	6.62	3.21
LAGUNA CARAPA	16.2	71.35	6.33	3.45
LARANJEIRAS DO SUL	14.47	77.62	5.27	2.09
LENCOIS	18.29	70.57	3.58	0.88
LINHARES	19.33	75.44	5.69	2.96
LINS	15.12	63.17	4.81	1.6
LUIZ EDUARDO MAGALHAES	16.23	64.9	5.51	2.48
LUZIANIA	13.97	62.64	5.42	2.6
Laguna - Farol de Santa Marta	16.17	80.09	8.23	5.47
MACAE	19.39	79.01	5.86	2.55
MACAJUBA	18.78	71.68	5.86	2.75
MACAPA	22.94	75.66	5.68	2.11
MACAU	20.57	65.91	8.54	4.59
MACAUBAS	14.72	54.07	5.18	2.22
MACEIO	21.4	78.8	5.49	2.65
MACHADO	14.59	71.19	3.5	1.31
MAJOR VIEIRA	13.75	82.41	4.52	2.03
MAL. CANDIDO RONDON	15.85	71.29	5.87	2.94
MANACAPURU	22.85	77.51	3.58	1.4
MANAUS	22.53	74.74	3.93	1.41
MANHUACU	15.4	75.36	4.6	2.0
MANICORE	22.75	80.23	4.51	1.38
MANTENA	17.58	69.64	3.5	0.87
MARABA	21.57	75.29	3.54	1.3
MARACAJU	16.16	66.53	4.31	1.18
MARAMBAIA	18.47	75.71	6.13	3.36
MARAU	19.21	71.77	5.07	2.03
MARECHAL THAUMATURGO	20.38	67.11	4.13	1.63
MARIA DA FE	13.1	81.6	2.64	0.63
MARIANOPOLIS DO TO	19.75	68.36	4.49	2.0
MARILANDIA	19.36	75.28	3.72	1.26
MARILIA	15.19	66.6	5.12	1.77
MARINGA	14.91	64.33	5.38	1.98
MATEIROS	15.04	60.32	6.38	3.49
MAUES	23.86	84.36	2.41	0.57
MEDICILANDIA	21.63	78.13	2.83	0.54
MINA DO PALITO	21.24	77.93	2.91	0.98

Table C.7 – Mean values for weather stations across the entire data collection period, part (g)

Station	Temperature - Dew Point (°C)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
MONTE ALEGRE	22.22	73.07	5.69	2.84
MONTE ALEGRE DE GOIAS	15.5	58.44	4.55	1.63
MONTE VERDE	11.01	79.62	4.5	1.42
MONTEIRO	15.97	60.12	6.27	2.78
MONTES CLAROS	13.96	57.34	4.09	1.67
MORADA NOVA	19.27	63.36	5.8	2.5
MORRETES	18.71	86.84	2.42	0.63
MORRINHOS	15.24	65.87	4.34	1.66
MOSSORO	21.06	68.3	7.2	3.46
MOSTARDAS	15.53	75.22	9.13	5.28
MURIAE	17.73	74.16	3.36	0.94
NATAL	22.32	76.85	8.29	3.95
NHUMIRIM	19.08	66.93	4.56	1.91
NITEROI	19.56	75.37	4.02	1.36
NOSSA SENHORA DA GLORIA	20.34	77.15	6.6	2.94
NOVA ALVORADA DO SUL	17.43	66.36	5.36	2.14
NOVA ANDRADINA	16.77	63.93	5.81	2.56
NOVA FATIMA	15.07	69.67	5.98	2.68
NOVA FRIBURGO	13.74	82.22	5.26	2.27
NOVA FRIBURGO - SALINAS	13.42	80.44	5.92	2.27
NOVA MARINGA	19.26	70.78	3.27	0.96
NOVA PORTEIRINHA (JANAUBA)	16.76	62.49	5.73	2.21
NOVA TEBAS	14.71	68.06	3.77	1.66
NOVA UBIRATA	19.1	70.08	5.23	2.53
NOVA VENECIA	18.74	75.1	4.79	2.15
NOVO ARIPUANA	23.97	86.16	4.31	0.79
NOVO HORIZONTE	13.28	75.25	7.3	3.23
NOVO REPARTIMENTO	20.4	72.07	3.97	1.61
OBIDOS	22.44	79.82	0.68	0.18
OEIRAS	17.81	60.42	5.06	2.06
OIAPOQUE	23.42	85.58	3.22	1.03
OLIVEIRA	14.55	69.45	5.5	2.74
OURICURI	15.75	54.58	6.42	2.93
OURINHOS	15.91	71.32	4.11	1.21
OURO BRANCO	14.67	73.72	5.26	2.13
PACAJA	21.54	71.49	2.49	0.8
PACARAIMA	14.8	77.5	2.31	1.13
PALMARES	20.89	80.13	4.88	2.17
PALMAS	18.58	62.9	4.3	1.49
PALMEIRA DAS MISSOES	13.6	73.87	5.81	2.41
PALMEIRA DOS INDIOS	19.77	73.39	5.22	2.0
PAMPULHA	14.33	64.48	5.3	2.03
PAO DE ACUCAR	18.46	62.61	4.95	1.8
PARACATU	14.93	61.38	4.79	2.17
PARAGOMINAS	22.38	77.51	3.84	1.27
PARANA	16.93	61.9	3.26	0.9
PARANAIBA	16.7	64.37	4.12	1.63
PARANAPOEMA	16.55	67.36	5.7	2.68
PARANATINGA	18.12	68.63	3.64	1.31
PARANOA (COOPA-DF)	14.79	68.47	5.1	2.24
PARATI	19.05	80.07	3.69	1.48
PARATY	19.28	81.63	3.64	1.5
PARAUNA	15.69	61.62	4.04	1.13
PARINTINS	23.0	77.0	2.44	0.89
PARNAIBA	23.11	78.04	6.65	3.03
PARQUE ESTADUAL CHANDLESS	21.36	81.1	2.64	0.58

Table C.8 – Mean values for weather stations across the entire data collection period, part (h)

Station	Temperature - Dew Point (°C)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
PATOS DE MINAS	14.11	64.22	5.0	2.08
PATROCINIO	14.58	72.06	3.66	1.15
PAULISTANA	15.8	51.14	7.4	3.22
PEDRO AFONSO	20.29	69.5	2.98	0.7
PEDRO GOMES	19.27	76.87	3.43	1.35
PEF ESTIRAO DO EQUADOR	23.54	82.05	2.1	0.7
PEF IPIRANGA	24.2	68.99	2.25	1.01
PEF YAUARETE	nan	nan	2.93	0.98
PEIXE	18.73	66.11	3.44	1.26
PETROLINA	15.85	52.49	7.32	3.38
PIATA	13.04	68.65	6.56	2.59
PICO DO COUTO	11.98	83.74	8.73	4.57
PICOS	17.44	54.69	5.9	2.22
PILAO ARCADO	16.62	52.87	6.53	2.82
PIRACICABA	15.51	70.2	4.85	1.84
PIRANHAS	19.37	66.49	6.65	2.65
PIRAPORA	15.83	64.22	3.95	1.31
PIRES DO RIO	15.28	65.4	3.28	0.82
PIRIPIRI	20.15	66.66	4.72	1.88
PIUM	21.44	79.04	3.52	1.28
PLACAS	21.02	74.5	2.97	0.95
PLANALTO	15.52	70.71	4.86	2.36
POCO VERDE	19.3	75.66	5.54	2.32
POMPEU	16.24	66.67	4.84	1.87
PONTA PORA	15.44	70.03	5.11	2.15
PONTES E LACERDA	18.71	70.08	3.83	1.16
PORANGATU	18.73	65.13	4.21	1.23
PORTO ALEGRE	15.4	76.88	4.75	1.48
PORTO ALEGRE - JARDIM BOTANICO	15.59	77.7	4.69	1.42
PORTO ALEGRE- BELEM NOVO	16.5	77.63	5.84	2.67
PORTO DE MOZ	24.05	85.03	3.38	1.06
PORTO ESTRELA	20.49	73.44	3.22	0.93
PORTO GRANDE	20.18	70.85	4.2	1.77
PORTO MURTINHO	17.6	65.54	4.63	1.57
PORTO SEGURO	20.34	82.67	4.17	1.44
PORTO VELHO	21.92	78.22	3.99	1.36
PORTO WALTER	22.18	80.29	3.5	1.44
POSSE	14.21	56.02	5.62	2.21
PRADOPOLIS	15.05	67.62	4.55	2.01
PREGUICAS	22.64	75.43	6.15	2.72
PRES. KENNEDY	18.95	75.29	6.64	3.77
PRESIDENTE FIGUEIREDO	22.76	79.64	2.78	0.71
PRESIDENTE KENNEDY	19.17	72.94	6.64	3.61
PRESIDENTE PRUDENTE	15.52	63.04	4.86	1.48
PRIMAVERA DO LESTE	15.19	55.28	5.42	2.5
Paty do Alferes - Avelar	16.56	77.0	3.55	1.23
QUARAI	13.14	72.39	5.79	2.42
QUEIMADAS	17.7	63.2	7.03	3.3
QUERENCIA	19.04	70.81	3.78	1.23
QUIXADA	22.68	75.62	5.7	2.27
QUIXERAMOBIM	18.26	59.45	6.17	2.16
RANCHARIA	16.03	73.0	4.07	1.48
RANCHO QUEIMADO	14.04	88.78	5.12	2.02
RECIFE	21.45	78.02	5.14	1.74
REDENCAO	21.31	67.02	3.84	1.49
REGISTRO	20.89	85.34	4.91	2.18

Table C.9 – Mean values for weather stations across the entire data collection period, part (i)

Station	Temperature - Dew Point ($°C$)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
RIO BRANCO	20.99	73.5	3.68	1.17
RIO BRILHANTE	16.22	67.46	3.91	1.34
RIO CLARO	16.79	79.66	4.2	1.62
RIO DE JANEIRO - FORTE DE COPACABANA	19.74	79.27	5.51	2.46
RIO DE JANEIRO - JACAREPAGUA	18.29	76.71	3.42	0.78
RIO DE JANEIRO - VILA MILITAR	19.0	75.49	3.72	1.17
RIO DE JANEIRO-MARAMBAIA	18.94	71.9	6.19	3.49
RIO DO CAMPO	15.11	81.78	3.31	1.0
RIO GRANDE	14.21	76.39	6.88	3.24
RIO NEGRINHO	14.44	86.79	3.7	1.16
RIO PARDO	14.93	77.08	5.41	2.49
RIO PARDO DE MINAS	15.13	71.01	4.8	2.06
RIO SONO	19.75	70.48	4.26	1.73
RIO URUBU	22.99	84.18	2.42	0.59
RIO VERDE	14.97	63.49	4.9	1.98
RONDON DO PARA	20.56	74.64	3.83	1.32
RONDONOPOLIS	18.18	66.73	3.94	1.33
ROSARIO OESTE	18.54	64.56	3.18	1.04
S. G. DA CACHOEIRA	22.58	81.17	1.98	0.62
S.J. DO RIO CLARO	19.28	71.16	2.81	0.9
SACRAMENTO	14.02	63.82	5.39	1.93
SALGUEIRO	17.64	58.78	6.07	2.32
SALINAS	15.59	62.98	3.89	1.26
SALINOPOLIS	22.32	74.73	6.89	2.03
SALTO DO CEU	18.78	71.14	4.28	1.35
SALVADOR	21.49	77.94	5.37	1.42
SALVADOR (RADIO FAROL)	22.66	85.83	4.56	1.34
SANTA CRUZ	18.53	63.63	6.68	2.69
SANTA FE DO ARAGUAIA	21.73	77.97	4.24	1.86
SANTA ISABEL DO RIO NEGRO	22.6	73.08	0.88	0.08
SANTA MARIA	15.2	78.61	5.06	2.04
SANTA MARIA DAS BARREIRAS	20.67	69.64	4.72	2.1
SANTA MARIA MADALENA	17.38	82.9	3.01	0.57
SANTA RITA DE CASSIA	16.16	56.91	4.48	1.8
SANTA RITA DO PARDO	16.76	70.57	5.27	2.63
SANTA ROSA	14.89	73.58	4.88	1.81
SANTA ROSA DO TOCANTINS	18.11	64.94	4.76	2.2
SANTA TERESA	15.42	82.38	5.36	1.85
SANTANA DO ARAGUAIA	19.89	68.28	3.4	1.19
SANTANA DO LIVRAMENTO	12.22	71.47	5.72	2.76
SANTAREM	22.75	77.87	4.07	1.41
SANTIAGO	13.68	73.6	6.05	2.33
SANTO ANTONIO DO LESTE	16.91	65.49	5.0	2.13
SANTO AUGUSTO	13.15	68.52	5.44	1.99
SAO BENTO	23.93	72.77	2.63	0.81
SAO BORJA	14.83	70.29	5.08	1.99
SAO CARLOS	14.25	67.86	4.69	1.58
SAO FELIX DO ARAGUAIA	16.07	52.49	4.33	1.32
SAO FELIX DO XINGU	21.45	77.14	3.92	1.66
SAO GABRIEL	14.34	74.08	5.31	2.07
SAO GABRIEL DO OESTE	16.38	67.05	5.17	1.61
SAO GONCALO	17.47	58.62	4.92	1.74
SAO JOAO DEL REI	14.39	74.06	5.16	2.35
SAO JOAO DO PIAUI	16.11	50.6	5.38	2.04
SAO JOAQUIM	10.37	80.24	5.9	2.27
SAO JOSE DO XINGU	20.54	71.81	3.76	1.6

Table C.10 – Mean values for weather stations across the entire data collection period, part (j)

Station	Temperature - Dew Point ($°C$)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
SAO LUIZ DO PARAITINGA	15.57	81.41	4.05	1.17
SAO LUIZ GONZAGA	14.27	69.57	6.22	2.75
SAO MATEUS	20.75	82.73	5.13	2.44
SAO MATEUS DO SUL	13.85	82.73	2.65	0.56
SAO MIGUEL ARCANJO	15.55	79.26	5.45	2.22
SAO MIGUEL DO ARAGUAIA	19.58	65.17	4.62	1.97
SAO MIGUEL DO OESTE	14.95	73.5	5.88	2.76
SAO PAULO - INTERLAGOS	15.91	82.14	5.2	1.98
SAO PAULO - MIRANTE	14.04	69.36	5.22	1.84
SAO PEDRO DO PIAUI	18.3	62.28	4.53	1.79
SAO RAIMUNDO NONATO	16.17	54.57	5.28	1.94
SAO ROMAO	16.57	64.61	3.92	1.33
SAO SEBASTIAO	19.8	72.2	7.52	4.13
SAO SEBASTIAO DO PARAISO	15.53	71.0	4.54	1.83
SAO SIMAO	16.23	64.81	4.63	1.85
SAO TOME	19.56	76.72	6.84	3.5
SAO VICENTE DO SUL	14.22	75.01	6.26	3.24
SAPEZAL	18.42	73.46	5.42	2.76
SAQUAREMA	19.22	77.3	4.78	1.87
SAQUAREMA - SAMPAIO CORREIA	19.02	77.59	4.47	1.7
SELVIRIA	18.52	75.65	6.32	2.81
SENHOR DO BONFIM	17.06	68.16	6.08	2.82
SERAFINA CORREA	13.32	78.25	3.7	1.25
SEROPEDICA-ECOLOGIA AGRICOLA	18.19	72.58	4.72	1.71
SERRA DOS AIMORES	18.71	76.46	5.61	2.8
SERRA DOS CARAJAS	18.9	72.26	6.09	2.83
SERRA NOVA DOURADA	17.62	58.64	5.18	2.65
SERRA TALHADA	15.35	55.65	6.57	2.49
SERRINHA	19.06	74.98	6.1	2.71
SETE LAGOAS	14.86	68.39	4.63	1.98
SETE QUEDAS	16.44	69.81	5.08	1.61
SIDROLANDIA	16.34	66.91	5.08	2.01
SILVA JARDIM	19.08	79.49	3.49	1.29
SILVANIA	14.87	67.66	3.53	1.11
SINOP	19.25	71.73	4.28	1.71
SOBRAL	20.11	67.25	4.55	1.57
SOLEDADE	13.1	76.49	7.5	3.89
SONORA	17.42	67.57	5.98	3.3
SOROCABA	15.13	73.5	4.66	2.06
SORRISO	18.56	66.4	3.76	1.41
SOURE	23.04	77.81	6.17	2.17
SURUBIM	18.53	70.69	6.54	3.17
Santa Vitoria do Palmar - Barra do Chui	13.75	80.53	7.56	4.55
TANGARA DA SERRA	18.39	70.77	4.38	2.02
TAUA	15.68	52.22	6.06	2.48
TAUBATE	15.82	74.58	4.19	1.91
TEOFILO OTONI	17.09	71.61	4.18	1.65
TERESINA	20.62	67.98	3.75	1.2
TERESOPOLIS	14.99	83.51	3.39	0.72
TERESOPOLIS-PARQUE NACIONAL	15.25	83.93	3.27	0.79
TEUTONIA	15.45	76.88	4.56	1.92
TIANGUA	18.74	77.35	7.56	3.8
TIMOTEO	16.74	71.6	3.37	0.87
TOME ACU	22.43	77.47	3.57	1.09
TORRES	15.82	78.98	5.83	2.81
TRAMANDAI	15.83	76.19	7.07	3.72

Table C.11 – Mean values for weather stations across the entire data collection period, part (k)

Station	Temperature - Dew Point (°C)	Relative Humidity (%)	Gust Wind (m/s)	Wind Velocity (m/s)
TUCURUI	22.38	77.09	3.02	0.6
TUPA	16.05	65.77	5.86	2.83
TUPANCIRETA	13.69	75.45	5.73	2.17
TURIACU	23.21	80.82	5.73	2.3
UAUA	17.38	66.39	7.25	3.61
UBERABA	15.59	68.87	5.66	2.28
UBERLANDIA	13.95	59.15	5.06	1.72
ULIANOPOLIS	nan	nan	nan	nan
UNA	20.82	83.77	3.84	1.24
UNAI	15.57	64.29	4.1	1.54
URUCARA	23.32	80.28	3.1	0.83
URUCUI	17.41	61.29	3.09	0.74
URUGUAIANA	14.07	71.95	6.16	2.96
URUSSANGA	15.73	77.61	3.13	0.89
VACARIA	12.02	81.36	6.45	3.22
VALENCA	19.09	80.0	4.16	1.59
VALENCA DO PIAUI	17.65	59.89	5.15	1.95
VALPARAISO	16.08	65.65	5.1	1.95
VARGINHA	14.06	70.67	4.5	1.73
VENDA NOVA DO IMIGRANTE	15.63	77.73	4.86	1.19
VENTANIA	14.11	79.04	6.57	3.36
VICOSA	15.96	79.99	2.96	0.81
VILA BELA DA SANTISSIMA TRINDADE	19.77	72.63	3.18	0.99
VILA MILITAR	17.82	70.87	3.95	1.38
VILA VELHA	20.07	80.47	6.06	2.96
VILHENA	18.12	69.48	4.98	1.9
VITORIA	19.45	74.81	5.03	1.71
VITORIA DA CONQUISTA	16.85	81.11	5.85	2.35
VOTUPORANGA	16.11	62.89	4.42	1.38
XANXERE	13.64	74.83	6.17	2.5
XEREM	18.71	80.28	3.01	0.82
XINGUARA	21.54	76.98	3.35	1.33
ZE DOCA	22.92	76.85	5.11	1.8

APPENDIX D – Appendix D

We presented below the collected data from our colaborative farmer in the city of Tatuí, state of São Paulo.

Table D.1 – Parameters gathered from a grain farmer in São Paulo State, part (a)

ID	Crop name	Family ID	Family	Average yield per hectare (sc/ha)	Cost per area (sc/ha)	Cover crop costs ($)	Sale price per unit	Usual planting dates - Begin (Week)	Usual planting dates - End (Week)	Init	Dev	Mid	Late	Cycle (Weeks)
1	Soybean	1	Legume	78.0	34.9	0.0	160	42	48	3	5	7	4	19
2	Summer corn	2	Grass	157.0	67.95	0.0	71	35	52	4	7	8	4	23
3	Winter corn	2	Grass	103.0	67.95	0.0	50	1	10	4	7	8	6	25
4	Sorghum	2	Grass	43.38	49.73	0.0	60	1	12	3	5	7	5	20
5	Wheat	2	Grass	37.87	26.68	0.0	66	10	20	3	9	10	4	26
6	Black Oats	2	Grass	0.0	0.0	150.0	0	9	22	0	0	0	0	14
7	Fodder Turnip	3	Mustard	0.0	0.0	142.5	0	9	39	0	0	0	0	12
8	Hairy vetch	1	Legume	0.0	0.0	436.0	0	9	26	0	0	0	0	10

Table D.2 – Parameters gathered from a grain farmer in São Paulo State, part (b)

ID	Crop name	%N Total	% P2O5 sol CNA+Água	%k2O SOL. WATER	S	B	Mn	Zn	Ca	Mg	COT	Dry Matter (kg/ha/ano)	Minimum Dry Matter (kg/ha/ano)	Total N Source (kg/ha)	Minimum Total N Source (kg/ha)	Nitrogen Scavenger	Soil Builder	Erosion Fighter	Weed Fighter	Lasting Residue	Commercial Crops
1	Soybean	20.0	92.5	90.67	19.39	0.25	0.5	0.5	0	0	0.0	0	0	0	0	0	0	0	0	0	Y
2	Summer corn	158.0	59.4	23.1	0.0	0.0	0.0	0.0	0	0	66.0	0	0	0	0	0	0	0	0	0	Y
3	Winter corn	158.0	59.4	23.1	0.0	0.0	0.0	0.0	0	0	66.0	0	0	0	0	0	0	0	0	0	Y
4	Sorghum	60.27	55.8	44.64	28.63	0.56	0.37	0.37	0	0	0.0	0	0	0	0	0	0	0	0	0	Y
5	Wheat	96.5	37.08	14.42	0.0	0.0	0.0	0.0	0	0	41.2	0	0	0	0	0	0	0	0	0	Y
6	Black Oats	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	0	0.0	2200-11000	2200	0	0	3	2	3	4	2	N
7	Fodder Turnip	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	0	0.0	2600-5600	2600	0	0	2	3	3	3	1	N
8	Hairy vetch	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	0	0.0	2600-5600	2600	100-220	100	0	1	3	2	1	N