GLOBAL STRONG SOLUTION OF THE EQUATIONS FOR THE MOTION OF A CHEMICAL ACTIVE FLUID

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ABSTRACT – By using the spectral Galerkin method, we prove a result on global existence in time of strong solutions for the motion of a chemical active fluid without assuming that the external forces decay with time. We also derive uniform in time estimates of the solution that are useful for obtaining error bounds for the approximate solutions.

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ABSTRACT. *By using the spectral Galerkin method, we prove a result on global existence in time of strong solutions for the motion of a chemical active fluid without assuming that the external forces decay with time. We also derive uniform in time estimates of the solution that are useful for obtaining error bounds for the approximate solutions.*

KEY WORDS: Chemical active fluid, global strong solutions, Galerkin method.

RESUMO: *SOLUÇÃO GLOBAL FORTE DAS EQUAÇÕES DO MOVIMENTO DE UM FLUIDO QUIMICAMENTE ATIVO. Usando o metodo de Galerkin espectral, provamos um resultado de existência global no tempo de soluções fortes para o movimento de um fluido quimicamente ativo sem supor que as forças externas decaem com o tempo. Também derivamos estimativas uniforme no tempo da solução que são utieis para obter limitações do erro para as soluções aproximadas.*

PALAVRAS CHAVE: Fluido quimicamente ativo, soluções globais fortes, metodo de Galerkin.

1. INTRODUCTION.

In this work we study global existence of strong solutions for the equations that describes the motion of a viscous-chemically-active fluid in a bounded domain $\Omega \subseteq I\!R^n, n = 2$ or 3, in the time interval $[0, T), 0 < T \leq +\infty$.

Let us denote by $u(t,x), p(t,x), \tilde{\theta}(t,x)$ and $\tilde{\psi}(t,x)$ the unknown velocity vector, the pressure, the temperature, and the degree of dissociation of the fluid at point x time t, respectively. Then the evolution equations in the Oberbeck-Boussinesq approximation are (to see Joseph [9]):

$$
\begin{aligned}
&\partial_t u + (u.\nabla)u - \nu\Delta u + \nabla p = j + (\tilde{\theta} + \tilde{\psi})g, \\
&\partial_t \tilde{\theta} + (u.\nabla)\tilde{\theta} - k_{\tilde{\theta}}\Delta\tilde{\theta} = f \\
&\partial_t \tilde{\psi} + (u.\nabla)\tilde{\psi} - k_{\tilde{\psi}}\Delta\tilde{\psi} = h \\
&div \;\; u = 0 \qquad (1.1)
\end{aligned}
$$

where $g(t,x), j(t,x), f(t,x)$ and $h(t,x)$ are source functions; $\nu > 0$ is the viscosity, the constants $k_{\tilde{\theta}}$ and $k_{\tilde{\psi}}$ are the thermal and solute diffusity, respectively.

On the boundary Γ, we assume that

$$u(t,x) = 0; \quad \tilde{\theta}(t,x) = \theta_1; \quad \tilde{\psi}(t,x) = \psi_1 \qquad (1.2)$$

where θ_1 and ψ_1 are known functions; and the initial conditions are expressed by

$$u(0,x) = u_0(x); \tilde{\theta}(0,x) = \tilde{\theta}_0(x); \tilde{\psi}(0,x) = \tilde{\psi}_0(x) \qquad (1.3)$$

where $u_0, \tilde{\theta}_0$ and $\tilde{\psi}_0$ are given functions on the variable $x \in \Omega$.

The expressions ∇, Δ and div, as usual, denote the gradiente, Laplace, and divergence operators, respectively; the i^{th} componente of $(u \cdot \nabla)u$ is given by $[(u \cdot \nabla)u]_i =$

$\sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j}$; $(u \cdot \nabla)\phi = \sum_{j=1}^{n} u_j \frac{\partial \phi}{\partial x_j}$, for $\phi = \widetilde{\theta}$ or $\widetilde{\psi}$.

When chemical reactions are absent ($\widetilde{\psi} \equiv 0$), the problem (1.1) - (1.3) is equivalent to the classical Boussinesq's problem (or Bernad's problem), which has been investigated by several authors; see for instance Hishida [8], Korenev [10], Morimoto [12], Shinbrot, Kotorynski [16] and references therein. Concerning the system (1.1) - (1.3), Gil's [6] studied the stationary model, Belov and Kapitonov [2], the stability of the solutions of the system (1.1) - (1.3) with different boundary conditions. They used linearization and fixed point arguments. The more construtive Spectral Galerkin method was used by Rojas-Medar and Lorca [13], [14] to obtain global in time of the weak solution for $n \geq 2$ and local in time of the strong solutions for $n = 2$ or 3. Also, regularity conditions for $t > 0$ were studied [14].

We observe that all known results on global existence of strong solutions for the system (1.1) - (1.3), as well as in the Boussinesq equations (see, for instance Hishida [8]) require some sort of decay in time of the associated external forces.

However, in the case of the classical Navier-Stokes equations ($\widetilde{\theta} \equiv \widetilde{\psi} \equiv 0$), this kind of decay requirement is not necessary (see, for instance, Heywood and Rannacher [7]). Therefore, one should be able to prove global existence without this decay condition in the case of equations (1.1) - (1.3).

This is indeed true, and we shall prove it under certain regularity assumptions on the initial data and external forces. This proof will be the main result of the present article. In particular, for the three - dimensional case, we will require smallness of the H^1-norm of the initial data as well as of the $L^\infty(0,\infty; L^2(\Omega))$ - norm of the forces. The two-dimensional problem is uniquely solvable for all $t \geq 0$ without any smallness restrictions.

Thus we rearch basically the same level of knowledge as the one in the case of the classical Navier-Stokes equations.

Also, we present a sequence of estimates for the strong solutions of (1.1) - (1.3) and their spectral approximations. These estimates are relevant because they are used in an essencial way to obtain uniform in time error bounds for the spectral approximations of (1.1) - (1.3). This will be the matter of another publication [15]. We observe that, as it is usual we will denote by C a generic positive constant depending only on Ω and the data of the problem.

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2. PRELIMINAIRES.

Let $\Omega \subseteq I\!R^n, n = 2$ or 3, be a bounded domain with boundary Γ of class $C^{1,1}$. Let $H^m(\Omega)$ be the Sobolev spaces on Ω with norm $||\cdot||_m$, $(\cdot,\cdot)$ denote the usual inner product in $L^2(\Omega)$ and $||\cdot||$ denote the L^2 norm on Ω . By $H_0^1(\Omega)$ is the completion of $C_0^\infty(\Omega)$ under the norm $||\cdot||_1$, the L^p norm on Ω is denoted by $||\cdot||_{L^p}, 1 \leq p \leq \infty$. If B is a Banach space, we denote by $L^q(0,T;B)$ the Banach space of the B-valued functions defined in the interval $(0,T)$ that are L^q-integrable in the sense of Bochner. The functions in this paper are either $I\!R$ or $I\!R^n$-valued and we will not distinguish them is our notations.

We shall consider the following spaces of divergence free functions

$$\begin{aligned} C_{0,\sigma}^\infty(\Omega) &= \{v \in C_0^\infty(\Omega) \mid div\ v = 0 \quad \text{in} \quad \Omega\} \\ H &= \text{closure of } C_{0,\sigma}^\infty \quad \text{in} \quad L^2(\Omega), \\ V &= \text{closure of } C_{0,\sigma}^\infty(\Omega) \text{ in } H^1(\Omega). \end{aligned}$$

We observe that the space V is characterized by

$$V = \{u \in H_0^1(\Omega), \ \ div\ u = 0\}.$$

The space $L^2(\Omega)$ has the descomposition $L^2(\Omega) = H \oplus H^\perp$, where $H^\perp = \{\phi \in L^2(\Omega) /$ exist $p \in H^1(\Omega)$ with $\phi = \nabla p\}$; it means that for every $v \in L^2(\Omega)$ there exists $v_1, v_2 \in L^2(\Omega)$ such that

$$v = v_1 + v_2$$

with $v \in H$ and $v_2 \in H^\perp$, or any may, for every $v \in L^2(\Omega)$ there exists $v_1 \in L^2(\Omega)$ and $p \in H^1(\Omega)$ such that the vector field v_1 is solenoidal (div $v_1 = 0$) and parallel with the boundary ($v_1.n = 0$ on Γ) and as same as it holds

$$v = v_1 + \nabla p$$

where p is defined as the solutions of

$$\begin{aligned} \Delta p &= 0 \quad \text{in} \quad \Omega, \\ \frac{\partial p}{\partial n} &= v.n \end{aligned}$$

(to see, Constantin and Foias [3] or Temam [17]).

We define the mapping $P : L^2(\Omega) \to H$ by $Pv = v_1$. Then the operator $A : H \to H$ given by $A = -P\Delta$ with domain $D(A) = H^2(\Omega) \cap V$ is called the Stokes operator. It is well known that the operator A is positive definite, self-adjoint operator and is characterized by the relation

$$(Aw, v) = (\nabla w, \nabla v) \quad \text{for all} \quad w \in D(A), v \in V.$$

The operator A^{-1} is linear continuous from H into $D(A)$, and since the injection of $D(A)$ is H is compact, A^{-1} can be considered as a compact operator in H. As an operator in H it is also self-adjoint. By a well know theorem of Hilbert spaces, there exists a sequence of positive numbers $\mu_j > 0, \mu_{j+1} \leq \mu_j$ and an orthonormal basis of $H, \{w_j\}_{j=1}^{\infty}$ such that $A^{-1} w_j = \mu_j w_j$.

We denote $\lambda_j = \mu_j^{-1}$. Since A^{-1} has range in $D(A)$ we obtain that

$$Aw_j = \lambda_j w_j \quad , \quad w_j \in D(A)$$

$0 < \lambda_1 < \cdots \leq \lambda_j \leq \lambda_{j+1} \leq \cdots \lim_{j\to\infty} \lambda_j = +\infty$ and $\{w_j\}_{j=1}^{\infty}$ are an orthonormal basis of H.

Therefore, $\{w_j|\sqrt{\lambda_j}\}_{j=1}^{\infty}$ and $\{w_j|\lambda_j\}_{j=1}^{\infty}$ form an orthonormal basis in V (doted of inner product $(\nabla u, \nabla v), u, v \in V$) and $H^2(\Omega) \cap V$ (doted of inner product $(Au, Av), u, v \in D(A)$), respectively. We denote by $V_k =$ span$[w^1, \ldots, w^k]$.

We observe that for the regularity properties of the Stokes operator, it is usually assumed that Ω is of class C^2; this being in order to use Cattabriga's results [4]. We use instead the stronger results of Amrouche and Girault [1] which implies, in particular, that when $Au \in L^2(\Omega)$ then $u \in H^2(\Omega)$ and $||u||_{H^2}$ and $||Au||$ are equivalent norms when Ω is of class $C^{1,1}$.

Similar considerations are true for the Laplacian operator $B = -\Delta : L^2(\Omega) \to L^2(\Omega)$ with the Dirichelet boundary conditions with domain $D(B) = H^2(\Omega) \cap H_0^1(\Omega)$ and we will denote $\varphi^k(x), \gamma_k$ by the eigenfunctions and eigenvalues of B, respectively. We denote by $H_k =$ span $[\varphi^1, \ldots, \varphi^k]$.

Before of to define strong solution, we transform to the problem (1.1) - (1.3) into another problem with homogeneous boundary value. In order to do it, we consider extensions θ_2 and ψ_2 of the functions θ_1 and ψ_1, respectively, such that

$$\begin{array}{lclcl} \partial_t\theta_2 - \Delta\theta_2 = 0 & ; & \partial_t\psi_2 - \Delta\psi_2 = 0 & \text{in} & (0,\infty)\times\Omega \\ \theta_2 = \theta_1 & ; & \psi_2 = \psi_1 & \text{on} & (0,\infty)\times\Gamma \\ \theta_2(0) \in H^1(\Omega) & ; & \psi_2(0) \in H^1(\Omega) & & \end{array} \tag{2.1}$$

where $\theta_2(0) = \theta_1(0)$ on Γ and $\psi_2(0) = \psi_1(0)$ on Γ. We know that problems (2.1) are uniquely solvable for suitable conditions for θ_1 and ψ_1 (see [11], [14] and references there in) with continuous dependence on the initial datas.

Now, we can transform the equations (1.1) - (1.3) by introduction the new variables $\theta = \tilde{\theta} - \theta_2$ and $\psi = \tilde{\psi} - \psi_2$, obtaining

$$\left.\begin{array}{lcl} \partial_t u + (u.\nabla)u - \Delta u + \nabla p & = & (\theta + \psi)g + g_1 \\ \partial_t \theta + (u.\nabla)\theta - \Delta\theta & = & f - u.\nabla\theta_2 \\ \partial_t \psi + (u.\nabla)\psi - \Delta\psi & = & h - u.\nabla\psi_2 \\ div\ u = 0 & \text{in} & (0,T) \times \Omega \end{array}\right\} \qquad (2.2)$$

$$u = 0 \quad ; \quad \theta = 0 \quad ; \quad \psi = 0 \quad \text{on} \quad (0,T) \times \Gamma \qquad (2.3)$$

$$u(0) = u_0 \quad ; \quad \theta(0) = \theta_0 \equiv \tilde{\theta}_0 - \theta_2(0) \quad ; \quad \psi(0) = \psi_0 \equiv \tilde{\psi}_0 - \psi_2(0) \qquad (2.4)$$

where $g_1 = (\theta_2 + \psi_2)g + j$. Here, without loosing generality, we have scaled the variables in order to the viscosity and coeficientes of diffusity to be one.

We observe that the problem (2.2) - (2.4) is equivalent to the problem (1.1) - (1.3); with this in mind, it is enough to study the problem (2.2) - (2.4).

Now, using the properties of P, we can reformulate problem (2.2) - (2.4) as follows find $(u, \theta, \psi) \in C([0,T]; V \times (H_0^1(\Omega))^2 \cap L^2(0,T; D(A) \times (D(B))^2), (\partial_t u, \partial_t \theta, \partial_t \psi) \in L^2(0,T; H \times (L^2(\Omega))^2) \quad (0 < T \leq +\infty)$ such that

$$\left.\begin{array}{l} (\partial_t u, v) + (u\nabla u, v) + (Au, v) = ((\theta + \psi)g, v) + (g_1, v), \ \ \forall v \in V \\ (\partial_t \theta, \varsigma) + (u\nabla\theta, \varsigma) + (B\theta, \varsigma) = (f, \varsigma) - (u\nabla\theta_2, \varsigma), \ \ \forall\varsigma \in H_0^1(\Omega) \\ (\partial_t \psi, \phi) + (u\nabla\psi, \phi) + (B\psi, \phi) = (h, \phi) - (u\nabla\psi_2, \phi), \ \ \forall\phi \in H_0^1(\Omega) \end{array}\right\} \qquad (2.5)$$

$$(u(0), \theta(0), \psi(0)) = (u_0, \theta_0, \psi_0). \qquad (2.6)$$

The above functions (u, θ, ψ) are called *strong solution* for the system (2.2) - (2.4).

The spectral Galerkin approximations for (u, θ, ψ) are defined for each $k \in \mathbb{N}$ as the solution $(u^k, \theta^k, \psi^k) \in C^2([0,T); V_k \times (H_k)^2) \cap C^1([0,T) \times \overline{\Omega})$ of

$$\left.\begin{array}{l} (\partial_t u^k, v) + (u^k\nabla u^k, v) + (Au^k, v) = ((\theta^k + \psi^k)g, v) + (g_1, v), \ \ \forall v \in V_k \\ (\partial_t \theta^k, \varsigma) + (u^k\nabla\theta^k, \varsigma) + (B\theta^k, \varsigma) = (f, \varsigma) - (u^k\nabla\theta_2, \varsigma), \ \ \forall\varsigma \in H_k \\ (\partial_t \psi^k, \phi) + (u^k\nabla\psi^k, \phi) + (B\psi^k, \phi) = (h, \phi) - (u^k\nabla\psi_2, \phi), \ \ \forall\phi \in H_k \\ u^k(0,x), = u_0^k(x), \theta^k(0,x) = \theta_0^k(x), \psi^k(0,x) = \psi_0^k, (x). \end{array}\right\} \qquad (2.7)$$

Here, u_0^k are the projections of u_0 on V_k, analogously, θ_0^k and ψ_0^k are the projections of θ_0 and ψ_0 on H_k, respectively.

By using these approximations, Rojas-Medar and Lorca [14] proved a local in time existence theorem for (2.2) - (2.4). Her results are the following.

Theorem 2.1 Let Ω be a bounded domain in $I\!R^n (n = 2$ or $3)$ with boundary Γ of class $C^{1,1}$. Suppose that

$$(\theta_2, \psi_2) \in L^\infty(0,T;(H^1(\Omega))^2); (u_0, \theta_0, \psi_0) \in V \times (H_0^1(\Omega))^2;$$
$$j \in L^2(0,T;L^2(\Omega)); g \in L^2(0,T;L^3(\Omega)); f \in L^2(0,T;L^2(\Omega))$$

and $h \in L^2(0,T;L^2(\Omega))$. Then, there exists $T_1 > 0$ with $T_1 \leq T$ such that the problem (2.2) - (2.4) has a unique strong solution in the interval $[0, T_1)$. ■

Also in [14] is proved

Theorem 2.2, Under the hypothesis of the above theorem and suppose the forces in theorem 2.1 satisfies $\int_0^T(||\partial_t g||^2 + ||\partial_t j||^2 + ||\partial_t f||^2 + ||\partial_t h||^2)ds < +\infty$ and the initial data satisfies $\int_0^T ||\partial_t \theta_2||^2 + ||\partial_t \psi_2||^2)ds < +\infty, u_0 \in D(A), \theta_0, \psi_0 \in D(B)$. Then the solutions (u, θ, ψ) obtained in Theorem 2.1 belongs to $C([0,T_1]; D(A) \times (D(B))^2)$.■

3. GLOBAL EXISTENCE IN THE THREE - DIMENSIONAL CASE.

We have the following result.

Theorem 3.1. Let $\Omega \subseteq I\!R^3$ with boundary Γ of class $C^{1,1}$. Suppose that $(\theta_2, \psi_2) \in L^\infty(0,\infty;(H^1(\Omega))^2)); (u_0, \theta_0, \psi_0) \in V \times (H_0^1(\Omega))^2$ and $g \in$

$L^\infty(0,\infty;L^3(\Omega)), j \in L^\infty(0,\infty;L^2(\Omega)), f \in L^\infty(0,\infty;L^2(\Omega)), h \in L^\infty(0,\infty;L^2(\Omega))$. Then, if $||u_0||_1, ||\theta_0||_1, ||\psi_0||_1, ||\theta_2||_{L^\infty(0,\infty;H^1(\Omega))}, ||\psi_2||_{L^\infty(0,\infty;H^1(\Omega))}, ||f||_{L^\infty(0,\infty;L^2(\Omega))}$, $||h||_{L^\infty(0,\infty;H^2(\Omega))}, ||j||_{L^\infty(0,\infty;L^2(\Omega))}$, and $||g||_{L^\infty(0,\infty;L^3(\Omega))}$ are small enough, the solution described in theorem 2.1 exists globally in time. Moreover, we have

$$\sup_{t\geq 0} ||\nabla u(t)||,\ ||\nabla\theta(t)||,\ ||\nabla\psi(t)||\} < +\infty; \tag{3.1}$$

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}(||Au(s)||^2 + ||B\theta(s)||^2 + ||B\psi(s)||^2)ds\} < +\infty \tag{3.2}$$

and

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}(||u_t(s)||^2 + ||\theta_t(s)||^2 + ||\psi_t(s)||^2)ds\} < +\infty \tag{3.3}$$

for all $\alpha > 0$. Also, the same kind of estimates hold unifomly in k for the Galerkin approximations.

Proof. We will combine arguments used by Rojas-Medar and Lorca [14] with of a variant of arguments used by Boldrini and Rojas-Medar [3] and Heywood and Rannacher [7]. The crucial estimate will be one for $||\nabla u^k(t)||^2 + ||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2$; and to obtain it, we proceed as follows: working as the proof of the local existence theorem (Theorem 2.1), for any $t \in (0,\infty)$, we have the estimates

$$\begin{aligned}\frac{d}{dt}||\nabla u^k(t)||^2 &+ ||Au^k(t)||^2 \leq C||\nabla u^k(t)||^6 + C(||\nabla\theta^k(t)||^2 \\ &+ ||\nabla\psi^k(t)||^2)||g||_{L^3}^2 + C||g_1(t)||^2;\end{aligned} \tag{3.4}$$

$$\frac{d}{dt}||\nabla\theta^k(t)||^2 + ||B\theta^k(t)||^2 \leq C||\nabla u^k(t)||^4||\nabla\theta^k(t)||^2 \tag{3.5}$$

$$+ \quad C||Au^k(t)||^2||\nabla\theta_2(t)||^2 + C||f(t)||^2;$$

$$\frac{d}{dt}||\nabla\psi^k(t)||^2 \quad + \quad ||B\psi^k(t)||^2 \leq C||\nabla u^k(t)||^4||\nabla\theta^k(t)||^2$$

$$+ \quad C||Au^k(t)||^2||\nabla\psi_2(t)||^2 + C||h(t)||^2. \tag{3.6}$$

Now, we put $R(t) = ||\nabla u^k(t)||^2 + ||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2$, the above inequalities imply for $R(t)$ the following differential inequality

$$\frac{d}{dt}R(t) \leq CR(t)^3 + CR(t)M - (1 - CM)(||Au^k(t)||^2 + ||B\theta^k(t)||^2 + ||B\psi^k(t)||^2) \tag{3.7}$$

where $M = \sup_{t\geq 0}\{||g(t)||^2_{L^3}; ||g_1(t)||^2; ||f(t)||^2; ||h(t)||^2; ||\nabla\theta_2(t)||^2; ||\nabla\psi_2(t)||^2\}$. Now, we observe that $\lambda_1||\nabla u^k(t)||^2 \leq ||Au^k(t)||^2$ and $\gamma_1||\nabla\phi(t)||^2 \leq ||B\phi(t)||^2$, so in the above inequality we obtain

$$\frac{d}{dt}R(t) \leq CR(t)^3 + CR(t)M + CM - C_1(1 - (M)R(t) \tag{3.8}$$

where $C_1 = \min(\lambda_1, \gamma_1)$.

Assume $M < \min\left\{\frac{1}{2C}, 1, \left(\frac{C_1}{6C}\right)^{3/2}\right\}$. We will show by contradiction that $R(t) \leq M^{1/3}$ for all $t \geq 0$ when $||\nabla u_0||^2 + ||\nabla\theta_0||^2 + ||\nabla\psi_0||^2 < M^{1/3}$.

In fact, suppose the opposite, then, there exist $T_2 > 0$ and $\varepsilon > 0$ such that $R(T_2) = M^{1/3}$ and $R(t) > M^{1/3}$ for $t \in (T_2, T_2 + \varepsilon)$. Then, due to our choice of M and estimate (3.8), we have

$$\begin{aligned} \frac{d}{dt}R(T_2) &\leq CM + CM^{4/3} + CM - \frac{C_1}{2}M^{1/3} \\ &< CM - \frac{C_1}{2}M^{1/3} = M^{1/3}(3CM^{2/3} - \frac{C_1}{2}) < 0 \end{aligned}$$

which is a contradiction. Thus, we have

$$R(t) \leq M^{1/3} \quad \text{for all} \quad t \geq 0.$$

From the estimate (3.7) we find

$$\frac{d}{dt}R(t) + \frac{1}{2}(||Au^k(t)||^2 + ||B\theta^k(t)||^2 + ||B\psi^k(t)||^2)$$

$$\leq CR^3(t) + CR(t)M + CM. \tag{3.9}$$

Multiplying the above inequality by $e^{\alpha t}$ and integrating the result, we have

$$e^{\alpha t}R(t) + \frac{1}{2}\int_0^t e^{\alpha s}(||Au^k||^2 + ||B\theta^k||^2 + ||B\psi^k||^2)ds$$
$$\leq C\int_0^t e^{\alpha s}(R^3 + RM + M + \alpha R)ds + R(0)$$

and so, from this, and the hypothesis of the initial data, we conclude the estimate (3.2). In continuation, we observe that

$$\begin{aligned} ||\partial_t u^k(t)||^2 &\leq C||\nabla u^k(t)||^6 + C(||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2||g(t)||^2 \\ &+ C||g_1(t)||^2 + ||Au^k(t)||^2; \end{aligned} \tag{3.10}$$

$$\begin{aligned} ||\partial_t \theta^k(t)||^2 &\leq C||\nabla u^k(t)||^2||\nabla\theta^k(t)||^2 + C||Au^k(t)||^2||\nabla\theta_2(\theta)||^2 \\ &+ C||f(t)||^2 + ||B\theta^k(t)||^2; \end{aligned} \tag{3.11}$$

$$\begin{aligned} ||\partial_t \psi^k(t)||^2 &\leq C||\nabla u^k(t)||^2||\nabla\theta^k(t)||^2 + C||Au^k(t)||^2||\nabla\psi_2(t)||^2 \\ &+ C||h(t)||^2 + ||B\psi^k(t)||^2. \end{aligned} \tag{3.12}$$

Thus, by using the above estimates, we conclud the estimate (3.3) for all $\alpha > 0$.

Hence, by standard one methods [17], these estimates enable us to assert that a solution exists. The uniqueness is proved as usual. ■

Corollary 3.2. The assumptions are those of Theorem 3.1, and we assume moreover that $g \in L^2(0,\infty;L^3(\Omega))$, $j \in L^2(0,\infty;L^2(\Omega))$ and $f,h \in L^2(0,\infty;L^2(\Omega))$.

Then, estimates (2.2) and (3.3) are valid for $\alpha = 0$.

Proof. We have the energy equalities,

$$||u(t)||^2 + \int_0^t ||\nabla u(t)||^2 = \int_0^t (g_1, u) + \int_0^t ((\theta+\psi)g, u) + ||u_0||^2;$$

$$||\theta(t)||^2 + \int_0^t ||\nabla \theta(t)||^2 = \int_0^t (f, \theta) - \int_0^t (u\nabla\theta_2, \theta) + ||\theta_0||^2;$$

$$||\psi(t)||^2 + \int_0^t ||\nabla \psi(t)||^2 = \int_0^t (h, \psi) - \int_0^t (u\nabla\psi_2, \psi) + ||\psi_0||^2.$$

By using the hypothesis and the estimates obtained in Theorem 3.1, we deduce that

$$\sup_{t\geq 0} \int_0^t (||\nabla u(s)||^2 + ||\nabla\theta(s)||^2 + ||\nabla\psi(s)||^2)ds < +\infty. \tag{3.13}$$

The differential inequalities (3.4) - (3.6), together with the estimates given in Theorem 3.1, implies

$$\begin{aligned}
&\frac{d}{dt}(||\nabla u^k||^2 + ||\nabla\theta^k||^2 + ||\nabla\psi^k||^2) + ||Au^k||^2 + ||B\theta^k||^2 + ||B\psi^k||^2 \\
\leq\ & CM||\nabla u^k||^2 + CM^{2/3}||g||^2_{L^3} + C||g_1(t)||^2 \\
+\ & CM^{4/3}||\nabla\theta^k||^2 + CM^{4/3}||\nabla\psi^k||^2 + C||f||^2 + C||h||^2 + C||Au^k||^2(||\nabla\psi_2||^2 + ||\nabla\theta_2||^2).
\end{aligned}$$

Since by hipothesis $||\nabla\psi_2||^2, ||\nabla\theta_2||$ are small enough, then we can conclude that

$$\begin{aligned}
&\int_0^t (||Au^k||^2 + ||B\theta^k||^2 + ||B\psi^k||^2)ds \\
\leq\ & CM\int_0^t ||\nabla u^k||^2 ds + CM^{2/3}\int_0^t ||g||_{L^3}^2 ds \\
+\ & C\int_0^t ||g_1||^2 ds + CM^{4/3}\int_0^t (||\nabla\theta^k||^2 + ||\nabla\psi^k||^2)ds \\
+\ & C\int_0^t (||f||^2 + ||h||^2)ds \\
\leq\ & C_1
\end{aligned}$$

for all $t \geq 0$, due to our hypothesis and the estimates (3.13).

Now, writer these estimates, together with the hypothesis, by using the inequalities (3.10) - (3.13) we obtain the estimates (3.3) for $\alpha = 0$, thus we conclude the result. This completes the proof of the corollary. ∎

Theorem 3.3. Suppose the forces in Theorem 3.1 satisfies $(\partial_t j, \partial_t g) \in L^\infty(0,T;(L^2(\Omega))^2); (\partial_t f, \partial_t h) \in L^\infty(0,T;(L^2(\Omega)^2), (\partial_t\theta_2, \partial_t\psi_2) \in L^\infty(0,\infty;(L^2(\Omega))^2)$ and the initial data $u_0 \in V \cap H^2(\Omega), \theta_0, \psi_0 \in H_0^1(\Omega) \cap H^2(\Omega)$. Then the solution obtained in Theorem 3.1 satisfies

$$\sup_{t\geq 0}\{||\partial_t u(t)||, ||\partial_t\theta(t)||, ||\partial_t\psi(t)||\} < +\infty; \tag{3.14}$$

$$\sup_{t\geq 0}\{||Au(t)||, ||B\theta(t)||, ||B\psi(t)||\} < +\infty; \tag{3.15}$$

and

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}(||\nabla\partial_t u(s)||^2 + ||\nabla\partial_t\theta(s)||^2 + ||\nabla\partial_t\psi(s)||^2)ds\} \tag{3.16}$$

for all $\alpha > 0$. ALso, the kind of estimates hold uniformly in k for the Galerkin approximations.

Proof. From the proof of Theorem 2.2, for any t, we have the estimates

$$\frac{1}{2}\frac{d}{dt}||\partial_t u^k||^2 + ||\nabla\partial_t u^k||^2 \leq C_\delta ||g||^2_{L^3}(||\partial_t\theta^k||^2 + ||\partial_t\psi^k||^2 + ||\partial_t\theta_2||^2 + ||\partial_t\psi_2||^2) + C_\delta||\partial_t g||^2(||\nabla\theta^k||^2 + ||\nabla\psi^k||^2 + ||\nabla\theta_2||^2 + ||\nabla\psi_2||^2) + C_\delta||\partial_t u^k||^2||u^k||^4_{L^6} + \delta||\nabla\partial_t u^k||^2 \quad (3.17)$$

$$\frac{1}{2}\frac{d}{dt}||\partial_t\theta^k||^2 + ||\nabla\partial_t\theta^k||^2 \leq C_\delta(||\theta_2||^4_{L^6} + ||\theta^k||^4_{L^6})||\partial_t u^k||^2 + C_\delta||Au^k||^2||\partial_t\theta_2||^2 + C_\delta||\partial_t f||^2 + \delta||\nabla\partial_t\theta^k||^2 + \delta||\nabla\partial_t u^k||^2 \quad (3.18)$$

$$\frac{1}{2}\frac{d}{dt}||\partial_t\psi^k||^2 + ||\nabla\partial_t\psi^k||^2 \leq C_\delta(||\psi_2||^4_{L^6} + ||\psi^k||^4_{L^6})||\partial_t u^k||^2 + C_\delta||Au^k||^2||\partial_t\psi_2||^2 + C_\delta||\partial_t h||^2 + \delta||\nabla\partial_t\psi^k||^2 + \delta||\nabla\partial_t u^k||^2 \quad (3.19)$$

Thus, by choosing δ appropriate and using Theorem 3.1, the function $R(t) = ||\partial_t u^k(t)||^2 + ||\partial_t\theta^k(t)||^2 + ||\partial_t\psi^k(t)||^2$ satisfies in $(0,\infty)$ the following estimate

$$\frac{d}{dt}R(t) \leq CR(t)M^2 + C(M^2+M) + CM||Au^k(t)||^2 \quad (3.20)$$

where $M = \sup_{t\geq 0}\{||g(t)||^2_{L^3}; ||\nabla\theta_2(t)||^2; ||\nabla\psi_2(t)||^2; ||\partial_t g(t)||^2; ||\partial_t f(t)||^2; ||\partial_t h(t)||^2; ||\partial_t\theta_2(t)||^2; ||\partial_t\psi_2(t)||^2; ||\nabla u^k(t)||^2; ||\nabla\theta^k(t)||^2; ||\nabla\psi^k(t)||^2\} < \infty$

Next, we multiply inequality (3.20) by $e^{\alpha t}$ obtaining

$$\frac{d}{dt}(e^{\alpha t}R(t)) \leq CM^2e^{\alpha t}R(t) + C(M^2+M)e^{\alpha t} + CMe^{\alpha t}||Au^k(t)||^2 + \alpha e^{\alpha t}R(t). \quad (3.21)$$

One can integrate (3.21) from 0 to t, obtaining

$$e^{\alpha t}R(t) - R(0) \leq (CM^2+\alpha)\int_0^t e^{\alpha s}R(S)ds + C(M^2+M)\int_0^t e^{\alpha s}ds$$

$$+ \quad CM \int_0^t e^{\alpha s} ||Au^k(s)||^2 ds. \tag{3.22}$$

On the other hand, we deduce from the inequalities (3.10), (3.11) and (3.12) that

$$R(0) \leq N$$

where N is independent of k, since $u_0 \in V \cap H^2(\Omega), \theta_0, \psi_0 \in H_0^1(\Omega) \cap H^2(\Omega)$. Consequently, by using the energy inequality (3.2), (3.22) implies

$$R(t) \leq N + C_2 \equiv K_2.$$

Consequently, we obtain (3.14).

Next, we note that estimates (3.17) - (3.19) imply

$$\frac{d}{dt} R(t) + ||\nabla \partial_t u^k(t)||^2 + ||\nabla \partial_t \theta^k(t)||^2 + ||\nabla \partial_t \psi^k(t)||^2 \leq CR(t)M^2 + C(M^2 + M) + CM||Au^k(t)||^2.$$

Now, the estimates (3.14) and (3.22), together with the above differential inequality, imply (3.16).

Finally, from the estimates obtained in Theorem 3.1 and setting $v = Au^k$ in (2.7) we have

$$\begin{aligned} ||Au^k(t)||^2 &\leq C||\nabla u^k(t)||^2 + C(||\nabla \theta^k(t)||^2 + ||\nabla \psi^k(t)||^2)||g(t)||_{L^3}^2 \\ &+ C||g_1(t)||^2 + C||\partial_t u^k(t)||^2 \\ &\leq K_3. \end{aligned}$$

Similarly, we can obtain

$$||B\theta^k(t)||^2 \leq K_4 \;, \; ||B\psi^k(t)||^2 \leq K_5$$

for all $t \geq 0$. This completes the proof. ∎

Analogously as in Corollary 3.2, we can prove.

Corollary 3.4. The assumptions are those of Theorem 3.3 and Corollary 3.2, and we assume moreover that $g_t, j_t, f_t, h_t \in L^2(0,\infty; L^2(\Omega))$. Then, the estimates (3.16) is valid for $\alpha = 0$.

4. GLOBAL EXISTENCE IN THE TWO - DIMENSIONAL PROBLEM.

We shall prove that for chemical active fluids it is possible to recover the classical result that for the usual Navier-Stokes equations, it is not necessary to assume smallness of initial data and external forces in the two dimensional case. In fact, we have the following result.

Theorem 4.1 Let Ω be a bounded domain of class $C^{1,1}$ in $\mathbb{R}^2$. Suppose that $\theta_2, \psi_2 \in L^\infty(0,\infty; H^2(\Omega)), u_0 \in V, \theta_0, \psi_0 \in H_0^1(\Omega),\ g \in L^\infty(0,\infty; L^p(\Omega))$, with $p > 2$ and $j, f, h \in L^\infty(0,\infty; L^2(\Omega))$. Then the solution described in Theorem 2.1 exists globally in time and satisfies

$$\sup_{t\geq 0}\{||\nabla u(t)||, ||\nabla\theta(t)||, ||\nabla\psi(t)||\} < +\infty; \tag{4.1}$$

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}\{||Au(s)||^2 + ||B\theta(s)||^2 + ||B\psi(s)||^2\}ds < +\infty \tag{4.2}$$

and

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}\{||u_t(s)||^2 + ||\theta_t(s)||^2 + ||\psi_t(s)||^2\}ds < +\infty \tag{4.3}$$

for every $\alpha > 0$.

Also, the same kind of estimates hold uniformly in k for the Galerkin approximations.

Proof. There hold the same remarks as the ones made in the proof of Theorem 3.1 (just after (3.12)) corning the fact that the estimates should first be derive for the approximates and then carried out to the limit.

From the proof of the local existence Theorem 2.1, for any $t \geq 0$, we have the equality

$$\frac{d}{dt}(||u^k(t)||^2 + ||\theta^k(t)||^2 + ||\psi^k(t)||^2)$$
$$+||\nabla u^k(s)||^2 + ||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2$$
$$= ((\theta^k + \psi^k)g, u^k) + (g_1, u^k) + (f, \theta^k) - (u^k\nabla\theta_2, \theta^k)$$
$$+(h, \psi^k) - (u^k\nabla\psi_2, \psi^k).$$

Consequently, by multiplying the above equation by e^{at} and recalling that $||\phi||^2 \leq C_\Omega||\nabla\phi||^2$ for $\phi E H_0^1(\Omega)$, we conclude

$$\frac{d}{dt}e^{at}(||u^k(t)||^2 + ||\theta^k(t)||^2 + ||\psi^k(t)||^2)$$
$$+\frac{1}{2}e^{at}(||\nabla u^k(t)||^2 + ||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2)$$
$$\leq \frac{1}{2}C_\Omega^2 e^{at}(||g_1||^2 + ||f||^2 + ||h||^2) + \frac{1}{2}C_\Omega^2 e^{at}||g||_{L^p}^4$$
$$+\frac{1}{2}C_\Omega^2 e^{at}(||\theta_2||_{H^2}^2 + ||\psi_2||_{H^2}^2).$$

for $0 < a \leq \dfrac{1}{4C_\Omega}$

The above inequality implies,

$$\begin{aligned}
& ||u^k(t)||^2 + ||\theta^k(t)||^2 + ||\psi^k(t)||^2 \\
+ \;& e^{-at}\int_0^t e^{as}(||\nabla u^k(s)||^2 + ||\nabla\theta^k(s)||^2 + ||\nabla\psi^k(s)||^2)ds \\
\leq \;& 2(||u_0||^2 + ||\theta_0||^2 + ||\psi_0||^2) \\
+ \;& C\sup_{t\geq 0}(||g_1||^2 + ||f||^2 + ||h||^2 + ||g||_{L^p}^4 + ||\theta_2||_2^2 + ||\psi_2||_2^2)
\end{aligned}$$

where C is a constant positive independent of k.

Also, working as in the proof of Theorem 2.1, we have

$$\frac{d}{dt}(||\nabla u^k(t)||^2 + ||\nabla\psi^k(t)||^2 + ||\nabla\theta^k(t)||^2)$$
$$\leq C||\nabla u^k(t)||^4 + C(||\nabla u^k(t)||^2 + ||\psi^k(t)||^2)$$
$$+C||\nabla u^k(t)||^2||\nabla\theta^k(t)||^2 + C||\nabla u^k(t)||^2||\nabla\psi^k(t)||^2$$

$$+C||\nabla u^k(t)||^2|| \; ||\nabla\theta_2(t)||^2 + C||\nabla u^k(t)||^2||\nabla\psi_2(t)||^2$$
$$+C(||g||^2_{L^p} + ||g_1||^2 + ||f||^2|| + ||h||^2 + ||\theta_2||^2_2 + ||\psi_2||^2_2).$$

Setting $R(t) = ||\nabla u^k(t)||^2 + ||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2$ in the last inequality, we have

$$\frac{d}{dt}R \leq CR^2 + C(||g||^2_{L^p} + ||g_1||^2 + ||f||^2 + ||h||^2 + ||\theta_2||^2_2 + ||\psi_2||^2_2).$$

Now, we observe that $CR^2 + C_1 \leq 2CR^2$ for $R \geq \left(\frac{C_1}{C}\right)^{1/2}$ where $C_1 = C\sup_{t\geq 0}(||g(t)||^2_{L^p} + ||g_1(t)||^2 + ||f(t)||^2 + ||h(t)||^2 + ||\theta_2(t)||^2_2 + ||\psi_2(t)||^2_2)$.

If we call $\ell^* = \max\left\{\left(\frac{C_1}{C}\right)^{1/2}, \; 1, ||\nabla u_0||^2 + ||\nabla\theta_0||^2 + ||\nabla\psi_0||^2)\right\}$, then either we have $0 \leq R(t) \leq \ell^*$ for all $t \geq 0$ or there exists some interval $[t_1, t_2]$, $t_2 > t_1$ for which $||\nabla u^k(t_1)||^2 + ||\nabla t_1\theta^k(t_1)||^2 + ||\nabla\psi^k(t_1)||^2 = \ell^*$ and for $t \in [t_1, t_2]$ it is true that

$$||\nabla u^k(t)||^2 + ||\nabla\theta^k(t)||^2 + ||\nabla\psi^k(t)||^2 \geq \ell^*.$$

Then, due to our choise of l^*, in this interval $[t_1, t_2]$ there holds $\frac{d}{dt}R \leq CR^2$ or, equivalently,

$$\frac{d}{dt}\ell n R \leq CR\,.$$

Multiplying the above inequality by e^{at} we have

$$\frac{d}{dt}e^{at}\ell n R \leq Ce^{at}R + ae^{at}\ell n R.$$

Now, we observe that there exists a positive constant d such that $\ell n R \leq d + dR$, consequently, using this and integrating the last inequality from t_1 to $t \in [t_1, t_2]$, we get

$$e^{at}\ell n R(t) - e^{at}\ell n R(t_1)$$
$$\leq (C + ad)\int_{t_1}^{t} e^{as}s(s)ds + ad\int_{t_1}^{t} e^{as}ds$$

so,

$$\ell n R(t) - e^{a(t_1 - t)}\ell n R(t_1)$$

$$\le (C+ad)e^{-at}\int_{t_1}^{t} e^{as}R(s)ds + ade^{-at}\int_{t_1}^{t} e^{as}ds$$

$$\le (C+ad)[2(||u_0||^2 + ||\theta_0||^2 + ||\psi_0||^2)$$

$$+C \sup_{t\ge 0}(||g_1||^2 + ||f||^2 + ||h||^2 + ||g||^4_{L^p} + ||\theta_2||^2_2 + ||\psi_2||^2_2)]$$

$$+d[1-e^{at_2}]$$

$$\le (C+ad)[2(||u_0||^2 + ||\theta_0||^2 + ||\psi_0||^2)$$

$$+C \sup_{t\ge 0}(||g_1||^2 + ||f||^2 + ||h||^2 + ||g||^4_{L^p} + ||\theta_2||^2_2 + ||\psi_2||^2_2)] + d$$

$$\equiv \overline{M}.$$

Consequently, since $-e^{a(t_1-t)}\ell n R(t_1) \ge -\ell n R(t_1)$, we have $\ell n \dfrac{R(t)}{R(t_1)} \le \overline{M}$, which implies, for all $t \in [t_1, t_2]$

$$||\nabla u^k(t)||^2 + ||\nabla \theta^k(t)||^2 + ||\nabla \psi^k(t)||^2 \le ||\nabla u^k(t_1)||^2 e^{\overline{M}} = \ell^* e^{\overline{M}}.$$

Since this is independent of t_1 and t_2, we conclude that for all $t \ge 0$, we have

$$||\nabla u^k(t)||^2 + ||\nabla \theta^k(t)||^2 + ||\nabla \psi^k(t)||^2 \le \max\{\ell^*, \ell^* e^{\overline{M}}\} = \ell^* e^{\overline{M}}.$$

The rest of analysis is now done exactly as in the tridimensional case. ■

Corollary 4.2. The assumptions are those of Theorem 4.1, and we assume moreover that $g \in L^2(0,\infty; L^p(\Omega)), p > 2, j, f, h \in L^2(0,\infty; L^2(\Omega))$. Then the estimates (4.2) and (4.3) are valid for $\alpha = 0$. ■

Analogously as in the Theorem 3.3, we can prove the following result.

Theorem 4.3. Suppose the forces in Theorem 4.1 satisfies $(\partial_t j, \partial_t g) \in L^\infty(0,\infty; L^2(\Omega)); (\partial_t f, \partial_t h) \in L^\infty(0,\infty; L^2(\Omega)); (\partial_t \theta_2, \partial_t \psi_2) \in L^\infty(0,\infty; L^2(\Omega))$ and the initial data $u_0 \in V \cap H^2(\Omega), \theta_0, \psi_0 \in H^1_0(\Omega) \cap H^2(\Omega)$. Then the solution (u, θ, ψ) obtained in the Theorem 4.1 satisfy

$$\sup_{t\geq 0}\{||\partial_t u(t)||, ||\partial_t\theta(t)||, ||\partial_t\psi(t)||\} < +\infty\ , \tag{4.4}$$

$$\sup_{t\geq 0}\{||Au(t)||, ||B\theta(t)||, ||B\psi(t)||\} < +\infty \tag{4.5}$$

and

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}(||\nabla\partial_t u(s)||^2 + ||\nabla\partial_t\theta(s)||^2 + ||\nabla\partial_t\psi(s)||^2)ds\} < +\infty\ , \tag{4.6}$$

for $\alpha > 0$. Also, the some kind of estimates hold uniformly is k for the Galerkin approximation. ■

Corollary 4.4. The assumptions are those of Theorem 4.3 and Corollary 4.2, and we assume moreover that $g_t, j_t, f_t, h_t \in L^2(0,\infty; L^2(\Omega))$. Then the estimate (4.6) is valid for $\alpha = 0$. ■

5. RESULTS ON THE PRESSURE

In a standard way we can obtain information on the pressure. In fact, we have

Proposition 5.1. Under the hypothesis of Theorem 3.1 or Theorem 4.1, if (μ,θ,ψ) is strong solution (2.2)-(2.4), there exists $p \in L^\infty(0,T,H^1(\Omega)/I\!R)$ for all $T > 0$, such that (u,θ,ψ,p) satisfies (2.2)-(2.4) a.e. and satisfies

$$\sup_{t\geq 0}\{e^{-\alpha t}\int_0^t e^{\alpha s}||p(s)||^2_{H^1(\Omega)}ds < +\infty \tag{5.1}$$

for all $\alpha > 0$.

Proof. We observe that (2.5) is equivalent to $Au = PF$ where $F = (\theta+\psi)g + g_1 - u\nabla u - u_t$. Now, our estimates for u,θ and ψ implies that $F \in L^2(0,T; L^2(\Omega))$ for all

$T > 0$, and, therefore Amrouche and Girault's results [1] imply that there is a unique $p \in L^2(0,T; H^1(\Omega)/\mathbb{R})$ such that

$$-\Delta u + \nabla p = F$$

and the following estimates holds

$$||p||_{H^1(\Omega)/\mathbb{R}} \leq c||F||$$

almost everywhere in $[0,T]$; now, the estimate (5.1) follows easily from the previous estimate and the estimates given in the above section. This completes the proof of the Proposition. ∎

Similarly, we can prove the following.

Proposition 5.2. Under the hypothesis of Theorem 3.3 or 4.3, if (u,θ,ψ) is a strong solution of (2.2)-(2.4), there exists $p \in L^\infty(0,\infty; H^1(\Omega)/\mathbb{R})$ such that (u,θ,ψ,p) satisfies (2.2)-(2.4) a.e. ∎

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