TESTING THE CONCEPT OF A PHOTON AS AN EXTENDED OBJECT IN A VARIATION OF FRANSON'S EXPERIMENT

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ABSTRACT – We describe a variation of Franson's Experiment to test a sub-class of the physical viewpoints which imagine that a photon is in some sense an extended object compared to the size of the apparatus, or which assume that there is a kind of back-action between the distant detectors. These are realistic theories which imagine that the collapse of the wave packet is physical. It will test such theories with any transmission velocity whatsoever. The sub-class includes theories for which the extended object or collapse is confined to the physical paths in the experimental apparatus and does not exist between the paths.

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ABSTRACT

We describe a variation of Franson's Experiment to test a sub-class of the physical viewpoints which imagine that a photon is in some sense an extended object compared to the size of the apparatus, or which assume that there is a kind of back-action between the distant detectors. These are realistic theories which imagine that the collapse of the wave packet is physical. It will test such theories with any transmission velocity whatsoever. The sub-class includes theories for which the extended object or collapse is confined to the physical paths in the experimental apparatus and does not exist between the paths.

I -INTRODUCTION

There exists physical viewpoints that a particle may in some sense be an extended object compared to the size of the apparatus. Such views permits one to invent local realistic explanations for the results of the polarization correlation experiments and the interference experiments of both second and fourth order. By a local theory we mean one with any finite velocity of transmission be it greater or less than the velocity of light, and we ignore any questions related to Lorentz invariance here.

The purpose of this article is to describe a variation of Franson's experiment that would test a sub-class of local realistic theories which assume that a particle is in some sense an extended object or that there is some sort of back action between the distant sides of the measuring apparatus. These are theories which imagine that there is a physical explanation for the collapse of the wave packet. The theories may have any transmission velocity whatsoever. The sub-class includes theories for which the extended object or collapse is confined to the physical paths in the experimental apparatus and does not exist between the paths.

Let us consider some informal examples of the types of theories that we have in mind. Consider Franson's Experiment[1,2,3] as shown in Figure 1, which is described in Section II.

1. We may imagine that when a photon pair is created in the non-linear crystal it is actually one object that spreads out in both the left and right arms including the two interferometers. When a photon is detected on one side of the apparatus it physically collapses the entire system resulting in two detection events. To be consistent with Aspect's Experiment[4], one would take this collapse to take place at superluminal velocities. This logically permits the information about phase and counter detection to be communicated from one side of the apparatus to the other.

2. Instead we might imagine the existence of a medium and that photons maintain their individual identity at all times. As the photons travel in the medium they cause some sort of global "vibration". Here the "vibration" is the global object. So when a photon is located just before a detector there exists a extended correlated vibration in both arms. When a photon is detected there occurs a physical collapse in the "vibration" which transmit information from one side of the apparatus to the other as in the previous case.

3. This example is not one of a extended object, but would still be tested in the below proposed experiment. Assume the photons of the photon pair have individual identity and also that they are small localized objects. Imagine that when a photon is detected it sends a signal along its previously traveled path and down the other arm. Thus information flows from one side of the apparatus to the other, but here there is no collapse, there is instead a kind of back-action.

There exist theories that imagine that a particle is extended that will not be tested by the proposed experiment as will become obvious below. For example, we may imagine a particle pair to be as in Item (1) above, but also filling in the entire space between the two sides of the apparatus and not just restricted to the possible physical paths of the particles.

II -REVIEW OF FRANSON'S EXPERIMENT

Franson's Experiment may be shown as in Figure 1 which we now review in the case of strictly ideal counters, mirrors, etc.. There is a source of photons of frequency ω with bandwidth $\Delta\omega$. We know the emission time only to within some time, $\Delta t_s = 1/\Delta\omega$, which is taken to be very large compared to any of the other times used in the below[5]. When a photon enters the non-linear crystal it may produces simultaneously, for all pratical purposes, two photons at frequency ω_r and ω_l with bandwidths $\Delta\omega_r$ and $\Delta\omega_l$ respectively by a process called parametric down conversion[6]. The index $l(r)$ represents the left (right) hand side of the apparatus in our notation. From conservation of momentum and energy and the crystal properties these pairs will be emitted in a cone of longer wavelength radiation. Slits, which are not shown in the figure, are placed in front of the crystal to obtain two fixed correlated beams (i.e., one of the possible sets of beams). By conservation of energy $\omega = \omega_r + \omega_l$, which may be satisfied in many different ω_r and ω_l. The bandwidths $\Delta\omega_r$ and $\Delta\omega_l$ may be much larger than $\Delta\omega$, and in practice they are usually restricted to a fixed value by putting bandwidth filters (not shown in the figure) at the windows where the beams exit from the crystal. We take $\Delta\omega_r$ and $\Delta\omega_l$ to be this fixed value. So there are exactly two photon exiting from the crystal, one in each beam, which were created simultaneously. Of course, most of the photons entering the crystal will not produce a pair of photons that can exit from the fixed slits. It is also assumed that the distances from the two windows to the two interferometers are equal and that the interferometers are identical.

Each right photon, ω_r, enters the interferometer with one arm much longer than the other arm. Let ΔL denote this difference in arm length. It is assumed that

$$\Delta L \gg c/\Delta\omega_r \qquad (1)$$

where c is the velocity of light. That is, the coherence length of the beam is much smaller than the difference in the arm lengths of the interferometer so that there will be no ordinary (i.e., second order) interference between the beams in the two arms of the interferometer. Each photon must ideally be detected at one of the counters R_1 or R_2 with a .5 probability. The situation is the same for each left photon, ω_1 on the left hand side of the apparatus. θ_l and θ_r are the phase angles of the phase shifters that are placed in the long arms of the interferometer as shown in the figure. We may adjust these to any desired phase angle.

We repeat ourselves. A photon leaves the source and enters the crystal. Some of these will create a photon pair with exactly one photon each traveling through the right and left interferometers. Ideally they each must then be detected in one of the counters on the left and right respectively.

What is measured in this experiment is the coincidence between the detection events in the counters on the left hand side with those on the right hand within some fixed time interval, T. There are four possible coincidences that can be measured. Let $p_{ij}(\theta_l,\theta_r)$ represent the probability of simultaneous detection events in the time interval T at the counters L_i and R_j, $i,j = 1,2$, at the phase angles θ_l and θ_r. It is desired that T satisfy the inequality

$$T \ll \Delta L/c \qquad (2)$$

There are four possible cases for the photons paths. The left photon of a pair takes the short path and the right the long path, vice versa, both photons take the short path, and both photons take the long path. We denote these cases by short-long, long-short, short-short, and long-long respectively. This inequality guarantees that we can distinguish between the cases (short-long, long-short) and (short-short, long-long) so there will be no quantum interference between them. However it is impossible to distinguish in principle and in practice between the cases long-long and short-short since the emission, Δt_s, is very large. Therefore there will be interference between the correlations from these two possibilities. This is called fourth order interference. If there were no quantum interference all the four $p_{ij}(\theta_l,\theta_r) = 1/4$ would be independent of θ_l and θ_r.

If we restrict p_{ij} only refer to the events composed of the pairs long-long and short-short, then quantum mechanics predicts that

$$p_{11}(\theta_l,\theta_r) = p_{22}(\theta_l,\theta_r) = 1/4\,(1 + \cos(\theta_r+\theta_l+\delta))$$
$$p_{12}(\theta_l,\theta_r) = p_{21}(\theta_l,\theta_r) = 1/4\,(1 - \cos(\theta_r+\theta_l+\delta)) \quad (3)$$

where δ is a fixed phase factor. A derivation of (3) is given in Appendix A.

In this article we are not interested in testing Bell's Inequality[7], but only in deviations from the quantum mechanical predictions as will be seen below. We very briefly touch on it only for the sake of completeness. If we attribute the value +1 and -1 to detections at the Counters 1 and 2 respectively, on either side of the apparatus then we may talk about the average value of the coincidences, $C(\theta_l,\theta_r)$, as a function of the phase angles. In terms of the p_{ij}, C is

$$C(\theta_l,\theta_r) = p_{11}(\theta_l,\theta_r) + p_{22}(\theta_l,\theta_r) - p_{12}(\theta_l,\theta_r) - p_{21}(\theta_l,\theta_r)$$
$$= \cos(\theta_l+\theta_r+\delta) \quad (4)$$

Bell's Inequality in the form

$$|\; C(\theta_l,\theta_r) - C(\theta_l,\theta'_r) + C(\theta'_l,\theta_r) + C(\theta'_l,\theta'_r)\;| \le 2 \quad (5)$$

may be derived in exactly the same manner as in the polarization correlation experiments. If we choose judiciously the values of these phase angles then Eq. (5) is violated by the quantum mechanical predictions given in Eq. (3) after using Eq. (4). For example, let

$$\theta_l = \pi - \delta,\ \theta_r = \pi/4,\quad \theta'_l = \pi/2 - \delta,\quad \text{and}\quad \theta'_r = 3\pi/4 \quad (6)$$

where we recall that δ is a known fixed value for a given apparatus.For

these values substituting the quantum mechanical predictions given by Eq. (3) violates Eq. (5). The three already performed experiments cannot test Bell's inequality since their T is not small enough to satisfy the right hand side of Eq. (2). So the cases (short-long, long-short) cannot be isolated from the other two in the sample. This leads to a factor of 1/2 in Eq. (3) which is sufficient to eliminate the violation of Eq. (5). The experiment however shows clearly the characteristic cosine oscillation in the correlations in complete agreement with the quantum mechanics predictions. We are unaware of any attempt to perform an Aspect type experiment (i.e. with time varying analyzers versus static) to actually try and test locality in a fourth order interference experiment.

III -THE EXPERIMENTAL PROPOSAL

We now describe a strictly ideal experiment which will test at least the types of local realistic theories described in the introduction. We will discuss some practical problems in the next section. Consider Figure 2, it is the same as Figure 1 except there is a very fast shutter in the right arm along with timing circuits between the detectors and the shutter. Assume that the distances from either of the detectors on the right hand side to the shutter along the short path are equal, and we represent it by R. Everything else in the experiment is as described in the previous section. The shutter permits us to compare the detection times with the time when the shutter is closed. The basic idea of the experiment is to choose a sub-ensemble for which the shutter was closed before the photons are detected thus blocking[8,9] any back communication or collapse. This choosing must be done in such a way that no information is obtained about which paths the photons traveled, so as not to destroy the interference.

It is easier to describe our ideal experiment in terms of the ideal quantum mechanical ensemble average[10]. Consider a large ensemble of absolutely identical experimental apparatus of Figure 2. In each of the individual experiments we prepare the source in the same one particle quantum mechanical state, ψ, at time t_0. So in each experiment a single photon may leave the source at some time undefined, Δt_s, which is very large as given above. We assume that the shutter is initially open when the state is first prepared and is then closed at some totally random time, t_s, which we know after the fact. In each apparatus there is only a certain probability that a photon pair will be produced in the crystal under the right conditions and exit from the slits (not shown in the figure). Let t_d be the detection time on the right hand side, which is defined for the cases that the right photon passed the shutter before it was closed. Since we only wish to consider the photons pairs that took the short-short or long-long paths as in the above, t_d is the simultaneous detection time on both sides of the apparatus. We also only consider the elements for which $t_d > t_s$, that is, the photons were detected after the shutter closed. Let S be the sub-ensemble which satisfies the above and the condition

$$R/c > t_d - t_s. \qquad (7)$$

Summarizing it means that S consists of the ensemble elements for which both photons took the long or short paths, the shutter closed after the right photon passed it and before the photons were detected, and Eq. (7) is satisfied. These conditions are clearly unambiguous in choosing our sub-ensemble.

It is easy to see that Eq. (7) guarantees that for this sub-ensemble, S, the shutter gives us no information of whether a photon took the long or short path[11]. There will then be fourth order interference for this sub-ensemble according to quantum mechanics.

Therefore for the sub-ensemble S, we obtain the same quantum predictions as in the Franson's experiment of Figure 1 as given by Eq. (3). Now since the shutter closed before the detection event then any physical collapse or back-action would be blocked by our ideal shutter. Therefore these theories cannot agree with quantum mechanics in this experiment. It is clear that the velocity of transmission of any imagined collapse or back action is irrelevant even if it were infinite[12]. That is, the experiment would test theories which imagined that something was flowing from one side of the apparatus to the other along the paths at any velocity whatsoever.

PRATICAL CONSIDERATIONS

We do not have sufficient laboratory experience to discuss in detail a real pratical experiment. We instead content ourselves with making several comments.

- The first critical question is the shutter switching time. In the experiment performed by Franson the distance from the source to the first beam splitter was 52 meters with a coincidence time window, T, of 3 ns. If we took R=50 meters, the distance from the shutter to a counter, then the time of flight would be about 166 ns. Since Pockel cells have a switching time, T_s, of several nano seconds, it would appear to be possible to isolate out the sub-ensemble of photon pairs that satisfy Eq. (7). On the other hand the already low coincidence rate in his experiment would be substantially reduced since a sub-ensemble must be isolated out and one may also have to insert a polarizer before the shutter.

- The second critical question is whether a Pockel cell (or any shutter) will actually block an extended object. We know from the delayed choice experiments that if we put a Pockel cell in one arm of an interferometer then there will be no interference for the other photons. If something extended exists here then it is being blocked. Also presumably if we put a Pockel cell in one arm of one interferometer in a Franson type experiment with appropriate timing, we will see no fourth order interference for the photons that travel the other path. We think that this experiment will test the most viable candidates for extended object type theories. Such theories have not been examined[13].

- In the above we took our shutter to open at a random time. This randomness is not essential. One could use, for example, a periodic shutter analogous to the rotating disk in the experiment of Rauch and Summhammer[14] (Here a rotating disk or any physical shutter would be orders of magnitude too slow to the best of our knowledge). The above analysis may be seen to still apply to the case of a periodic shutter.

- The idea of this article would be applicable to a polarization correlation experiment in principle. In practice, this may not be the case since one needs to use a very fast shutter which didn't involve polarization effects. This would eliminate the use of Pockel or Kerr cells.

- A comment considering alternate locations of the initial triggering of any physical wave collapse is made in a footnote[15,16].

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APPENDIX A

A derivation of Eq. (3) is presented here. Let ψ represent the normalized product state of a photon pair at the crystal exit. Since we in principle can't distinguish between the cases where both photons travel the long paths or the short paths our wave function will have the formal form

$$\psi = \psi_l(\text{long})\, \psi_r(\text{long}) + \psi_l(\text{short})\, \psi_r(\text{short}) \qquad \text{(A1)}$$

at any of the counter pairs L_2 and R_2. This may be expressed more concretely at a given pair of counters by using the fact that the wave function changes by a factor of i on reflection, remains the same on transmission[17] and is changed by $\exp(i\theta)$ by a phase shifter. So, for example, if ψ_r represents the wave function just before the first beam splitter on the right hand side, then we may think of the wave function on the right hand side that travels the long interferometer arm as becoming

$$\frac{1}{\sqrt{2}}\; i\; \exp(i\theta_r)\; i\; \frac{1}{\sqrt{2}}\; \exp(i\, \omega_r\, t_{lo})$$

$$=1/2\, \exp\left[\, i\,(\theta_r + \omega_r\, t_{lo})\,\right] \psi_r \equiv \psi_r(\text{long}) \qquad \text{(A2)}$$

and the part that travels the short interferometer arm as becoming

$$\frac{1}{\sqrt{2}}\; \frac{1}{\sqrt{2}}\; \exp(\, i\, \omega_r t_{sh}\,)\, \psi_{sh}$$

$$=1/2\, \exp(\, i\, \omega_r\, t_{sh}\,)\, \psi_r \equiv \psi_r(\text{short}) \qquad \text{(A3)}$$

where t_{lo} and t_{sh} are the times it take a photon to traverse the long and short arms to the counters respectively. Substituting Equations

(A2), (A3) and similar expressions for ψ_l(long) and ψ_l(short) in (A1) we obtain

$$\psi = 1/4 \left\{ \exp\left[i(\theta_l + \theta_r + \delta) \right] + 1 \right\} \psi_l \, \psi_r \, \exp[i\omega t_{sh}], \qquad (A4)$$

where ψ now represents the joint wave function at the detectors L_2 and R_2 only. $\delta = (\omega_l + \omega_r)\,(t_{lo} - t_{sh}) = \omega\, \Delta L/c$ which is a constant phase factor for a given apparatus. Then the probability of a joint detection at Counters L_2 and R_2 as a function of the phase angles is

$$p_{ll}(\theta_l,\theta_r) = |\psi|^2 = 1/4 \left[1 + \cos(\theta_r + \theta_l + \delta) \right] \qquad (A5)$$

This expression contains a factor of two to normalize it to include in our sample only the short-short and long-long cases. The other correlations in (3) be derived similarly.

FIGURE CAPTIONS

Figure 1: This is Franson's experiment with the design given in Ou, Zou, Wang and Mandel, except we have included four counters. See Section II for detailed information.

Figure 2: This is the same as Figure 1 except there is a shutter in the right arm which is closed at random times. There is also a timing circuit which allows us to compare the detection times with the shutter closing times. See Section III for detailed information.

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[8]A. Shimony, "Controllable and Uncontrollable Non-Locality", *Proc. Int. Symp. Foundations of Quantum Mechanics*, Tokyo, pp. 225, (1983).

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[10]It will be clear that discussion applies to the real laboratory time average without modification as long as the average time interval between the photon emissions at the source is sufficiently large.

[11]For the detections events which violate Eq. (7) but satisfy our other requirements, we know that the corresponding photons must have taken the longer path. Therefore there will be no fourth order interference for these.

[12]One may mean different things when one says the velocity of transmission of the collapse to be infinite. (i) One may mean that it is always finite in a fixed situation but varies in different physical situations with no upper limit. This is the usual sense of the word, infinite, that is used in mathematics. (ii) One may mean that something is actually physically being transmitted at an infinity velocity, that is instantaneously. One occasionally uses it in this sense in mathematics, as an object in and of itself. That is, as an element in a set, but this is always accompanied by very special mathematical care. (iii) One may mean it as a short hand way to express the concept that separated object have a global nature. Despite there physical separation in space they are somehow part of the same thing. What happens to one instantly affects the other, but nothing is being transmitted through space and time between them. Obviously, our experiment would examine realistic theories with infinite velocities of transmission only in first two senses.

[13]The only experiment that might claim to have partially examined such theories is that of Aspect with the time varying analyzers. One might argue that the commutators blocked the extended object or back action. Even accepting this, it would only be the case for transmission velocities near that of light. Our proposal is valid for any transmission velocity, even an infinite one.

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[15]In the above we have been assuming that this collapse is triggered at the detectors. The question of where the collapse is trigger has been discussed by Ryff (see reference below). For example, it could be taken to be at the last beam splitter just before the detectors. It is only our intention here to say that one could tighten up the criteria given in Eq. (7) to guarantee that the shutter closed before the photon reached the interferometer and still not give us information about which paths the photons traveled. That is, our experiment would test theories which assumed the collapse didn't necessarily take place at the detectors. The criteria that would replace Eq. (7) is

$$\Delta t_{long} < t_d - t_s < R/c$$

where Δt_{long} is the time a photon takes to travel the long path from the first beam splitter to the detector. The distance from the shutter to the first beam splitter would have to be sufficiently large to satisfy this inequality.

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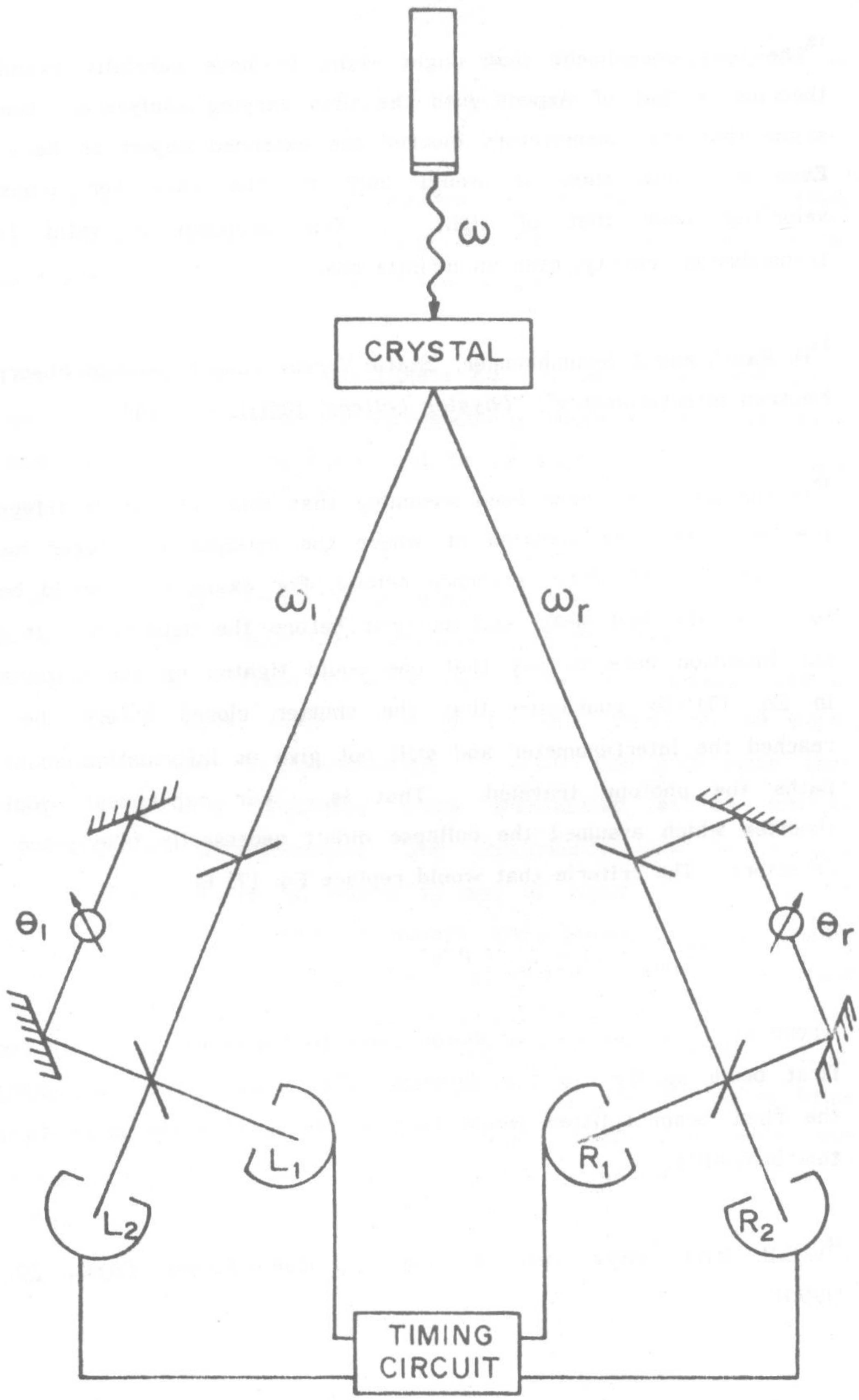

ω
CRYSTAL
ω_l
ω_r
θ_l
θ_r
L_1
L_2
R_1
R_2
TIMING CIRCUIT

FIGURE 1

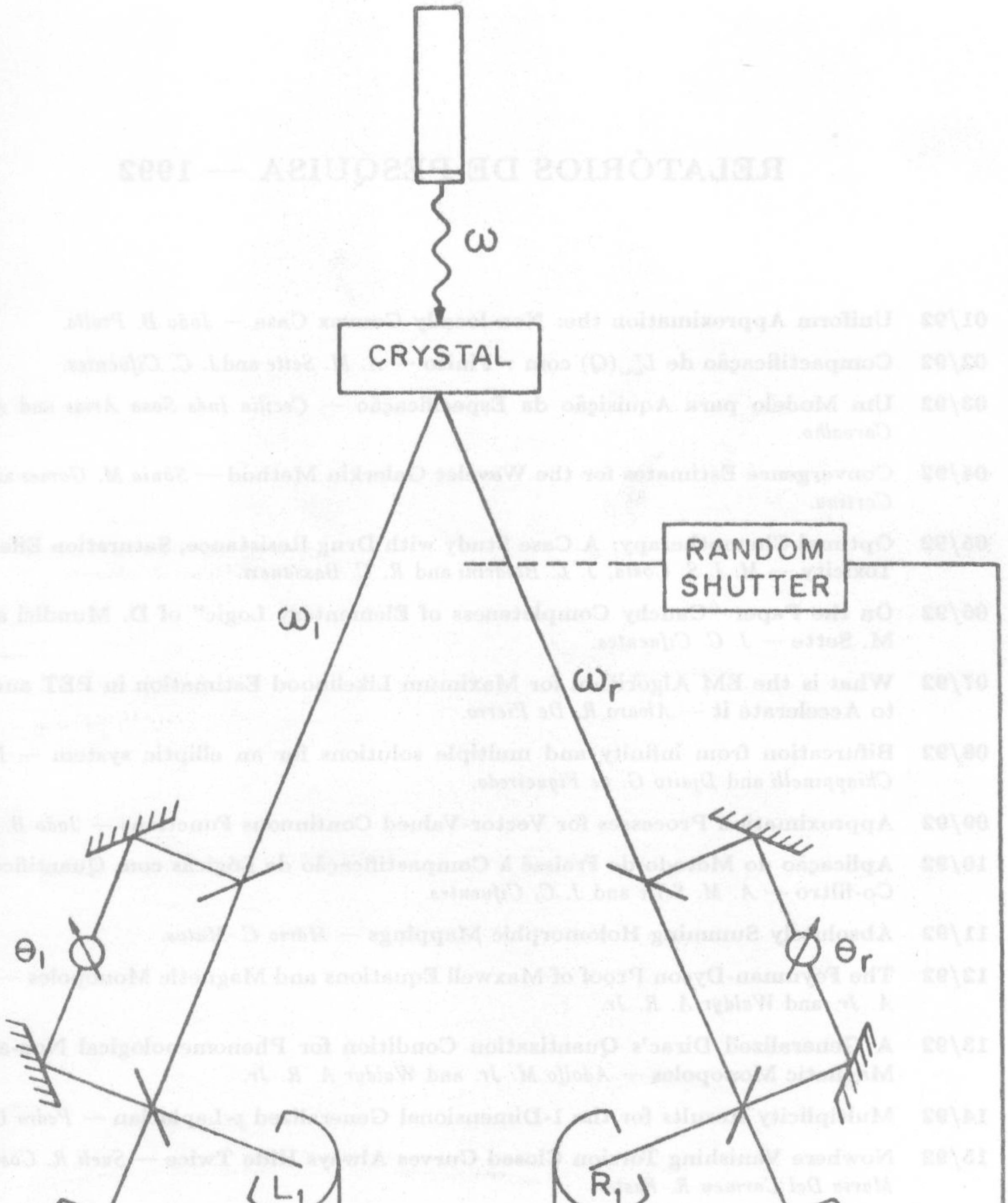
ω
CRYSTAL
RANDOM
SHUTTER
ω_1
ω_r
Θ_1
Θ_r
L_1
L_2
R_1
R_2
TIMING
CIRCUIT

FIGURE 2

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