DISAPPEARANCE OF THE NUMERICALLY IRRELEVANT SOLUTIONS (NIS) IN NON-LINEAR ELLIPTIC EIGENVALUE PROBLEMS

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In this note, we study the disappearance of the numerically irrelevant solutions, for non-linear elliptic eigenvalue problem:

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Where Ω is a smooth bounded domain of the plane and the function f changes its sign and does not satisfy the so-called area conditions.

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"Disappearance of the numerically irrelevant solutions (NIS) in non-linear ellipitic eigenvalue problems"

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Abstract

In this note we study the disappearance of the numerically irrelevant solutions, for non-linear elliptic eigenvalue problem:

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0. Introduction

The numerically irrelevant solutions "NIS", occur in the approximation by finite differences of the solutions of a class of non-linear elliptic eigenvalue problems, when one proceeds in the context of the Topological Degree theory.

The type of non-linear problem, indicated in the abstract is

$$(0.1) \qquad -\Delta u = \lambda f(u) \quad \text{in} \quad \Omega,\ u = 0 \quad \text{ou} \quad \partial\Omega.$$

where Ω is a bounded smooth domain of the plane, λ a real positive parameter and $f : [0, \infty) \to I\!R$ a function satisfying the following conditions:

$$(0.2) \quad \begin{cases} \text{a)} & f \text{ is locally Lipschitz continuous} \\ \text{b)} & f \text{ has exactly 2m non} - \text{negative simple zeros} \\ & s_0 = 0 < s_1 < \ldots < s_{2m-1}, \text{ where it change its} \\ & \text{sign, as follows} \\ & (-1)^i f(s) > 0 \text{ for } s \in (s_i, s_{i+1}), \ i = 0, 1, \ldots, 2 \\ & (m-1). \end{cases}$$

we will say that the function f in (0.2), satisfies the positive area condition on $[s_{2i-1}, s_{2i+1}]$ where $i = 1, \ldots (m-2)$, if

$$(0.3) \qquad F(s_{2i+1}) > F(s_{2i-1}) \quad \text{with} \quad F(t) = \int_0^t f(s)ds.$$

The discrete analogue of (0.1) obtained by finite difference, with n suitable grid points at $\overline{\Omega}$ is

$$(0.4) \qquad Ax = \lambda h^2 \tilde{f}(x), \quad x \in I\!R^n$$

Thus A is a M-matrix ([15], [16]) arising in the discretization of the differential operator $-\Delta$, h is the mesh size and $\tilde{f}(x) = (f(x_1), \ \ldots, f(x_n))$ is the Nemitskii operator associated with the scalar function f.

Peitgen, Schmitt and Saupe ([11]), made a careful investigation of the solutions of (0.4) using the Topological Degree and Global Bifurcation theories when the nonlinearity f essentially satisfies (0.2). The first two authors ([12]) gave a classification of the NIS in three types and prove briefly for the one dimensional case, the disappearance of the first type as the mesh size $h \to 0$. The NIS of the first type accur when $F(t) < 0$ for all $t \in (s_2, \ s_{2m})$ and its norm is between s_3 and s_{2m-1} (section 4, [12]).
In this note we will give alternative Proofs for some of their propositions [1.7; 1.8]. Our proofs are simpler and hold even under weaker assumptions. Mantaining the same context we show the existence of the NIS, as well as the disappearance of the first type of NIS, for two dimensional case.
Section 1 is devoted to study the existence of the NIS and the section 2 is devoted to show the disappearance of the NIS wich have norm between s_{2i} and s_{2i+1} and $F(s_{2i+1}) \leq F(s_1)$. (This is a optimal condition [5]).

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1.- Existence of numerically irrelevant solutions

Notations 1.1

$$I\!R^n_+ = \{x \in I\!R^n : x_i \geq 0 \ \forall i = 1, \ldots, n\}, \quad \overset{\circ}{I\!R}{}^n_+ = \text{interior } (I\!R^n_+)$$

$$||x||_\infty = \max\{|x_i|;\ i = 1, \ldots, n\}, \quad ||x|| = (\sum_{i=1}^{n} |x_i|^2)^{1/2}$$

Definition 1.2

A real $n \times n$ matrix; $A = [a_ij]$ is said to be a M-matrix if:

(1.2)
- a) $a_{ii} > 0,\ a_{ij} \leq 0$ for $i \neq j$
- b) $\sum_{\substack{j=1 \\ j\neq i}}^n |a_{ij}| \leq a_{ii}$, where the inequality is strict at least one i (Hence $\sum_{j=1}^n a_{ij} \geq 0$ with strict inequality for some i)
- c) A is irreducible.

we state without proof the following proposition. The reader can see the proof in [16].

Proposition 1.3

If $A = [a_{ij}]$ is a M-matrix. Then
a) A is nonsingular and it has a unique positive eigenvalue λ_1 of multiplicity one and its correponding eigenvector belong to $\overset{\circ}{I\!R}{}^n_+$.
b) $A^{-1}(I\!R^n_+) \subset I\!R^n_+$ and $A^{-1}(I\!R^n_+ - 0) \subset \overset{\circ}{I\!R}{}^n_+$. □

The next proposition is a consequence of the Topological Degree Theory in the finite dimensional case. It will be needed in the proofs of the subsequent propositions.

Proposition 1.4

Let $f : I\!R \to I\!R$ be a continuous function, $\tilde{f}$ as in (0.4) and $A = [a_{ij}]$ is a $n \times n$ matrix.
Let V be open bounded subset in $I\!R^n$.
Suppose that $0 \notin \tilde{f}(\partial V)$ and deg $(-\tilde{f},\ V, 0) \neq 0$. then there exists a $\overline{\lambda} \geq 0$ such that the equation

$$(1.3) \qquad Ax = \lambda \tilde{f}(x), \quad x \in I\!R^n$$

has a solution $x_\lambda \in V$ for all $\lambda \geq \overline{\lambda}$.

Proof

There exists positive constants R, τ such that

$$||Ax|| \leq R \quad \text{and} \quad ||\tilde{f}(x)|| \geq \tau \quad \forall x \in \partial V.$$

Hence with $\overline{\lambda} > R/\tau$ we have $||Ax|| < \lambda ||\tilde{f}(x)||$ $\forall \lambda \geq \overline{\lambda}$ and $\forall x \in \partial V$.
Writing $t = 1/\lambda$: $tAx - \tilde{f}(x) \neq 0 \quad \forall t \in [0,\ 1/\overline{\lambda}]$ and $\forall x \in \partial V$ then deg $(tA - \tilde{f},\ V, 0)$ =deg $(-\tilde{f},\ V, 0) \neq 0$. In view of this, for each $\lambda \geq \overline{\lambda}$ there exists a solution x_λ of (1.3) that belongs to V. □

Proposition 1.5

Let $f : I\!R \to I\!R$ be a continuous function, that has exactly k zeros $s_1 < s_2 < \ldots < s_k$. Moreover we assume that f changes its sign at the zeros i.e. there exists $\varepsilon > 0$ such that.
$f(t)f(\tau) < 0$ if $s_i - \varepsilon \leq t < s_i < \tau \leq s_i + \varepsilon$. Then
a) The Nemitskii operator associated with f, has k^n zeros, which are $\tilde{f}^{-1}(0) = f^{-1}(0) \times \ldots \times f^{-1}(0)$ (n times).

b) For each $z \in \tilde{f}^{-1}(0)$ and for any open bounded neighborhood V of z with $(\overline{V} - \{z\}) \cap \tilde{f}^{-1}(0) = \phi$, then deg $(-\tilde{f},\ V, 0) = \pm 1$.

Proof

When in addition f is a C^1 function, then deg $(-\tilde{f},\ V,0) = (-1)^n$ sgn (det $\tilde{f}'(z)) = (-1)^n\ \Pi_{j=1}^n$ sgn $(f'(s_{ij}))$, where $z = (s_{i_1},\ldots,s_{i_n})$, $i_j \in \{1,\ldots,k\}$.
Hence deg $(-\tilde{f},\ V,0) = \pm 1$.
For the general case see ([11]).

Corollary 1.6

With the same hypotheses of proposition 1.5 it can be shown that for each $z \in \tilde{f}^{-1}(0)$ there exists solutions x_λ of (1.3) such that $x_\lambda \to z$ as $\lambda \to +\infty$.

Proof

Since $\tilde{f}^{-1}(0)$ is a finite discrete subset of $\mathbb{R}^n$, open bounded neighborhood can be taken in such way that they are pairwise disjoint . The rest follows easily from this fact. □
In the next two propositions we will show the behaviour of the solutions of (0.4) in regard to the nonlinearity f. The hypothesis needed are weaker than those used in lemma 2.7 [11].

Proposition 1.7

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, such that $f(s) > 0$ for $s < 0$. Let $A = [a_{ij}]$ be a M-matrix. Then if x is a solution of (0.4) it follows that $x \in \mathbb{R}^n_+$.

Proof

Let $x = (x_1,\ldots,x_n)$ be a solution of (0.4) for a $\lambda > 0$ fixed.
Let $x_i = \min\ \{x_j : j = 1,\ldots,n\}$. Then

$$x_i(\sum_{j=1}^n a_{ij}) \geq \sum_{j=1}^n a_{ij}x_j = \lambda h^2 f(x_i)$$

As A is a M-matrix, x_i can not be negative. □

Proposition 1.8

Let f be as in (0.2) and define $f(s) > 0$ for $s < 0$. Let $A = [a_{ij}]$ be a M-matrix. Then there is no solution x of (0.4) such that $||x||_\infty = \theta$ where $\theta > 0$ and $f(\theta) \leq 0$.

Proof

Suppose that (0.4) has a solution x with $||x||_\infty = \theta$ where $\theta > 0$ and $f(\theta) \leq 0$.
Since f is locally Lipschitz continuous, it follow that there exists $\sigma > 0$ such that $f(s)+\sigma s$ is strictily increasing on $[0,\ \theta]$. Hence taking $w = (\theta,\ \theta, \dots, \theta)$ we have:

$$(A + \lambda h^2 \sigma I)(w - x) \geq \lambda h^2(\tilde{f}(w) + \sigma w - (\tilde{f}(x) + \sigma x)) \in \mathring{I\!R}^n_+.$$

By proposition 1.3 it follows that $\theta > x_i\ \ \forall i = 1, \dots, n$ contradicting the assumption. (This is the discrete version of lemma 6.2 [1]). □

Peitgen and Schmitt (theorem 3.2 [12]) give the following proposition which can be shown using the preceeding propositions.

Proposition 1.9

Let A, f be as in proposition 1.8. Then for each large λ the equation (0.4) has a least $(2m)^n$ solutions in $\mathring{I\!R}^n_+$ and they are close to the zeros of $\tilde{f}$ which are in $\mathring{I\!R}^n_+$. Furthermore these solutions have the following properties:
a) for each $i = 1,\ 2, \dots, 2m$ there are $i^n - (i-1)^n$ solutions with norm close to s_i.
b) There are $(i+1)^n - (i+1)^n$ solutions x such that $s_i < ||x||_\infty < s_{i+1}$ for $i = 2,\ 4,\ 6, \dots, 2(m-2)$.
c) There is at least one solution x such that $0 < ||x||_\infty < s_1$
d) There is no solution x such that $s_i < ||x||_\infty < s_{i+1}$ for $i = 1,\ 3,\ 5, \dots, 2m-1$.
e) There are $(2m)^n - (2m-1)^n$ solutions x such that $||x|_\infty > s_{2m}$.

Proof

By corollary 1.6 there exist $\overline{\lambda} > 0$ such that for all $\lambda \geq \overline{\lambda}$, the equation (0.4) has solutions for each small ball centered at each zero of $\tilde{f}$ that belong to $\overset{\circ}{\mathbb{R}}{}^n_+$. Hence for each $\lambda \geq \overline{\lambda}$ the number of solutions is equal to the number of zeros of $\tilde{f}$ in $\overset{\circ}{\mathbb{R}}{}^n_+$.
But $B_j = \{z \in \tilde{f}^{-1}(0) \cap \overset{\circ}{\mathbb{R}}{}^n_+ : ||z||_\infty \leq s_j\}$ has the cardinal $\#(B_j) = j^n$.
Hence
a) For each $\lambda \geq \overline{\lambda}$, the equation (0.4) has $\#(B_i - B_{i-1}) = i^n - (i-1)^n$ solutions with norm close to s_i.
b) For each $\lambda \geq \overline{\lambda}$, (0.4) has $\#(B_{i+1} - B_{i-1}) = (i+1)^n - (i-1)^n$ solutions with norm between s_i and s_{i+1}, $i = 2, 4, \ldots, 2(m-2)$.

The rest in similar (part d follows from proposition 1.8). □.

2- Disappearance of NIS in the nonlinear elliptic eigenvalue problems.

First we will study the one dimensional case, that is the NIS for nonlinear elliptic eigenvalue problem that we give below.

$$(2.1) \qquad -u'' = \lambda f(u) \text{ in the interval } (0,\ 1) \text{ and } u(0) = u(1) = 0$$

where $f : \mathbb{R} \to \mathbb{R}$ is a continuous function.
It is a known fact that u is a solution of (2.1) if only if u is a solution of the following integral equation:

$$(2.2) \qquad u(t) = \lambda \int_0^1 G(t,\ s) f(u(s)) ds$$

where $G(t,\ s)$ is the Green function for the operator $-d^2/td^2$ subject to Dirichlet boundary conditions on $[0,\ 1]$.
(2.3) Let $\pi_n = \{t_k = \dfrac{k}{n+1} : k = 0,\ 1, \ldots, n+1\}$ be a partition or grid of interval $[0,\ 1]$. Then the discrete analogue of (2.1) is

$$(2.4) \begin{cases} \dfrac{-u(t_{k+1}) + 2u(t_k) - u(t_{k-1})}{h^2} = \lambda f(u(t_k)), & k = 1,\ 2, \ldots, n \\ u(t_0) = u(t_{n+1}) = 0, \quad \text{where } h = 1/(n+1). \end{cases}$$

Letting $u(t_k) = x_k$, we have

$$(2.5) \qquad \begin{cases} -x_{k+1} + 2x_k - x_{k-1} = \lambda h^2 f(x_k), \ k = 1, \dots, n \\ x_0 = x_{n+1} = 0 \end{cases}$$

which is writen as $Ax = \lambda h^2 \widetilde{f}(x)$ and as before, we see that A is a M-matrix and $\widetilde{f}$ the Nemitskii operator associated with the scalar function f.
Next proposition is the discrete analogue of the equivalence that there is between the problems (2.1) and (2.2).

Proposition 2.1

Let $G(t,\ s)$ be as in (2.2) and $\overline{z} = (z_0, \dots, z_{n+1})$, $\overline{y} = (y_0, \dots, y_{n+1})$. Then $\overline{y}$ is solution of (2.5) if only if $\overline{y}$ is solution of the following equation

$$(2.6) \qquad y_k = h \sum_{i=1}^{n} G(t_k,\ s_i) z_i, \ k = 0,\ 1, \dots, n+1.$$

Proof

It is easy to verify that

$$(2.7) \qquad -G(t_{k+1},\ s_i) + 2G(t_k,\ s_i) - G(t_{k-1},\ s_i) = h^{-1}\delta_{ik}, \ \forall i, \\ k = 1,\ 2, \dots, n.$$

From this and the symmetry of $G(t,\ s)$, everything follows. □

Proposition 2.2

If $\overline{x} = (x_0, \dots, x_{n+1})$ is a solution of (2.5), then

$$|x_{k+1} - x_k| \le h(\max_{0 \le s \le \rho} |f(s)|), \quad \forall\ k = 0, 1, \dots, n$$

where $\rho = ||x||_\infty$

Proof

As $\overline{x}$ is solution of (2.4)

$$\begin{aligned} |x_{k+1}-x_k| &= h|\sum_{i=1}^{n}[G(t_{k+1},\ s_i)-G(t_k,\ s_i)]f(x_i)| \\ &\leq \frac{1}{2}h^3(\max_{0\leq s\leq \rho}|f(s)|)\ [(k+1)k+(n-k)(n-k+1)] \\ &\quad \forall\, k=0,\ldots,n. \end{aligned}$$

Hence the resolut follows. □

Let Π_n be as in (2.3). For each solution $\overline{x}=(x_0,\ldots,x_{n+1})$ of (2.5) we define the interpolant function

$$(2.8)\ x(s)=\sum_{i=1}^{n}x_i\varphi_i(s),\ \text{where}\ \varphi_i(s)=\frac{1}{h}\begin{cases} s-t_{i-1} & \text{if}\ t_{i-1}\leq s\leq t_i \\ t_{i+1}-s & \text{if}\ t_i\leq s\leq t_{i+1} \\ 0 & \text{esle where}\end{cases}$$

let us note that $x(s)$ is a continuous function such that $x(0)=x(1)=0$ and $||x||_\infty=\max_{0\leq t\leq 1}|x(t)|=||\overline{x}||_\infty$. Using the similar ideas of the paper of Lees and Schultz ([9]), we will prove the next proposition.

Proposition 2.3

Let $\{\Pi_m\}$ be a partitions sequence of the interval [0, 1]. Suppose that for each Π_m the corresponding discrete problem (2.4) has a solution $\overline{x}^m=(a_0^m,\ldots,x_{m+1}^m)$ such that $||\overline{x}^m||_\infty\leq R$, with R independet of m. Then
a) The sequence of interpolant functions x^m for $\overline{x}^m$, described in (2.8), has a convergent subsequence, converging uniformly in [0, 1] to a continuous function x^0.

b) x^0 is a solution of (2.1) and $||x^0||_\infty\leq R$.

Proof

a) Let $\Pi_m=\{t_k=kh,\ k=0,\ 1,\ldots,m+1\}$ be a partition, $\overline{x}^m$ and x^m as in the hyphotesis. Then for $t,\ s\in[0,\ 1]$ with $t_k\leq t,\ s\leq t_{k+1}$ by proposition 2.2 we have

$$|x^m(t) - x^m(s)| = |\frac{x^m_{k+1} - x^m_k}{h}||t - s| \leq (\max_{0 \leq s \leq R} |f(s)|)|t - s|$$

Hence the same inequality holds for all $t,\ s\varepsilon[0,\ 1]$.
Taking into account this last inequality and the fact that $||x^m||_\infty \leq R$ for all m; we can use the Arzela-Ascoli theorem and assume that there exists a subsequence (denoted in the same way) x^m and a continuous function x^0 such that $x^m \to x^0$ uniformly on $[0,\ 1]$.

b) We now prove that x^0 is a solution of (2.2). For this purpose will be sufficient to show that

$$x^0(t) = T[x^0](t) = \lambda \int_0^1 G(t,\ s) f(x(s)) ds \quad \text{for all} \quad t \in [0,\ 1].$$

For $t = 0,\ 1$ it is immediate. Then we take a $t \in (0,\ 1)$.
Since f is uniformly continuous on $[0,\ R]$ and $G(t,\ \cdot)$ has a L^1-norm uniformly bounded by $1/8$, we have that
(2.8) $T[x^m] \to T[x^0]$ uniformly on $[0,\ 1]$, in particular $T[x^m](t) \to T[x^0](t)$.
Our next objective is to show that $T[x^m](t) \to x^0(t)$.
Fixed $t \in (0,\ 1)$, let $\varepsilon > 0$.
Since $G(t,\ \cdot)\ f(x^m(\cdot))$ is a continuous function on $[0,\ 1]$ and the meshzise $||\Pi_m|| \to 0$, it follows that there is a positive number m_2 [depending on $t,\ \varepsilon$], such that

(2.9) $$|T[x^m](t) - \sum_i hG(t,\ s_i) f(x^m(s_i))| < \varepsilon/5 \ \ \forall m \geq m_2.$$

As the sequence $\{\Pi_m\}$ is uniform, for each Π_m, there exist a unique $t_k \in \Pi_m$ such that $t \in [t_{k_m},\ t_{k_m} + h)$ and $t_{k_m} < t_{k_{m+1}} < \ldots < t < \ldots < t_{k_m} + h$, i.e. $t_{k_m} \to t$.
Since $G(\cdot\ ,\ \cdot)$ is Lipschitz continuous on $[0,\ 1] \times [0,\ 1]$, there exists a $m_3 > 0$ such that

(2.10) $$|\sum_i h[G(t,\ s_i) - G(t_{k_m},\ s_i)] f(x^m(s_i))| < \varepsilon/5 \ \ \forall m \geq m_3$$

Considering now the continuity of x^0 and the uniform convergence of x^m to x^0 on $[0,\ 1]$, there is another positive number m_4 such that

(2.11) $$|x^m(t_{k_m}) - x^0(t)| < \varepsilon/5 \ \ \forall m \geq m_4.$$

Using (2.9) and (2.11) we have then that $T(x^m](t) \rightarrow x^0(t)$ and by (2.8) $T[x^0](t) = x^0(t)$. □

According to Laetsch [10] or Browr and Budin [3] if u is a solution of (2.1) then u also satisfies

$$[u'(t)]^2 = \lambda[F(||u||_\infty) - F(u(t))] \quad \text{for all} \quad t \in (0, 1) \tag{2.12}$$

Hence if $F(s_{2i+1}) < F(s_1)$ then (2.1) has no solutions u such that $s_{2i} < ||u||_\infty < s_{2i+1}$. Thus the numerical solutions x of (2.5) will be NIS (of the first type).

In the final part of the one dimensional case we now show the disappearance of the NIS, when the meshzise convergs to zero.

Proposition 2.4

Let f be as in (0.2) and $F(s_{2i+1}) \leq F(s_1)$ for some $i = 1, 2, \ldots, (m-2)$. Let $\lambda_* = inf\{\lambda > 0$: for every grid Π_m, (2.5) has a solution $\overline{x}$ with $s_{2i} < ||\overline{x}||_\infty < s_{2i+1}\}$. Then $\lambda_* = +\infty$.

Proof

Let us suppose by a contradiction that $\lambda_* < +\infty$. Then there is a $\lambda < \lambda_*$ such that for every grid Π_m (2.5) has a sequence $\overline{x}^m$ of solutions with $s_{2i} < ||\overline{x}^m|| < s_{2i+1}$. Letting $R = s_{2i+1}$ in the proposition 2.3 then there exist a solution x^0 of (2.1) with $s_{2i} < ||x^0||_\infty < s_{2i+1}$ which is a contradiction to (2.12). □

In order to study the disappearance of the NIS in the bidimentional case, one has to require properties on the domain Ω and the finite difference Scheme, that will give us the discrete analogue (0.4) with all necessary features described in the preceding propositions. With this purpose in mind we take those restrictions stated in Allgower and Jeppson [2] or Forsythe and Wasow [7].

On the other hand. Theorem 1 of a paper of Dancer and Schimitt ([5]) that to be given below, has the same performance of the equality (2.12). According to theorem 1 and the section 6 of [5], we state without proof the following proposition:

Proposition 2.5

Let f be as in (0.2) and suppose that it does not satisfy the positive area conditions on $[s_1, s_{2i+1}]$ where $i = 1, \ldots, (m-2)$, i.e. $F(s_{2i+1}) \leq F(s_1)$. Then (0.1) has no positive solution u such that $s_{2i} \leq ||u||_\infty \leq s_{2i+1}$ □

According to [2], $\overline{x} = (x_1, \ldots, x_n)$ will be a slolution of the equation (0.4) i.e.

$$A\overline{x} = \lambda h^2 \tilde{f}(\overline{x})$$

if only if $\overline{x}$ is solution of

$$x_k = \lambda h^2 \sum_{i=1}^{n} G_h(P_k,\ Q_i) f(x_i) \tag{2.13}$$

where $x_k = u(P_k)$ and $G_h(P_k,\ Q_i) = a_{ki}^{-1}$ are the components of the matrix A^{-1}, called Green's matrix ([2], [7]).

A similar fact is know for the continuous case i.e. u is a solution of (0.1) if only if u is a solution of

$$u(P) = \int_\Omega G(P,\ Q) f(u(Q)) dQ. \tag{2.14}$$

(under assumptions that f has sufficient smoothness and that Ω satisfies some conditions which assure the existence of the Green's function G). Next we state without proof the following proposition ([7]).

Proposition 2.6

Let Ω be a bounded, finitely connected, open region in the plane, with a four times continuously differentiable boundary $\partial\Omega$. Let $G(P,\ Q)$ the Green's function and $G_h(P,\ Q)$ as in (2.13). Then

$$G(P,\ Q) - G_h(P,\ Q) = 0(h) \tag{2.15}$$

uniformly for P in the grid Ω_h if Q is bounded away from $\partial\Omega$ independently of h and $||P - Q|| \geq (const)\sqrt{h} > 0$. □

From all this, one can to get now a similar proposition to 2.2 and 2.3, because they are independent of the dimension.
Hence we have the following proposition.

Proposition 2.7

Under the assumptions of proposition 2.5. The numerical solutions of (0.4) such that

$$s_{2i} < \|\overline{x}^m\|_\infty < s_{2i+1}$$

are NIS and disappears when the meshzise convergs to zero. □

Remark 2.8

It is a interesting problem to look for a relation between the meshsize $h = \dfrac{1}{m+1}$ and the parameter λ for each solution x^m of (0.4), when the nonlinearity f does not satisfy the so-called area condition and when f satisfies it.

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