THE STATUS OF THE PRINCIPLE OF RELATIVITY

W. A. Rodrigues Jr.
and
Q. A. Gomes de Souza

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Instituto de Matemática, Estatística e Ciência da Computação
Universidade Estadual de Campinas
13.081, Campinas, S.P.
BRASIL

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THE STATUS OF THE PRINCIPLE OF RELATIVITY

W. A. Rodrigues Jr.[1] and *Q. A. Gomes de Souza*[2]
[1] Instituto de Matemática, Estatística e Ciência da Computação
IMECC- UNICAMP,C.P. 6065, 13081 (Campinas) S.P. - BRASIL.

[2] Instituto de Física "Gleb Wataghin"
UNICAMP, C.P. 6165, 13081, (Campinas), S.P. - BRASIL.

Abstract: We discuss some of the recent results we obtained in our studies concerning the status of the so called "Special Principle of Relativity" (SPR) and the "General Principle Relativity" (GPR).

To start we must agree on the mathematical and physical meaning of these principles.

Now, take three books at random on Special, or General Relativity, or Relativistic Field Theory as e.g., Anderson [1], Friedman [2] and Bergman [3]. You will find the following statements as being the SPR.

PR_1: All (Special Relativistic) Theories have the Poincaré Group as its invariance (or symmetry) group (Anderson).

PR_2: All inertial frames are physically equivalent or indistinguishable (Friedman).

PR_3: The laws of all physical theories are represented by mathematical equations that have the same form, i.e., are covariant in all inertial frames (Bergman).

Now, go on and try to read and compare the concepts of invariance as used, e.g., by Weinberg [4], Ohanian [5], Bergamn [3], Anderson [1], Treder et al [6], etc. You will become very confused since no agreement exists. Also what is the *precise meaning* of physical equivalence of all inertial reference frames in PR_2?

In order to obtain a rigorous mathematical and physical meaning of the SPR and the GPR and to see if these are really true laws of Nature we have recently [7,8] give a general formulation of spacetime theory T as a "species of structure" in the sense of Bourbaki together with a physical interpretation of the structure. It is then possible to clarify the concepts of passive and active covariance of T under the action of the manifold mapping group (G_M). For each T we define also an *invariance* (or *symmetry) group* $G_I T$, and in general $G_I T \neq G_M$. This group $(G_I T)$ is defined once we realize that for each $\tau \in Mod\ T$ each explicit geometrical object defining the structure can be classified in *absolute* or *dynamical.* All spacetime theories possess also *implicit* geometrical objects that do not appear explicitly in the structure. These implicit objects are not absolute nor dynamical. Among them are the *reference frame fields*, i.e., time-like vector fields $X \in TU, U \subseteq M$, where M is a 4-dimensional manifold which is part of ST, for each $\tau \subset Mod\ T$, called spacetime. We give a physically motivated definition of *equivalent reference frames* and introduce the concept of the *equivalence group of a class of reference frames of kind X according to T, $G_X T$*. We define that T admits a *Weak Principle of Relativity* (WPR) only if $G_X T \neq$ identity for some X. If $G_X T = G_I T$ for some X we say that T admits a *Strong Principle of Relativity* (PR).

We also define the precise meaning of the *covariance group*(*) of a system of differential equations in $\mathbf{R}^4$. We introduce also Maxwell–Lorentz Electrodynamics T_{LME} that has a model,

$$\tau_{LME} = \langle M, g, D, F, J, \{\sigma, m, e\}\rangle$$

where (M, g, D) is a Lorentzian manifold modelling space-time, $\{\sigma, m, e\}$ is the set of all charged particles, $F \in \Lambda^2(T^*M)$, $J \in \Lambda^1(T^*M)$ and the *proper axioms* are

$$\mathbf{R}(D) = 0 \ , \ D(g) = 0 \ , \ (\delta F = -J \ ; \ dF = 0)$$

(*) We analyse some of these notions generalizing the approach of Anderson for the first time in [9] where $\mathbf{R}$ is the Riemann tensor. These axioms imply that $mD_{\sigma_*}\sigma_* = e\sigma_* \rfloor F$ [10,11].

[It is interesting to quote that we can identify a Lorentzian metric in the Newtonian spacetime structure and then we can also formulate Maxwell's equation in intrinsic form. These result as above. However the motion's equation is not $mD_{\sigma_*}\sigma_* = e\sigma_* \rfloor F$. It contains a new term coupling of σ_* with the absolute vector fielf of Newtonian spacetime.[11]]

The absolute objects of T_{MLE} are D and g. From this we can show that it holds,

Proposition (A): (i) The *invariance group* of T_{MLE} is the Poincaré group.

(ii) The *covariance group* of the *standard formulation* of T_{MLE} is the Poincaré group.

(iii) The *equivalence group* of the class of inertial frames according to T_{MLE} is the Poincaré group.

It is the coincidence of the these groups in the Proposition (A) that generated the non trivial confusion concerning the concepts of covariance, invariance and the proper meaning of a Principle of Relativity in a spacetime theory T. In particular, from Proposition (A) we can show the equivalence of $\underline{\mathrm{PR}_1}$, $\underline{\mathrm{PR}_2}$ and $\underline{\mathrm{PR}_3}$ [7,8].

Now, we said that each spacetime theory T possess intrinsic geometrical objects that do not appear explicity in the structure. The most important kind of *intrinsic* objects are the reference frames fields (rf) in $\langle M, g, D\rangle$. A rf is *defined* as time-like vector field $Q \in TU$, $U \subseteq M$ such that each integral line is an observer and such that

$$g(Q, Q) = 1$$

Given $U \subseteq M$, there are, of course, an infinity of charts *(coordinate systems)* $\langle x^\mu \rangle : U \to \mathbf{R}^4$ of the maximal oriented atlas of M covering U. We have,

Definition: A chart in $V \subset U \subseteq M$ is said to be a naturally adapted coordinate system to a reference frame $Q \in TU$ (nacs/Q) if in the natural coordinate base of TV associated with the chart the space-like components of Q are null.

Observation: Note that given a rf Q there are in general an infinity of $\langle$ nacs/$Q\rangle$. Each $\langle$ nacs/$Q\rangle$ define a coordinate gauge [12, 13].

For a given spacetime theory T we can use all possible coordinate gauges. Coordinates after all are labels; not the physics!

Obviously in certain particular problem the use of a specific *coordinate gauge* can simplify its resolution. In particular in Special Relativity (SR) the Einstein-Lorentz coordinate gauge (t, x^i) where t are measured by a set of standard clocks at rest in Q and x are distances along three orthogonal directions have a preferred status [14, 15]. But in SR we can use other coordinate gauges like Marinov gauge or Galileo gauge [13].

It is very important that we can give an intrinsic classification of reference frames Q according to their acceleration, rotation, shear and expansion, and according to their synchronizability [14]. We have

Proposition: Let $\mathcal{U} \in TU$, $U \subseteq M$ be a rf in $\langle M, g, D\rangle$ and let $u = g(\mathcal{U},\ )$ be the one-form field ($u \in T^*U$) physically equivalent to $\mathcal{U}$. Then,

$$(Du)_x = a \otimes u + \sigma + \omega + \frac{1}{3}\theta h$$

where, $a = g(A,\ )$. $A = (D_u\mathcal{U})_x$ is the *acceleration;* $\omega \in H_x^* \otimes H_x^*$ is an antisymmetric tensor called rotation tensor; $\sigma \in H_x^* \otimes H_x^*$ is a tensor with zero trace relative to $h_x = g|_{H_x}$, called the *shear tensor;* $\theta = (div\ u)_x$ is called the expansion and,

$$T_xM = [\mathcal{U}_x] \bigoplus [\mathcal{U}_x]^1 \ ; \ [\mathcal{U}_x]^1 = H_x$$

H_x is called the rest space of the *instantaneous observer* (i.o.)*. The direct sum is called the associated orthogonal decomposition. h_x is the metric in H_x determined by the instantaneous observer $(x, \mathcal{U}_x)$ and $h_x(X,Y) = g_x(pX, pY)$, $\forall X, Y \in T_xM$; $p : T_xM \to H_x$.

In a natural coordinate basis we have

$$\omega_{\mu\nu} = \frac{1}{2}(u_{\sigma;\tau} - u_{\tau;\sigma})h^\sigma_\mu h^\tau_\nu$$

(*) An i.o. at $x \in M$ is a pair $(x, \mathcal{U}_x)$.

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\sigma;\tau} + u_{\tau;\sigma})h^\sigma_\mu h^\tau_\nu - \frac{1}{3}\theta h_{\mu\nu}$$
$$\theta = u^\alpha_{;\alpha} = div\, u$$

It is important to observe that if we choose in T_xU a moving frame $(\mathcal{U}, X_1, X_2, X_3)$ we get

$$a^i = \gamma^i_{00} \; ; \; \omega_{ij} = \frac{1}{2}(\gamma^0_{ij} - \gamma^0_{ji}) \; ; \; \sigma_{ij} = \frac{1}{2}(\gamma^0_{ij} + \gamma^0_{ji}) - \frac{1}{3}\theta\delta_{ij} \; ; \; \theta = \gamma^j_{j0}$$

which shows clearly the physical meaning of the components of the connection in a moving frame.

Observation: We note that the rotation tensor has a very simple definition in terms of differential forms, being equivalent in physical contain to a vector field Ω orthogonal to $\mathcal{U}$. Indeed, let $u = g(\mathcal{U}, \ )$ and $\widetilde{\Omega} = du \wedge u$.

Definition: The rotation vector associated to $\mathcal{U}$ is

$$\Omega = \widehat{g}(*(du \wedge u), \ )$$

where $\widehat{g}$ is the metric of the contangent bundle.
The classification of reference frames according to their synchronizatibility is:

Let be $\alpha = g(Q, \ )$. We have the

Definiton: [14, 16]

(i) Q is locally synchronizable if and only if $d\alpha \wedge \alpha = 0$.

(ii) Q is locally proper-time synchronizable if and only if $d\alpha = 0$.

(iii) Q is synchronizable if there are mappings $f : M \to \mathbf{R}$ and $x^0 : M \to \mathbf{R}$, such that $f > 0$ and $\alpha = f dx^0$.

(iv) Q is proper-time synchronizable if and only if $\alpha = dx^0$.

It is clear that (ii) $\Rightarrow$ (i), and (iv) $\Rightarrow$ (ii) and the reciprocals are valid only locally.

Definition: When Q is synchronizable (proper-time syncronizable) whatever function x^0 like in Definition 3 is called a time function (proper-time function).

The reference frames Q introduced above are mathematical instruments. This means that a given frame does not need to have a material support in all points of the world manifold. An example will ilustrate this point. Let $V_4 = (M, g, D)$ be a flat Lorentzian manifold, namely Minkowski spacetime. Let $i = \partial/\partial t$ be an inertial frame(*), defined of course for all $x \in M$. Let now (t, r, ϕ, z) be the cilindrical coordinates naturally adapted to i. Then g is

$$g = dt \otimes dt - dr \otimes dr - r^2 d\phi \otimes d\phi - dz \otimes dz.$$

Let

$$Q = (1 - \omega^2 r^2)^{-1/2} \frac{\partial}{\partial t} + \omega (1 - \omega^2 r^2)^{-1/2} \frac{\partial}{\partial \phi}$$

be a reference frame defined in $U \equiv (-\infty < t < \infty\ ;\ 0 < r < \frac{1}{\omega}\ ;\ \ 0 \leq \phi < 2\pi, -\infty < z < \infty)\ \ (U \subset M)$.

Then

$$\alpha = g(Q,\ ) = (1 - \omega^2 r^2)^{-1/2} dt - \omega r^2 (1 - \omega^2 r^2)^{-1/2} d\phi$$

$$d\alpha \wedge \alpha = \frac{-2\omega r^2}{(1 - \omega^2 r^2)^{1/2}} dt \wedge dr \wedge d\phi \neq 0.$$

It follows that Q is not synchronizable. The rotation vector associated to Q is $\Omega = \frac{1}{2}\widehat{g}(*(d\alpha \wedge \alpha),\ \ ) = (1 - \omega^2 r^2)^{-1/2} \omega\ \partial/\partial z$ (where $\widehat{g}$ is the metric of the cotangent bundle T^*U , $U \subseteq M$). This means that Q is rotating with constant angular velocity ω relative to the z-azis of i. Now, Q can be *materialized* in $U \subset M$ by a solid rotating disc, but it is obvious that in U the frame i, *cannot have material support*. The rf Q defined by eq.(12) is also an example where it does not exist a (nacs/Q) such that the time-like coordinate of the system can have the meaning of proper-time registred by standard clocks at rest in Q , $\forall x \in U$.

The above discussion shows very clearly that, in general, different reference frames cannot be physically equivalent. In particular, the *General Principle of Relativity* cannot have the meaning of physical equivalence of all reference frames. (We will return to the GPR below).

Here we want to look again at PR_1 : $\langle$ All physical systems have the

(*) For an inertial frame it is $Di = 0$. Inertial frames exist iff $\mathbf{R}(\mathbf{D}) = 0$

Poincaré Group as its invariance group). We can show [17] that in order for PR_1 to be valid *no* inertial reference frame can couple with the fields describing a given physical system. If one inertial frame couples, then all must couple, otherwise a breakdown of *Lorentz invariance* occurs.

The so called Lorentz Aether Theories [12,13](*) are theories which contain a spacetime structure given by the 4-uple $\mathcal{A} = \langle M, g, D, V \rangle$ where $\langle M, g, D \rangle$ is Minkowski space and V is a preferred time-like vector field (a reference frame) that couples with the other geometrical objects that define a given particular Lorentz Aether Theory. (LAT)

In LAT, due to the coupling of V it is possible to devise *experiments* for obtaining an *internal synchronization* procedure in an inertial frame Q moving with respect to V that is *absolute*. The precise meaning of this statement is the following. Let (T, x) be a (nacs/V) and (t_M, x_M) a (nacs/Q). Then given two events $e_1, e_2 \in \mathcal{A}$,

$$T(e_1) = T(e_2) \Rightarrow t_M(e_1) = t_M(e_2) \qquad (\alpha)$$

Observation: It is obvious that if in Q we use as a (nacs/Q) (t_E, x_E), the standard Einstein-Lorentz coordinate gauge of Special Relativity, then

$$t_E(e_1) \neq t_M(e_2) \qquad (\beta)$$

In [12, 13] we showed that there are several possible hypothesis in LAT for the angular velocity of the roto-translational motion of solid bodies that implies in the absolute synchronization (eq(α)).

Even more, already in [20] we showed that in LAT it is to be expected that solid bodies when in roto-translational motion does not go instantaneously in their Lorentz-deformed version. This point has also been considered by Atkins [21] and more recently by Winterberg [22]. The issue has been also discussed by Cavalleri [23] who even said that there is an experiment being done (he don't say where ...) that finally will permits us to distinguish LAT from SR.

Here we must also mention that there are two experiments already done that use a solid in roto-translational motion. These are Marinov's

(*) See also for a recent review [28] and for experimental proposals and discussion of rotor experiments see [19] coupled mirror experiment [24] and Sherwin's experiment [25]. As all of you know Marnov's claims that he realizes an absolute synchronization, then disproving Special Relativity. Sherwin's [25] experiment is based on ideas similar to the ones behind Marinov's experiment and do not show any breakdown of Lorentz invariance. This is probably the experiment "quoted" by Cavalleri. If Marinov's experiment is correct it shows the breakdown of the "Special Principle of Relativity" (be it PR_1, PR_2 or PR_3) since it is an *obvious* theorem within SR that there does not exist any internal synchronization procedure that realizes *absolute synchronization.* Even more it can be also shown that (in SR) it does not exist any "Lorentz invariant clock" in nature [26].

We now analyse General Relativity (GR) and the *General Principle of Relativity* (GPR).

In all spacetimes $\langle M, g, D\rangle$ (modelling gravitational fields) of GR inertial reference frames do not exist. Indeed for i to be inertial: $Di = 0 \Leftrightarrow \mathbf{R(D)} = \mathbf{0}$ and $\mathbf{R(D)} \neq \mathbf{0}$ in GR.

Suppose we try to present the GPR as the statement,

GPR_3: The laws of all physical theories are represented by mathematical equations that have the same form i.e., are generally covariant in all possible reference frames (Bergman, [3], Torreti [27]).

GPR_3 has no physical contain since as shown by Kretschmann [28] any (spacetime) Theory can be made covariant by introducing some extra absolute objects.

Consider now the statement,

GPR_2: All r.f. are physically equivalent from the point of view of the laws that describe all physical phenomena nature.

GPR_2 is obvious a false statement since the above discussion shows that in general different reference frames are not equivalent.

Indeed in [17] and [29] we show that there are models of GR where even locally inertial reference frames $i_\ell(D_{i_\ell} i_\ell = 0\ ,\ \ d\alpha_{i_\ell}\wedge\alpha_{i_\ell} = 0\ ,\ \ \alpha_{i_\ell} = g(i_\ell,\ ))$

are not physically equivalent. Anderson [1] states the GPR as the statement,

GPR_1: The invariance group of GR is the manifold mapping group G_M.

With this statement and taking into account that in GR there are no absolute objects then the laws of "GR Theories" can be represented by "few" generally covariant equations involving only the dynamical objects of the theory. GPR_1 is then of great heuristical value.

Nevertheless our discussion in [7,8] shows that for *general invariance* to be true it is necessary that "identical boundary conditions" can be realized for the set of differential equations representing a given physical phenomena in all reference frames. If this is realized in nature it is something that pure mathematics can said anything. Only real experiments can show if *general invariance* is true or no.

We conclude this paper with two observations:

(A) There are now several papers in the literature that study the possibility of a breakdown of Lorentz invariance, e.g.,

(i) A. Zee, "Perhaps proton decay violates Lorentz invariance", Phys. Rev. D 25, 1864-1867 (1982).

(ii) H.B. Nielsen, "Lorentz Non-Invariance", Nuclear Phys. B 211, 289-296 (1983).

Many other possibilities and even some experimental data suggesting the breakdown of Lorentz invariance is discussed by Santilli (Il Grande Grido - Ethical Probe on Einstein's Fellows in USA, Hadronic Press 1984).

(B) There are now several experiments in electrodynamics that have been presented at the Conference "Foundations of the Mathematics and Physics in the XX Century, Perugia, Sept. 27-29 (1989)" by Papas, Graneau, Wesley suggesting that the electrodynamics is not Lorentz invariant.

It is then possible that we are finding the limit of validity of the SPR. Also no GPR exists in Nature!

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