PRIVILEGED REFERENCE FRAMES IN GENERAL RELATIVITY

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ABSTRACT. In previous paper [Rodrigues et al (1)], we characterized a space time theory T as the theory of a species of structure in the sense of Bourbaki (2). We succeded in this way in giving precise mathematical and physical meaning to the concepts, of covariance and invariance of T and in introducing the fundamental notion of equivalent reference frames, which are time-like vector fields $X \in TU$, $U \subseteq M$, where M is part of the space-time substructure ST(1,2) for each $\tau \in$ ModT. In particular we showed in the quoted reference that a space-time theory T admits a Principle of Relativity only if $G_X T$ (the equivalence group of the class of reference frames of kind X according to T) is different from the identity for some X. Here, after remembering the definition of reference frames appropriated for relativistic space-time [Rodrigues and Faria-Rosa (3) and also (1)] , we prove that there are models of General Relativity with a canonically privileged reference frame (cprf). The precise meaning of the cprf is given through Propositions 3 and 4. We show that the cprf can be physically distinguished from any other reference frame with the performance of mechanical experiments (Proposition 4). Although the predicted effects are perhaps very small to be detected within present technology, our results show that no Principle of General Relativity (meaning Physical equivalence of all reference frames) holds for General Relativity. Of particular importance is that even locally inertial reference frames are not equivalent.

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Abstract: In previous paper [Rodrigues et al (1)], we characterized a space time theory T as the theory of a species of structure in the sense of Bourbaki (2). We succeded in this way in giving precise mathematical and physical meaning to the concepts, of covariance and invariance of T and in introducing the fundamental notion of equivalent reference frames, which are time-like vector fields $X \in TU, U \subseteq M$, where M is part of the space-time substructure ST(1,2) for each $\tau \in \mathrm{Mod}\,T$. In particular we showed in the quoted reference that a space-time theory T admits a Principle of Relativity only if $G_X T$ (the equivalence group of the class of reference frames of kind X according to T) is different from the identity for some X. Here, after remembering the definition of reference frames appropriated for relativistic space-time [Rodrigues and Faria-Rosa (3) and also (1)], we prove that there are models of General Relativity with a canonically privileged reference frame (cprf). The precise meaning of the cprf is given through Propositions 3 and 4. We show that the cprf can be physically distinguished from any other reference frame with the performance of mechanical experiments (Proposition 4). Although the predicted effects are perhaps very small to be detected within present technology, our results show that no Principle of General Relativity (meaning physical equivalence of all reference frames) holds for General Realativity. Of particular importance is that even locally inertial reference frames are not equivalent.

1. THE CONCEPT OF REFERENCE FRAMES IN RELATIVISTIC SPACE-TIME THEORIES

Let T be a space time theory as defined in (1). Let ModT be the class of all models of T. T is said to be a relativistic theory if each $\tau \in$ ModT contains a substructure ST = $\langle M,g,D\rangle$ that is a relativistic space-time, as defined in (1,2) and in Sachs and Wu (4). We remember here that g is a Lorentz metric and D is the Levi-Civita connection of g on M.

Definition 1. Let ST be a relativistic space-time. A moving frame, in $x \in M$, is an orthonormal basis for the tangent space T_xM.

Proposition 1. Let $Q \in TM$ be a time-like vector field such that $g(Q,Q) = 1$. Then for each $x \in M$ there exists in a neigbouhood U of x three space-like vector fields such that together with Q determine a moving frame for each $x \in$ $\in U$. (The proof is trivial)

Definition 2. A particle in ST is a pair (m,σ) where σ : $R \supset I \to M$ is a future pointing causal curve [1,3,4] and $m \in [0,+\infty)$ is the rest mass. When $m = 0$ the particle is called a photon . When $m \in (0,+\infty)$ the particle is said to be a material particle. σ is said to be the world line of the particle.

The relation between $m \neq 0$ and σ is given by:

Proposition 2. For each pair (x,m), $x \in M$, $m \in (0,+\infty)$, there exists a unique curve σ: $R \supset I \to M$ with closed image $\sigma(I)$ such that $\forall u \in I$ $\sigma_* u$ is future pointing and $g(\sigma_* u, {}_* u) = m^2$.

The proof of Proposition 2 can be constructed as an extension of Proposition 0.21 of (4). Quantify $\sigma_* u$ is called the momentum of the particle.

Definition 3. An observer in $\langle M,g,D\rangle$ is a future pointing time-like curve γ: $\mathbb{R} \supset I \to M$ such that $g(\gamma_* u, \gamma_* u) = 1$. The inclusion parameter $I \to \mathbb{R}$ in this case is called the proper time along γ, which is said to be the world line of the observer.

Observation 1. The physical meaning of proper time is discussed in details in (3) which deals with the theory of time in relativistic theories.

Definition 4. An instantaneous observer is an element of TM, i.e., a pair (z,Z), $z \in M$, and $Z \in T_zM$ is a future pointing time-like vector.

The Proposition 1 together with the above definitions suggests:

Definition 5. A reference frame in $ST = \langle M,g,D\rangle$ is a time-like vector field in TU, $U \subseteq M$ such that each one of its integral lines is an observer.

2. CANONICALLY PRIVILEGED FRAMES IN GENERAL RELATIVITY

Einstein's General Relativity T_E is a theory of the gravitational field [Weinberg (5) and also (4)]. Then a typical model $\tau \in \mathrm{Mod}\,T_E$ is of the form $\tau = \langle M,g,D, T,\{(m,\sigma)\}\rangle$, where $ST = \langle M,g,D\rangle$ is a relativistic space-time and $T \in \sec T^*M \times T^*M$ is said to be the energy momentum tensor. T represents the material and energetic content of space-time, including contributions from all physical fields (with exception of the gravitational field) and particles. For what follows we do not need knowing the explicit form of T. The proper axioms of T_E are:

$$D(g) = 0; \; G = Ric - \frac{1}{2} Sg = T \qquad (1)$$

G is said to be the Einstein tensor. Ric is the Ricci tensor, S is the scalar curvature. The equation of motion of a particle (m,σ) that moves only under the influence of gravitation is:

$$D_{\sigma_*} \sigma_* = 0 \qquad (2)$$

ST is in general not flat, which implies that (in general) there do not exist inertial reference frames i, i.e., reference frames such that $(Di)_x = 0 \; \forall x \in M$. The reference frames more similar to the inertial reference frames of flat space-time are given by:

Definition 6. A reference frame $i \in TU, U\; M$ is said to be locally inertial if $D_{i_l} i_l = 0$ and $d\alpha_l \wedge \alpha_l = 0$

$$\forall x \in U \text{ and } \alpha_l = g(i_l,).$$

Observation 2. In (1,3) we classify an arbitrary reference frame Q either according to its synchronizability or according to the decomposition of $D\alpha, \alpha = g(Q,)$. This last decomposition shows that the reference frames can be characterized for each $\tau \in \mathrm{Mod}\,T_E$ according to their acceleration, rotation, shear and expansion in an absolute way (1, 3,4). It follows that in general different reference frames cannot be physically equivalent according to the definition of physical equivalence presented in (1,3). From this it follows that there does not exist a Principle of General Relativity in T_E.

Now, the physical universe we live in is well represented by metrics of the Robertson-Walker-Friedman type (6). In particular a very simple space-time structure $ST = \langle M,g,D\rangle$ that represents the main properties observed is formulated as follows: Let $M = R^3 \times I, I \subset R$ and $R: I \to (0,\infty), t \to R(t)$ and define g in M (considering M as a subset of R^4) by:

$$g = dt \otimes dt - R(t)^2 \Sigma dx^i \otimes dx^i, \ i = 1,2,3. \tag{3}$$

Then g is a Lorentzian metric in M and $\partial/\partial t$ is a time-like vector field in (M,g). Let $\langle M,g,D\rangle$ be oriented in time by $\partial/\partial t$ and space-time oriented by $dt \wedge dx^1 \wedge dx^2 \wedge dx^3$. Then $\langle M,g,D\rangle$ is a relativistic space-time for $I = (0,\infty)$. We have the

Proposition 3. Let $V \in TM$ be a future pointing time-like vector field, $g(V,V) = 1$, and an eigenvector of Einstein's tensor G in the sense that

$$G(V,) = fg(V,) \tag{4}$$

for some real function $f: M \to R$. Then $V = \partial/\partial t$.

Proof: First we need to calculate the Einstein's tensor G for the metric given by eq.(3). We get:

$$G = 3\dot{R}^2 R^{-2} dt \otimes dt \ ; \ \dot{R} = \frac{d}{dt} R \tag{5}$$

Using eq.(5) in eq.(4) we obtain:

$$3\dot{R}^2R^{-2}dt(V)dt = fg(V,) \tag{6}$$

and then for each $x \in M$ we have

$$dt(V)dt = ag(V,)\ ;\ a = f(x)R^2/3\dot{R}^2 \tag{7}$$

In eq.(7) $a \in \mathbb{R}$. Also $dt(V) = 0$ since dt and V are time-like. Then $dt = bg(V,\ )$; $\mathbb{R} \supset b \neq 0$, which implies $(V)_x = e(\partial/\partial t)_x$ for some $\mathbb{R} \supset e \neq 0$ and since V is normalized and future pointing then $e = 1$. Since the above argument is valid $\forall x \in M$, then $V = \partial/\partial t$.

Observation 3. Proposition 3 appears in a particular form and in a very different context in (4).

We shall say that $V = \partial/\partial t$ is canonically preferred or privileged in the mathematical sense that it can be defined, in the particular $\tau \in \mathrm{Mod}T_E$ above, only in terms of the metric tensor g and the time orientation, without any reference to structures that $\mathbb{R}^4$ possesses but M does not.

We can show very easily that $V = \partial/\partial t$ is a locally inertial reference frame. It is canonically privileged in the physical sense that there are no other reference frame physically equivalent to it, as proved in Proposition 4. Of course, we must only show that any other locally inertial reference frame is not equivalent to V.

3. PHYSICAL NON EQUIVALENCE OF LOCALLY INERTIAL REFERENCE FRAMES

Proposition 4. In the space-time defined by eq.(2) which is a model of T_E locally inertial reference frames are not physically equivalent.

The proof of Proposition 4 can be obtained immediately with the methodology of (1). Here we prove the validity of the proposition, following Rodrigues (6), i.e., by showing that there are mechanical experiments that can distinguish between two locally inertial frames, $V = \partial/\partial t$ and

$$Z = \left(\frac{R^2+u^2}{R}\right)^{1/2} \partial/\partial t + \frac{u}{R}\,\partial/\partial x^1 \qquad (8)$$

In eq.(8) R is the function defined by eq.(2) and $\mathbb{R} \supset u \neq 0$ is a constant with the physical meaning of coordinate velocity of Z relative to V. To prove Proposition 4 we need the concept of naturally adapted coordinate system to a reference frame Q(nacs/Q). This concept has been originally introduced in (1,3). We have:

Definition 7. Let $Q \in TU, U \subseteq M$. A chart in U of the maximal oriented atlas of M is said to be a (nacs/Q) if in the natural coordinate basis of TU associated with the chart the space-like components of Q are null.

Proof of Proposition 4: (i) We start by finding a (nacs/Z). To do that we note that, if γ is an integral curve of Z, then its parametic equations can be written as

$$\frac{d}{dt}\, x^1 \circ \gamma = \frac{u}{R(R^2+u^2)^{1/2}}, \; x^2 \circ \gamma = 0; \; x^3 \circ \gamma = 0 \qquad (9)$$

(The direction $x^1 \circ \gamma$ is obviously arbitrary). We then choose for (nacs/Z) the coordinate functions $(t', x^{1'}, x^{2'}, x^{3'})$ given by:

$$x^{1'} = x^1 - u\int_0^t dt\, \frac{1}{R(R^2+u^2)^{1/2}}; \; x^{2'} = x^2$$

$$x^{3'} = x^3; \; t' = \int_0^t dt\, R(R^2+u^2)^{-1/2} - ux^{1'} \qquad (10)$$

We then get:

$$g = dt'\otimes dt' - \bar{R}(t')^2\{[\frac{1-v^2(1-\bar{R}^2)}{1-v^2}]\, dx^{1'}\otimes dx^{1'} + dx^{2'}\otimes dx^{2'} +$$

$$dx^{3'}\otimes dx^{3'}\} \qquad (11)$$

where $\bar{R}(t') = R(t(t'))$ and $v = R(\frac{d}{dt} x^1 o \gamma)_{t=0} = u(1+u^2)^{-1/2}$ is the initial metric velocity of Z relative to V.

(ii) The solution of the equation of motion for a free particle (m,σ) in V, with the initial conditions: $x^i o\sigma(0) = 0$, $i = 1,2,3$ and $\frac{d}{dt} x^i \; o\sigma(0) = u\, \partial/\partial x^i$ and $\frac{d}{dt} x^j o\, \sigma(0) = 0$, $j \neq i$, is given by an equation analogous to eq.(9). The accelerations are such that $\frac{d^2}{dt^2} x^j \, o\, \sigma(t) = 0$ for $j \neq i$.

(iii) The solution of the equation of motion for a free particle (m,σ') in Z can be found in the case $R(t) = 1 + At + \ldots$, $A \ll 1$. We get for motion in the $(x^{1'}, x^{2'})$ plane:

(a)

If $\bar{R}(\frac{d}{dt} x^{2'} o\sigma')_{t=0} = 0$; $R(\frac{d}{dt} x^{1'} o\sigma')_{t=0} = v$; $\bar{R} \frac{d}{dt} x^{1'} o\sigma\, (t') = v_1'$,

the measured acceleration in Z will be:

$$\frac{d^2}{dt^2}(\bar{R}\, x^{1'} \; o\, \sigma'(t')) = -A(1-v^2)^{1/2} \; v_1'(1 - v_1'^2)^{1/2} \; ;$$

$$\frac{d^2}{dt^2}(\bar{R}\, x^{2'} \; o\sigma'(t')) = 0 \; ; \qquad (12)$$

(b)

If $\bar{R}(\frac{d}{dt} x^{1'} o\sigma')_{t=0} = 0$; $R(\frac{d}{dt} x^{2'} o\sigma')_{t=0} = v$; $\bar{R} \frac{d}{dt} x^{2'} o\sigma'(t') = v_2'$,

the measured acceleration in Z will be:

$$\frac{d^2}{dt^2}(\bar{R}\, x^{1'} \; o\, \sigma'(t')) = v\, Av_2'/(1 - x^2)^{1/2} \; ;$$

$$\frac{d^2}{dt^2}(\bar{R}\, x^{2'} \; o\, \sigma'(t')) = -Av_2'(1 - v_2'^2)/(1 - v^2)^{1/2} \; . \qquad (13)$$

From (ii) and equations (12) and (13) is follows that V and and Z are not physically equivalent.

4. CONCLUSIONS

We noticed that reference frames in $ST = \langle M,g,D\rangle$ can be classified according to their acceleration, rotation, shear and expansion (these concepts being absolute in the sense that they depend only on D) or according to their synchronizability. This means that, given two arbitrary reference frames, they are not in general physically equivalent according to the definitions of (1,3). In this paper , even without using the methodology of (1,3), we succeded in proving (Proposition 4) that there are models of T_E in which even locally inertial reference frames are not equivalent. We conclude that no Principle of Relativity (in the sense of the physical equivalence of all reference frames) hold for T_E. All reference frames are however mathematically equivalent , a trivial consequence of the mathematical structure associated with T_E (general covariance according to the methodology of (1)). Of course , physical equivalence and mathematical equivalence are completely different concepts, it being a misunderstanding trying to associate general covariance with a Principle of Relativity

According to Einstein (7,8,9,10,11,12,13,14,16)General relativity is a theory of the aether! However, Einstein had the wrong opinion that it would be impossible to associate a privileged reference frame to the aether and then that his "relativistic aether would not violate the Principle of Relativity. For more details on this point see Kostro (17)

In a letter by Einstein to Lorentz (17) we read:

"... the general relativity theory is nearer to an aether hypothesis than is special relativity theory. However, this new aether theory would not violate the principle of relativity, because its state $g_{\mu\nu}$ = aether would not be of a rigid body in an independent state of motion, but its state of motion would be a function of position determined via material processes."

We see that the origin of Einstein's wrong statement is the fact that he did not know how to characterize mathematically reference frames. If he knew that a reference frame must be characterized by a time-like vector field $Q \in TM$, as done above, he would realized from the decomposition of

$\mathfrak{p}\alpha$, $\alpha = g(Q, \;)$ that in General Relativity reference frames (in general) do not have the properties of rigid bodies. From our analysis in (1,3) and also in Rodrigues and Tiommo (18,19), Maciel and Tiommo (20) and Witenberg (21), we arrive at the conclusion that even breakdown of Lorentz invariance is to be expected in experiments involving the coupling of light and the roto-translational motion of solid bodies.

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