POLARIZED PARTITION RELATIONS OF HIGHER DIMENSION

Walter Alexandre Carnielli
and
Carlos Augusto Di Prisco

RELATÓRIO TÉCNICO Nº 12/88

ABSTRACT. We consider polarized partition relations concerning partitions into an infinite number of pieces and also partitions defined on products of higher dimension. We use an infinite version of the method of induced coloring which is frequent in Finite Ramsey Theory. Sufficient conditions on cardinals $\lambda_1, \lambda_2, \ldots, \lambda_n, \beta$ are given in order to satisfy the polarized partition relation

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_n \end{bmatrix} \to \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_n \end{bmatrix}^{1,1,\ldots,1}_{\beta}$$

It is shown that the simplest infinite dimensional polarized partition relations fail under the assumption of the axiom of choice, and that under certain large cardinal hypotesis, there are valid polarized partition relations defined on the union of all the finite dimensional powers of a cardinal.

Universidade Estadual de Campinas
Instituto de Matemática, Estatística e Ciência da Computação
IMECC – UNICAMP
Caixa Postal 6065
13.081 - Campinas, SP
BRAZIL

Junho – 1988

The systematic study of polarized partition relations was initiated by Erdös and Rado in [ER]. Sierpnski had already obtained some results in [Si]. Later on, Erdös, Hajnal and Rado pursued this study in another article [EHR], investigating mainly polarized partition relations defined in products of dimension 2. In this note we will discuss some cases of partitions of higher dimension using a method inspired in the finite Ramsey Theory (see [G.R.S]).

Definition: Let $\kappa, \lambda, \alpha, \beta$ and δ be cardinals. The partition symbol

$$\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_n\end{bmatrix} \to \begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_n\end{bmatrix}^{k_1,k_2,\dots,k_n}_{\delta}$$

means that for every $F: [\lambda_1]^{k_1} \times [\lambda_2]^{k_2} \times \dots \times [\lambda_n]^{k_n} \to \delta$, there are sets $H_1 \subset \lambda_1$, $H_2 \subset \lambda_2, \dots, H_n \subset \lambda_n$ with $|H_i| = \alpha_i$ for $i=1,\dots,n$, and $\xi \in \delta$, such that $F''([H_1]^{k_1} \times [H_2]^{k_2} \times \dots \times [H_n]^{k_n}) = \{\xi\}$.

Proposition (Monotonicity): If $\lambda_i \leq \lambda'_i$, $\alpha_i \leq \alpha'_i$, $k'_i \leq k_i$ for all i, $1 \leq i \leq n$, and $\delta' \leq \delta$, then

$$\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_n\end{bmatrix} \to \begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_n\end{bmatrix}^{k_1,k_2,\dots,k_n}_{\delta} \quad \text{implies} \quad \begin{bmatrix}\lambda'_1\\ \lambda'_2\\ \vdots\\ \lambda'_n\end{bmatrix} \to \begin{bmatrix}\alpha'_1\\ \alpha'_2\\ \vdots\\ \alpha'_n\end{bmatrix}^{k'_1,k'_2,\dots,k'_n}_{\delta'}$$

Proof: Follows immediately from the definition.□

It is also easy to show that if $n \leq m$,

$$\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_m\end{bmatrix} \to \begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_m\end{bmatrix}^{k_1,k_2,\dots,k_m}_{\delta} \quad \text{implies} \quad \begin{bmatrix}\lambda'_1\\ \lambda'_2\\ \vdots\\ \lambda'_n\end{bmatrix} \to \begin{bmatrix}\alpha'_1\\ \alpha'_2\\ \vdots\\ \alpha'_n\end{bmatrix}^{k'_1,k'_2,\dots,k'_m}_{\delta'}$$

The first infinite case of a multidimentional polarized partition fails, indeed,

$$\begin{bmatrix}\aleph_0\\ \aleph_0\end{bmatrix} \to \begin{bmatrix}\aleph_0\\ \aleph_0\end{bmatrix}^{1,1}$$

is easily seen to be false by defining $F(i,j)=0$ if $i \leq j$ and $F(i,j)=1$ otherwise.

In this paper we will restrict our attention to the case in which $k_1=k_2=\dots=k_n=1$, and in this case we may omit the exponents in the partition symbol.

We start by proving a lemma concerning products of dimension two (i.e. $n=2$)

Lemma: Let κ, λ be infinite cardinals such that $2^\kappa < \operatorname{cof}\lambda$, then, for any $\delta < \operatorname{cof}\lambda$,

$$\begin{bmatrix} \kappa \\ \lambda \end{bmatrix} \rightarrow \begin{bmatrix} \kappa \\ \lambda \end{bmatrix}_{\delta}.$$

Proof: Let $F:\kappa\times\lambda\rightarrow\delta$ be given. Define an equivalence relation $\sim$ on λ by $\alpha\sim\beta \iff \forall\xi<\kappa\ F(\xi,\alpha)=F(\xi,\beta)$. The number of equivalence classes is given by $|^{\kappa}\delta|=\delta^{\kappa}=2^{\kappa}<\operatorname{cof}\lambda$. Therefore, there is an equivalence class B of cardinality λ.

Define now $G:\kappa\rightarrow\delta$ by $G(\xi)=F(\xi,\alpha)$ for any $\alpha\in B$. Since $\operatorname{cof}\kappa>\delta$, there must be a set $A\subset\kappa$, $|A|=\kappa$, on which G is constant. Clearly, $A\times B$ is homogeneous for F. □

This result improves Lemma 4.2.6 of [W] which only deals with partitions into two pieces.

We will now consider polarized partition relations of higher dimension, the previous proof is extended to this case.

Lemma: Suppose $\lambda_1, \lambda_2, \ldots, \lambda_n$ are infinite cardinals such that $\operatorname{cof}\lambda_{i+1}>2^{\lambda_i}$ for $i=1,2,\ldots,n$. Then

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \rightarrow \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}^{1,1,\cdots,1}_{\beta}$$

for all $\beta<\operatorname{cof}\lambda_1$.

Proof: The proof is by induction on $n\geq2$. For $n=2$ we have the result by the previous proposition.

Suppose the result holds for $n=k$, and let $F:\lambda_1\times\lambda_2\times\ldots\times\lambda_{k+1}\rightarrow\beta$. As before, we define an equivalence relation $\sim$ on λ_{k+1} by:

$\alpha\sim\beta \iff$ For all $(\alpha_1, \alpha_2, \ldots,\alpha_k)\in\lambda_1\times\lambda_2\times\ldots\times\lambda_k$, $F(\alpha_1, \alpha_2, \ldots,\alpha_k,\alpha) = F(\alpha_1, \alpha_2, \ldots,\alpha_k,\beta)$.

The number of equivalence classes is the cardinality of ${}^{(\lambda_1\times\lambda_2\times\ldots\times\lambda_k)}\beta$, which is $|{}^{(\lambda_k)}\beta| = 2^{\lambda_k}$. By our hypothesis, $2^{\lambda_k} < \operatorname{cof}(\lambda_{k+1})$, and thus there is a class $H_k\subset\lambda_k$ of cardinality λ_k.

Define now $G:\lambda_1\times\lambda_2\times\ldots\times\lambda_k\rightarrow\beta$ by $G(\alpha_1, \alpha_2, \ldots,\alpha_k) = F(\alpha_1, \alpha_2, \ldots,\alpha_k,\alpha)$ for any $\alpha\in H_k$.

By the inductive hypothesis, there are $H_1\subset\lambda_1$, $H_2\subset\lambda_2$, ..., and $H_k\subset\lambda_k$, with $|H_i|=\lambda_i$ for $i= 1,2,\ldots,k$ such that G is constant on $H_1\times H_2\times\ldots\times H_k$.

Then $H_1\times H_2\times\ldots\times H_k\times H_{k+1}$ is homogeneous for F. □

Theorem: If κ is a strongly inaccessible cardinal,

Lemma: Let κ, λ be infinite cardinals such that $2^\kappa < \operatorname{cof}\lambda$, then, for any $\delta < \operatorname{cof}\lambda$,

$$\begin{bmatrix} \kappa \\ \lambda \end{bmatrix} \to \begin{bmatrix} \kappa \\ \lambda \end{bmatrix}_{\delta}.$$

Proof: Let $F: \kappa \times \lambda \to \delta$ be given. Define an equivalence relation $\sim$ on λ by $\alpha \sim \beta \iff \forall \xi < \kappa\ F(\xi, \alpha) = F(\xi, \beta)$. The number of equivalence classes is given by $|^{\kappa}\delta| = \delta^\kappa = 2^\kappa < \operatorname{cof}\lambda$. Therefore, there is an equivalence class B of cardinality λ.

Define now $G: \kappa \to \delta$ by $G(\xi) = F(\xi, \alpha)$ for any $\alpha \in B$. Since $\operatorname{cof}\kappa > \delta$, there must be a set $A \subset \kappa$, $|A| = \kappa$, on which G is constant. Clearly, $A \times B$ is homogeneous for F. □

This result improves Lemma 4.2.6 of [W] which only deals with partitions into two pieces.

We will now consider polarized partition relations of higher dimension, the previous proof is extended to this case.

Lemma: Suppose $\lambda_1, \lambda_2, \ldots, \lambda_n$ are infinite cardinals such that $\operatorname{cof}\lambda_{i+1} > 2^{\lambda_i}$ for $i = 1, 2, \ldots, n$. Then

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \to \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}^{1,1,\cdots,1}_{\beta}$$

for all $\beta < \operatorname{cof}\lambda_1$.

Proof: The proof is by induction on $n \geq 2$. For $n=2$ we have the result by the previous proposition.

Suppose the result holds for $n=k$, and let $F: \lambda_1 \times \lambda_2 \times \ldots \times \lambda_{k+1} \to \beta$. As before, we define an equivalence relation $\sim$ on λ_{k+1} by:

$\alpha \sim \beta \iff$ For all $(\alpha_1, \alpha_2, \ldots, \alpha_k) \in \lambda_1 \times \lambda_2 \times \ldots \times \lambda_k$, $F(\alpha_1, \alpha_2, \ldots, \alpha_k, \alpha) = F(\alpha_1, \alpha_2, \ldots, \alpha_k, \beta)$.

The number of equivalence classes is the cardinality of ${}^{(\lambda_1 \times \lambda_2 \times \ldots \times \lambda_k)}\beta$, which is $|{}^{(\lambda_k)}\beta| = 2^{\lambda_k}$. By our hypothesis, $2^{\lambda_k} < \operatorname{cof}(\lambda_{k+1})$, and thus there is a class $H_k \subset \lambda_k$ of cardinality λ_k.

Define now $G: \lambda_1 \times \lambda_2 \times \ldots \times \lambda_k \to \beta$ by $G(\alpha_1, \alpha_2, \ldots, \alpha_k) = F(\alpha_1, \alpha_2, \ldots, \alpha_k, \alpha)$ for any $\alpha \in H_k$.

By the inductive hypothesis, there are $H_1 \subset \lambda_1$, $H_2 \subset \lambda_2, \ldots$, and $H_k \subset \lambda_k$, with $|H_i| = \lambda_i$ for $i = 1, 2, \ldots, k$ such that G is constant on $H_1 \times H_2 \times \ldots \times H_k$.

Then $H_1 \times H_2 \times \ldots \times H_k \times H_{k+1}$ is homogeneous for F.□

Theorem: If κ is a strongly inaccessible cardinal,

$$\begin{bmatrix}\kappa\\ \kappa\\ \vdots\\ \kappa\end{bmatrix} \to \begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_n\end{bmatrix}_\beta \quad \text{for all } \alpha_1, \alpha_2, \ldots, \alpha_n, \beta \ll \kappa \text{ and } n \in \omega.$$

Proof: Take $\lambda \ll \kappa$ such that $\alpha_1, \alpha_2, \ldots, \alpha_n < \lambda$ and $\beta < \mathrm{cof}\lambda$ (in fact, we can take λ to be regular)

Put $T(\delta) = (2^\delta)^+$, and in general, $T^{m+1}(\delta) = T(T^m(\delta))$ for each cardinal δ. By the lemma we have

$$\begin{bmatrix}\lambda\\ T(\lambda)\\ \vdots\\ T^{n-1}(\lambda)\end{bmatrix} \to \begin{bmatrix}\lambda\\ T(\lambda)\\ \vdots\\ T^{n-1}(\lambda)\end{bmatrix}_\beta$$

The monotonicity of the partition relation and the inaccessibility of κ imply the desired result. □

We would like to consider now products of infinite dimension. With the axiom of choice, the simplest infinite dimensional polarized partition relations are false:

Theorem (AC): For all $\kappa \geq 2$,

$$\begin{bmatrix}\kappa\\ \kappa\\ \vdots\end{bmatrix} \not\to \begin{bmatrix}2\\ 2\\ \vdots\end{bmatrix}_2.$$

Proof: Define an equivalence relation on κ^ω by

$f \sim g \iff \exists m\ f{\downarrow}m = g{\downarrow}m$ (where $f{\downarrow}m = f \upharpoonright \{n \in \omega \mid n \geq m\}$).

Pick one element of each equivalence class, and given $f \in \kappa^\omega$, denote by $\hat{f}$ the chosen element from $[f]_\sim$, the class of f.

Given $f \in \kappa^\omega$, let n_f be the least $m \in \omega$ such that $f{\downarrow}m = \hat{f}{\downarrow}m$. Define a partition $F: \kappa^\omega \to 2$ by

$$F(f) = \begin{cases} 0 \text{ if } n_f \text{ is even} \\ 1 \text{ if } n_f \text{ is odd} \end{cases}$$

Let $\{H_i \mid i \in \omega\}$ be a collection of subsets of κ such that $|H_i| \geq 2$ for each $i \in \omega$. We will show that the product $\Pi_{i \in \omega} H_i$ is not homogeneous for F.

Let $f \in \Pi_{i \in \omega} H_i$. The functions f and $\hat{f}$ coincide from n_f on: $f(k) = \hat{f}(k)$ for all $k \geq n_f$. Define $g \in \Pi_{i \in \omega} H_i$ by $g(n) = f(n)$ for $n \neq n_f$ and $g(n_f)$ = some element of H_{n_f} different from $f(n_f)$. Clearly, $g \sim f$ (and thus $\hat{f} = \hat{g}$), but $n_g = n_f + 1$. Thus $F(f) \neq F(g)$. □

Nevertheless, some polarized partition relations defined on the union of all the finite dimensional products of a cardinal κ can be interesting.

First, we state without proof the following technical lemma:

Lemma: Given measurable cardinals $\kappa_1 < \kappa_2$ and measures μ_1 and μ_2 on κ_1 and κ_2 respectively, the filter on $\kappa_1 \times \kappa_2$ generated by the sets $C \subset \kappa_1 \times \kappa_2$ satisfying
$\mu_1(\{\alpha \in \kappa_1 \mid \mu_2(\{\beta \in \kappa_2 \mid (\alpha,\beta) \in C\})\}) = 1$
is a κ_1-complete ultrafilter. □

Definition: The partition symbol

$$\begin{bmatrix}\kappa\\ \kappa\\ \vdots\end{bmatrix} \overset{<\omega}{\rightarrow} \begin{bmatrix}\lambda_0\\ \lambda_1\\ \vdots\end{bmatrix}_2$$

means that for every function $F: \bigcup_{n \in \omega} \kappa^n \rightarrow 2$, there are sets $H_1, H_2, \ldots, H_n, \ldots$ all subsets of κ such that for all $i \geq 1$, $|H_i| = \lambda_i$ and for all $n \geq 1$ F is constant on $H_1 \times H_2 \times \ldots \times H_n$.

Theorem: Suppose there is an ω-sequence $\kappa_0 < \kappa_1 < \ldots < \kappa_n < \ldots$ of measurable cardinals below a cardinal κ, then

$$\begin{bmatrix}\kappa\\ \kappa\\ \vdots\end{bmatrix} \overset{<\omega}{\rightarrow} \begin{bmatrix}\lambda_0\\ \lambda_1\\ \vdots\end{bmatrix}_2$$

for any sequence of cardinals $\lambda_0, \lambda_1, \ldots$ such that $\lambda_i < \bigcup_{n \in \omega} \kappa_n$ for every $i \in \omega$.

Proof: We will show $\begin{bmatrix}\kappa\\ \kappa\\ \vdots\end{bmatrix} \overset{<\omega}{\rightarrow} \begin{bmatrix}\kappa_0\\ \kappa_1\\ \vdots\end{bmatrix}_2$

From here the statement of the theorem by picking the appropriate subsequence of the sequence of measurable cardinals.

Let $F: \bigcup_{n \in \omega} \kappa^n \rightarrow 2$. Let μ_i be a measure on κ_i. We will construct sets $H_0, H_1, \ldots$ with the required properties. For this purpose, we will define sets A_i^n for all $n \in \omega$ and $i \leq n$, satisfying $A_i^{n+1} \subset A_i^n$ for $i \leq n$ $(n \in \omega)$; and for every n, $A_i^n \subset \kappa_i$ and $\mu_i(A_i^n) = 1$.

For every $i \in \omega$, the set H_i will be defined by $H_i = \bigcap_{n \geq i} A_i^n$.

Consider $F \upharpoonright \kappa_0$. One of the sets $\{\alpha < \kappa_0 \mid F(\alpha) = 0\}$ or $\{\alpha < \kappa_0 \mid F(\alpha) = 1\}$ has measure one with respect to μ_0. Call it A_0^0, and let i_0 be the constant value of F on A_0^0.

To define A_0^1 and A_1^1, notice that for each $\alpha\in A_0^0$, $\{\beta\in\kappa_1 | F(\alpha,\beta)=0\}$ has μ_1-measure one or $\{\beta\in\kappa_1 | F(\alpha,\beta)=1\}$ has μ_1-measure one. In the first case we call α a 0-point, and in the second we say it is a 1-point. The set A_0^1 is the set of 0-points or the set of 1-points, whichever has measure one with respect to μ_0. Let i_1 be the appropriate value, i.e. A_0^1 is the set of i_1-points in A_0^0 and $\mu_0(A_0^1)=1$.

Now, for each $\alpha\in A_0^1$, $\mu_1(\{\beta\in\kappa_1 | F(\alpha,\beta)=i_1\})=1$, put

$A_1^1 = \bigcap_{\alpha\in A_0^1} \{\beta\in\kappa_1 | F(\alpha,\beta)=i_1\}$. Note that since $\kappa_0 = |A_0^1| < \kappa_1$ and μ_1 is κ_1-complete, $\mu_1(A_1^1)=1$.

We proceed inductively. Suppose we have defined $A_0^k, A_1^k, \dots, A_k^k$ such that F is constant on $A_0^k \times A_1^k \times \dots \times A_i^k$ and $\mu_i(A_i^k)=1$ for every $i\leq k$.

Classify the k+1-tuples of $A_0^k \times A_1^k \times \dots \times A_k^k$ in two classes: tuples of type 0 and tuples of type 1 according to which of the sets, $\{\beta\in\kappa_{k+1} | F(\alpha_0, \alpha_1, \dots, \alpha_k, \beta)=0\}$ or $\{\beta\in\kappa_{k+1} | F(\alpha_0, \alpha_1, \dots, \alpha_k, \beta)=1\}$ has measure one with respect to μ_{k+1}. Let $i_{k+1}\in\{0,1\}$ be such that the set of tuples of type i_{k+1} of $A_0^k, A_1^k, \dots, A_k^k$ has measure one with respect to the measure $\mu_0 x \mu_1 x \dots x \mu_k$ on $\kappa_0 \times \kappa_1 \times \dots \times \kappa_k$. We thus have: $\mu_0(\{\alpha_0\in A_0^k \mid \mu_1(\{\alpha_1\in A_1^k | \dots \mu_k(\{\alpha_k\in A_k^k | \mu_{k+1}(\{\beta\in\kappa_{k+1} | F(\alpha_0, \alpha_1, \dots, \alpha_k, \beta) = i_{k+1}\})=1\})=1 \dots \})=1\})=1$.

For each tuple $(\alpha_0, \alpha_1, \dots, \alpha_i) \in A_0^k \times A_1^k \times \dots \times A_i^k$ $(i<k)$, call $B^k_{(\alpha_0, \alpha_1, \dots, \alpha_i)} = \{\alpha_{i+1}\in A_{i+1}^k | \mu_{i+2}(\{\alpha_{i+2}\in A_{i+2}^k | \dots \mu_{k+1}(\{\beta\in\kappa_{k+1} | F(\alpha_0, \dots, \alpha_i, \alpha_{i+1}, \dots, \alpha_k, \beta) = i_{k+1}\})=1\}) \dots \})=1\}$. And for $(\alpha_0, \alpha_1, \dots, \alpha_k)$, $B^k_{(\alpha_0, \alpha_1, \dots, \alpha_k)} = \{\beta\in\kappa_{k+1} | F(\alpha_0, \alpha_1, \dots, \alpha_k, \beta)=i_{k+1}\}$.

We put $A_0^{k+1} = \{\alpha\in A_0^k | \mu_1(B_\alpha^k)=1\}$,

$A_1^{k+1} = \bigcap_{\alpha\in A_0^{k+1}} B_\alpha^k$.

$$A_2^{k+1} = \bigcap_{(\alpha_0, \alpha_2) \in A_0^{k+1} \times A_2^{k+1}} B^k_{(\alpha_0, \alpha_2)}.$$

and in general,

$$A_i^{k+1} = \bigcap_{(\alpha_0, \alpha_1, \dots, \alpha_{i-1}) \in A_0^{k+1} \times \dots \times A_{i-1}^{k+1}} B^k_{(\alpha_0, \dots, \alpha_{i-1})} \text{ (for } i \leq k)$$

finally,

$$A_{k+1}^{k+1} = \bigcap_{(\alpha_0, \alpha_1, \dots, \alpha_k) \in A_0^{k+1} \times \dots \times A_k^{k+1}} B^k_{(\alpha_0, \dots, \alpha_k)}.$$

By the completeness property of each μ_i, for all $i \leq k+1$, $\mu_i(A_i^{k+1}) = 1$.

To complete the proof, put, for each $i \in \omega$, $H_i = \bigcap_{n \geq i} A_i^n$. Clearly, for every $i \in \omega$, $\mu_i(H_i) = 1$ and for every $n \in \omega$, $F'' H_0 \times H_1 \times \dots \times H_n = \{i_n\}$. □

We would like to point out that finite dimensional polarized partitions with finite parameters on the right hand side of the arrow have finite solutions, that is, if $c_1, c_2, \dots, c_n, \beta$ are finite cardinals, then there is a finite cardinal k such that

$$\begin{bmatrix} k \\ k \\ \vdots \\ k \end{bmatrix} \to \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}_\beta .$$

As an example, k=5 is a solution for the partition property

$$\begin{bmatrix} k \\ k \end{bmatrix} \to \begin{bmatrix} 2 \\ 2 \end{bmatrix}_2$$

and some k, $8 \leq k \leq 20$ is a solution for

$$\begin{bmatrix} k \\ k \\ k \end{bmatrix} \to \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}_2$$

(see, for example, [Ca]).

REFERENCES

[Ca] Carnielli, W. A.: Ramsey type theorems for cubes. Technical Report, University of Campinas

[ER] P. Erdös and R. Rado: A partition calculus in set theory. Bull Amer. Math. Soc. 62 (1956) 427-489.

[EHR] P. Erdös, A. Hajnal and R. Rado : Partition relations for cardinal numbers. Acta Mat. Acad. Sci. Hungar. 16 (1965) 93-196.

[G.R.S] Graham, R.L., B.L. Rothschild and J.H. Spencer :Ramsey Theory. John Wiley & Sons, New York (1980).

[Si] W. Sierpinski: Concernant l'hypothese du continu. Acad. Royale Serbe Bull. de l'Acad. Sc. Math. et Naturelles, Ser. A Sc. Math. et Phys. 1 (1933) 67-73.

[W] N. Williams: Combinatorial Set Theory. North Holland (1977).

Walder Alexandre Carnielli
Universidade Estadual de Campinas
Instituto de Matemática
Caixa Postal 6065
13.081 Campinas, SP., Brasil

Carlos Augusto Di Prisco
Instituto Venezolano de Investigaciones Científicas
Apartado 21.827, Caracas 1020–A,
Venezuela and Universidad Central de Venezuela.

RELATÓRIOS TÉCNICOS 1988

01/88 **A Linear Continuous Transportation Problem** - *Enrique D. Andjel, Tirtha L. Lopes and José Mario Martínez.*

02/88 **A Splitting Theorem for Complete Manifolds With Non-Negative Curvature Operator** - *Maria Helena Noronha.*

03/88 **Mathematical Physics of the Generalized Monopole without String** - *W. A. Rodrigues Jr., M. A. Faria-Rosa, A. Maia Jr. and E. Recami.*

04/88 **A Family of Quasi-Newton Methods with Direct Secant Updates of Matrix Factorizations** - *José Mario Martínez.*

05/88 **Rotation Numbers of Differential Equations. A Framework in the Linear Case** - *Luiz San Martin.*

06/88 **A Geometrical Theory of non Topological Magnetic Monopoles** - *Marcio A. Faria-Rosa and Waldyr A. Rodrigues Jr.*

07/88 **Cosmic Walls and Axially Symmetric Sigma Models** - *Patricio S. Letelier and Enric Verdaguer.*

08/88 **Verificação do Nível de Enlace do Protocolo X-25** - *Célio C. Guimarães e Edmundo R. M. Madeira.*

09/88 **A Numerically Stable Reduced-Gradient Type Algorithm for Solving Large-Scale Linearly Constrained Minimization Problemas** - *Herminio Simões Gomes and José Mario Martínez.*

10/88 **On Integral Bases of Some Ring of Integers** - *Nelo D. Allan.*

11/88 **Generating Inexact-Newton Methods Using Least Change Secant Update Procedures** - *José Mario Martínez.*