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Characterization of sandwiched piezoelectric transducers - a complement for teaching electric circuits

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Abstract

This paper shows a method for simplifying the equivalent electric circuit of sandwiched piezoelectric transducers. The analysis of the transducers is performed using elementary electric network theory. The simplicity of the method enables students of introductory semesters of Physical courses to understand the operation principles of sandwiched piezoelectric transducers. Besides, the physical properties of the transducer's pieces relative to the longitudinal vibration mode are determined. The paper's purpose can be used as complement for teaching fundamentals of piezoelectricity and electric networks. The experimental procedure is simple too and the instruments employed in the measurements are found in low cost teaching laboratories. The results obtained with the tested transducers show that the parameters found have similar values to those obtained from Brazilian suppliers. Moreover, the fact of using a technological application is motivating and encourages the development of interdisciplinary activities.

Keywords: Circuit analysis, equivalent circuits, frequency measurement, piezoelectric transducers.

Resumen

Este trabajo muestra un método para simplificar la equivalencia de un circuito eléctrico de transductores piezoeléctricos intercalados. El análisis de los transductores es realizado usando la teoría elemental de red eléctrica. La simplicidad del método permite a los estudiantes de semestres básicos de cursos de Física entender los principios operación de intercalado de transductores piezoeléctricos. Además, se determinan las propiedades físicas de las piezas de los transductores en relación con el modo de vibración longitudinal. El propósito del artículo puede ser usado como complemento para los fundamentos de enseñanza de redes eléctricas y piezoeléctricas. El procedimiento experimental es demasiado simple y los instrumentos empleados en las mediciones son encontrados en los laboratorios de enseñanza de bajo costo. Los resultados obtenidos con los transductores probados muestran que los parámetros encontrados tienen valores similares a los obtenidos de proveedores Brasileños. Por otra parte, el hecho de utilizar una aplicación tecnológica es motivante y fomenta el desarrollo de actividades interdisciplinarias.

Palabras clave: Análisis de circuitos, circuitos equivalentes, medición de frecuencia, transductores piezoeléctricos.

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I. INTRODUCTION

In a previous paper [1] has presented a method to characterize piezoelectric plates vibrating in thickness mode. That method allows that students of first semesters of Physics course learn basic fundamentals of piezoelectricity and understand the equivalent electric circuits used to study piezoelectric plates vibrating in thickness mode.

In many applications, the transducers are composed of piezoelectric ceramics and metallic parts. A voltage source supplies these transducers with alternating and sinusoidal signals. Depending on the frequency of this voltage signal, a vibration mode is excited. Usually, by contingency of the design, the vibration of the piezoelectric transducers is concentrated only in one direction. For this reason, the one-dimensional treatment is enough to give a good description of the behaviour of these transducers. Several one-dimensional modelling of piezoelectric transducers are found in the literature [2, 3, 4, 5, 6, 7, 8, 9]. These modelling can be based on equivalent electric circuits. Mason's equivalent [10] is the most well known electric circuit used for this purpose.

Nowadays, the full 3D analysis of piezoelectric transducers has been explored in the literature using the powerful tool of the finite element method [11, 12, 13]. Even so, one-dimensional modelling of equivalent electric circuits continues being a good option for investigating the physical behaviour of the piezoelectric transducers. Also, the simplicity of the one-dimensional modelling is attractive as teaching activity.

The methods for electric circuit analysis are studied in physics courses [14, 15]. The procedures based on electric network theory are easy, well spread and systematized**.** Encouraging the use of the theory of electricity in technological applications attracts considerable attention due to the motivational impact that should bring to the student.

This paper uses an example of piezoelectric transducer to show that the boundaries for teaching of the electricity's laws can be dealt as a wide-ranging way. The electric representation of mechanical loads is connected in the previously developed circuit for the piezoelectric plate [1]. The longitudinal vibration of the piezoelectric sandwich can be investigated with the new equivalent electric circuit. By using elementary concepts of network electric circuits theory and a simple experimental procedure, elastic, electrical and piezoelectric parameters of the main pieces of a sandwiched piezoelectric transducer are calculated.

II. EQUIVALENT ELECTRIC CIRCUITS

Equivalent electric circuits are largely employed for studying piezoelectric transducers [16]. Fig. 1 shows a typical configuration of a sandwiched piezoelectric transducer that is used in this paper. This transducer is symmetrical and its center is considered clamped. For this reason, the analysis of the transducer can be performed in only one of its halves. The ceramics are piezoelectric rings. The metallic pieces are cylindrical and have a central hole. In addition, only the longitudinal fundamental vibration mode, along the axis x_3, is considered because the transducer has length (along x_3) larger than the diameter.

In according to the development employed in the early paper [1], the simplification of Mason's electric circuit yields the circuit showed in Fig. 2. The procedure to obtain this circuit is not repeated here.

Eqs. (1) and (2), respectively, show the expressions for voltage source and electric impedance correspondent to mechanical part.

$$V_{Thel} = V_1 , \tag{1}$$

$$Z_{Thel} = \frac{-jZ_0 \cot\left(\frac{\omega}{v_c} l_c\right)}{n^2} - \frac{1}{j\omega C_0} , \tag{2}$$

where

V_1 is the voltage applied to the ceramic's electrodes;

v_c is the wave propagation velocity in the piezoelectric medium;

l_c is the thickness of the one piezoelectric ceramic;

C_0 is the clamped capacitance of the piezoelectric ceramic;

n is given in Eq. (3):

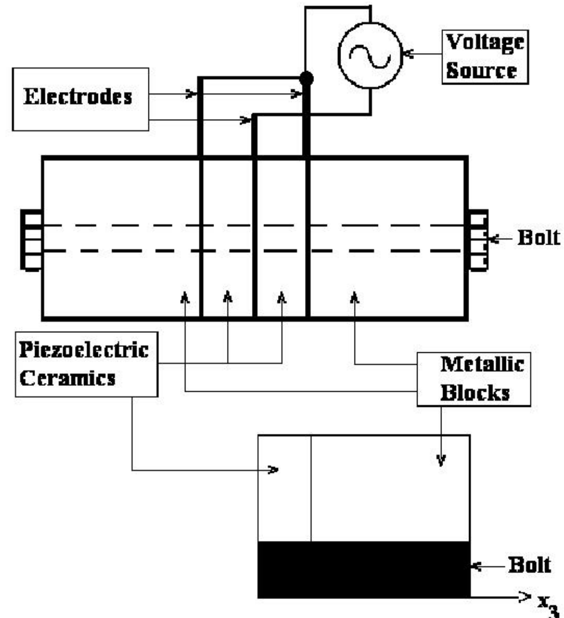

FIGURE 1. Schematics of sandwiched piezoelectric transducers.

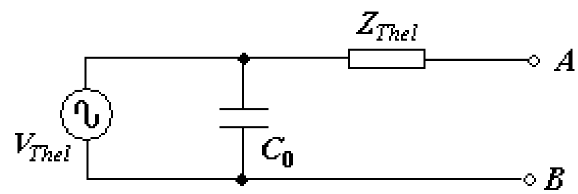

FIGURE 2. The simplified version of the equivalent electric circuit of the piezoelectric ceramic.

$$n = h_{33} C_0 , \tag{3}$$

h_{33} is the piezoelectric coefficient;

$\omega = 2\pi f$ is the angular frequency and f is the frequency;

$Z_0 = Z_c A_c$;

Z_c is the acoustic impedance of the ceramic;

A_c is the area of the flat surface of the ceramic.

The equivalent electric circuit of a metallic piece is showed in Fig. 3.

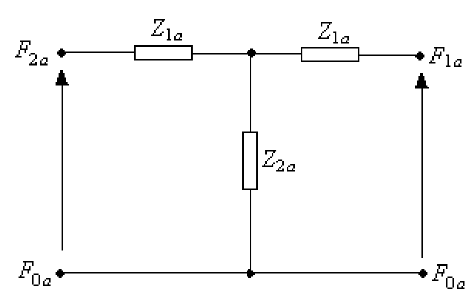

FIGURE 3. Equivalent electric circuit for a passive element.

The mechanical impedances of the metallic pieces are given by Eqs. (4) and (5) [17]:

$$Z_{1a} = jZ_{0a} \tan\left(\frac{\omega l_a}{2v_a}\right), \quad (4)$$

$$Z_{2a} = -jZ_{0a} \csc\left(\frac{\omega l_a}{v_a}\right), \quad (5)$$

where

v_a is the wave propagation velocity in the metallic medium;

l_a is the thickness of the metallic piece;

$Z_{0a} = Z_a A_a$;

Z_a is the acoustic impedance of the metallic piece;

A_a is the area of the flat surface of the metallic piece.

The port F_{0a}-F_{2a} is connected to the terminals A and B from the circuit of Fig. 2. The port F_{0a}-F_{1a} is short-circuited because this face of the passive element is in contact with air. The mechanical impedance of the air is near zero.

The mechanical impedance of the metallic piece (Z_L), "seen" by mechanical port (A and B) from circuit of Fig. 4, is given by Eq. (6).

$$Z_L = jZ_{0L} \tan(\frac{\omega}{v_L} l_L), \quad (6)$$

where

v_L is the wave propagation velocity in the metallic medium;

l_L is the thickness of the metallic piece;

$Z_{0L} = Z_{Lj} A_L$;

Z_{Lj} is the acoustic impedance of the metallic load;

A_L is the area of the flat surface of the metallic load.

The new circuit is showed in Fig. 4. Transducers represented by this model vibrate only in the longitudinal direction and are considered lossless.

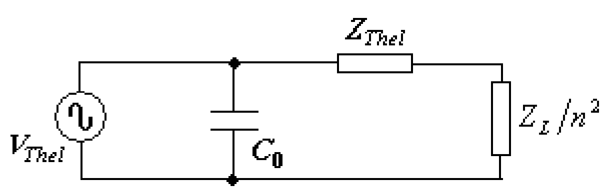

FIGURE 4. Equivalent electric circuit of the analysed piezoelectric transducer.

In the circuit shown in Fig. 4, the mechanical branch has two reactances in serial. The current flux through this branch reaches a maximum value at resonances, that is, at frequencies whose phasorial adding of the reactances is null ($Z_{Thel} = -Z_L/n^2$). In this case, the flat face of the transducer has maximum vibration velocity.

The resonance is calculated by solving Eq. (7).

$$\frac{n^2}{\omega_r C_0} = -Z_0 \cot\left(\frac{\omega_r}{v_c} l_c\right) + Z_{0L} \tan\left(\frac{\omega_r}{v_L} l_L\right), \quad (7)$$

where

ω_r is the angular resonance of the transducer.

By other hand, the reactances of the mechanical branch are in parallel with the capacitance C_0. At anti-resonance, the reactances of C_0 and the mechanical branch are same magnitude and opposite signals. Similar to the behaviour of a LC tank circuits, the electric impedance of the transducer is infinity at anti-resonance. The anti-resonance is determined by solving Eq. (8).

$$0 = -Z_0 \cot\left(\frac{\omega_a}{v_c} l_c\right) + Z_{0L} \tan\left(\frac{\omega_a}{v_L} l_L\right), \quad (8)$$

where

ω_a is the angular anti-resonance of the transducer.

The transducer showed in Fig. 1 is symmetrical. Hence, the circuit presented in Fig. 4 represents one half of the transducer and this is enough to evaluate the whole system. Non-symmetrical transducers must be evaluated with two circuits.

The fully characterization of the transducer is done in two steps. Firstly, the piezoelectric ceramics are characterized using Z_L=0. This step has been done in the early paper [1] and allows determine only the physical parameters of the ceramics. In the second step, regarding the characteristics of the ceramics previously determined, the metallic piece of the sandwiched transducer is characterized too.

The physical parameters of metallic pieces are determined through of Eqs. (9), (10) and (11). The wave propagation velocity is obtained solving Eq. (9). Eq. (9) is derived by dividing Eq. (8) by Eq. (7).

$$\Gamma = \frac{\tan\left(\frac{\omega_a}{v_L} l_L\right)}{\tan\left(\frac{\omega_r}{v_L} l_L\right)}, \quad (9)$$

where

$$\Gamma = \Gamma_a/\Gamma_r,$$

$$\Gamma_r = \frac{n^2}{\omega_r C_0} + Z_0 \cot\left(\frac{\omega_r}{v_c} l_c\right),$$

$$\Gamma_a = Z_0 \cot\left(\frac{\omega_a}{v_c} l_c\right).$$

Then the acoustic impedance and Young's modulus of the same piece are calculated with (10) and (11), respectively.

$$Z_a = \rho_L v_L , \quad (10)$$

where ρ_L is the density of the metallic piece.

$$Y_L = \rho_L v_L^2 . \quad (11)$$

III. EXPERIMENTAL PROCEDURE

This section describes the experimental procedure used to perform the fully characterization of the transducer. The piezoelectric ceramics used in the transducers were manufactured by Thornton-Inpec Co. Brazil. A sample of 20 piezoelectric rings was used. The dimensions of these rings are: thickness is 6.3mm, inner diameter is 12.6mm and outer diameter is 38.0mm.

The same procedure of the previous paper [1] was employed for determination of resonance and anti-resonance of the thickness mode and capacitance (in 1kHz).

Three sandwiched transducers were mounted with two piezoelectric ceramics and metallic pieces, annular shaped, made of an aluminium alloy. The radial dimensions of these pieces are the same of the ceramics and the lengths are: 32.5, 35.0 and 37.5mm. Hereafter, these transducers are called T1, T2 and T3, respectively. Through the central hole of the transducers there is a bolt that pre-stresses the set. The mechanical pre-stress level is closely 30MPa. A calibration curve is previously determined. References [18-20] give major details about the procedure to impose this level of mechanical pre-stress. The mechanical influence of the bolt is not considered in the modelling. Using the same metallic piece, each transducer was mounted 5 times with two ceramics randomly chosen.

IV. EXPERIMENTAL RESULTS

The average results obtained with the sample of the piezoelectric rings are: mass (m = 47.37g), resonance (f_r = 317.91kHz), anti-resonance (f_a =343.22kHz) and clamped capacitance (C_0 = 2.62nF). In addition, the volume and density of the ceramics were calculated, $V = 6.36\times10^{-6}$m^3 and ρ_c = 7463.5kg/m^3, respectively.

Additional characteristics of the piezoelectric ceramics are obtained from the same procedure reported in the early paper [1]. They are given in Table I.

TABLE I. Wave propagation velocity, acoustic impedance, elastic coefficient, dielectric permittivity and piezoelectric coefficient of the ceramics used in the experiments.

v_c (m/s)	Z_c (kg/m^{2}s)	c_{33}^D (N/m^2)	ε_{33}^T (F/m)	h_{33} (N/C)
4322	32.2×10^6	13.9×10^{10}	16.4×10^{-9}	16.9×10^8

Table II shows the experimental values of length, mass and density of the aluminum alloys employed in the transducers.

Table III shows the average value of the resonances and anti-resonances of the transducers from the first longitudinal mode.

Finally, in Table IV are found the values of the mechanical parameters of the metallic pieces of each transducer obtained from Eqs. (9), (10) and (11). Eq. (9) is solved using conventional numerical methods implemented in Matlab, for instance.

TABLE II. Experimental values of length and mass and density calculation of the metallic pieces employed in the transducers.

Transducer	Length (mm)	Mass (g)	Density (kg/m3)
T1	32.5	88.900	2709.9
T2	35.0	95.641	2707.1
T3	37.5	99.618	2631.7

TABLE III. Average values of resonance and anti-resonance of the transducers.

Transducer	Resonance (kHz)	Anti-resonance (kHz)
T1	40.50	42.20
T2	36.30	38.30
T3	35.41	36.42

TABLE IV. Wave propagation velocity, acoustic impedance and elastic coefficient (Young modulus) of the aluminum pieces used in the sandwiched transducers.

	v_L (m/s)	Z_L (kg/m^{2}s)	Y_L (N/m^2)
T1	5486	13.6×10^6	74.6×10^9
T2	5362	13.3×10^6	71.2×10^9
T3	5463	13.2×10^6	71.9×10^9

V. DISCUSSION AND CONCLUSION

A new equivalent electric circuit for the sandwiched transducers is presented. This circuit is very simple and can be used as complement for teaching electric networks. The analysis of the circuit uses elementary physics concepts and thus, the subject is accessible to the students of first semesters of Engineering or Physics courses.

The experimental data necessary to the determination of the transducers parameters are obtained through a very simple procedure (the resonance method [20]) and, in classes can be employed according to the sophistication pattern of the laboratory. The experimental set up for performing the resonance method includes conventional function generator and oscilloscope. The resonance method is employed since a long time and it is considered a solid method to make characterizations of piezotransducers. The results obtained with the characterization of the metal part of the transducers are close to aluminum alloys found in tables of Brazilian suppliers [21].

The direct analogy with electrical circuits helps to understand the concepts of resonance and anti-resonance in piezoelectric devices. The resonance is related to the current in the mechanical branch, while the anti-resonance is consequence from the resultant effect of the parallel association between mechanical and electrical branches.

The calculation of the currents and voltages throughout the simplified circuit can be performed in a very simple way. This allows the instructor does a deeper investigation in the new circuit in order to determine the vibration velocities and the mechanical forces. In the mechanical branch, the current divided by n is the velocity of vibration. The voltage on Z_L multiplied by n is the mechanical force in the interface ceramic-aluminium.

The results show that the method adds the possibility to insert hybrid devices, such as piezoelectric devices, in the learning process of electric networks. This paper shows that a metal part can be physically characterized by means of electrical measurements on a piezoelectric resonator. This underscores the possibility to relate the theories of electricity and mechanics, which should be a novelty for students of introductory semester. Furthermore, the use of a piezoelectric device employed in several technological applications motivates the students. The instructor can define the level of technical deepening and the complexity of the transducer choosing among single ceramics and sandwiched transducers.

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