

Extreme values of ET_0 at Piracicaba, Brazil, for designing irrigation systems¹

Valores extremos de ET_0 em Piracicaba, Brasil, para o dimensionamento de sistemas de irrigação

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HIGHLIGHTS:

ET_0 for designing the system capacity was selected considering the system lifespan, irrigation interval, and risk of failure.

As higher the anticipated irrigation system lifespan, higher is the return period needed to attain a low risk of failure.

Gumbel distribution showed to be adequate to characterize the frequency distribution of maximum ET_0 values.

ABSTRACT: Irrigation system capacity is typically defined by analyzing probabilities of non-exceedance of evapotranspiration. The use of mean monthly values of ET_0 may lead to underestimation of the required capacity, whereas use of maximum daily values may result in overestimation of required capacity. This study had the following objectives: (1) to analyze a 30-year series of daily ET_0 data from Piracicaba, SP, Brazil, to evaluate the suitability of the Gumbel distribution for estimating the maximum values of ET_0 organized in periods of up to 30 days; (2) to determine probable maximum values and to select ET_0 values considering the irrigation interval and the risk of failure in terms of irrigation system capacity. Daily data from 1990 to 2019 were used to calculate ET_0 using the Penman-Monteith model. The Gumbel distribution fitted to the data and was suitable for characterizing the frequency distribution of the maximum ET_0 . The probable ET_0 for designing irrigation systems can then be estimated based on the expected lifespan, irrigation interval, and return period of ET_0 maximum values. The higher the anticipated irrigation system lifespan, the higher the return period needed to attain a low risk of failure. Using the average of maximum ET_0 values alone leads to underestimation of system capacity and a high risk of failure in terms of irrigation system capacity.

Key words: extreme values type I distribution, confidence interval, irrigation system design, risk of failure, Gumbel distribution

RESUMO: A capacidade de sistemas de irrigação é tipicamente definida assumindo probabilidades de não-excedência de valores de evapotranspiração. O uso de valores médios mensais de ET_0 pode conduzir a subdimensionamento da capacidade, enquanto valores máximos diários podem resultar em superdimensionamento. Este estudo teve os seguintes objetivos: (1) analisar uma série de 30 anos contendo dados diários de ET_0 em Piracicaba, SP, Brasil, avaliando a aptidão da distribuição de Gumbel para a estimativa de valores máximos de ET_0 organizados em períodos de até 30 dias; (2) determinar valores máximos prováveis e selecionar valores de ET_0 considerando o intervalo entre irrigações e o risco de falha em termos de capacidade do sistema de irrigação. Dados diários, de 1990 a 2019, foram utilizados para calcular a ET_0 pelo modelo de Penman-Monteith. A distribuição de Gumbel se ajustou aos dados e foi adequada para caracterizar a distribuição de frequência de valores máximos de ET_0 . A ET_0 provável para o projeto de sistemas de irrigação foi estimada com base na vida útil do sistema, intervalo entre irrigações e período de retorno de valores máximos de ET_0 . Quanto maior for a vida útil do sistema de irrigação, maior deve ser o período de retorno adotado para reduzir o risco de falha. O uso da média dos valores máximos de ET_0 conduz a subdimensionamento e alto risco de falha em termos de capacidade do sistema de irrigação.

Palavras-chave: distribuição de valores extremos do tipo I, intervalo de confiança, projeto de sistema de irrigação, risco de falha, distribuição de Gumbel

INTRODUCTION

In irrigated agriculture, accurate quantification of reference evapotranspiration (ET_0) is fundamental for estimating crop water requirements, designing irrigation systems, and managing water resources properly (Pereira & Frizzzone, 2005; Ababaei, 2014; Silva et al., 2015). In the design of irrigation systems, the peak water requirement dictates the minimum capacity for supply pipes, pumps, and open channels to sustain potential crop growth (Hoffman et al., 2007).

The most conservative method of designing irrigation systems aims to provide sufficient capacity for the peak period of crop water consumption. This peak for various crops may occur at different times during the growing season (USDA, 1997). Rainfall and stored soil moisture can be neglected when determining system capacity. The use of mean monthly values of ET_0 may lead to underestimation of irrigation system capacity, whereas the use of maximum daily values may result in overestimation of capacity (Saad et al., 2002). The system capacity is typically designed so that the probabilities of non-exceedance of ET_0 range from 75 to 95% (with a return period from 4 to 20 years), depending on the value of the intended crop (USDA, 1993).

In addition, the length of the analyzed period has a substantial effect on the values of ET_0 estimated for a specific probability of non-exceedance (Hoffman et al., 2007). As the maximum ET_0 values change over time and according to the length of the analyzed period, studies on frequency and risk analysis should be conducted, and the values should be presented in terms of probability.

The extreme value type I distribution (i.e., the Gumbel distribution) has shown great importance in several research areas. It has been reliably applied in the statistical analysis of variables related to hydrological phenomena (Beijo et al., 2005; Kang et al., 2015; Pereira & Caldeira, 2018; Gómez et al., 2019; Ye et al., 2020) and studies on maximum ET_0 (Silva et al., 2015).

In this study, the following hypotheses were assessed: (1) the Gumbel distribution is suitable for estimating the maximum values of ET_0 organized in periods of up to 30 days; (2) some of the traditional recommendations for selecting the peak water consumption for the design of irrigation systems lead to a high risk of failure in terms of irrigation system capacity.

The following objectives were addressed in this study: (1) analysis of a 30-year series of ET_0 daily data from Piracicaba, SP, Brazil, in order to evaluate the suitability of the Gumbel distribution for estimating the maximum values of ET_0 organized in periods of up to 30 days; (2) determining probable maximum values of ET_0 and selecting the appropriate values for fulfilling water requirements during the irrigation system lifespan, considering the irrigation interval and the risk of system incapacity.

MATERIAL AND METHODS

The study was conducted using data from the conventional meteorological station of the Escola Superior de Agricultura “Luiz de Queiroz” University of São Paulo, Piracicaba, SP, Brazil. The station is located at the geographical coordinates

22° 42' 30" S and 47° 38' 00" W at an altitude of 546 m, with a mesothermal-type climate (Cwa) according to the Köppen-Geiger classification. The Cwa climate type is characterized by dry winters and an average annual rainfall of 1300 mm occurring mostly in the summer, i.e., from January to February (LER, 2020).

Daily data from a 30-year period (1990 to 2019) were used to calculate the daily ET_0 using the FAO-56 Penman-Monteith model (Allen et al., 1998). The following data were available at the weather station (LER, 2020) for computing ET_0 : solar radiation, air relative humidity, wind speed, and temperature (minimum, mean, and maximum).

All the steps for calculating ET_0 complied with the procedures recommended by FAO-56 (Allen et al., 1998). Daily net radiation was obtained from the net shortwave radiation and the net longwave radiation. The net shortwave radiation was calculated based on the solar radiation and an albedo of 0.23. The net longwave radiation was calculated based on the maximum and minimum values of air temperature, actual vapor pressure, solar radiation, altitude, and extraterrestrial radiation. The soil heat flux was neglected because its value was relatively small compared to the net radiation at daily intervals. Vapor pressure and saturation vapor pressure deficit were obtained using the mean values of air temperature and relative air humidity. The psychrometric constant was calculated from the local atmospheric pressure, which was obtained based on the altitude at which the weather station was located.

The period analyzed was from August to December of each year, because it presented the highest water deficit values for the region. This interval corresponded to the peak period of water consumption for planning irrigation systems. For the 30-year series of data, a monthly water balance considering rainfall and ET_0 was determined as a preliminary investigation. Although deficits could occur during the entire period analyzed, the highest values of deficit were identified in August (82.7 mm) and September (59.4 mm).

Based on daily data, the ET_0 values were grouped and summed into clusters (periods) of 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25, and 30 days using the following procedure: i) Starting with the first year of the series (1990), the accumulated ET_0 values were obtained for consecutive periods (e.g., for the five-day periods of August 1-5, 2-6, 3-6, ..., 27-31 through to December). The same process was applied to the other periods in each year of the historical series. ii) In each study period, the highest accumulated ET_0 value for a single cluster within each year was subjected to frequency analysis.

The Gumbel frequency distribution model was adjusted, and its probability density function (PDF) for the maximum ET_0 values is given by Eq. 1 (Abdi, 2014; Cotta et al., 2016).

$$f(x, \mu, \beta) = \frac{1}{\beta} \exp \left\{ - \left(\frac{x - \mu}{\beta} \right) - \exp \left[- \frac{(x - \mu)}{\beta} \right] \right\} \quad (1)$$

where:

- β - scale parameter ($\beta > 0$);
- μ - position parameter ($-\infty < \mu < \infty$); and,
- x - random variable associated with the maximum ET_0 values of the period ($-\infty < x < \infty$).

The cumulative probability function is given by Eq. 2 (Lee et al., 2020).

$$P(X \leq x) = F(x) = \exp \left\{ -\exp \left[-\left(\frac{x - \mu}{\beta} \right) \right] \right\} \quad (2)$$

where:

$P(X \leq x)$ - probability of obtaining a maximum ET_0 x ; and,
 μ - mode of the distribution.

In addition, the variance of the distribution is $\beta^2\pi^2/6$; the median is $\mu - \beta \ln(2)$; and the kurtosis is 12/5. The maximum likelihood estimators (MLE) for the parameters μ and β were obtained using Eqs. 3 and 4 (Cremoneze, 2016; Ozonur et al., 2020).

$$\hat{\beta} = \bar{x} - \frac{\sum_{i=1}^N x_i e^{-x_i/\hat{\beta}}}{\sum_{i=1}^N e^{-x_i/\hat{\beta}}} \quad (3)$$

$$\hat{\mu} = -\hat{\beta} \ln \left(\frac{\sum_{i=1}^N e^{-x_i/\hat{\beta}}}{N} \right) \quad (4)$$

where:

e - basis of Neperian logarithm;
 x - sample mean; and,
 N - sample size.

The MLE does not have explicit expressions that require iterative numerical calculations to estimate the first β . In this case, the Newton-Raphson iterative procedure is recommended, where $\beta_{\text{initial}} = 0.78$ is the sample standard deviation.

Probable ET_0 values were obtained for return periods of 2, 4, 5, 10, and 20 years, which corresponded to probabilities of exceedance of 0.50, 0.25, 0.20, 0.10, and 0.05, respectively. The return period T_R , defined as the period in which an event is expected to equal or exceed the value at least once, on average, is expressed as:

$$T_R = \frac{1}{P(X \geq x)} = \frac{1}{1 - F(x)} = \frac{1}{F'(x)} \quad (5)$$

where:

T_R - return period (years);
 $F(x)$ - probability of obtaining a maximum $ET_0 \leq x$; and,
 $F'(x)$ - probability of obtaining a maximum $ET_0 > x$.

For the estimated values of the Gumbel distribution parameters and probable maximum ET_0 values, the confidence interval (CI) was set as $\alpha = 0.05$, representing a 95% confidence level. To calculate the two-tailed CI, the t-distribution was used with $v = N - 1$ degrees of freedom. The standard error of the estimate (Bonamente, 2017) is expressed as:

$$CI = \hat{y} \pm t_{\alpha/2, v} \frac{s}{\sqrt{N}} \quad (6)$$

where:

y - estimated value;
 s - sample standard deviation;
 N - sample size;
 $t_{\alpha/2, v}$ - critical value of the t-distribution at the level of significance; and,
 α and v - degrees of freedom.

The Kolmogorov-Smirnov test (K-S) was used to determine whether the Gumbel distribution fit to the data. For this purpose, D_{sup} was used as the statistic of the K-S test for a given cumulative distribution function. D_{sup} corresponded to the maximum value of a set of vertical distances between the empirical distribution function and the cumulative function of the theoretical frequency distribution. At $p \leq 0.05$, D_{sup} was compared with the quantile $D_{(1-\alpha)}$, or $D_{\text{threshold}}$, given in the quantile table for the K-S test statistic (Bonamente, 2017).

Although other distribution frequencies might fit to the data, comparison of various distribution frequencies was not an objective of this study. Only the Gumbel distribution was evaluated.

Curves of probable maximum ET_0 as a function of irrigation interval were plotted to support the selection of ET_0 values for design purposes. For irrigation system design, the allowable number of days between irrigation events (i.e., the irrigation interval) usually determines the length of the analyzed periods to obtain the probable ET_0 (Hoffman et al., 2007).

For each probable ET_0 value, there is a corresponding return period that enables estimation of the risk of failure to satisfy the peak water requirement. Considering the expected irrigation system lifespan, the risk of failure can be estimated as the probability of an event being exceeded at least once every N years using Eq. 7.

$$J = 1 - [1 - F'(x)]^N \quad (7)$$

where:

J - risk of failure or probability that ET_0 will be exceeded at least once every N years (-); and,
 N - irrigation system's expected lifespan (years).

RESULTS AND DISCUSSION

Table 1 presents the statistical analysis of the maximum ET_0 values organized in clusters corresponding to periods not longer than 30 days. The ET_0 values from August 1 to December 31 were obtained by summing the maximum daily values within the period analyzed.

The percentage variation between the maximum and minimum ET_0 values in relation to the maximum value (x_{var}) showed a large range of values (19.30 to 33.54%). The difference between the maximum (x_{max}) and minimum (x_{min}) values of ET_0 indicated a relatively large dispersion. The coefficients of variation (CV) ranged from 6.21 to 9.39%. Overall, the data

Table 1. Statistics of maximum ET_0 values observed for 30 years (1990 to 2019) in various clusters (n, days) in Piracicaba

n (days)	x	Me (mm)	Mo	CV (%)	x_{max} (mm)	x_{min} (mm)	x_{var} (%)	A	C
2	13.56	13.52	13.07	6.86	15.84	12.05	23.91	0.129	4.568
3	19.73	19.62	19.02	6.43	22.40	19.07	23.68	0.262	4.236
4	25.61	25.55	25.28	6.21	28.13	22.70	19.30	0.121	3.385
5	31.40	31.33	31.04	7.21	34.85	27.59	20.85	0.100	3.096
6	36.96	37.05	36.99	7.48	41.15	31.79	22.74	-0.092	3.236
7	42.37	42.80	42.66	8.00	47.37	36.43	23.10	-0.372	3.249
8	47.83	47.84	47.95	7.94	54.05	40.73	24.64	-0.009	3.543
9	53.22	53.70	53.81	7.90	60.66	44.37	26.86	-0.342	3.965
10	58.41	59.03	59.19	8.14	67.55	47.03	30.38	-0.389	4.565
15	84.18	85.01	84.93	8.44	96.95	67.80	30.07	-0.352	4.406
20	109.00	110.38	110.91	8.80	124.94	85.37	31.67	-0.432	4.770
25	133.41	134.32	136.62	9.26	156.87	104.26	33.54	-0.221	4.562
30	156.72	157.11	157.87	9.39	183.57	122.36	33.34	-0.080	4.805

n - Number of days grouped in the period or cluster size; x - Sample mean of maximum ET_0 values in 30 years; Me - Median; Mo - Mode; CV - Coefficient of variation; x_{max} - The highest value of maximum ET_0 in the historical series; x_{min} - The lowest value of maximum ET_0 in the historical series; x_{var} - Percentage variation between maximum and minimum values of ET_0 in relation to the maximum value; A - Coefficient of skewness; C - Coefficient of kurtosis

dispersion indicated the need for the frequency analysis of ET_0 values to be presented in probabilistic terms, as suggested by Silva et al. (2015) and Souza et al. (2019).

From the skewness coefficients (A), it was observed that the distributions of maximum ET_0 in clusters of 2-5 days were slightly skewed toward the right. Skewness within ± 0.2 is classified as weak (Bonamente, 2017). For clusters of 6-30 days, the skewness was toward the left, indicating the predominance of the lowest values of observations. Skewness toward the left indicates a distribution in which the mean is less than the mode; skewness toward the right indicates that the mean is greater than the mode.

The kurtosis (C) values were greater than 3.0. Only for the 5-day cluster was the kurtosis close to 3.0, a value characteristic of the normal distribution (Bonamente, 2017). This indicates that the data presented a frequency curve that was more closed and tapered than the normal distribution and with a higher peak, denoting a high concentration of values around the center of the distribution.

In general, the mean, median, and mode of ET_0 values were similar, and the skewness coefficient was low, indicating that these events can be studied by a normal distribution, as pointed out by Silva et al. (1998), Hoffman et al. (2007), Silva et al. (2014), and Silva et al. (2015). According to a study by Souza et al. (2019), although descending values of ET_0 adjusted

better to the normal density function (51.4%), approximately half of the 10-day periods over a year adjusted to the other four PDFs; this indicates the need to consider other functions in climate studies aimed at predicting maximum values of ET_0 .

Table 2 lists the values of the Gumbel distribution parameters for the various clusters, and the D_{sup} values of the K-S statistic. For all clusters, the deviation of the D_{sup} from the K-S statistic at a 5% significance level was lower than the critical value $D_{Threshold(0.05; 30)} = 0.248$. Therefore, the Gumbel distribution fitted to the data and was suitable for analyzing maximum values of ET_0 organized in periods of up to 30 days.

The parameter values (μ and β) are presented with their respective CIs for $\alpha = 0.05$, with lower limits (LL) and upper limits (UL). For example, for an accumulated period of two days, there is a 95% probability that the intervals (12.801, 13.436) and (0.572, 1.042) contain the parameters μ and β , respectively.

Previous studies indicated that several probability distributions may fit to the ET_0 data series. Silva et al. (1998) analyzed ET_0 at Cruz das Almas, BA, Brazil, in periods not longer than 30 days and verified that Normal, Log-normal, and Beta distributions fitted to the data. Saad et al. (2002) investigated the frequency distribution of ET_0 in September from 1950 to 1990 in Piracicaba, SP, Brazil. According to the authors, September often corresponds to the peak period of

Table 2. Values of the parameters of the Gumbel distribution and D_{sup} values of the Kolmogorov-Smirnov (K-S) test, referring to the analysis of maximum ET_0 values in various clusters (n, days)

n (days)	Parameter μ	CI for μ		Parameter β	CI for β		D_{sup} K-S
		LL	UL		LL	UL	
2	13.118	12.801	13.436	0.807	0.572	1.042	0.078
3	19.104	18.629	19.578	1.207	0.928	1.569	0.096
4	24.824	24.236	25.412	1.496	1.143	1.957	0.104
5	30.270	29.436	31.105	2.122	1.616	2.785	0.109
6	35.588	34.572	36.604	2.585	1.832	3.337	0.109
7	40.679	39.419	41.939	3.205	2.272	4.138	0.116
8	45.941	44.528	47.354	3.594	2.548	4.641	0.099
9	51.106	49.472	52.739	4.155	2.945	5.365	0.107
10	56.023	54.133	57.914	4.808	3.729	6.202	0.107
15	80.614	77.822	83.406	7.101	5.494	9.178	0.097
20	104.153	100.271	108.036	9.876	7.000	12.751	0.114
25	127.259	122.433	132.084	12.273	8.700	15.846	0.129
30	149.368	143.562	155.173	14.766	11.445	19.049	0.127

$D_{critical(0.05; 30)} = 0.248$ (Source: Bonamente, 2017); n - Number of days grouped in the period or cluster size; LL - Lower limit; UL - Upper limit

water consumption by crops sowed from April to May. They analyzed clusters organized in periods of 5, 10, 15, and 30 days and verified that Beta and Normal distributions fitted to the data. In Petrolina, PE, Brazil, Gamma and Normal distributions presented the best goodness-of-fit for ET_0 values clustered in periods of less than one month (Silva et al., 2015). In Pinhais, PR, Brazil, the Normal distribution fitted to the ET_0 data organized in a 10-day period (Souza et al., 2019).

The aforementioned studies analyzed average values of ET_0 . However, in the current study, focus was given to the maximum ET_0 . Pereira & Frizzzone (2005) analyzed the maximum annual values of ET_0 at Nova Odessa, SP, Brazil, and assessed the goodness-of-fit of Normal, Log-Normal, Gamma and Gumbel distributions. Gumbel and Log-Normal distributions presented the best fitting based on the K-S test and D_{sup} . The authors also identified that the water deficit was more intense from August to November in this region.

Table 3 presents the maximum accumulated ET_0 values for several probabilities and return periods. The maximum ET_0 values for a 50% probability (2-year return period) were similar to the average of the maximum values, x (Table 1), with a maximum difference of 1.4% for a cumulative period of 25 days. This is possible because the distribution of the observed values is close to the normal distribution, as reported by Silva et al. (2014) and Silva et al. (2015).

The maximum ET_0 values estimated at a 25% probability of exceedance ($T_R = 4$ years) are larger than x by up to 7.1% (30-day period), with the smallest difference (4.1%) occurring with the 2-day period.

For any given return period, ET_0 expressed in $mm\ day^{-1}$ decreased as the length of the analyzed period increased. As an example, for the return period of four years, the maximum ET_0 decreased from $7.06\ mm\ day^{-1}$ (2-day period) to $5.59\ mm\ day^{-1}$ (30-day period), which is a reduction of 20.8%. Saad et al. (2002) examined the average values of ET_0 and verified a decrease in ET_0 as the length of the analyzed period increased. Likewise, Hoffman et al. (2007) mentioned that the probable ET_0 decreases as the length of the averaging period increases.

Figure 1A presents the risk of failure to satisfy the peak water requirement as a function of the return period chosen for estimating the maximum values of ET_0 and the expected irrigation system lifespan (Eq. 7). Figure 1B presents the maximum values of ET_0 as a function of the irrigation interval and return period. For irrigation system design, the irrigation interval usually indicates the length of the analyzed period that should be chosen to obtain the probable ET_0 (Hoffman et al., 2007).

For example, consider a high-frequency irrigation system, with an expected lifespan of 15 years, designed for a 2-day irrigation interval. As shown in Figure 1A, the risk of failure to satisfy the peak water requirement is 100.0, 98.7, 96.5, 79.4, and 53.7% for return periods of 2, 4, 5, 10, and 20 years, respectively. In this situation, the risk of failure corresponds to the probability of exceedance of the maximum ET_0 at least once every 15 years. For a 2-day irrigation interval, Figure 1B provides maximum ET_0 values of 6.7, 7.1, 7.2, 7.5, and 7.8 $mm\ day^{-1}$ for the return periods of 2, 4, 5, 10, and 20 years, respectively. The peak water requirement increased by 15.6%

Table 3. ET_0 values ($mm\ period^{-1}$) in various clusters (n, days), probability levels, and return periods (T_R , years) determined by the Gumbel distribution

n (days)	Probability of obtaining a maximum $ET_0 > x$, $F'(x)$				
	0.50	0.25	0.20	0.10	0.05
	($T_R = 2$ years)	($T_R = 4$ years)	($T_R = 5$ years)	($T_R = 10$ years)	($T_R = 20$ years)
	ET_0 ($mm\ period^{-1}$)				
2	13.42 [13.07;13.42]	14.12 [13.65;14.60]	14.33 [13.81;14.85]	14.94 [14.27;15.60]	15.52 [14.71;16.33]
3	19.55 [19.04;20.06]	20.61 [19.91;21.1]	20.91 [20.15;21.68]	21.82 [20.85;22.79]	22.69 [21.51;23.87]
4	25.37 [24.74;26.00]	26.69 [25.52;27.56]	27.07 [26.11;28.02]	28.19 [26.97;29.41]	29.27 [27.78;30.75]
5	31.05 [30.15;31.95]	32.91 [31.67;34.16]	33.45 [32.09;34.82]	35.04 [33.30;36.79]	36.57 [34.45;38.70]
6	36.49 [35.50;37.48]	38.82 [37.83;39.81]	39.51 [38.41;40.60]	41.43 [40.44;42.42]	43.47 [42.48;44.46]
7	41.82 [40.61;43.03]	44.65 [43.44;45.86]	45.53 [44.36;46.70]	47.95 [46.74;49.16]	50.27 [49.06;51.48]
8	47.23 [45.87;48.59]	50.38 [49.02;51.74]	51.49 [50.11;52.85]	54.08 [52.72;55.44]	56.76 [55.40;58.12]
9	52.59 [51.09;54.09]	56.23 [54.73;57.73]	57.31 [55.75;58.87]	60.52 [59.12;62.12]	63.56 [62.6;65.06]
10	57.79 [55.76;59.81]	62.01 [59.27;64.76]	63.24 [60.24;66.23]	66.84 [63.06;70.63]	70.30 [65.72;74.89]
15	83.22 [80.23;86.21]	89.46 [85.39;93.53]	91.27 [86.83;95.71]	96.59 [90.98;102.21]	101.71 [94.89;108.52]
20	107.74 [104.31;111.17]	116.05 [112.62;119.48]	118.61 [115.39;121.83]	126.55 [123.118;129.98]	134.12 [130.69;137.55]
25	131.59 [127.17;136.01]	142.21 [137.79;146.63]	145.49 [141.21;149.77]	154.99 [150.57;159.41]	163.52 [159.41;167.94]
30	154.78 [148.57;160.99]	167.77 [159.33;176.20]	171.52 [162.32;180.72]	182.60 [170.97;194.22]	193.23 [179.14;207.31]

Values in square brackets refer to the limits of the confidence interval, with $\alpha = 0.05$, for the estimated ET_0 at the specified probability level; n - Number of days grouped in the period or cluster size

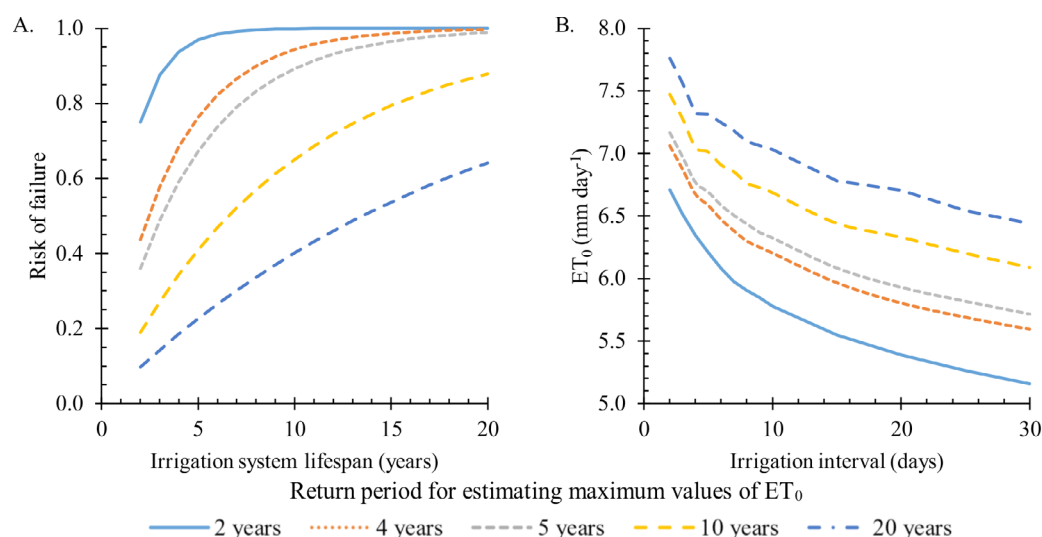


Figure 1. Risk of failure to satisfy the peak water requirement as a function of the return period chosen for estimating maximum values of ET_0 and the expected irrigation system lifespan (A); Recommended ET_0 for Piracicaba, SP, Brazil, as a function of irrigation interval and return period (B)

from the 2- to 20-year return period. In addition, considering a 2-day irrigation interval and a 10-year return period, there is a 95% probability that the maximum ET_0 is in the range of 7.1 to 7.8 mm day⁻¹ (Table 3).

Therefore, results shown in Figure 1A supports decision-making regarding which return period should be selected for estimating maximum ET_0 , and Figure 1B enables the definition of the maximum ET_0 based on the irrigation interval and return period (or risk of failure). In addition, Figure 1A shows that the higher is the irrigation system lifespan, the higher is the return period necessary to attain a low risk of failure.

The return period assumed for a project depends on several factors, such as the user's capacity to assume risk, economic value of the irrigated crops, climate of the region, and the availability of water resources (Bernardo et al., 2019; Souza et al., 2019). Investment and operational costs are sensitive to the irrigation system capacity, so it may not be economical to design a system to cater to abnormally high peak-use rates that are expected to occur only rarely (Hoffman et al., 2007).

Under typical supplementary irrigation conditions, the economics of irrigation projects fail to justify the selection of probability levels greater than 90% ($T_R = 10$ years). According to Souza et al. (2019), the adopted values for probability of non-exceedance often range from 50% ($T_R = 2$ years) to 75% ($T_R = 4$ years), depending on the economic implications associated with the project. However, this study demonstrated that, considering an irrigation system with an expected lifespan of 15 years designed for a 2-day irrigation interval, the risk of failure was 100% for $T_R = 2$ years and 98.7% for $T_R = 4$ years.

Return periods longer than 10 years may be chosen only for irrigated crops of high economic value or those overly sensitive to water deficit (Doorembos & Pruitt, 1984; Saad et al., 2002). The USDA (1993) recommends that the system capacity should be defined based on a 90% probability of non-exceedance ($T_R = 10$ years), but also mentions that typical design probabilities range from 75 to 95% (T_R from 4 to 20 years) depending on the value of the intended crop.

Considering a 10-year return period for the design of an irrigation system (10% probability that the event will be exceeded at least once every 10 years, Table 3), the ET_0 values for the project could be at most 14.2% higher (30-day period, Table 1) and at least 9.2% higher (4-day period) than the average of maximum values (x). For a 2-day period, the ET_0 value would be 9.2% higher than that of x . The use of the average values of ET_0 for any period leads to underestimation of irrigation system capacity, as pointed out by Silva et al. (2015).

CONCLUSIONS

1. The type I Gumbel probability distribution was suitable for characterizing the frequency distribution of maximum ET_0 values for Piracicaba, SP, Brazil, between August and December in periods of 2 to 30 days.
2. The probable ET_0 for designing irrigation systems can be estimated based on the system expected lifespan, irrigation interval, and return period of the maximum ET_0 . The higher is the lifespan of the irrigation system, the higher is the return period necessary to attain a low risk of failure in terms of irrigation system capacity.
3. For irrigation systems with an expected lifespan of 15 years, which are designed for a 2-day irrigation interval, and the maximum ET_0 of which is estimated considering a 10-year return period, the peak value of ET_0 is 7.5 day⁻¹. The probability of exceedance of the maximum ET_0 at least once every 15 years (i.e., the risk of failure) was 79.4%.
4. The maximum ET_0 values estimated at 50% probability were close to the average of the maximum values, and using the average of maximum values may lead to underestimation of irrigation system capacities with a high risk of failure.

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