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INSTITUTO DE FILOSOFIA E CIÊNCIAS HUMANAS

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BEYOND MONISM AND PLURALISM IN LOGIC

ALÉM DE MONISMO E PLURALISMO EM LÓGICA

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Orientadora: Itala Maria Loffredo D'Ottaviano

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RESUMO

Pluralismo lógico é uma família de propostas unidas sob a tese de que existe mais de uma lógica boa ou correta. Do outro lado, monismo lógico é a tese de que existe apenas Uma Única Lógica Verdadeira. A rivalidade entre essas duas posições até agora dominou o debate sobre a variedade de lógicas − o fato de que existem muitos sistemas lógicos −, mas argumentamos que essa aparente dicotomia é falha: existem muitas opções além de pluralismo e monismo. A fim de cumprir esse objetivo estruturamos esse trabalho em três capítulos. Primeiro, estabelecemos uma fundamentação formal para lidar com conceitos como validade e consequência lógica. Segundo, fazemos uma apresentação substancial acerca de diversas propostas presentes na literatura que tentam responder nossa pergunta-guia: existe mais de uma lógica correta? Por fim, tendo em vista o frequente uso do conceito de relativismo, dedicamos um capítulo para examinar um modelo de relativismo epistêmico e então, como trabalho autoral, sugerimos uma interpretação e reconstrução do modelo no contexto de filosofia da lógica a fim de organizar e analisar a discussão apresentada no segundo capítulo dentro de um modelo versátil e intuitivo.

Palavras chave: Lógica; Pluralismo; Monismo.

ABSTRACT

Logical pluralism is a family of views united under the thesis that there is more than one good or correct logic. Alternately, logical monism is the thesis that there is only One True Logic. The rivalry between these two positions has so far dominated the debate over the variety of logics − the fact that there are many logical systems−, but we argue that this apparent dichotomy is flawed: there are many options beyond pluralism and monism. In order to fulfill this objective, we structured this work in three parts. First, we establish a formal foundation for dealing with concepts such as validity and logical consequence. Second, we make a substantial exposition about several articulations present in the literature that aim to answer our guiding question: is there more than one correct logic? Finally, in view of the recurrent use of the concept of relativism, we dedicate a chapter to examine a model of epistemic relativism and then, as an authorial proposal, we suggest an interpretation and reconstruction of the model in the context of philosophy of logic in order to organize and analyze the discussion presented in the second chapter within a versatile and intuitive framework.

Key words: Logic; Pluralism; Monism.

List of Symbols

$a, a_1, a_2, ..., a_n, ..., b, b_1, b_2..., b_n...$	individual constants
$x, y, z, w, x', y', z', w', x'', ...$	individual variables
u, v	syntactical variables which vary through expressions
A, B, C, D...	syntactical variables which vary through formulas
a, b, c, d...	syntactical variables which vary through terms
x, y, z, w, ...	syntactical variables which vary through variables
f, g	syntactical variables which vary through function symbols
p, q	syntactical variables which vary through predicate symbols
e	syntactical variable which varies through constants
$\Sigma, \Gamma, \Delta, ...$	sets of formulas

$\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, ...$	structures
$\neg$	negation
$\vee$	disjunction (or)
$\wedge$	conjunction (and)
$\rightarrow$	material implication (implies)
$\leftrightarrow$	double implication
$=$	equivalence
$\top$	tautology (top)
$\bot$	contradiction (bottom)
$\forall$	universal quantifier
$\exists$	existential quantifier
$\vdash$	syntactic consequence
$\models$	semantic consequence
$\langle , \rangle$	ordered pair

Contents

Introduction

A well-known and celebrated fact is that there are many kinds of different logics. The impressive emergence of non-classical logics had a strong impact on the traditional monist view of logic, which has taken classical logic as the One True Logic. By the end of the $19th$ to the beginning of the $20th$ century implicit classical ideals were overcome, not only in logic, but also in many human endeavors, such as science and art. It was indeed a period of drastic change that keeps producing new and unprecedented ideas to this day. Many suggest that this turn in the history of logic was motivated by the study of non-euclidean geometry and the audacious questioning of the so called "logical thought principles"[1], as described bellow:

> The aurora of non-classical logics is due, according to our survey and analysis, to at least two conjunctural aspects present in the theoretical framework of the beginning of the 20th century. The first stems from the mathematical environment of the 19th century and the theoretical endeavor that followed the crisis in the foundations of mathematics at the end of that century. [...] The second is linked to the resumption of the previous logical tradition, particularly the legacy of Aristotle's logic, under a refined and updated logical perspective.[2] (GOMES; D'OTTAVIANO, 2017, p. 281).

This growing plethora of both classical and non-classical logics came accompanied with new concerns, naturally. What can be considered a logic? Is there more than one correct logic? How can they validate incompatible arguments? Is logic universal? In light of so many diverse approaches on how to reason about the world, the status of classical logic

[1] The law of non-contradiction, the law of excluded middle and the law of identity

[2] Original text: "A aurora das lógicas não clássicas deve-se, segundo nosso levantamento e análise, a pelo menos dois aspectos conjunturais vigentes no marco teórico do princípio do século XX. A primeira deriva-se do ambiente matemático do século XIX e do empreendimento teórico que se seguiu à crise dos fundamentos da matemática ao final daquele século. [...] A segunda vincula-se à retomada da tradição lógica anterior, particularmente, do legado da lógica de Aristóteles, sob uma perspectiva lógica hodierna e apurada."

changed. Questions about validity and logical consequence began to arise and disturb (in a good sense, we hope) logicians, philosophers and mathematicians.

A discussion with chief role here is about logical monism versus logical pluralism. Monism asserts that there is a One True Logic, classical or not. Pluralism states that we can have more than one correct logic. The first notable difference is on how logic is adjectivated: as true or correct, or even as good or legitimate in some formulations. They are clearly not the same thing, since truthness brings an ontological weight to the discussion, while correctness do not. This justifies us to not define logical pluralism in direct opposition to monism. This topic has its roots in Willard Quine and Rudolf Carnap, mainly due to their takes on the meaning of logical terminology, but it started to get more attention with Beall & Restall 2006's book, *Logical Pluralism*, making it a trend in philosophy of logic to this day. The present state of affairs denote a multiplicity of theses about the variety of logics and in order to discuss them we have dedicated the first chapter to set a logical foundation to this work. Given that, our discussion proceeds to an attempt of finding a common framework to discuss logical variety. So let us make a brief survey of the chapters.

Chapter One, "Formal concepts", starts out with a hard problem: what is logic? Our answer is essentially given to draw a line between the wide conception of logic and the specific notion of contemporary formal logic. After that we carefully and minutely introduce textbook definitions to characterize both syntactic and semantic aspects of a formal system. We have two main reasons to do so: first, to elucidate how malleable it is to specify a logic and what changes from classical to non-classical formulations. Second, the discussion carried out in the next chapters demand that some of the concepts be as clear as possible, such as the logical and non-logical divide, truth conditions, structures, models and the leading notions of logical consequence and validity.

Chapter Two, "Perspectives on the variety of logics", is dedicated to present a substantial overview of the present thesis concerning logical plurality. First, we offer the recurrent distinctions useful for most arguments, such as pure and applied logics and the notion of a canonical application for logic. Also, we present the three main challenges: the Meaning-variance Thesis, the Collapse Argument and the Substantial Logical Pluralism. A methodological remark is that the approaches presented were chosen by criteria like

informative and educational aspect, use by the philosophical community, innovative feature and theoretical integrity. We present in detail Beall & Restall case-based pluralism, Shapiro's eclectic orientation, Gillian's truth-bearer pluralism, da Costa's contextualism and Priest's logical monism. Also, we surface Hjortland's intra-theoretic pluralism, Logical Ecumenism, Batens' relativism, Read's monism and Varzi's relativism.

Chapter Three, "Brave New Relativism", is a reconstruction of the extensive concept of relativism which is embedded in popular formulations of logical pluralism, such as when it is said that 'a logic is adequate *relatively* to a certain domain'. Relativism and logic sounds like an unexpected combination, since logic gives us tools to reason objectively and rigorously, with means to clearly decide what is valid or not. Contrastingly, relativism is often associated with a possible trivializing or arbitrary aspect. Nonetheless, we argue that this combination is not readily disposable, and relativism can be a reasonable and useful concept when trying to make sense of the varieties of logic. We employ Kusch's model of epistemic relativism as a start point to develop a broad framework for our discussion on logical plurality. Our main effort was to adapt the epistemic terms of the original model to terms in context of philosophy of logic, which required changes in the assumptions, by removing or adding features. Thus, the third chapter is an original work that present relativism by a new light in order to organize and understand the philosophical debates around the existence of many logics. Finally, by the end of the chapter and in our final remarks we address some open questions and future work.

Formal concepts

This chapter is dedicated to introduce some basic concepts necessary for our discussion. The first section aims to give a general notion of what is understood by 'logic', which is the opening problem we face when talking about many logics. The second section is dedicated to establish some definitions we adopt regarding the syntax and semantics of a logical system.

2.1 What is logic after all?

To begin with the most fundamental concept, and likely the most problematic one, what does "logic" means? This question can be answered mainly by historical, logical, philosophical or mathematical approaches. We do not intend to give an all-encompassing answer to what logic is (if there is such a thing), but we shall present few articulations on how logic is characterized.

There are many ways to characterize logic, taking it by a mere formal tool to evaluate valid inferences, taking it as means to grasp reality and truth, associating it to norms on how we should reason and judge and so on. We recognize that it would be desirable to expose Leibniz and Kant's conceptions of logic, since they represent a major influence on many recent characterizations, but for the moment they are not under the scope of this work.

In what follows we shall present some different characterizations of logic, starting with da Costa: "The term 'logic' is ambiguous, since it is currently used in many senses; however, by the very way we equate the problem that interests us, it appears that by logic we mean formal logic (pure or theoretical)."[1](1994, p. 1). Also, da Costa offers an

[1]Original excerpt: "O termo 'lógica' é ambíguo, já que se utiliza correntemente em diversas acepções; porém, pela própria maneira a equacionarmos o problema que nos interessa, verifica-se que por lógica queremos significar

interesting view on how logic relate to reason. When trying to establish the relations between them, he recognizes a dogmatic and a dialectic stance. The first coincides with a more conservative view of logic, that it has a univocality in its interpretations, while the second has a pluralistic tendency, where we can choose what logical principles we adopt as much as physicists choose the geometry that best fits a domain of physics. This pluralistic tendency is made clear in several passages of da Costa's book (COSTA, 1994), and we shall see it in detail in the next chapter.

D'Ottaviano and Feitosa (2020, p. 25) propose a general and broad definition for the concept of logic, as logics being characterized as pairs constituted by an arbitrary set (without the usual requirement of dealing with formulas of a formal language) and a Tarskian consequence operator.

Definition 2.1. A *Tarskian consequence operator* on a set A is a function $C : \mathscr{P}(A) \to \mathscr{P}(A)$ such that, for every $X, Y \subseteq A$:
(i) $X \subseteq C(X)$;
(ii) $X \subseteq Y \to C(X) \subseteq C(Y)$;
(iii) $C(C(X)) \subseteq C(X)$.

Observe that when the consequence operator is closed under a well-defined substitution function, it is said to be structural.

Definition 2.2. A logic **A** is a pair *(A, C)*, where the set A is the domain of **A** and C is a Tarskian consequence operator on A.

This definition could be even more general in order to encompass wider forms of pluralism, so that logics could be characterized by pairs constituted by sets and consequence relations, allowing abductive or inductive characterizations as well as any non-Tarskian consequence relations.

Following their definition, logics constructed over formal languages are considered logical systems.

Definition 2.3. A *logic system* defined over L is a pair **L**=(L, C), where L is a formal language and C is a structural consequence operator in the free algebra For(L) of the

a lógica formal (pura ou teórica)."

formulas of L.

Thus, according to this view a logic can be any kind of mathematical structure that fulfills the definition, such that even topological spaces could be identified as logics. Whereas, a logical system would require the entities to be a set of logical formulas, which we shall define below (Definitions 1.16 and 1.17). We think that the value of this definition lies in its generality, but for our purposes along this work we shall think of logic in the strict sense of logical systems.

There are many ways to characterize formality and to say that a logic is formal[2], although the term "formal logic" is usually loosely employed in the literature over the last and present century by philosophers such as Bertrand Russell, Gottlob Frege and Alfred Tarski. The general idea is that logic is not concerned with matter, or particular relations, but with form and general relations.

What is understood by 'logic' can and does change over time, being re-signified and permeated with disagreement, such that we could easily consider this to be the hardest question. Thus, it makes sense why, to this day, there are plenty of disagreement about the subject. For instance, there is a well known discussion over the legitimacy of second-order (and higher order) logics. In this matter, Quine famously stated that second-order logic is set theory in sheep's clothing (1986, p. 66-68) and, more recently, Kneale & Kneale (1962) also argued that logic can be extended only to first-order quantification, since second-order logic was shown to be incomplete as a further consequence of the renowned Gödel's incompleteness theorems. By contrast, Shapiro (1991) argues in defense of second-order logics, accompanied by George Boolos (1975) and many others.

For now, we shall present two other articulations of what is logic. The first is given by Shoenfield, from whom we use some definitions in the next section, and the second is given by Beall & Restall, since they are the traditional exponents of logical pluralism. We present them as mere illustrations on how one can grasp the idea of logic, i.e., we are not advocating them.

> Logic is the study of reasoning; and mathematical logic is the study of the type of reasoning done by mathematicians. (SHOENFIELD, 1967, p. 1).

[2]See MacFarlane (2000), Chapter Three.

> Logic is about consequence. Logical consequence is the heart of logic; it is also at the centre of philosophy and many theoretical and practical pursuits besides. [...] Logic, whatever it is, must be a tool useful for the analysis of the inferential relationships between premises and conclusions expressed in arguments we actually employ. If a discipline doesn't manage this much, it cannot be logic in its traditional sense. (BEALL; RESTALL, 2006, p. 8).

Observe that those definitions are compatible with D'Ottaviano and Feitosa's broad definition, as sets with consequence relations.

We can also apprehend logic as the investigation of mathematical aspects of formal languages; or as a normative pursuit, such as that by argumentation and rationality theory; or assigning to it metaphysical significance, which consist to a great extent in the investigation of what are logical truths; or we can even see logic as the historically traditional view that takes it as regarding the laws of thought. Here we have only shown the literature closer to our perspective, although the question of what is logic has a never-ending condition.

2.2 Definitions

The majority of definitions presented below can be found with little or no modifications in Shoenfield's book *Mathematical logic* (1967). We tend to follow his order of presentation and the definitions are aimed to characterize Classical First-Order Logic, although it is possible to use them to also characterize non-classical logics, and we shall indicate how when suitable.

2.2.1 Syntax

Definition 2.4. Any finite sequence of symbols of a language is called an *expression* of that language. We shall designate the language of a formal system F by $L(F)$, but the definition is presented latter.

Definition 2.5. Each axiom shall be a *formula* of the language of the formal system.

Definition 2.6. Each *rule of inference* states that under certain conditions, one formula, called the conclusion of the rule, can be inferred from certain other formulas, called the hypotheses of the rule. They enable us to conclude theorems from the axioms.

Definition 2.7. (i) The axioms of F are *theorems* of F;

(ii) If all of the hypotheses of a rule of F are theorems of F, then the conclusion of the rule is a *theorem* of F.

In order to give a more precise definition for theorems we can use the *generalized inductive definition*: Let S_0 be the set of axioms; these are the formulas which can be seen to be theorems on the basis of (i). Let S_t be the set of formulas which are conclusions of rules whose hypotheses are all in S_0; these are some of the formulas which can be seen to be theorems on the basis of (ii). Let S_2 be the set of formulas which are conclusions of rules whose hypotheses are all in S_0 and S_i; these are also theorems on the basis of (ii). In this way, we can construct sets $S_3, S_4, ...$ Let S_w be the set of formulas which are conclusions of rules whose hypotheses are all in at least one of $S_0, ..., S_i, ...$; these are again theorems by (ii). We continue in this way until no new theorems can be obtained by (ii); and we then have all of the theorems.

Definition 2.8. A rule in a formal system F is *finite* if it has only finitely many hypotheses.

Definition 2.9. Let F be a formal system in which all the rules are finite. By a *syntactical proof* in F we mean a finite sequence of formulas, each of which either is an axiom or is the conclusion of a rule whose hypotheses precede that formula in the proof.

If A is the last formula in a proof P, we say that P is a proof of A. We shall write $\vdash_F A$ as an abbreviation for A is a theorem of F. When no confusion results, we omit the subscript F.

We are going to define a first-order language with names and variables (Definition 1.14), but before that it is wothy observing that, unlike a name, which has only one meaning, a variable has many meanings. A formula containing syntactical variables (or metalinguistic variables, or metavariables) has many meanings, one for each assignment of an expression as a meaning to each syntactical variable occurring in the formula. If we assert such a formula, we are claiming that all of its meanings are true. Also, syntactical variables are not symbols of the language being discussed.

Notation. We shall use boldface letters as syntactical variables. In particular, **u** and **v**

will be syntactical variables which vary through all expressions, and **A, B, C** and **D** will be syntactical variables which vary through formulas.

Further notation, such as variables, $\forall$ and $\exists$, can be found in the List of Symbols and we shall omit for simplicity purposes

First-Order Theories

In order to formalize the concept of a first-order theory we need some elementary notions and results of set theory.

Truth functions

Definition 2.10. We select two objects, T and F, which we call *truth values*. The truth value T is called designated.

It doesn't matter what these objects are, so long as they are distinct from each other. We then assign a truth value to each formula as follows: we assign T to a formula to be considered as true and F to a formula to be considered as false.

In non-classical logics we can define more than two truth-values and obtain a many-valued logic, as long as we establish what are the designated values, i.e., the truth-values preserved in valid inferences. For instance, in Kleene's K3 (1952) logic we have three truth-values: True, False and Unknown and only the first is designated. In Łukasiewicz logic (1970) we have three truth values (later it was developed for many values) where only True is designated, so False and Possible are undesignated. In D'Ottaviano and da Costa's system J3 (1970), the truth-values are $1, 0$ and $1/2$, where 1 and $1/2$ are designated and 0 undesignated. Also, in Asenjo-Priest's Logic of Paradox (LP) (1979) there is True, Paradoxical and False, where the first two are designated. Worthy noting that we could employ a bivalent semantics for K3, LP and other many-valued logics.

Definition 2.11. A *truth function* is a function from the set of truth values to the set of truth values.

Following, we introduce the equations for each connective adopted in our language. Here we present only $\wedge$, $\vee$, $\rightarrow$ and $\neg$, which are the most common.

Definition 2.12. Connectives truth-functions

Negation. If **A** has the truth value a then $\neg\mathbf{A}$ has the truth value $H_\neg(a)$ where $H_\neg$ is the unary truth function defined by

$$H_\neg(\mathbf{T}) = \mathbf{F},\ H_\neg(\mathbf{F}) = \mathbf{T}.$$

Conjunction. There is a binary truth function $H_\wedge$ such that if a and b are the truth values of **A** and **B** respectively, then $H_\wedge(a, b)$ is the truth value of $A \wedge B$. This truth function is described by the equations:

$$H_\wedge(\mathbf{T},\ \mathbf{T}) = \mathbf{T},$$
$$H_\wedge(\mathbf{T},\ \mathbf{F}) = H_\wedge(\mathbf{F},\ \mathbf{T}) = H_\wedge(\mathbf{F},\ \mathbf{F}) = \mathbf{F}.$$

Disjunction. If a and b are the truth values of **A** and **B** respectively, then the truth value of $A \vee B$ is $H_\vee(a, b)$, where $H_\vee$ is the binary truth function defined by:

$$H_\vee(\mathbf{T},\ \mathbf{T}) = H_\vee(\mathbf{T},\ \mathbf{F}) = H_\vee(\mathbf{F},\ \mathbf{T}) = \mathbf{T},$$
$$H_\vee(\mathbf{F},\ \mathbf{F}) = \mathbf{F}.$$

Implication. With $\rightarrow$ we mean *if... then*, so that $\mathbf{A} \rightarrow \mathbf{B}$ means *if* $\boldsymbol{A}$*, then* $\boldsymbol{B}$:

$$H_\rightarrow(\mathbf{T},\ \mathbf{T}) = H_\rightarrow(\mathbf{F},\ \mathbf{T}) = H_\rightarrow(\mathbf{F},\ \mathbf{F}) = \mathbf{T},$$
$$H_\rightarrow(\mathbf{T},\ \mathbf{F}) = \mathbf{F}.$$

For *equivalence*:

$$H_=(\mathbf{T},\ \mathbf{T}) = T$$
$$H_=(\mathbf{F},\ \mathbf{F}) = T$$
$$H_=(\mathbf{T},\ \mathbf{F}) = F$$
$$H_=(\mathbf{F},\ \mathbf{T}) = F$$

Since they are functions, those truth-functions can have several algebraic properties, such as associativity, reflexivity, transitivity, monotonicity, idempotence, among others.

Some of these symbols can be defined in terms of the others (we call that *interdefinability* of connectives). For instance, all of them can be defined in terms of $\neg$ and $\vee$.

Definition 2.13. An *individual variable* can mean any individual, but its meaning remains fixed throughout any one context[3].

A formula containing an individual variable has many meanings, one for each assignment of an individual as meaning to each individual variable in the formula. Since individual variables are the only variables which will occur in our languages, we shall call them simply variables.

First-order languages

Definition 2.14. A *first-order language* has as symbols the following:

A. the variables
$x, y, z, w, x', y', z', w', x'', ...;$

B. for each n, the n-ary function symbols and the n-ary predicate symbols;

C. the symbols $\neg, \vee$ and $\exists$.

A 0-ary function symbol is called a *constant*. A function symbol or a predicate symbol other than $=$ is called a *nonlogical symbol*; other symbols are called *logical symbols*.

We shall see that this distinction is important for the relativism put forth by Varzi (2002).

The parentheses and commas are used to indicate grouping, but we could also simply write $\vee\mathbf{AB}$ instead of $\mathbf{A} \vee \mathbf{B}$.

Definition 2.15. We define the *terms* by the generalized inductive definition:

A. a variable is a term;

B. if $\mathbf{u}_1, ..., \mathbf{u}_n$ are terms and $\mathbf{f}$ is a n-ary function symbol, then $\mathbf{fu}_1, ..., \mathbf{u}_n$ is a term.

Definition 2.16. An *atomic formula* is an expression of the form $pa_1, ...a_n$ where $\mathbf{p}$ is n-ary predicate symbol.

Definition 2.17. We define the *formulas* by the generalized inductive definition:

[3]"Context" is used to talk about formal systems. For instance, Shoenfield argues that in an analysis text we can have names for real numbers and use variables that can mean any real number, since they vary through them.

A. an atomic formula is a formula;

B. if **u** is a formula, then ¬ **u** is a formula;

C. if **u** and **v** are formulas, then ∨ **uv** is a formula;

D. if **u** is a formula, then ∃**xu** is a formula.

Definition 2.18. An occurrence of **x** in **A** is *bound* in **A** if it occurs in a part of **A** of the form ∃ **x B**; otherwise, it is *free* in **A**.

We use $b_x[a]$ to designate the expression obtained from **b** by replacing each occurrence of **x** by **a**; and we use $A_x[a]$ to designate the expression obtained from **A** by replacing each free occurrence of **x** by **a**. Using induction on the length of **b** and **A**, we easily prove that $b_x[a]$ is a term and that $A_x[a]$ is a formula.

2.2.2 Semantics

We now turn to a precise description of the semantics of first-order languages. As already indicated, a meaning for a first-order language consists of a universe and a meaning of the appropriate sort for each nonlogical symbol.

Structures

Definition 2.19. Let L be a first-order language. A *structure* $\mathfrak{A}$ for L consists of the following things:

(a) A nonempty set $|\mathfrak{A}|$, called the *universe* of $\mathfrak{A}$. The elements of $|\mathfrak{A}|$ are called the *individuals* of $\mathfrak{A}$.

(b) For each n-ary function symbol **f** of L, an n-ary function $\mathbf{f}_{\mathfrak{A}}$ from $|\mathfrak{A}|$ to $|\mathfrak{A}|$. (In particular, for each constant **e** of L, $\mathbf{e}_{\mathfrak{A}}$ is an individual of $\mathfrak{A}$)

(c) For each n-ary predicate symbol **p** of L other than =, an n-ary predicate $\mathbf{p}_{\mathfrak{A}}$ in $|\mathfrak{A}|$.

Let $\mathfrak{A}$ be a structure for L. For each individual a of $\mathfrak{A}$, we choose a new constant, called the *name* of a. It is understood that different names are chosen for different individuals.

The first-order language obtained from L by adding all the names of individuals of $\mathfrak{A}$ is

designated by $L(\mathfrak{A})$. We use **i** and **j** as syntactical variables which vary through names.

An expression is *variable-free* if it contains no variables. We shall now define an individual $\mathfrak{A}(\mathbf{a})$ of $\mathfrak{A}$ for each variable-free term **a** of $L(\mathfrak{A})$. The definition is by induction on the length of **a**. If **a** is a name, $\mathfrak{A}(\mathbf{a})$ is the individual of which **a** is the name. If **a** is not a name, then (since it is variable-free) it must be $\mathbf{fa}_1...\mathbf{a}_n$ with **f** a function symbol of L. We then let $\mathfrak{A}(\mathbf{a})$ be $\mathbf{f}_{\mathfrak{A}}(\mathfrak{A}(\mathbf{a}_1), ..., \mathfrak{A}(\mathbf{a}_n))$.

Definition 2.20. A formula **A** is *closed* if no variable is free in **A**. (This means that **A** has only one meaning.)

We shall now define a truth value $\mathfrak{A}(\mathbf{A})$ for each closed formula **A** in $L(\mathfrak{A})$. The definition is by induction on the length of **A**.

Definition 2.21. If **A** is $\mathbf{a} = \mathbf{b}$, then **a** and **b** must be variable-free (since **A** is closed). We let

$$\mathfrak{A}(\mathbf{A}) = \mathbf{T} \text{ iff } \mathfrak{A}(\mathbf{a}) = \mathfrak{A}(\mathbf{b})$$

(i.e., iff $\mathfrak{A}(\mathbf{a})$ and $\mathfrak{A}(\mathbf{b})$ are the same). If **A** is $\mathbf{pa}_1, ..., \mathbf{a}_n$, where **p** is not $=$, we let

$$\mathfrak{A}(\mathbf{A}) = \mathbf{T} \text{ iff } \mathbf{p}_{\mathfrak{A}}(\mathfrak{A}(\mathbf{a}_1), ..., \mathfrak{A}(\mathbf{a}_n))$$

(i.e., iff the n-tuple $(\mathfrak{A}(\mathbf{a}_1), ..., \mathfrak{A}(\mathbf{a}_n))$ belongs to the predicate $\mathbf{p}_{\mathfrak{A}}$. If **A** is $\neg\mathbf{B}$, then $\mathfrak{A}(\mathbf{A})$ is $H_{\neg}(\mathfrak{A}(\mathbf{B}))$. If **A** is $\mathbf{B} \vee \mathbf{C}$, then $\mathfrak{A}(\mathbf{A})$ is $\mathbf{H}_{\vee}(\mathfrak{A}(\mathbf{B}), \mathfrak{A}(\mathbf{C}))$. If **A** is $\exists\mathbf{xB}$, then $\mathfrak{A}(\mathbf{A}) = \mathbf{T}$ iff $\mathfrak{A}(\mathbf{B}_x[\mathbf{i}]) = \mathbf{T}$ for some **i** in $L(\mathfrak{A})$.

It is clear that $\mathfrak{A}(\mathbf{A} \to \mathbf{B}) = H_{\to}(\mathfrak{A}(\mathbf{A}), \mathfrak{A}(\mathbf{B}))$, and similarly for $\wedge$ and $\leftrightarrow$.

If **A** is a formula of L, an $\mathfrak{A}$-instance of **A** is a closed formula of the form $\mathbf{A}[\mathbf{i}_1, ..., \mathbf{i}_n]$ in $L(\mathfrak{A})$.

Definition 2.22. A formula **A** of L is *valid* in $\mathfrak{A}$ if $\mathfrak{A}$(**A'**) = **T** for every $\mathfrak{A}$-instance **A'** of **A**. In particular, a closed formula **A** of L is valid in $\mathfrak{A}$ iff $\mathfrak{A}(\mathbf{A}) = \mathbf{T}$.

Logical Axioms and Rules

The following are the logical axioms and rules of inference of first-order theories.

Certain formulas of L are valid simply because of the meaning of the logical symbols; i.e., they are valid in *every* structure for L. For example, $x = x$ has this property. Such formulas are said to be *logically valid*. Certain of our axioms, called the *logical axioms*, will be logically valid. The others, called the *nonlogical axioms*, will be valid because of particular properties of the structure $\mathfrak{A}$.

Definition 2.23. We say that $\mathbf{A}$ is a *logical consequence* of a set Γ of formulas if it is a consequence of Γ because of the meaning of the logical symbols, i.e., if $\mathbf{A}$ is valid in every structure for L in which all of the formulas in Γ are valid.

We might expect our rules also to divide into two classes: *logical rules*, in which the conclusion is a logical consequence of the hypotheses, and *nonlogical rules*, in which the conclusion is a consequence of the hypotheses only because of special properties of $\mathfrak{A}$.

A *propositional axiom* is a formula of the form $\neg\mathbf{A} \vee \mathbf{A}$.

A *substitution axiom* is a formula of the form $\mathbf{A}_x[\mathbf{a}] \to \exists\mathbf{x}\mathbf{A}$.

An *identity axiom* is a formula of the form $\mathbf{x} = \mathbf{x}$.

An *equality axiom* is a formula of the form:

$$\mathbf{x}_1 = \mathbf{y}_1 \to ... \to \mathbf{x}_n = \mathbf{y}_n \to \mathbf{f}\mathbf{x}_1...\mathbf{x}_n = \mathbf{f}\mathbf{y}_1...\mathbf{y}_n$$

or of the form:

$$\mathbf{x}_1 = \mathbf{y}_1 \to ... \to \mathbf{x}_n = \mathbf{y}_n \to \mathbf{p}\mathbf{x}_1...\mathbf{x}_n \to \mathbf{p}\mathbf{y}_1...\mathbf{y}_n$$

Every classical logic is based on those axioms, but non-classical, such as paracomplete and paraconsistent logics, deny some of those axioms and rules.

Definition 2.24. A *logical axiom* is a formula which is a propositional axiom, a substitution axiom, an identity axiom, or an equality axiom.

Definition 2.25. A *first-order theory*, or simply a *theory*, is a formal system T such that:

(i) the language of T is a first-order language;

(ii) the axioms of T are the logical axioms of $L(T)$ and certain further axioms, called the *nonlogical axioms*;

(iii) the rules of T are the expansion rule, the contraction rule, the associative rule, the cut rule, and the $\exists$-introduction rule.

Definition 2.26. By a *model* of a theory T, we mean a structure for $L(T)$ in which all the non-logical axioms of T are valid. A formula is *valid* in T if it is valid in every model of T; equivalently, if it is a logical consequence of the nonlogical axioms of T.

Theorem 1.23. If T is a theory, then every theorem of T is valid in T.

Definition 2.27. A formula $\mathbf{A}$ is a *tautology* if it is a tautological consequence of the empty sequence of formulas, i.e., if $\mathcal{V}(\mathbf{A}) = \mathbf{T}$ for every truth valuation $\mathcal{V}$.

Definition 2.28. A formula $\mathbf{A}$ of T is *undecidable* in T if neither $\mathbf{A}$ nor $\neg\mathbf{A}$ is a theorem of T; otherwise, $\mathbf{A}$ is *decidable* in T.

Definition 2.29. A theory T is *inconsistent* if every formula of T is a theorem of T; otherwise, T is consistent.

Definition 2.30. A theory T is complete if it is consistent and if every closed formula in T is decidable in T. Restating: a theory T is complete if for every closed formula $\mathbf{A}$ in T, exactly one of $\mathbf{A}$ and $\neg\mathbf{A}$ is a theorem of T.

A series of important results relative to first-order theories can be found in Chapters 3 and 4 from Shoenfield (1967). An important result of first-order theories are the completeness theorems:

Theorem 1.24. Completeness Theorem, 1st. Form. A formula $\mathbf{A}$ of a theory T is a theorem of T iff it is valid in T.

Theorem 1.25. Completeness Theorem, 2nd. Form A theory T is consistent iff it has a model.

These two versions are sometimes referred as *weak* and *strong* completeness, respectively. The weak version states that all tautologies from a theory T (every valid formula) are

theorems of T, such that $\vDash \phi$ implies $\vdash_T \phi$ for any formula ϕ. The strong version states that $\Gamma \vDash \phi$ implies $\Gamma \vdash_T \phi$ for any set $\Gamma \cup \{\phi\}$. If we take $\Gamma = \emptyset$, then the the strong version implies the weak version.

The Completeness theorems concern the relation of provability of theorems and tautologies in first-order theories. But another result of great importance not only for logic but for any formal system, such as computational algorithms, are the Incompleteness Theorems. They show the limitations of what can (and cannot) be proved inside an axiomatic theory which is recursively enumerable, consistent, and able to express some Peano arithmetic. But to explain them we would need a large exposition of definitions on recursive functions, Church's thesis, decidability, representability and others. Thus, for a detailed exposition see Chapter 6 from Shoenfield (1967).

Perspectives on the variety of logics

> A successfully chosen name is a bridge between scientific knowledge and common sense, between new experience and old habits. The conceptual foundation of any science consists of a complicated network of names of things, names of ideas, and names of names. It evolves itself, and its projection on reality changes. (MANIN; ZILBER, 2010, p. 9)

This chapter is dedicated to organize and present some theses that attempt to make sense of the variety of logics. The questions they are trying to answer include: is there a correct logic? If so, only one or many? Are they equally correct? How can we compare the different logics? Are they really disagreeing? Is there an all-encompassing logic? Is logic topic-neutral? And so on.

We take logical pluralism as a start point of discussion, but later it becomes clear that we also cover other perspectives beyond logical pluralism and monism, such as logical nihilism, universalism, instrumentalism, contextualism and logical relativism (which earned a whole chapter). The panorama on pluralism is given to help us recognize what is generally understood by the term and how it easily becomes a thorny discussion. Then, we offer some conceptual tools to guide us, such as the distinction between local and global pluralism given by Haack (1978), the distinction on pure and applied logics as well as a canonical application, given by Priest (2005) and the most popular problems that logical pluralists faces: the Meaning-variance Thesis, the Collapse Argument and the pursuit for a Substantial Logical Pluralism.

The second section is a detailed exposition of several perspectives on the variety of logics, namely instrumentalism, logical nihilism, logical universalism, Beall & Restall's case-based

pluralism, Shapiro's eclectic perspective, Russell's truth-bearer pluralism, da Costa's contextualism, and Priest's logical monism. There are many more approaches that we could examine, but would be impracticable to address them all. Nonetheless, we dedicated a subsection to acknowledge Hjortland's intra-theoretic pluralism, logical ecumenism, Batens' contextualism, Read's monism and Varzi's relativism. By the end of the chapter we have attempted to offer a diagram to identify how the different perspectives on the variety of logics answer to the questions they are all interested.

A considerable source of confusion between the different approaches we dealt with are the names given to them. These names (tags, labels and so on) proliferate very quickly, especially in such a recent and trendy topic as logical pluralism. This justifies our epigraph and motivates us on finding a common framework for all of them in the next chapter.

3.1 What is pluralism?

3.1.1 Pluralism in Philosophy

Pluralism, variety, multiplicity, plethora: those are some of the words employed when talking about the diversity of a given phenomenon, idea, concept, explanations and so on, and we will use them interchangeably. Pluralism is a feature of many subject areas, such as pluralism about well-being, mathematics, archaeology, values, science, religion and politics. We can say that pluralism is a big family of views that share a common thesis: there are many kinds of a given phenomenon.

To recognize different apprehensions of the same phenomenon requires *tolerance* in a very general sense, and that is the first step for pluralism. When one faces difficulties to unify different explanations about the same phenomenon, it seems reasonable to find a way to do not dismiss all of them in virtue of only one. Take for instance the case of scientific pluralism.

> Pluralism, as an explicit program in philosophy of science, emerged from an increasing frustration with the limitations of unifying frameworks in the light of the disunified reality of scientific practice. (LUDWIG; RUPHY, 2021).

That put, it seems less controversial to adopt a pluralist approach. It should be clear, however, that pluralism and tolerance are different ideas, despite the former being always

accompanied by the latter. As already mentioned in the introduction, Carnap is considered as one of the predecessors on the discussion around tolerance about different logics, with his famous expression "In logic, there are no morals. Everyone is at liberty to build his own logic, i.e. his own form of language, as he wishes." (Carnap, 1937, §17). Being tolerant doesn't mean one *adopt* all the different existent perspectives, neither that she doesn't have a preferred one, she merely tolerates their development. Thus, one can be tolerant and not a pluralist, but not the other way around.

If we want to go a little deeper in that point, we can argue that the Principle of Tolerance put forward by Carnarp was thought to deal with scientific frameworks, and the tolerance would be about how to formulate and develop different frameworks, but not on how to choose the framework, since the choice would be of pragmatical import. Following Arroyo & da Silva (2022), a better excerpt from Carnap to illustrate the Principle of Tolerance would be:

> [1] Let us grant to those who work in any special field of investigation the freedom to use any form of expression which seems useful to them; [2] the work in the field will sooner or later lead to the elimination of those forms which have no useful function. (CARNAP, 1950, p. 40).

So that we have two parts, the first one was presented with our previous Carnap's quote and represents what is usually associated to tolerance in Carnap, while the second one would deal with a choice of the best framework, and is usually omitted.

3.1.2 Pluralism in Logic

To be pluralist requires stronger commitments with what is accepted. Logical pluralism is the general claim that there are many different and equally correct logics. By 'many' we mean at least two, by 'different' we mean that the different logics validate different sets of inferences, i.e., they are not merely verbal variants of one another. The 'equally correct' bit is a little more complicated. For now, we mean that there is no way to tell if one logic is more 'correct' than another, they are correct relative to something, such as a goal or domain, in the same sense we could employ the term 'adequate'. We could also use terms like 'good' and 'legitimate' to mean merely that different logics meet the requirement of what a logic should be. But a difference shall be pointed out when we use the term 'true' instead of all others, since it carries a stronger claim about truthness of a

theory. On account of this we shall not use "true" to define logical pluralism in order to avoid ontological concerns, although logical monists often employ that term, since they are usually ontologically committed to the chosen logic.

Logical monism is usually portrayed as a direct opposition to pluralism, but we shall see that this is not accurate. Our first apprehension is: logical monism asserts that there is only One True Logic, be it classical or not, so that we have a set of universally valid logical laws. This attitude is considered traditional since it was held for a long period of time.

A last remark is about the relation between logical pluralism and methodological pluralism. Those two names are sometimes associated but they represent distinct fields of discussion. Methodological pluralism is the thesis that logic has many methodologies, such as abductivism and predicativism. Therefore, the pluralism is about how its practice is heterogeneous in a way that two logicians could use different methodologies. Tajer (2022) argues that it is possible to perceive methodological pluralism as a scientific pluralism as long as we have a "coexistence of different (and sometimes incompatible) research programmes inside one discipline." (TAJER, 2022, p. 194). It is an interesting topic in philosophy of logic, but doesn't concern our interests in this dissertation.

3.1.3 Pure, Applied and Canonical

A useful distinction for us is about pure and applied logics and a canonical application to logic. The canonical application part was proposed mainly by Priest (2005), and the distinction between pure and applied can be found already in da Costa's works. They help us to understand and differentiate weak and stronger versions of pluralism, in which the latter aims to not merely state that there are many things identified as logics.

Pure logics, as the name suggests, are the logics without extra-logical applications. They just exist as logics. In this sense, it is undeniable that there are many pure logics. In fact, according to Priest, pluralism about pure logics is a 'no-brainer' in the sense that it is straightforwardly true; considering that, we could even extrapolate and say that Priest is a "pluralist about pure logics", although it would be only a trivial statement that says that exists many logics such that they are all "well-defined mathematical structure with a proof-theory, model theory" (PRIEST, 2005). Nonetheless, it is indeed possible to

disagree that this is a trivial fact, as does Bueno (2002), who argues that the development of many logics is at least a relevant sociological fact, as much as the development of non-Euclidean geometry. If we assume that only applied logics can ensue rivalry, then it seems that there is no conflict between Priest and logical pluralists, the conflict emerges only when Priest claims a canonical application, explained later in this section.

Continuing, theoretical pluralism occurs when one apply a pure logic to some extra-logical domain that is not canonical. One could think of a "theoretical monism", but that only makes sense if we assume that all extra-logical applications of logic are the canonical one, which seems unreasonable and contradicts the initial formulation. Thus, we can only be theoretical pluralists, with two main variations: global or local. The first asserts that *all* domains have at least two different equally good logics, whereas the second asserts that there are different domains of which two different logics equally apply, not necessarily to the same domain. Therefore, we could have many logics that does not compete about the same domains. The local theoretical pluralism represents the most common form of logical pluralism. A terminological remark is that sometimes pure logics are referred as 'logical systems', in contrast to 'logical theories', that are applied to something, other than just being a well-defined mathematical structure, proof theory, model-theory or algebra.

The last distinction here is about a canonical application of logic. This is a quite traditional view and also the reason why many logicians were monists about logic in the past. A canonical application of logic is a privileged extra-logical application, such that it has the power to determine a best logic: one that captures the canonical application,. For instance, Priest takes the canonical application to be deductive reasoning. We could also call the adoption of a canonical application "Telic Monism", since it assumes that logic has only one goal / application.

Let us for moment agree that the canonical application of logic is the analysis of deductive reasoning. Then, taking Priest as an example, the best logic that captures deductive reasoning, and therefore the One True Logic, would be the Logic of Paradox. His idea is that since the logical arguments are rough translations of our natural language, then there is always a way to disambiguate terms and decide which interpretation is the correct one.

But, of course, we can deny that idea and say that logic has many different canonical applications (Telic Pluralism) or even that it doesn't have any privileged goal, which

brings back a kind of theoretical pluralism. Also, it seems arbitrary to privilege one application over others and say that this is the canonical one and everything else doesn't matter, but we understand that sometimes one needs to make a dogmatic move.

3.1.4 Local and Global Pluralism

As we already mentioned, under the idea of logical pluralism lies many different theses which demand different assumptions. That is why we could redundantly talk about a "plurality of pluralisms", as done by Wylie (2015). If there are many kinds of pluralism, we can find some subcategories to place them and try to organize the discussion, as done by Susan Haack (1978) with a distinction between *global* and *local* pluralism. The difference lies in the universality of logical laws. In a local pluralism we have different correct logics for different domains.

> The local pluralist relativises the extra-systematic ideas of validity and logical truth, and hence the idea of the correctness of a logical system, to a specific area of discourse; an argument isn't valid, period, but valid-in-*d*. (HAACK, 1978, p. 223).

Global pluralism, however, take the logical principles to be valid in *any* domain (global) and that there are at least two *all-purpose* logics equally correct (pluralism). Using this distinction we could reformulate our first apprehension of logical pluralism to regard the range (or scope) of domains in which different logics apply to.

> [...] the global pluralist denies either that the classical and deviant logician are really using 'valid'/ 'logically true' in the same sense, or else that they are really disagreeing about one and the same argument/statement. (HAACK, 1978, p. 223).

Haack mentions yet another version of global pluralism, which can be associated with the meaning-variance thesis, since it asserts that arguments in different logics, even though typographically identical, have different meanings.

3.1.5 Meaning-variance Thesis

If one concedes that different logics have different meanings associated to the logical terms, then we could say that they are not really disagreeing, since they are just talking about different arguments. Here lies the *meaning-variance thesis* (or the *change of meaning objection*), which is usually attributed to Quine (1986).

> My view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation, '¬', 'not'; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions in the form '$p \wedge \neg p$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject. (Quine, 1986, §81).

The meaning-variance thesis is a central problem for any kind of logical pluralism and, therefore, constitutes one of the main challenges to them. For this argument to work we have to accept the premise that an interesting logical pluralism needs to have a substantial logical disagreement, where the logics share the same interpretation of logical terms. We shall address this subject several times from now on. An that takes us to the next issue: is logical pluralism uninteresting?

3.1.6 Is logical pluralism uninteresting?

Logical pluralism has an appeal to our intuitions, it seems obvious that there are many logics, therefore we should accept it until a contrary proof appears. But this is only a recent perception, since monism was the traditional view for a long period of time. This kind of appeal can be called *The Argument from Appearances*, but of course we should be cautious with appearances and it is a rather subjective 'argument' (if we can call it an argument at all). An upshot of the superficiality of this view is what Caret (2021) calls 'Mammal pluralism', where pluralists are merely stating a trivial fact: there are many kinds of mammals, such as dogs and cats, analogously there are many kinds of logics, for instance classical and paraconsistent. But, he argues that using pluralism to describe this would be a "pointless proliferation of jargon", in addition to being a boring thesis, although obviously true.

The seek for an interesting logical pluralism is a job for any defender of this kind of thesis. In order to establishes some criteria to what counts as a good pluralism, Cook (2010) proposed the Substantial Logical Pluralism (**SLP**). The first step is to settle a formal language and a vocabulary to define the desired notions of logical consequence. Then, Cook establishes four criteria that are considered desirable in a pluralistic formulation and necessary for any interesting form of pluralism. In his words:

> Once we realize that logics are merely models of the consequence relation in natural language, we might wonder whether there could be two logics such that (i) the logics apply to the same language, and involve the same interpretation of the logical / nonlogical divide; (ii) the logics are incompatible – that is, they validate different sets of inferences; (iii) the logics are, all things considered, equally good models of logical consequence in natural language; and (iv) there is no third logic such that this logic is a better model of logical consequence than the two competing logics. (COOK, 2010, p. 501) .

It is questionable if those are good criteria, but nevertheless they serve well to one purpose: to organize the characteristics of different articulations on the variety of logics.

The first criteria avoids the meaning-variance thesis, since we fix the meanings and interpretations of our terms. The second guarantee logical disagreement, since the logics competing are different. The third guarantees that the logics are competing with each other about the same domain, in the case of Caret, the logical consequence the context of natural language. Finally, the fourth establishes the *equal validity* criteria, that the two competing logics are the best models we can have, there is no way to untie them. We will see in Chapter Three that this assumption is problematic and, at best, defines a kind of relativism.

This proposal is a good standard for comparison between the different pluralistic thesis, and we will use it in the next sections. Nonetheless, it doesn't impliy that non-SLP pluralisms are uninteresting. In fact, there are forms of pluralism that change the logical vocabulary, such as the logical relativism by Varzi (2002), the truth-bearer pluralism by Russell (2008) and even the Carnapian pluralism (1937).

3.1.7 The Collapse Argument

The Collapse Argument (or Collapse Problem), is a very common objection towards pluralist approaches, in a very broad sense. The general idea is that when a pluralist tries to harmonize or reconcile different logics they end up collapsing them. This collapse can happen in any direction: up, down or to the sides. This means that a collapse can lead to universalism (up), nihilism (down) or something in between (sides), such as having only two logics as the best ones. Several authors have formulated versions of the collapse argument, such as Bueno and Shalkowski (2009), Keefe (2014) and Read, who describe it

as follows:

> [...] suppose there really are two equally good accounts of deductive validity, K_1 and K_2, that β follows from α according to K_1 but not K_2, and we know that α is true. Is β true? [...] It follows K_1-ly that β is true, but not K_2-ly. Should we, or should we not conclude that β is true? The answer seems clear: K_1 trumps K_2. (READ, 2006, p. 194-195).

The idea is that there is always a way to determine which logic is better: the one that answer the normative questions. Therefore, a pluralist wouldn't always have equally good logics. It is explicit, however, that this argument presupposes that logic is normative, so one could deny it and go on. Unfortunately, it is not that easy, since normativity plays a central role in many apprehensions of logical consequence, such as the one put forward by Beall & Restall.

The Collapse Argument intended to claim logical monism can be illustrated with diagrams, so that each circle represents a logic accepted as correct and at the intersection of all accepted logics there is only one logic, the best one. Therefore, monism. But it is not clear what would result out of the intersection of all accepted logics, and even if we could call it a 'logic'. Imagine that only the rule of elimination of negation is left, could that be considered a logic? Or worst, imagine if we get nothing left, thus we would have trivialism, or universalism.

It seems that what is at stake here is the choice of a best logic, so that the collapse argument would be responsible for it, but it takes only normativity as criteria, whereas in fact we can have many others. So it makes sense to think on how can we choose a logic. So far we have already mentioned several times the possibility of choosing a best logic, either for a specific domain, or to a canonical application. Priest (2005) points out some useful criteria to help the choice of the better logical system, such as: theoretical integrity, adequacy to the data, simplicity, explanatory power, avoid ad hocness, unity and fruitfulness. The opposites of each of them would be the undesirable features.

Those criteria are useful for pluralists too, since they do not necessarily determine a best logic for *every* domain, as we shall see with the Principle of Uniqueness by da Costa. And even if they do, if there is no such thing as a canonical application then pluralists do not

have to worry, since the choice of logics would be governed by different applications, at least for local pluralists.

Furthermore, there can be other criteria beyond the pragmatical ones, such as the domain (or case, or context) in which the logic is applied, as suggests Bueno (2009). This is a common strategy for different kinds of logical pluralism. After all, the question of theory choice doesn't segregate pluralists and monists, but this is another intricate topic of which we will not delve into.

3.2 One, Many, All or None

3.2.1 Logical nihilism and universalism

Let us begin by the edges of our discussion. If we ask: how many logical laws are there? We could answer none. Or all of them. Or just some. But in this section we shall work just with the two first responses.

The first tends to logical nihilism: the thesis that there is no universal logical law. This sounds pretty controversial, since it goes against many monist and pluralistic views on logic. It seems to left us with nothing to work with, to destroy the logic endeavor. But it hides an assumption: to say that no logical law is *universal* is to say that there is no domain neutral logical law, one that applies to every possible domain we consider. Furthermore, it means that among all the legitimate, or correct, logics there is no common logical law, i.e., the intersection of all logical laws of all logics is empty.

In that sense it is not as much as controversial as first it seemed, and it recall us the collapse problem, since the intersection of all legitimate logics would originate only one true/correct logic. But there is a difference between 'logic' and 'logical laws', i.e., to have a logical law doesn't imply we have a logic. That is why nihilism is a stronger claim: the intersection must be empty in the sense that not a single logical law remains and, consequently, no logic.

Logical universalism is the thesis that all logical laws are valid and, therefore, we inevitably create a trivial statement. But it is not a direct opposition of nihilism, i.e. the idea that we can have different logical laws valid in different logics but there is no common logical law among all of them. Since anything would follow we can also call this view "trivialism".

Those two kinds of approaches are used in the literature to illustrate possible problems of a thesis that tries to explain the plurality of logics, since they are usually taken to be both trivial. But here we consider that only logical universalism represents a serious threat, since it deflates the number of logics to a point that anything goes. Regarding logical nihilism, we think it is a problem only in the sense that it is hard to achieve, but it doesn't evolve into triviality and, thus, can be a reasonable position.

3.2.2 Instrumentalism

It is very common to refer to logic as a *tool*. Whether for thinking, for deductive reasoning, or even to more unorthodox applications, such as electronic circuits, to formalize possible worlds, to work with grammatical categories, among many other inquiries. Instrumentalists argue that this is all logic is about: pragmatic adequacy to a chosen goal.

Logical instrumentalism asserts that logics must be accepted and employed in accordance to a established goal, but it doesn't guarantee that there are at least two logics useful for the same purpose, neither that they are correct or true in any sense. It can be seen as a simple theory that ends abruptly in its philosophical considerations, since there is nothing more to logic than its utility for a given domain. Instrumentalists are not concerned about correctness or truth of a logic. The 'correct logic' is merely a choice and is not determined by an objective reality, but only internally to the system. This could lead us to important discussions about realism and anti-realism in logic, that are often interconnected with the whole them of this dissertation. Nevertheless, that would represent an effort we chose not to follow for now, but it is a part of a future work.

One could ask if instrumentalism is a form of pluralism, monism or other theory. Our answer is that this would be too much of a stretch of terminology. Since instrumentalists are not concerned about correctness then they would fall out of many formulations of pluralism, even if they accept different logics as useful. Also, an instrumentalist could say that there is only one useful logic and, therefore, look like a monist. But again, the commitment involved with the accepted logic is different.

Instrumentalism is a well known approach in philosophy, so it is easier to understand its problems and virtues. A virtue, in this case, is that it makes sense of the plurality of logics. The most obvious problem is that it doesn't explain *why* it makes sense; certain theories

work for certain purposes and others do not, but there is no explanation on how the relationship between empirical facts and logical facts is done. For a instrumentalist it does not matter if a logical theory is consistent with reality, just that it works for pragmatical purposes, so in analogy to science it is like accepting the theories that currently work. For instance, today it could be general relativity and quantum mechanics, but in the beginning of the 19th century a instrumentalist about scientific theories would accept the Bohr's theory of atom, even though it was shown to be inconsistent later. So, either a logical theory is consistent or not (in a very general sense) is not a concern for instrumentalism, all it needs to fulfil is pragmatical usefulness.

3.2.3 Case-based pluralism

In this section we present the case-based logical pluralism developed by Beall & Restall. It is a standard example of logical pluralism and has been studied by many authors. Their logical pluralism revolves around the notions of validity and logical consequence, which are built on the Generalized Tarski Thesis (GTT):

> An argument is $valid_x$ if and only if, in every $case_x$ in which the premises are true, so is the conclusion. (BEALL; RESTALL, 2006, p. 29).

The variety of logics is originated from the vagueness of the notions of deductive logical consequence and validity, which are specified by what they call *cases*. In this way, where validity is defined as truth-preservation over these cases, we can have different logics within the same formal language, but with a different background class of interpretations. By preserving the language Beall & Restall aim to preserve the meaning of the truth-bearers (sentences, statements, ...).

At first sight their conception of validity seems simple, but they impose classical restrictions of what an *admissible* notion of logical consequence needs to meet. First, it must be necessary, formal and normative. Necessary because the truth of the premises necessitates the truth of the conclusion. Formal because the matter / content of the argument is irrelevant, only its logical form. Normative because invalid arguments are somehow wrong with respective to how we ought to reason. This is a very simplified explanation of these constraints, but they offer a comprehensive explanation of which of them in their book *Logical Pluralism* (2006).

Second, it must be a Tarskian relation, that is, according to **Definition 1.1.**, the logical consequence is reflexive, monotonic and transitive. Reflexive because for any set of formulas $\Gamma, \Gamma \vdash \Gamma$. Transitive because given three sets of formulas Γ, Δ, Σ if $\Gamma \vdash \Delta$ and $\Delta \vdash \Sigma$ then $\Gamma \vdash \Sigma$. And monotonic since the addition of premises to an argument doesn't changes its conclusion. That is, if $\Gamma \vdash \Delta$, then for any Σ we have $\Sigma \cup \Gamma \vdash \Delta$.

In short, case-based logical pluralism is the thesis that there are at least two different admissible instances of validity given by the GTT. But, considering the restrictions imposed, their pluralism doesn't encompasses some alternative accounts of validity, such as in non-reflexive, non-transitive and non-monotonic logics, as well as proof-theoretic approaches. This ends up making their pluralism quite narrow and, some claim (BUENO; STEI), too classical. Their response to this is that non-transitive or irreflexive logics "are logics by courtesy and by family resemblance" (2006, p. 91).

The main examples of validity are when we take *cases* as i) Tarskian models[1] or possible worlds to characterize classical logic; ii) constructions − such as stages − to characterize intuitionistic and constructive logics ; iii) incomplete or inconsistent situations to characterize relevant or paraconsistent logics.

An important question is what exactly they mean by "cases", since there is no clear definition to that, only examples. This is one of the many objections to case-based pluralism and they reply most of them in the worthy reading chapter *General Objections*. They offer an analogy to cases as truth conditions one can specify to the logical consequence relation, but it doesn't guarantee that these conditions generate admissible precisifications of validity, that is, we can have cases that form non-admissible notions of validity. This is where they draw the line to what is a legitimate logic or not, although they claim to take a liberal position regarding what counts as cases.

Now we shall try to show how this kind of pluralism is sometimes seen as a form of relativism. To begin with Burgess (2010), he argues that Beall & Restall pluralism only makes sense if we have a previous application in mind and, if that is the case, then their pluralism would be best understood as a relativism, although they deny that designation.

> Perhaps pluralism *is* relativism but relativism of such a harmless kind that to

[1] In the sense of model-theory.

> use that word to promote it would dramatise the position too much. (BURGESS, 2010, p. 521).

What Burgess means by "harmless" is not clear and, we shall argue, it is symptomatic of a literature that is not dedicated to further investigate the concept of relativism. Also, his attempt to dismiss logical pluralism enlightens a point that we further discuss below. He argues that "For pluralism to be true, one logic must be determinately preferable to another for one clear purpose while determinately inferior to it for another." (2010, p. 521). Having a preferable logic means we have a best logic, even if it is relative to a specific domain, and that resembles a monist view in the sense that, assuming the logics are domain neutral, we always have a way to elect a best logic. But it is not monist, it is just a weak pluralism: there are many logics that can be correctly applied to different domains, each of them having a preferable one. This is just a local pluralism regarding pure and applied logics.

To obtain monism we would need to show that this doesn't constitute only pure and theoretical applications, but also that logic has a canonical application and, therefore, a correct logic. However, Beall & Restall make it clear that "there is no canonical account of $cases_x$ to which GTT appeals" (2000, p. 36), so we think Burgess' argument that case-bases pluralism collapses into monism doesn't follow, although we agree that their pluralism is indeed a form of relativism.

Another objection in favor of a collapse into monism asserts the universal quantifier in the GTT, "every $case_x$" requires that we have a unique logic at the intersection of all admissible specifications of the GTT, resulting in a monism. But Beall & Restall argue that the intersection would hardly be a logic, in fact, they guess the only inference left on the intersection of all legitimate logics would be the law of identity.

Up to now we have shown some problems Beall & Restall's pluralism faces: it can be too narrow and too classical, it is not clear what counts as 'cases', it can be a form of relativism, it collapses into logical monism. But there is still important questions left: how they avoid the meaning-variance thesis? Do they guarantee logical disagreement?

Case-base pluralism establishes a formal language for every logic, the only aspect that changes is the specification of the vague notion of validity. Thus, there is only one language

and only one notion of validity, but it is vague. This avoids in part the meaning-variance, but it doesn't assure logical disagreement, since the different logics they obtain using the GTT are not *always* competing about the same cases, by definition. In other words, the different logics their pluralism accept are not rivals, just different. If this is really a fault for logical pluralism is not a solved matter. For Beall & Restall it is not a problem, since pluralism emerges from the different cases and it doesn't matter if there is always rivalry (in the sense of competing about the same domain) and how it is disclosed. Differently, according to the SLP from Caret it is indeed a problem, since it violates the last two assumptions about rivalry and equal validity.

3.2.4 Eclectic perspective

In this section we present the perspective endorsed by Shapiro (2014). The starting point of his analyses is to set out that logic is about rigorous deductions and how they can be used in legitimate mathematical theories. His strategy is to show that classical logic would render inconsistent some interesting mathematical theories and, therefore, we have to accept non-classical logics, such as intuitionistic and paraconsistent ones[2]. Hence, his logical pluralism stems from a mathematical pluralism.

The initial idea of this eclectic orientation is to argue that validity and logical consequence are *relative* to structures and, therefore, this kind of pluralism can be understood as some kind of *folk-relativism*, a designation introduced by Crispin Wright and inspired by the motto "There is no such thing as simply being ϕ." (2008). It means that something (ϕ) can only be evaluated relatively to another thing, and that is the general understanding of relativism. A very simple schema, the General Relativistic Schema (GRS), is given by Swoyer (2003): Y is relative to X[3]. This means that Y can be judged only when relative to some independent X, such as etiquette relative to social groups. For now this brief introduction to relativism suffices, but we are going to explore this concept further and in detail in the next chapter.

With that in mind, Shapiro's orientation is more comprehensive than Beall & Restall's, since he doesn't impose classical restrictions to what a logical consequence needs to be. Instead, he follows the Hilbertian slogan: consistency implies existence. This is the strong

[2]Shapiro dedicates the third chapter of his book to illustrate some of the mathematical theories he is interested in, such as Heyting arithmetic and smooth infinitesimal analysis.

[3]This is an initial attempt to capture relativism, but it can clearly overgenerate unwanted cases of relativism.

version, but Shapiro mentions that consistency can also be understood as simply non-triviality, so it can also encompasses paraconsistent logics. That way, as long as a mathematical theory is consistent relative to a logic, it is a legitimate mathematical theory worthy studying and, since mathematical pluralism entails logical pluralism, we obtain an eclectic approach. But pluralism and folk-relativism are not the same, as the first is stronger, since it requires equal validity between at least two consequence relations.

The eclecticism comes from the many ways one can be a folk-relativist about logic, Shapiro mentions four: i) logical consequence as cluster concepts; ii) logic-as-model[4] approach; iii) logical consequence relative to logical terminology; and iv) logical consequence as a vague, but unique concept.

The first takes logical consequence and validity as a cluster that unites different notions that go under the same name and that "there are different, mutually incompatible, but equally legitimate ways to sharpen or further articulate the intuitive notion(s) of logical consequence"(2014, p. 17). Since they are modelling different aspects, they are not competing and there is no sense saying one is more correct than another. To exemplify Shapiro mentions nine characterizations of logical consequence, which can be *modal*, with respect to possible worlds; *linguistic*, relative to terminology or to the interpretation of a language; *semantic*, when relative to truth; and *epistemic*, with respect to deductive reasoning and what is rational or not or even relevant or not. All those notions of logical consequence are not necessarily disjointed, i.e., completely distinct, but some of them are indeed different and incompatible and that is why it is a cluster, polysemous concept.

Second, if we take a logical system analogously to mathematical models then we are modelling sentences, instead of mathematical objects. Those sentences come from the natural language and different logics will be used to model different aspects of it, for instance, to refute or deduce arguments or to regulate beliefs. As long as the different logics are models to different aspect of natural language, we can have some models better than others, i.e., one can be simpler but less accurate and another more cumbersome but more accurate. Nonetheless, he argues that we cannot talk about a "more correct" model, but we can evaluate them among criteria such as simplicity, *ad hocness*, explanatory power etc. To obtain a Substantial Logical Pluralism we would need to show that at least two models are equally good in the sense they meet the same criteria and can not be untied.

[4]In the general sense, not model-theoretical.

Otherwise, it is "just" folk-relativism.

Third, validity and logical consequence can be relative to our choice of logical terms, i.e., we can have different logical and non-logical terminology which will determine what is a sentence and its content. This means that an argument can be valid in respect to some logical terms and invalid in another, such as the case of first and second-order logics or even first-order with or without identity, and this view allows both to be legitimate logics. Also, it allows both model-theoretic and proof-theoretic studies (which Beall & Restall dismiss, for instance).

If we want pluralism in this sense, then we have to assume that there is no absolute logical and non-logical distinction, so that not every sentence has a unique logical form. It doesn't mean, however, that any distinction is a good one − in fact, that is what generally is disagreed about in the literature. Then, negation, conjunction, disjunction or other connectives we want could not all be described with certainty from the natural language to a formal language and, furthermore, we would not have a sharp boundary between what is a logical term or not. Here it seems hard to avoid the meaning-variance thesis, since we are literally changing the interpretation of our language to obtain different logics, but it doesn't necessarily implies that this would be a bad pluralism.

The fourth way to be a folk relativist is to take the notion of validity and logical consequence as unique, but vague and with borderline cases of validity. Vagueness means that logical consequence can be sharpened by different forms, such as conversational contexts. Then, validity would be a "fixed, but unspecified aspect of the intuitive notion" (2014, p. 57). That alone doesn't entails pluralism, since vagueness can be eliminated, as suggests Priest when talking about how the natural language can be uniquely interpreted into a logical terminology. But, if we deny that then we can discuss vagueness.

A borderline case can happen when the same argument can be valid or invalid with respect to different notions of validity. We can assume different stances on that. As Shairo notes, an epistemicist argues that the vague argument can be sharpened and has a correct veridict over its validity, but we can not know which. Therefore, the epistemicist is a monist in this regard, although they do not indicate which is the correct logic, just that it exists. The second option for Shapiro is to use many-valued approaches, where we can have a degree of truth for vague statements. For instance, a person with few strands

of hair would be closer to be bald than to be hairy, but would not be precisely either one.

Yet another folk-relativist option is to argue that vagueness is context-sensitive, so that our predicates would be always relative to some context, even if some, but not all, are valid or invalid in all cases. This covers indexical and non-indexical contextualism. The first take the content of the predicates as varying relatively to contexts, while the latter take the meaning of the predicates as fixed, but with their extensions varying in different contexts.

Then, Shapiro makes clear his own version of folk-relativism, which takes logical consequence relative to structures. He exemplifies his case with Heyting arithmetic plus Church's thesis, intuitionistic analysis and smooth infinitesimal analysis, each of wich requires a non-classical logic to make sense. Since they are all mathematically interesting theories, we should accept the different and mutually logics they employ, provoking a pluralist approach towards logic (at least, to the logics of mathematics).

One last thing to say about Shapiro's approach is the useful notion of *super validity*: an argument is super valid when it preserves truth in *all* interpretations or structures. It helps us to understand better the distinction between local and global pluralism, since only in the latter we would have logical laws that are super valid, therefore, a nihilism about logical laws would be impossible. We will see that this distinction is crucial to explain how logical pluralism is peculiarly understood by Priest. To Shapiro and most logical pluralists, however, the concern is not about this non-context-sensitive notion.

3.2.5 Truth-Bearer pluralism

Pluralism about logic generally works with the notions of validity and logical consequence, but Gillian Russell's truth-bearer pluralism changes this pattern. Instead, she proposes that what generates a plurality is the variety of kinds of truth-bearers. In general they are taken for granted to be sentences or propositions, but this is not well justified, argues Russell (2008).

First of all, if we take an argument to be a pair composed by a set of premises and a conclusion, we can assume that they determine the validity of an argument accordingly to its truth-values. So, in that sense they are some kind of truth-bearer, but it is not clear what kind, we want to know what the arguments are made of. The usual candidates,

according to Russell, are "[...] sentences, propositions, characters, statements, utterances, occurrences of sentences, beliefs and judgements." (RUSSELL, 2008, p. 596).

To find better candidates to make sense of logical practice, she works with some problems from philosophy of language, such as how to deal with validity of arguments with indexicals like "I am here now" and co-referential names like 'Hesperus' and 'Phosphorus'. Depending on what are our truth-bearers the validity of our arguments changes. To illustrate, we will use two of her examples:

(1) Hesperus is bright

Phosphorus is bright

(2)

I am here now

If truth-bearers are propositions isolated then only (1) is valid, since 'Hesperus' and 'Phosphorus' are co-referential names. But (2) can express many different propositions that are not valid in every context, since its reference changes. If we take truth-bearers to be sentences paired with characters, then (2) is valid, since the character determines that the content of 'I' is relative to contexts and its agents[5]. Thus, if we obtain different sets of valid inferences when changing the kind of truth-bearer, we have different logics and consequently, if they are both legitimate kinds of truth-bearer, we must assume a pluralist position towards the multiplicity of the logics originated.

Yet another example is if our truth-bearers are possible worlds, then classical and modal logics would be equally good choices. It doesn't mean, though, that we cannot evaluate which is better. They are only equally good in the sense that both are legitimate logics. This leaves us with some questions if this kind of logical pluralism is genuine in the sense of **SLP**. It seems that it is not, since when telling if an argument is valid or not we are answering to different questions, but at the same time we are accepting that different logics are equally good logics in the sense they work for their purposes. To be a monist in that case would requires one to fix the 'right' truth-bearer, if there is such a thing, but that is not the case here.

[5] As Russell puts it: "Character is meant to be the component of the meaning of an expression that determines its content relative to different contexts, and which competent speakers of the language know (at least tacitly.)"

3.2.6 da Costa contextualism

In his book *Ensaios sobre os Fundamentos da Lógica* (1994), da Costa discusses several issues about philosophical practice and rationality, especially on how logic and reason are related. His views are strongly grounded on what he calls *scientific philosophy* (*filosofia científica*), which is a methodology to philosophy guided by four main methods: "1. semiotic analysis; 2. the use of special sciences; 3. the historical exemplification; 4. the elaboration of hypothetical models." [6] (1994, p. 12).

From that he derives the *dialectical* position concerning reason and logic, which deviates from the so called *dogmatic view*, endorsed by philosophers such as Quine. Summarily, the dialectical position defended by da Costa takes experience to play a central role in reason, which can be exercised and determined by logic. Meanwhile, the dogmatic position take the logical laws to be constituted by the formal principles of reason and independent of experience. But the main difference that interests us here is that the dialectical position allows many logics to be rational, while the dogmatic postulates the univocality of logic, i.e., the ideia that there is only one logic, although it can have little variations in its possible formalization.

Thus, da Costa explicitly state a plurality thesis: "There is no unique logic. In principle, there are several, all legitimate from the rational point of view."[7](1994, p. 18). One of his analogies is about the use of different geometries for different physical theories as much as we can use different logics for different domains of investigation.

To make sense of the recent developments of logic and how they are related to reason, da Costa first establishes three pragmatical principles of reason, they are (1994, p. 46):

A. Systematization: reason is always expressed by a logic.

B. Uniqueness: each context has a unique underlying logic.

C. Adequacy: the best underlying logic for a context is the one that better adapts to it.

[6] Original text: "1. a análise semiótica; 2 . o recurso às ciências especiais; 3. a exemplificação histórica; 4. a elaboração de modelos hipotéticos."

[7] Original text: "Não há uma única lógica. Em princípio, existem várias, todas lícitas do ponto de vista racional."

For instance, he argues that classical logic is the underlying logic for traditional mathematics. But da Costa recognizes that these principles can stand a monist approach to logic, that is, we could have only one logic, so that by itself doesn't guarantee his pluralistic view. Later, he adds a fourth principle of constructivity, that is attached to how intuitionistic arithmetic take account of the regularities of the intuitive and creative aspect of reason.

Those pragmatical principles delimit the choice of logical systems for a given rational purpose, they offer the pragmatical dimension of logic and mathematics. Given that, we shall try to understand his pluralistic view. Although he uses the expression "logical pluralism" it is just to refer to the fact that there are many logics different from the traditional classical one, so it is a weaker sense of the term. What is also interesting is that he recognizes that, as long as we use different logics for different purposes, we have a kind of relativism and, interestingly, it doesn't happen only with change of domain, but also because logic is dialecticizable, as much as scientific theories changes, logic changes: "Between physical laws, to mention one example, and logical laws there is only a difference of degree regarding their content of reality, but not a difference of nature."[8](1994, p. 114). This aspect about revision in logic is further and interestingly investigated by Arenhart (2021) and da Costa and Arenhart (2018).

In agreement with the pluralism advocated by Beall & Restall, Shapiro and others, da Costa also uses the concept of logical consequence to guide the discussion. He argues that non-classical logics encompass different notions of logical consequence and validity. One of the examples given to illustrate is the Schrödinger logic that "dialectizes" (changes) the classical inference of identity so that it loses universality. But da Costa's pluralism seems more liberal insofar it seems to accept logical systems with different formal languages, different semantics and different relations of logical consequence.

We shall call his form of pluralism of *Contextual Pluralism*, since the different logics are obtained by the use of them in different rational contexts, specifically scientific ones. One of the reasons why da Costa is considered a logical pluralist is motivated by the following:

> However, at no time should it be thought that pragmatic principles unequiv-

[8]Original text: "Entre as leis físicas, para mencionar um exemplo, e as leis lógicas, há unicamente uma diferença de grau, no tocante a seu conteúdo de realidade, e não uma diferença de natureza."

> ocally determine a logical system, even in a restricted domain of knowledge. On the contrary, the reader will be convinced that several logics fits for the same rational-objective situation, and that, even more, what we will call the *relativity norm* holds: there is almost no logical principle that cannot be derogated from, in the sense of that there is a reasonable system of logic in which it doesn't generally hold.[9] (1994, p. 124).

It could seem that Priest agrees with da Costa, especially concerning the uniqueness and adequacy principle, since we could think that all contexts have a best suited logic. But those principles of reason do not unequivocally determine a single logic, so that more than one can be applied to the same domain. In that sense we could identify da Costa's pluralism with a local pluralism and to check whether Priest differs from him is about finding a canonical application to logic where there is only one best suited logic.

As much as we understand da Costa's philosophy, the closest we have to a canonical application is the use of logical laws to make rational sense of scientific practice and, ultimately, to follow the pragmatical principles of reason. Since many logics achieve that, it justifies logical pluralism. But it doesn't seem to be a Substantial Logical Pluralism, as a consequence of the principles of adequacy and uniqueness. To conclude, there is an even stronger assertion made by Da Costa, that of the *relativity norm* in which no logical law is safe to be universally and *a priori* valid, and even if that was the case, it is hard to see how that could be a logical system at all[10]. This reminds us of the weak version of logical nihilism: there is not enough universal logical laws to characterize a proper logic. The next section hopefully will clarify some of Priest's similarities with da Costa.

3.2.7 Priest's Logical Monism

Graham Priest is probably the most prominent defender of logical monism, and for that he endorses most of the objections directed to pluralistic approaches, such as the meaning-variance thesis, the collapse problem and the uninteresting pluralism. In order for monism to work, two initial suppositions are needed. The first and foremost is that logic has a canonical application, and that is deductive reasoning. Second, the natural language,

[9] Original text: "Porém, em momento algum, deve-se pensar que os princípios pragmáticos determinam univocamente um sistema lógico, mesmo em área restrita do conhecimento. Ao contrário, o leitor se convencerá que várias lógicas dão conta da mesma situação racional-objetiva, e que, mais ainda, vale o que chamaremos de *norma da relatividade*: quase não há princípio lógico algum, que não possa ser derrogado, no sentido de que há um sistema razoável de lógica no qual ele não vale em geral."

[10] A discussion relating da Costa's scientific methodology and the relativity norm can be found in Arenhart's article "Newton da Costa on hypothetical models in logic and on the modal status of logical laws" (2018)

our vernacular, can be unambiguously determined by a formal language, including the notions of validity and logical consequence. Thus, validity is not a vague concept, as some pluralists suggests, but rather a unique notion that is captured by the One True Logic – which for Priest is the Logic of Paradox (from now on, **LP**)(1979).

The main idea of the **LP** is to accomodate semantic paradoxes, such Liar-like ones, and to make sense of dialetheism, the thesis that there are true contradictions. In order to so, Priest gets rid of the Principle of Non-Contradiction, which was already denied by philosophers such as da Costa. The advantage of his logic, he claims, is the intuitive semantics. But allowing contradictions without trivializing the system has a high price, and for Priest it means to abandon the truth-preservation of useful inferences such as Modus Ponens, Disjunctive Syllogism, Modus Tollens and Reductio Ad Absurdum. Nonetheless, he argues that these rules preserve validity in classical contexts, i.e., when the truth-values are classical, those inferences are valid, or as he baptizes them 'quasi-valid'.

The reason for that is due to the possibility of sentences to be both true and false, called 'paradoxical' sentences. The classical truth-values are understood as 'true only' and 'false only'. That said, we can construct the truth tables for each connective and check the validity of inferences. For instance, the *ex falso quodlibet* (or Principle of Explosion) can not be used as a rule of deduction, only when dealing with classical contexts. Differently, da Costa's paraconsistent logics establishes a difference between *well-behaved* and *ill-behaved* formulas, where the first is defined to work under the Principle of non-contradiction and thus is valid in classical contexts, so that all classical inferences follows for them. But for Priest introducing operator of ill and well-behaving formulas is not a good idea, since we could produce strengthened liar paradoxes.

The taxonomy presented in the beginning of this chapter, about pure and applied logic as well as theoretical pluralism, is adopted from Priest (2005) and is essential to make sense of his monism. He judges as uninteresting ('no-brainer') any kind of pluralism that emerges from pure logics and logics applied to non-canonical applications (theoretical). Moreover, even when we have different formal languages for different logics, such as classical and modal ones, he argues that they are not really disagreeing, since we can identify if one of them is an extension of the other and, therefore, not rivals.

When explaining da Costa's argument for pluralism, Priest says that da Costa talks about

different kinds of objects requiring different properties and also different logical operators. But he argues that validity is truth-preservation in *all* situations, in the sense of super-validity mentioned before. What he concludes is just that those laws that are valid in some domains and invalid in others are just contingent properties of their respective domains. As long as different logical operators have different truth-conditions, they have different meanings and, therefore, given that the vernacular is unambiguous, we can choose which of the meanings is the correct one.

If we really had two correct apprehensions of connectives, say, conjunction, then we would need two different symbols for the two kinds of conjunction and that would be pluralism about connectives. But the problem would not be logic, but the ambiguity of our natural language and that doesn't constitute logical pluralism. He then goes on to say what exactly are monists and pluralists disagreeing about, since it seems that both of them can identify "a core of universally correct inferences" (2005, p. 203). But this is grounded in the belief that we must only evaluate the variety of logics by the lens of an allegedly canonical application. So there is many pure logics and there is many applied logics, but it seems to not matter if we use different logics for different applications, i.e., a local pluralism. That helps us understand why Priest is a mathematical pluralist, but not a logical one. The main idea of mathematical pluralism is:

> [...] there is a plurality of pure mathematical investigations such that none of them can be reduced to the others ¯ in the sense in which it is standardly assumed that all mathematics can be reduced to ZFC.[11] (PRIEST, 2021, p. 40).

It is analogous to theoretical pluralism, so that each context of application can have a best suited mathematical theory that is evaluated using some criteria, such as adequacy to data, little or, preferably, non *ad hocness*, explanatory power, simplicity, theoretical integrity, fruitfulness, unity and so on. All that matters is that the different mathematics are equally good and legitimate structures.

As much as pluralism about pure logics and theoretical pluralism are true, so is mathematical pluralism. But Priest (2021) argues, against Shapiro, that there is a difference between those two, such that mathematical pluralism doesn't entail logical pluralism.

[11] 'ZFC' stands for Zermelo–Fraenkel set theory with the Axiom of Choice.

The canonical application of logic is about truth-preservation *simpliciter*, and not merely truth-in-a-structure.

Summing up, his idea of mathematical pluralism is about pure mathematical theories, while logical pluralism needs to be about the canonical application. Thus, pluralism about pure logics and theoretical pluralism are trivially true for him, but they are independent of mathematical pluralism − here lies the difference from Shapiro's perspective.

He recognizes that one could think it is not fair to compare mathematical and logical pluralism as he does, since we are using different standards. That is, if mathematical pluralism is about pure mathematical theories then logical pluralism should be about pure logical theories. But Priest claims that this works only in accordance to Shapiro and other logical pluralists who take mathematical pluralism about pure mathematics to entail logical pluralism in the broadest sense. His divergence is solely in the fact that logic requires a canonical application to logic, of which results the only One True Logic and, therefore, justifies his idea of logical monism. In other words, his idea of logical pluralism is not about pure or applied (theoretical) logical theories, but about deductive reasoning, the canonical application.

Our take here is that this monism is not as polemical as it sounds and it is actually pretty much in agreement with many pluralistic approaches, the main difference is the use of a canonical application to justify a privilege when choosing a logic.

3.2.8 Approaches Aplenty

We once again emphasize that there are numerous approaches to make sense of logical variety and it would be impracticable to discuss all of them in this work. Nonetheless we would like to briefly mention some more of them, for the sake of acknowledgment. Namely: Hjortland's intra-theoretic pluralism, Logical Ecumenism, Batens' relativism, Read's monism and Varzi's relativism.

Hjortland's intra-theoretic pluralism

The idea of an intra-theoretic pluralism is mainly to get rid of the meaning variance problem by taking a single logical system and formal language, but multiple relations of logical consequence. For that, Hjortland (2013) argues that the logical connectives in

different logical theories have the same meaning. The only thing that change from one logic to another (and gives rise to pluralism) is how the truth-values are formulated, in a way that the same argument can be valid for one logic and invalid for another. An illustrative case is for **LP** and Kleene's 3-valued logic (**K3**): the designated values for **LP** are both the 'true only' and the 'paradoxical', so that the truth-values are $\mathcal{V}_{LP} = \{t, p, f\}$ and the designated values are $\mathcal{D}_{LP} = \{t, p\}$. Differently, in **K3** we have only true and false, such that the truth-values are $\mathcal{V}_{K3} = \{t, i, f\}$ and the designated values are $\mathcal{D}_{K3} = \{t\}$. Thus, they yield different valid inferences.

It is important to note, however, that as much as it avoids meaning variance, it also doesn't fit into a **SLP** pluralism − in fact, this is seen as an advantage for Hjortland −, since we do not need to rely in the existence of two equally good theories. About the collapse problem, if one equates logic to interpretation and language, but not how the truth-values are treated, then we could see this pluralism as a monism which permits different logics inside the same system, but we think that would be an abuse of vocabulary.

Logical Ecumenism

Ecumenism aims to offer a pacific environment where two distinct logics can co-exist, usually classical and intuitionistic logics, since they were the ones which Prawitz (2015) worked with in the later called *ecumenical systems*. The name 'logical ecumenism' was coined by Dag Prawitz and shares the religious meaning of it, with the analogy of different religions celebrated in the same ecumenical church, as much as we could use different logics within the same system, sharing some of its connectives:

> In this system, the classical logician and the intuitionistic logician would share the universal quantifier, conjunction, negation, and the constant for the absurd (*the neutral connectives*), but they would each have their own existential quantifier, disjunction, and implication, with different meanings. (MARIN et al., 2020, p. 1).

Although it is very similar to the whole pluralist endeavor, the context and origins of ecumenism are different. The debate around this tends to be more formal in the sense of trying to find the logical relations between different elements of different formal systems, so as to include one in another, even if there are losses on the way. For instance, studying how to adapt classical proof systems or classical quantifiers into a constructive system.

An interesting upshot of this perspective is the possibility of a dual collapse between pluralism and monism, such that one would annihilate the other. First, take a logic L_1 chosen by a monist as the correct one. If we manage to express another logical system L_2 into this one, by defining its terms, then it seems that the monist would have to recognize some of the legitimacy of L_2. Second, and more obviously, if there is only one system, even if it is capable of expressing different logics, then we would have some kind of logical monism with a universal logic.

But we think that this allegedly annihilation is only apparent, since it doesn't consider the different kinds of logical pluralism and monism. Let us take Priest's monism for instance. He recognizes that there are a plurality of logics, pure and theoretical, and the possibility of one logic extending the other. Nonetheless he doesn't consider himself as pluralist, since those logics, while being used, are not as good as **LP** for the canonical application. Even if all inferences of **LP** can be expressed in classical terms, the validity of them changes. Only merely verbal variants of **LP** would corroborate this annihilation idea, but then it would be a rather trivial 'plurality' of logics.

Batens' Contextualism

The perspective advanced by Diderick Batens (1985; 1990) encompasses much of the literature on paraconsistency and thus we consider his views to be reasonably comparable with Priest and da Costa's previously mentioned ideas. His kind of pluralism is reasoned in a similar way to da Costa as much as they relate logic and scientific reasoning, so there is a whole philosophy behind their proposal. Batens argues that different terms of a language have different meanings for different 'contexts', which he defines as *communication situations*, or *problem-solving* situations. Those situations include a set of individuals, a problem to be solved, some statements taken as correct, the relevant data and a methodology.

His main difference from Priest is that the meanings change completely from one context to another, so that there is no singular correct meaning, nor meanings that only change over time or even meanings with different precisifications. There are simply different and local meanings for different contexts, so that we can not compare them to say which is better, and that is partially why Batens' view can be considered relativistic.

To what extent we can say the same for logical theories is not crystal clear, but we have some safety from the statement "The meaning of logical terms is just as context dependent as the meaning of referring terms" (1985, p. 341). He holds a dynamical character of logics that consists in the change of inferential process for different contexts, and not the total replacement of logics. It means that the problem-solving situations are corrected via a local change in the logic, but only in the problematic context, like a band-aid. It is only a local change. That way, we could avoid some unwanted consequences − inconsistencies −, while maintaining the richness of different logical theories.

Furthermore, Batens deny the possibility of a Global Paraconsistency (1990), where a paraconsistent logic, such as advocated by Priest, can not express everything of natural language. For instance, Batens argues that the notion of rejection can only be expressed by classical negation, thus paraconsistent negation, although has its virtues, pays for it with inexpressibility. Yet another criticism to Priest derives from the use of a classical meta-theory to describe a paraconsistent theory, and that entail a metalinguistic inconsistency. Both of those criticisms are answered by Priest (2005, p.199) using his usual resources that we have seen exhaustively so far: distinction between pure logics, theoretical pluralism and a canonical application.

Read's Relevance Monism

The monism proposed by Stephen Read (2006) relies on much criticism to Beall & Restall case-based pluralism, mainly about the collapse problem and the limits of the classical constraints, including the use of classical meta-theories to analyse non-classical logics. According to him there is only One True Logic and that is relevance logic.

The purpose of logic is to "distinguish the valid inferences from the invalid ones." (2006, p.209). Thus, we can not have more than one way to do so, and the purpose is always set in relevance logics, yielding only one verdict to the truth of the conclusion. Also, since impossible cases can not be true, they are not relevant and therefore there are no true contradictions, and that would assure relevance logics to encompass all cases of truth-preservation, thus, of validity.

Varzi's Relativism

Achille Varzi (2002) puts forth a logical relativism in which different relations of logical consequence are obtained relative to the logical and non-logical boundary. Since there are at least two legitimate ways to do it, there is more than one correct logic. He is not afraid of the label 'relativism', and argues that there is no such thing as super-valid arguments in formal languages, neither that there is a common core of logical laws that constitutes a "minimal logic", then it avoids the collapse problem.

For instance, Varzi claim that we could disagree over the identity predicate as a logical constant and that would led us to different logics. A useful distinction is made between Tarskian Relativism and Carnapian Relativism. The first would be the relativism of what is considered to be a logical term and, furthermore, what interpretation we give to them, i.e., it is possible to accept the identity predicate as a logical term, but employ a different semantics, such that identity can be transitive or not. The Carnapian Relativism, inspired by the principle of tolerance, is about the change in the meaning of the logical terms, not just the choice of them. Varzi then argues that both kinds of relativism are sound and that the first entails the second. It is clear that his view doesn't fit into **SLP**, since the formal language is not fixed. It looks like this kind of relativism faces the change of meaning objection, since we are literally changing the logical divide and giving different interpretations for symbols, but the disagreement is not only in the level of the meaning of logical terms, it is also prior to that, in the level of what is considered to be logical or not.

3.3 Final remarks

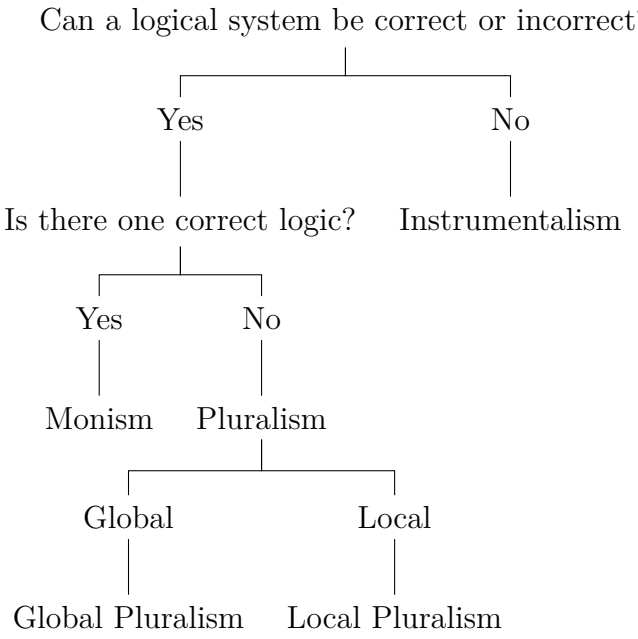
In this chapter we dedicate ourselves to offer an overview of the current thesis that aim to explain the variety of logics. We have stressed that not all of them are pluralists, and that is why this chapter is not about logical pluralism only, although it ends up organizing some of the discussion around it.

Let us make a quick recap. Our main questions is if there is a unique correct logic, or if there are many correct logics. This question clearly divides monism and pluralism, although those views do not exhaust the possibilities. Furthermore, how can one compare the different logics and make sure they are really disagreeing with each other? This

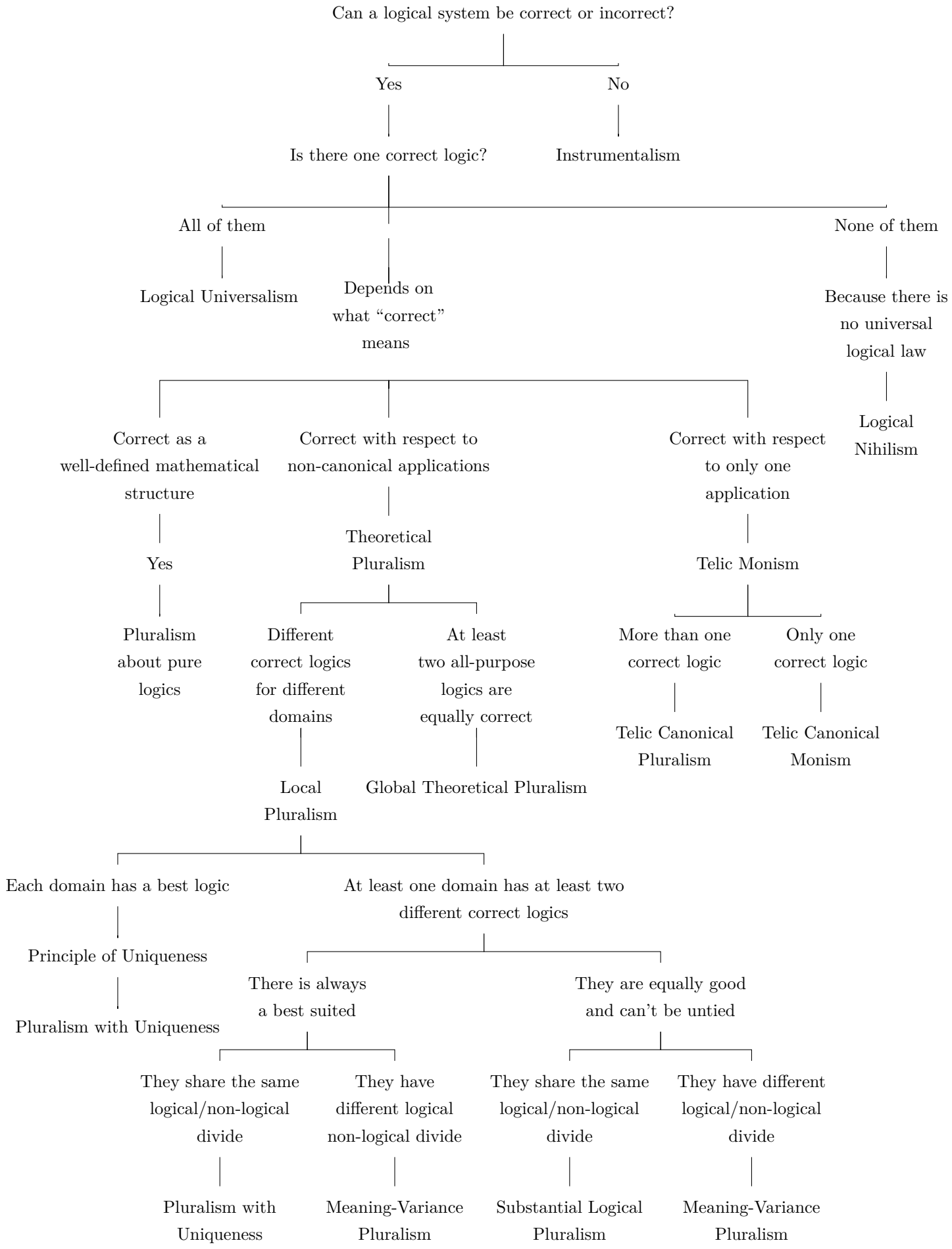
encompasses the main problems we have presented: the meaning-variance, the collapse problem and the SLP proposal. They have served us to evaluate the different articulations presented, as well as did the distinctions between global and local pluralism and of pure and applied logics.

Instrumentalism was the first articulation presented since it is an early version of logical pluralism, empty of philosophical worries and quite plain. Then, we proceed to apparent absurd positions: nihilism and universalism. We have argued that although universalism is indeed trivial, this is not the case for nihilism. Then, we started with some "authors-associated" articulations, beginning with the most prominent form of logical pluralism, that of Beall & Restall; Shapiro's eclectic perspective that explains many kinds of pluralism and, as he calls, folk-relativism; Truth-bearer pluralism that challenges the usual constructions of pluralism by logical consequence; da Costa's pluralism that has a rich philosophy embedded; and, finally, Priest's monism, whom we extensively cite through this dissertation. In an attempt to save ourselves from omission we have also dedicated a section to mention some more approaches, but unfortunately there are many more which we haven't had the chance to present here.

We are aware that all those articulations are hard to put together and to identify how each of them are related. The following diagram by Haack is a way to identify some of the articulations we have presented.



It is a useful diagram but since it was done before most of our discussion it doesn't contemplate the current intricate debate around the variety of logics. This motivates us to sketch the following diagram.


Can a logical system be correct or incorrect?
Yes
No
Is there one correct logic?
Instrumentalism
All of them
None of them
Logical Universalism
Depends on what "correct" means
Because there is no universal logical law
Logical Nihilism
Correct as a well-defined mathematical structure
Correct with respect to non-canonical applications
Correct with respect to only one application
Yes
Theoretical Pluralism
Telic Monism
Pluralism about pure logics
Different correct logics for different domains
At least two all-purpose logics are equally correct
More than one correct logic
Only one correct logic
Telic Canonical Pluralism
Telic Canonical Monism
Local Pluralism
Global Theoretical Pluralism
Each domain has a best logic
At least one domain has at least two different correct logics
Principle of Uniqueness
There is always a best suited
They are equally good and can't be untied
Pluralism with Uniqueness
They share the same logical/non-logical divide
They have different logical non-logical divide
They share the same logical/non-logical divide
They have different logical/non-logical divide
Pluralism with Uniqueness
Meaning-Variance Pluralism
Substantial Logical Pluralism
Meaning-Variance Pluralism

Some remarks on the diagram can be made. First, there are different branches leading to the same kind of view. For instance, the meaning-variance pluralism can be achieved either with the Principle of Uniqueness or with Equal Validity. It doesn't mean they are the same kind of pluralism, though. Second, the term "correct" is being used with commas because one can change it to mean "true", "legitimate" or "good" or any other term suitable for the use here, although the change of terms could entail different ontological commitments.

Third, the labels "Telic Canonical Monism" and "Telic Canonical Pluralism", "Global Theoretical Pluralism" and "Pluralism with Uniqueness" are baptized by us and not present in the literature adopted. Fourth, there are some possibilities not taken into account. Telic Pluralism would be about a logic correct with respect to many applications, but then it fits as a theoretical pluralism. Second, to say *no* to the statement that there is a well defined mathematical structure is to deny formal logic, and this doesn't concerns us here.

We hope this diagram can be useful to synthesize the perspectives we have exposed in this chapter, as well as other different perspectives that may emerge or that weren't mentioned here.

Brave New Relativism

4.1 Overview

In this chapter we shall present the historically old concept of relativism by a fresh perspective. The first step is to make a brief construction of the notion and how it was broadly used so far in the philosophical literature and, then, we start delineating what is understood by relativism in logic. Given that, we examine Martin Kusch's standard model of (epistemic) relativism (2020) (*K's model*). This model[1] is being updated to this day and has the help of many different kinds of philosophers, such that it constitutes a relevant topic for the contemporary philosophical community. Another virtue is its objectivity, with clear characteristics and great applicability. We shall introduce K's model along with our proposed interpretation in the context of philosophy of logic, which consists in an authorial work first presented in this dissertation. The idea behind our interpretation is to offer a framework to understand and discuss, within a unified foundation, the perspectives presented in the last chapter, as well as (ideally) any perspective that tries to make sense of logical variety. Our prospect is to take a first step in the direction of organizing the currently thorny discussion on logical pluralism, monism, relativism and other "isms".

4.1.1 Relativism in philosophy

The core idea of any relativistic account is to take something − experiences, reality, truth, moral values and so on − to be relative to something else − individuals, cultures, language, laws of nature and so on. This is a direct opposition to absolutism, the claim that there is an absolute truth or standard. Being relativistic, therefore, would be accepting anything whatsoever? Our answer is a clear no and by the end of this chapter we hope to have

[1] "Model" in the general sense, not model-theoretical.

offered a more comprehensive view of relativism.

As suggests Baghramian & Carter (2022), relativism is a two-edged thesis: "Defenders see it as a harbinger of tolerance and the only ethical and epistemic stance worthy of the open-minded and tolerant. Detractors dismiss it for its alleged incoherence and uncritical intellectual permissiveness.". This fissure is even more evident when we change the subject where relativism is applied: ethics, science, metaphysics, aesthetics, religion, epistemology and, as we shall see, even logic.

It is well known that relativism is a polemical idea widely used in several areas of philosophy, although it is not as well developed as other popular concepts in philosophy. This apparent paradox can be stated as following:

> Although relativistic motifs have always played a significant role in philosophy, their systematic investigation - and thus the explicit formulation of different forms and strengths of relativism - is a child only of the twentieth century. (KUSCH, 2020a).

The major worry with relativistic accounts seems to be that when we take a fact X relative to a fact Y, the choice of what is Y can be artificial, or even trivial. Artificial because we have to accept a referential for comparison between facts, but the criteria for the referential seems to be arbitrary or dogmatic. Trivial since if there is no universal standard for comparison, then anything goes. But there is a gap here between 'anything goes' and 'something goes relative to other something' and even 'something is correct (or true, justified, moral, ugly, etc) relative to another something'. It is necessary to establish some limits to what are our facts and referential. Merely stating that a fact X is relative to Y doesn't say much, but saying that X is *correct* relative to Y implies some judgement and that is what is normally associated with the permissiveness of relativism.

So let us begin by the most common approaches. Take relativism to be a schema which instantiates an X factor relative to a Y factor, as in the general relativistic schema given by Cook (2010): "One is a relativist about a particular phenomenon X if and only if one thinks that the correct account of X is a function of some distinct set of facts Y". The variants X and Y can be interpreted in a great variety of ways, that can be summarized by ontological, alethic, epistemological, moral and gustatory relativism. The gustatory

one is probably the easiest to grasp: something is tasty relative to someone's tastes.

One more didactic example is of gustatory relativism: a given food is delicious relative to an individual. For instance, açaí[2] is delicious to the author of this work, but it tastes like dirt (not delicious) for one of the author's friends. We have four possible combinations for it, so we could have a relativistic monism about food where there is only one set X of delicious food relative to i) different people or ii) an individual. Or we could have a relativistic pluralism with different sets of delicious food relative to iii) different people or iv) an individual.

4.1.2 Relativism in logic

First of all, the logical relativism we are going to discuss here is not about the anthropological claim that different cultures subscribe to different kinds of logic in contrast to the so called "Western logic". This is a very rich topic of research, but it is not of our concern now.

Applying relativism in logic may seem like a transgression, since logic is associated with rigor and objectivity, contrary to usual conceptions of relativism. But we can not simply ignore the relativistic aspects in our debate. To begin with, we must delimit what kind of thing plays the role of X and Y in the aforementioned general relativistic scheme. While moral relativism talks about moral values and societies, logical relativism has other kinds of key notions, such as validity, truth, truth-bearers, arguments, logical laws, domains of investigation and so on. Depending on what is taken to be our object X and what is our referential Y, different kinds of logical relativism emerges. If logical consequence is relative to cases, in the sense of Beall & Restall, then we have a case-based pluralism that is also relativistic. However, as we have seen in the last chapter, they do not agree with this label, even if some authors, such as Burgess, suggests that Beall & Restall pluralism is also a kind of relativism. Thus, we must be careful when labeling the current views on logical plurality.

Giving all that, one could ask: what is the difference between pluralism and relativism? This is a tough question, since we first need to fix what is pluralism and what is relativism. At the end of this chapter we will get back to this question, with the tools we judge

[2] Açaí is a popular fruit in Brazil, native to eastern Amazonia.

necessary to answer it. For now, it suffices to mention two answers. The first is given by Shapiro and he claims that the difference is merely terminological (2014, p. 96), so that one can choose whatever they prefer. On the other hand, Caret directly disagrees with that: "[...] this is not only a poor choice of rhetoric, it genuinely misrepresents the evidential import of the data." (CARET, Colin R, 2021, p. 14).

Another terminology used to denote views similar to relativism is *contextualism*. The notion is adapted from philosophy of language and the general idea is that:"[...] the validity of an argument depends on the subject matter or the frame of reference or some other context of evaluation." (BEALL et al., 2019). Then, the law of identity can be valid taking classical contexts, but invalid in non-reflexive contexts. As we have seen in the last Chapter, Shapiro goes even further and draws the useful distinction between *Indexical Contextualism* and *Non-indexical Contextualism*. To recap, the difference between them is about what varies from one context to another, the content (Indexical) or the extension (Non-indexical) of a given term.

Applying this distinction can help us understand a difference between relativism and pluralism. The logical pluralist is making a stronger assertion in which the same argument can have different and equally adequate contents or extensions for the same context, and not just that they vary from one context to another.

Putting it another way, an argument can be valid and invalid in the same context according to different logics; for instance, one can use classical and intuitionistic logics to analyse consistent contexts and conclude that the law of the excluded middle is only classically valid, but it doesn't affect the consistency, so both of these logics are equally adequate to deal with consistent contexts, even if they disagree about the validity of arguments. This justifies one to say that a local pluralism where every domain has a unique best logic can be considered a form of contextualism.

4.2 Kusch's model of (epistemic) relativism

In this extensive section we will present the model of epistemic relativism proposed by Martin Kusch along with our interpretation of it. The model of epistemic relativism has some key-characteristics (assumptions, elements) that can be combined in order to pro-

duce a relativistic *stance*[3]. They are not individually necessary and sufficient conditions, and sometimes they may be redundant. It is important to note that the epistemic model of relativism was constructed with the work of several philosophers − Kusch mentions twenty-eight of them[4].

Since the model is presented in several of Kusch's works, we choose to begin with the order presented in the book *Relativism in the Philosophy of Science* (2020), and then insert along the way the additional assumptions, presented in the Introduction of *The Routledge Handbook of Philosophy of Relativism* (2020).

4.3 An interpretation for philosophy of logic

We do not intend to provide a homophonic translation and/or interpretation to K's model. Rather, we shall use it as an inspiration, ignoring some aspects and adding others as means to find a good framework in the context of philosophy of logic to discuss the different perspectives on the variety of logics.

4.3.1 Dependence

The first assumption is dependence. It states that "A belief has an epistemic status (e.g. 'epistemically justified' or 'knowledge') only relative to epistemic standards." (2020, p.2). Before that, Kusch (2019) made a distinction between i) Regularism, where the epistemic standards are a "system of epistemic principles"; and ii) Particularism, where the epistemic standards are a "coherent bundle of precedents (or paradigms)".

The first challenge for us is how to interpret the epistemological vocabulary: belief, epistemic status and epistemic standards. Our aim is to say:

> The logical status of a truth-bearer is relative to a given logical system.

Thus we can loosely associate epistemic status with logical status, belief to truth-bearers

[3] "Stance" is a techinical term used by Kusch, from Bas van Fraassen's "The Empirical Stance" (2002), and it means that "[...] some philosophical positions are primarily not 'doctrines' but 'stances', that is, bundles or systems of values, emotions, policies and preferences" (KUSCH, 2019, p.5).

[4] Maria Baghramian (2019), Barry Barnes and David Bloor (1982), Simon Blackburn (2005), David Bloor (2011), Paul Boghossian (2006), Adam Carter (2016), Lorraine Code (1995), Annalisa Coliva (2010), Hartry Field (2009), Steven Hales (2014), Gilbert Harman (1996), Christopher Herbert (2001), Barbara Herrnstein Smith (2018), Max Kölbel (2004), John MacFarlane (2014), Duncan Pritchard (2009), Gideon Rosen (2001), Richard Rorty (1979), Carol Rovane (2013), F. F. Schmitt (2007), Markus Seidel (2014), Harvey Siegel (1987), Sharon Street (2011), David Velleman (2015), Bernard Williams (1981), Michael Williams (2007), Timothy Williamson (2015) and Crispin Wright (2008).

and epistemic standards to a given logical system. We shall make an effort to keep this association further, for the sake of coherence. This formulation allows us to present logical systems both from a semantic point of view, and from a proof-theoretical one, since the logical status can be validity but also can be provability.

We shall make some comments considering the semantic approach, which takes validity as matter of truth-preservation. Usually we mean truth-preservation in a structure, but if one (like Priest) want it to be truth-preservation *simpliciter*, then one could think that this assumption of dependence doesn't necessarily hold, since the logical status of a formula would be fixed along all cases and given before the adoption of a logical system. But it doesn't seem to follow, since the adoption of a logical system is the first move which then determines what "rules" the truth-preservation *simpliciter*. The difference is that the logical status, assuming monism, would be fixed, so the relativism would be just a way of phrasing it. It is relative to a logical system, but there is only one logical system.

Still on this topic, it is worthy of attention how da Costa's understand of scientific rationality relate to the question of truth in a system and truth *simpliciter*. Following Peña:

> da Costa regards the epistemological status of logical theses as twofold. On the one hand, they are formal truths, independent of experience, which, in order for them to be true, depend only on their being proved within a particular formal system (syntactical approach) or, whenever possible, on their holding in all models of that system (semantical approach). On the other hand, those theses are also scientific hypotheses about reality. (PEÑA, 1982, p. 8).

As a matter of course, da Costa is talking about the epistemological status of *logical thesis*, while we are talking about *propositions* (or truth-bearers to be more general), not thesis. Nonetheless, we think it is possible do draw an analogy between them and the formulation as given by Kusch (not ours). As formal truths, the dependence assumption would hold completely, at least taking the regularist position. When the logical theses are seem as scientific hypotheses about reality, we can think of a particularist view, taking into account the epistemic achievements of the past.

One last thing before continuing is that this assumption may seem trivial or straightforward for many logicians since, as Steinberger (2019, p. 438) puts "[..] no one – classical

or intuitionistic logician alike – has ever disputed the claim that the law of double negation elimination is valid in classical but not in intuitionistic logic.". What can make a difference is how one understands logical status and logical system. At least about logical status as validity we have seen in Chapter Two that there are many ways to specify it, taking it as a cluster concept, a vague concept, a unique and well defined notion or even as super-validity.

Another form to think of dependence is through the assumption called "Self-vindication" by Kusch: "Every system or bundle is such that it vindicates as true or correct all beliefs formed by relying on its norms or precedents." (KUSCH, 2020b, p. 5). Here we interpret that as:

Every logical system is such that it vindicates the logical status of its truth-bearers by relying on its syntactic and/or semantic counterparts.

This idea is very similar to our first formulation, only that here we talk about syntax and semantics, while the first only mentions a logical system, which can be seen as signal of generality over self-vindication.

4.3.2 Plurality

The formulation given by Kusch states that "There is (has been, or could be) more than one set of standards in the same domain; the standards of different sets can conflict." (2020, p. 3). Those sets are abbreviated with 'S'. For now we shall ignore the temporal aspects of it, for the sake of simplicity. We want to state that:

There can be multiple notions/specifications of validity, even if they conflict with each other.

It should be clear that by 'multiple' we mean at least two, by 'adequate' we do not mean *equally* adequate but merely fitting, and by 'conflict' we mean that different notions of validity validate different sets of inferences. There is also the possibility that one notion of validity is an extension of the other or that they are the same, in which cases they do not conflict. We consider plurality to be an undeniable aspect for the majority of pluralistic views about logic and also for many monists, since we are not saying that there many *correct* notions of validity, only that there at least two admissible notions/specifications

of validity.

But we can go further and define *strong plurality*:

There can be multiple notions of validity equally adequate to the same domain, even if they conflict with each other.

This distinction enables us to separate theoretical pluralists from advocates of the Principle of Uniqueness. The first would accept it as long as there is an extra-logical application with two equally adequate logics, while the second wouldn't accept it, since even if there are two logics adequate for the same domain, one of them is better than the other. How one apply the PU can change, for instance, a monist could apply the principle only to logics about the canonical application, but accept theoretical pluralism. In that case the monist would accept strong plurality only when analysing logics for a canonical application.

Just as a reminder, an additional supposition that precedes plurality is that of tolerance, which we have already briefly mentioned. The general idea is that logical theories other than one's own must be tolerated. We think that this is a fundamental premise of a healthy logical practice.

4.3.3 Conflict

This is a crucial assumption to decide about genuine disagreement in logic, which is usually taken to be desrable. Kusch defines it as

> Epistemic verdicts, based on different S, sometimes exclude one another. This can happen either ...
> (a) because the two S license incompatible answers to the same question, or
> (b) because the advocates of one S find the answers suggested by the advocates of another S unintelligible.(KUSCH, 2020a, p. 3).

The epistemic verdicts here are identified with the validity of formulas, which varies from one logic to another.

Different logics validate different sets of formulas, which sometimes exclude one another. This can happen either ...

(a) because the two logics validate incompatible sets of formulas, or
(b) because the advocates of one logic find the sets of formulas validated by the advocates of another logic unintelligible.

From (a) we can start formulating a genuine disagreement, since we would be dealing with the same set of formulas, disagreeing only whether it is valid or not. Kusch calls it "ordinary disagreement", but as it s formulated, it doesn't escape the meaning-variance thesis, since we could have different meanings attached to the formula in the different logics. A logical monist can argue that one of the logics is the correct one, and that the remainder is wrong. A logical pluralist usually desires to meet this condition, since it seems to be the first step to a genuine disagreement, a requirement for SLP.

This kind of conflict can be solved by some criteria to elect a better logic, as done by Priest, or to elect two equally good logics, like some pluralists do. The conflict can also be empty, if we take the different logics applying to different domains and, therefore, not competing among each other, even if they are mutually intelligible.

From (b) it is clear that pluralists fall into meaning-variance (and, as we shall see, incommensurability), but it is left for they to decide if this is really a requirement for a reasonable pluralistic perspective. A logical monist can also sustain (b) by arguing that any logic other than the correct one is simply unintelligible.

This characterization of conflict is very useful and, for that reason, we think that this assumption deserves a deeper study in a future work. Usually conflict is associated to a inconsistent set of things, such that two given factors X and Y can not be both true at the same time. But we could also obtain conflict in cases where two things can not be both false at the same time. The question to define what exactly is conflict is a very intricate one and so far we have mentioned only two types of it: (a) ordinary disagreement and (b) incommensurability. But we propose two other types: (c) substantial and (d) faultless disagreement.

Faultless disagreement is an assumption originally suggested by Kusch as follows:

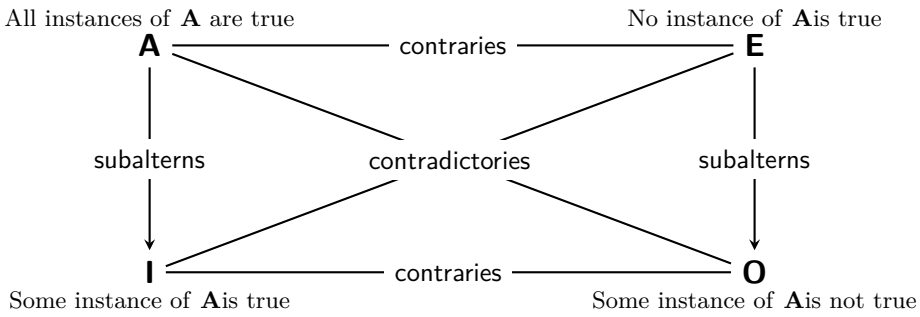
> If two epistemic subjects, committed to different epistemic systems or bundles, disagree over an epistemic issue, and if their differing views are based on their respective epistemic systems or bundles, then their disagreement is faultless:

> neither side can be faulted for their positions on the issue. (KUSCH, 2020b, p. 4).

Here it is interpreted as: If advocates of two different logics disagree over the validity of a formula, and if their differing views are based on their respective logical systems, then their disagreement is faultless. The difference between faultless disagreement and incommensurability is that in the first the advocates of one logic are able to understand advocates of other logics, they just aren't talking about the same things.

Substantial disagreement can be seen as a subtype of ordinary disagreement in the form of two different logics that share the same logical/non-logical divide and interpretation. We define it that way: substantial disagreement occurs when two different logics that share the same logical interpretation disagree about validity over the same domain. We established this to refer to the disagreement proposed by the Substantial Logical Pluralism.

To extend a little bit here, we could use the square of oppositions to explore the main kinds of conflict. The square of oppositions is a good didactic tool, extensively studied in the literature and with well established virtues and vices, beyond its visual appeal, although we are aware that it has some problems, such as how to deal with non-existent entities, empty terms or facts that are not only true or only false. Anyway, to adapt the square we replace the subject (S) by an arbitrary formula **A** and the predicate (P) by 'is true', such that we would obtain:



We would have two main senses of conflict, that of contrariness and of contradiction. The first asserts that two things cannot be both true at the same time, but can be both false; the second asserts that two things cannot be both true nor both false at the same time.

Take two logics, a classical one L_c, and a intuitionistic one L_i. Some instances, but not

all, of the Law of Excluded Middle (LEM) − $\mathbf{A} \vee \neg \mathbf{A}$ − can be true in both logics, since all instances of LEM are true in L_c (A) and some instance of LEM are true in L_i (I), therefore we would have subalternation. But some instance of LEM can not be false in both logics, since in L_c all instances are true (A) and in L_i some instance of LEM is not true (O), thus we would get a contradiction. To obtain contrariness we would need two logics with a formula that is valid in one and contradictory in the other.

4.3.4 Conversion

The formulation given by Kusch treats conversion as:

> In some cases, switching from one S to another has the character of a 'conversion': that is, the switch is underdetermined by S, evidence, or prior beliefs, and is experienced by the converting X as something of a leap of faith.'(2020, p. 3).

It is strange to think of that in the context of philosophy of logic since the idea of conversion above is closer to a religious or cult conversion than to some simple change in the set of standards (S), which we are taking to be logics.

The concept is hand in hand with Kuhn's structure of scientific revolutions (1970), where in face of anomalies the normal science (in accordance to the current paradigm) collapses into a crisis until there is a complete change (a scientific revolution) from the accepted theories and beliefs to a whole new paradigm, incompatible to the previous one and impossible to compare. This is the motivation for the conversion assumption.

In the context of philosophy of logic this could represent − among many other things − a strong meaning-variance, such that everything, but the current logic being used, is meaningless. It is not just to say that two logics have different meanings, but that the terms of one logic are *inapplicable* to another. We could not even try to translate logics, since they are incommensurable. In this case, the conversion means that the previous state is completely rejected, it is not a mere change of views, it is a refusal from the first.

What does it have to do with monism, pluralism or relativism? We think that a logical monist could take some advantage here, if they consider that logic is revisable (a.k.a. if they accept anti-exceptionalism), but that still there is only one true logic. That would

be the 'normal logic', in analogy to normal science in a Kuhnian sense. Anything other than that can not be compared or can be seen as anomalies. One problem with that is about the notion of truth, since the idea of approaching the 'true theory' is not sound in analogy to Kuhn, and there is no way to say which paradigm is truer, since they are incommensurable. So a logical monist could bear conversion by abandoning the previous one true logic and adopting another one.

An interesting comparison made by Kusch is with Feyerabend's *Against Method* (1975). We could embrace the "epistemic anarchism" by rejecting a unifying, absolute and universal paradigm and claiming instead different and incompatible scientific theories, so that anarchists would be "[...] committed "pluralists" fighting for the coexistence of conflicting scientific theories." (2020, p. 14). This can be put in clear analogy with philosophy of logic, where the anarchist would be the logical pluralist insisting that there are different and equally legitimate logics − this even makes the pluralists more poetically defensible.

But we do not need to buy the whole Khun's theory of scientific revolutions, so that the conflict between advocates of different logics could be something else other than unintelligible, they could also be incompatible. Following this path we could even interpret conversion as a concept of translation between logics, and each concept would arise different kinds of conversions:

Switching from one logical system to another can be done by i) conversion or ii) by a concept of translation between logics.

We could adopt the concept of translations of processes of inferences, or translation of logical terms, or translations of valid formulas and so on. The possibility of expressing one logic inside another is usually characterized as a virtue of the bigger logic and can even lead us to the collapse problem, since if we could find a logic that express in itself all other legitimate logics, so we could elect it as the one correct logic.

The relativistic aspect here is that the choice of the paradigm, or of the source logic, is extra-logical, otherwise, it would be circular. In the context of philosophy of logic this means that there is some level of arbitrariness in the choice of a preferred logical theory, although we are not denying some kind of preponderance between them.

Since conversion is more a possibility than a characteristic of a theory, we shall frequently omit it in our further analysis. We think that conversion in the first sense (a) is something only monists anti-exceptionalists would agree with. Regarding the second sense, it will depend on the concept of translation adopted and if it makes sense to a logical pluralist. For instance, logical ecumenism can be seen as a practice of translating classical systems into constrictive ones, therefore having (b) conversion in its heart, but we are not entirely sure if advocates of logical ecumenism would agree with that.

4.3.5 Comparability

The original name Kusch uses to describe this assumption is 'Symmetry', but we think that 'Comparability' serves better to our purposes. The initial idea is to characterize the kinds of symmetries between sets of standards, and this is the core of relativism.

> Different S are symmetrical in that they all are:
>
> (a) based on nothing but local, contingent, and varying causes of credibility (LOCALITY);
>
> (b) impossible to rank except on the basis of a specific S (NONNEUTRALITY);
>
> (c) impossible to rank since the evaluative terms of one S are inapplicable to another S (NONAPPRAISABILITY);
>
> (d) equally true or valid (EQUAL VALIDITY)." (KUSCH, 2020b, p. 3).

If we want to compare logical systems, each sub-assumption can characterize one way of comparing, so that different logical systems are comparable in that they are:

(a) based on nothing but local, contingent, and varying causes of extra logical justification (LOCALITY);

(b) impossible to rank except on the basis of a specific logic (NONNEUTRALITY);

(c) impossible to rank since the evaluative terms of one logic are inapplicable to another logic (NONAPPRAISABILITY);

(d) equally true or valid (EQUAL VALIDITY).

The first important change is the removal of "all" in the first phrase "in that they all

are", since we do not want to make statements of comparability between *every* logic, only locally, with the logics being compared.

Locality directly denies absolutism, in a way that we compare and question other logics. In Kusch's model he argues that locality can be combined with nonneutrality, so that in our interpretation we could always assume one logic to evaluate another, which is what usually happens. This is also stated by the Non-absolutism assumption, in its original form "None of these systems or bundles is absolutely correct."(2020, p. 3). But here it is interpreted as: no logic is absolutely correct. It can be seen both as one of the first steps to relativism as well as a possibility to include normativity in our framework, since there could be a sense of being correct.

Nonappraisability is a stronger assertion that can be seen as Kuhn's incommensurately, in the sense that it is impossible to compare the different logics, so that we could only 'compare' a logic to itself.

Equal validity is the most polemical assumption and, according to Kusch, is wrongly associated with relativists, despite being denied by "card-carrying relativists". It is also a source of disagreement between monists and pluralists about logic:

> Relativism equals pluralism plus the claim that each of the plurality is, in some relevant sense, equally good. Pluralists sometimes sail very close to the relativist wind though: "There are different, equally good ways of... [giving an account of validity]; they are different, equally good logics" (Beall and Restall (2000)). (PRIEST, 2005, p. 194).

What Priest means by 'equally good' is vague enough to mean both equal validity as well as a weaker sense of just two logics being equally good as well-defined mathematical structures, which we shall call *weak* equal validity.

The Substantial Logical Pluralism requires equal validity as long as it asks for two logics within the same language and interpretation, validating different inferences and being *equally* good models of logical consequence, in the strong sense that there is no third best logic. We think that this is a problematic requirement, since a local pluralism can use criteria to elect each context's best logic and still be a pluralism. Maybe there are such two logics equally good in every aspect for a given domain, but we think that to demand

it from a logical pluralist is too radical.

We shall distinguish two other types of equal validity: local and global. Local equal validity occurs when different logics are equally valid for the same domain, while global validity occurs when different logics are equally valid for all domains. Summing up, we have: *weak*, *local* and *global* equal validity.

The relativism as is usually and poorly conceived would take Nonappraisability accompanied by Equal validity, so that all set of standards would be valid and we could not compare them. Kusch calls it "Vulgar relativism", and that would be a form of radical relativism, where there is no preponderance between different logical systems. Besides, Kusch argues that the "card-carrying relativists" usually deny this vulgar relativism, since they want to be able to compare, criticize and judge different sets of standards, like how different cultures judge what is right or wrong.

There are others assumptions mentioned by Kusch (2020), such as *semantic relativity*, that encompasses the indexical and non-indexical contextualism we have exposed in the last chapter; *metaphysical commitment* and *underdetermination*, that regards realism versus anti-realism questions, factualism versus anti-factualism and if a theory is determined by the laws of nature or not; *contingency*, to analyse historical aspects of a system; and *arbitrary choice* to identify the kind of 'anything goes' relativism.

4.4 Applying the proposed framework

One of the motivations for this proposed framework of logical relativism is to use it to identify what are the assumptions adopted in the different articulations presented in Chapter Two from a neutral point of view. We shall examine each of them, but identify only the assumptions we are more certain of.

Instrumentalism can be compatible with dependence, plurality, strong plurality, ordinary, substantial and faultless disagreement and comparability by locality.

Logical nihilism can be seem as compatible with dependence, plurality, strong plurality, ordinary conflict without resolution, potentially substantial disagreement and comparability with locality and equal validity.

Logical universalism can be compatible with dependence, plurality, strong plurality, ordinary disagreement, conflict and comparability with locality and equal validity.

Those three views are harder to characterize since they are very general and have their own variations. For instance, a logical nihilist can concede that there are different logics that equally apply to the same domain but that they don't share the same logical interpretation, so that substantial disagreement is not necessary. This is why we used the idea of compatibility of assumptions, instead of acceptance/adoption.

Case-based Pluralism can accept dependence, strong plurality, ordinary and substantial conflict, comparability by nonneutrality and a weak sense of equal validity: that the different logics are equally good for their own purposes.

Shapiro's eclectic orientation has a varying combination depending on the four kinds of folk-relativism and his own version. All of them assume dependence, plurality, ordinary conflict and comparability by nonneutrality. The first and second kinds of folk-relativism assume faultless disagreement, since the different logics do not compete about the same domain. The first can assume comparability by nonappraisability. and the second comparability by locality and nonneutrality. The third is compatible with faultless disagreement and comparability by locality and nonappraisability. The fourth supports the possibility of substantial disagreement and comparability by nonneutrality. Shapiro's own version takes logical consequence relative to mathematical structures so that it is compatible with strong plurality if there is a logic equally adequate for the same mathematical structure, substantial disagreement, since the logical interpretation is the same and comparability by locality, having as the extra-logical justification the relation to mathematical structures.

Truth-bearer pluralism can accept dependence, plurality, ordinary, unintelligible and faultless conflict, since the change from one logic to another is made by a change in the truth-bearers and comparability by nonappraisability.

da Costa's contextualism can accept dependence, plurality, ordinary conflict, possibly substantial and faultless disagreement, since the different logics can have different logical interpretations and notions of validity, conversion (remind the discussion about revisability) and comparability by locality and nonneutrality.

Priest's monism can accept dependence, plurality, ordinary and substantial disagreement (solved by criteria), conversion and comparability by nonneutrality and locality.

There are some important aspects of the theses mentioned which we would like to better represent with our framework, such as the use (or not) of the same language for different logics; the plurality that comes from different sources, like validity, truth-bearers, or truth; better grasp how we can use the conversion assumption, since it has a strong epistemological take about the role of epistemic agents, although logic is usually discussed without it[5]; identify the relations between different assumptions (if they are compatible or not, mandatory or not); specify issues regarding the formal aspects of logic, such as the choice of semantics, of formal language, of constraints to logical consequence and of ontological concerns.

This framework for logical relativism can be explored and developed in future works. Since our purpose is to offer a unified framework of discussion, we are not concerned in assuming any special perspective now, nor to defend any kind of relativism. Instead, the framework can be used to make it possible the discussion between advocates of different thesis on the variety of logics, avoiding misunderstandings between them and organizing this rather complex debate.

Given that, a note about personal taste is that the name of this framework (*framework of logical relativism*) doesn't do justice to it, since we can formulate thesis far distant from usual apprehensions of relativism. In fact, we can define anti-relativism mainly by denying non-absolutism, nonappraisability and equal validity. Also, as we have seen, we can identify inside the framework forms of monism, pluralism, instrumentalism and so on.

A final comment is about a question that may be underlying all of our discussion: is there an ideal combination of assumptions, one that give rise to the best way to capture and explain the variety of logics? We do not have a conclusive response to this, although there are hints of what assumptions we should assume. The answer depends on our inclination: to monism or to pluralism. Here we tend to a pluralistic account, and for that we should avoid all its main problems. Meaning-variance by denying unintelligible conflict and faultless disagreement; The Collapse Problem by denying absolutism and, maybe, normativity; The Uninteresting Pluralism problem by denying faultless disagreement and

[5]There is a new trend in Philosophy of Logical Practice that can elucidate this question at some point.

nonappraisability. But we do not think that we must achieve a Substantial Logical Pluralism, since we disagree with the assumption that an interesting logical pluralism must fulfill all four requirements, in special the last one, of equal validity. Else, if one has a monistic tendency, then they should especially avoid strong plurality, equal validity and nonneutrality.

Summing up, we have presented different perspectives on how to make sense of logical variety and then we have proposed a framework to discuss all of them. We don't deny or endorse any perspective on the variety of logics, nor do we propose any ideal combination of assumptions. All we propose is a framework of discussion, based on a model of epistemic relativism, to organize, identify, discuss and evaluate different theories about logical plurality.

Final Remarks

How to make sense of the variety of logics? This question has guided most of our inquiry here, to the point that we tried to "make sense of the attempts that make sense of logical variety". In better and simpler words: our aim was to understand a very recent and thorny discussion around many perspectives on the variety of logics, and we did so by presenting a framework to identify, discuss and evaluate them. But what is there to make sense of? What can we securely call 'logic' and is it possible for more than one logic to be good, adequate, correct or even true?

These are the initial questions which we began to answer by its foundations, so in the first chapter we assemble a basis for discussion, making clear what are our elementary assumptions about logic, validity, truth and so on. That way we have offered a general take on what is logic and then we adopted and adapted definitions from Shoenfield's textbook on mathematical logic. The main ambition was to make clear how one can construct a logic from zero and what changes from classical to non-classical logics. This is of significant importance since the main question of our work is a relation between these different kinds of logics. Furthermore, the approaches we have presented in Chapter Two and the formulations we have offered in Chapter Three needed concepts like what is a logical law, what are truth values, truth-bearers and functions, what is a structure and a model, what is consistency of a system and, most importantly, what is validity and logical consequence.

The second chapter consists in a comprehensive overview on several thesis regarding logical variety. We have given priority to those with enlightening effect, such as nihilism, universalism and instrumentalism; with wide familiarity, such as Beall & Restall case-based pluralism, Shapiro's eclectic orientation and Priest's logical monism; with heterogeneous formulations, such as Gillian's truth-bearer pluralism; and with a wholeness theory, such

as da Costa's contextualism. All of these perspectives share those virtues with each other. Even so, they do not deflate the debate and that is why we also have mentioned other theses worthy of attention, but we know that we have left out many others. We hope that this chapter has a informative and organizational impact, elucidating that logical pluralism is a loose umbrella for many different articulations and logical monism isn't a direct opposition to it. We can go beyond this dispute with many other approaches, such as instrumentalism, logical universalism and nihilism, ecumenism, relativism and many other "isms" we manage to conceive.

The third chapter was the hardest to construct, but also the most rewarding. Having noticed the importance of the expression "relative to" in our discussion, we decided to dedicate special attention to the used and abused concept of relativism. The first step was reconstructing the notion in order to get rid of its prejudices and biases. Then we had the fortunate chance to be inspired by Kusch's model of epistemic relativism, which is pretty recent and assembled by a long list of philosophers of many different areas, which is perceived by us as a great virtue. Our goal was to interpret and adapt this model in the context of our logical-philosophical discussion. The main challenge was how to move from epistemic concepts, like 'subject', 'belief' and 'judgement' to logical concepts, like 'formula', 'logical law' and 'validity'. This has a cost of having to remove or add new aspects to the assumptions. We recognize that our proposed framework, which we also call a framework for logical relativism, can be further developed in order to enclose and explain with more rigor the current perspectives on logical variety, but we also think that this is an imminent outcome of an incipient idea towards a method to organize, characterize and inspect different articulations within the philosophy of logic using epistemology as starting point.

A lot of future work can be done from now on. The discussion revolving logical pluralism is abundant and prosperous, such that we hope to witness the creation of new perspectives on the subject. Furthermore, it has numerous dividing branches. For instance, we could explore the study of normativity and anti-exceptionalism in logic, which are recurrent topics for many authors discussed here. We could take an ontological inclination to the debate and explore questions regarding realism and pragmatism about logic. Another very interesting possibility is the study of Philosophy of Logical Practice (PLP), which is increasingly popular and makes total sense in the context of our discussion, for instance,

in order to better grasp how advocates of different logics are disagreeing with each other; the focus turns to the epistemic agent (in this case, the logician), instead of the usual idealized logical concepts.

This final remarks were the last things I wrote and I kept thinking “How can I write about everything without specifically writing about anything?”. In the end, I conclude that this was only the pain to say farewell to this special collection of words.

References

ARENHART, Jonas Rafael Becker. Newton da Costa on hypothetical models in logic and on the modal status of logical laws. **Axiomathes**, Springer, p. 1–21, 2021. Cit. on p. 49.

ARROYO, Raoni Wohnrath; DA SILVA, Gilson Olegario. Taking models seriously and being a linguistic realist. **Principia: an international journal of epistemology**, 2022. Cit. on p. 31.

BAGHRAMIAN, Maria. The virtues of relativism. In: OXFORD UNIVERSITY PRESS, 1. ARISTOTELIAN Society Supplementary Volume. [S.l.: s.n.], 2019. v. 93, p. 247–269.

BAGHRAMIAN, Maria; CARTER, J. Adam. Relativism. In: ZALTA, Edward N. (Ed.). **The Stanford Encyclopedia of Philosophy**. Spring 2022. [S.l.]: Metaphysics Research Lab, Stanford University, 2022. Cit. on p. 63.

BATENS, Diderik. Against global paraconsistency. **Studies in Soviet Thought**, JSTOR, v. 39, n. 3/4, p. 209–229, 1990. Cit. on pp. 55, 56.

BATENS, Diderik. Meaning, acceptance and dialectics. In: CHANGE and progress in modern science. [S.l.]: Springer, 1985. p. 333–360. Cit. on pp. 55, 56.

BEALL, JC; RESTALL, Greg. **Logical Pluralism**. [S.l.]: Oxford University Press, 2006. Cit. on pp. 19, 40, 41.

BEALL, JC; RESTALL, Greg. Logical pluralism. **Australasian journal of philosophy**, v. 78, n. 4, p. 475–493, 2000. Publisher: Taylor & Francis. Cit. on p. 42.

BEALL, JC; RESTALL, Greg; SAGI, Gil. Logical Consequence. In: ZALTA, Edward N. (Ed.). **The Stanford Encyclopedia of Philosophy**. Spring 2019. [S.l.]: Metaphysics Research Lab, Stanford University, 2019. Cit. on p. 65.

BOCHEŃSKI, Józef M. **A history of formal logic**. [S.l.: s.n.], 1961.

BORKOWSKI, L et al. On three-valued logic. **Selected works. BORKOWSKI, L.(ed.). Orth-Holland Publishing Company: Amsterdan-London**, p. 87–88, 1970. Cit. on p. 21.

BUENO, Otávio. Can a paraconsist theorist be a logical monist?, 2002. Cit. on pp. 33, 41.

BUENO, Otávio; SHALKOWSKI, Scott A. Modalism and logical pluralism. **Mind**, Oxford University Press, v. 118, n. 470, p. 295–321, 2009. Cit. on pp. 36, 38.

BURGESS, JA. Review of JC Beall and Greg Restall, Logical Pluralism. **Philosophy and phenomenological research**, v. 81, n. 2, p. 519–522, 2010. Cit. on pp. 41, 42.

CARET, Colin R. Why logical pluralism? **Synthese**, Springer, v. 198, n. 20, p. 4947–4968, 2021. Cit. on pp. 35, 65.

CARET, Colin R. The Collapse of Logical Pluralism has been Greatly Exaggerated. **Erkenntnis**, v. 82, n. 4, p. 739–760, Aug. 2017.

CARNAP, Rudolf. Empiricism, semantics, and ontology. **Revue internationale de philosophie**, JSTOR, p. 20–40, 1950. Cit. on p. 31.

CARNAP, Rudolf. **Logical syntax of language**. [S.l.]: Routledge, Reprinted in 2007, 1937. Cit. on p. 36.

CHANG, Hasok. Relativism, perspectivism and pluralism. In: THE Routledge handbook of philosophy of relativism. [S.l.]: Routledge, 2019. p. 398–406.

COOK, Roy T. Let a Thousand Flowers Bloom: A Tour of Logical Pluralism. **Philosophy Compass**, p. 13, 2010. Cit. on pp. 35, 36, 63.

COSTA, Newton da. **Ensaio sobre os fundamentos da lógica**. [S.l.]: Hucitec, 1994. Cit. on pp. 16, 17, 48–50.

COSTA, Newton da; ARENHART, Jonas R Becker. Full-blooded anti-exceptionalism about logic. **The Australasian Journal of Logic**, v. 15, n. 2, p. 362–380, 2018. Cit. on pp. 49, 50.

D'OTTAVIANO, Itala; COSTA, Newton da. Sur un problème de Jaskowski. C.R. Acad. Sc. Paris 270A, p. 1349–1353, 1970. Cit. on p. 21.

D'OTTAVIANO, Itala; FEITOSA, Hercules de Araujo; MRAS, GM; WEINGARTNER, P; RITTER, B. Translations Between Logics: A Survey. **Philosophy Of Logic And Mathematics**, Walter De Gruyter Gmbh, p. 71–90, 2020. Cit. on p. 17.

EKLUND, Matti. Making sense of logical pluralism. **Inquiry**, Taylor & Francis, 2017.

ERICKSON, Evelyn Fernandes. **An Investigation of Logical Pluralism and B-entailment**. 2016. MA thesis – Universidade Federal do Rio Grande do Norte.

FEYERABEND, Paul. **Against method: Outline of an anarchistic theory of knowledge**. 2010. ed. [S.l.]: Verso Books, 1975. Cit. on p. 73.

FIELD, Hartry. Pluralism in logic. **The Review of Symbolic Logic**, Cambridge University Press, v. 2, n. 2, p. 342–359, 2009.

GABBAY, Dov M; WOODS, John. **Greek, Indian and Arabic Logic**. [S.l.]: Elsevier, 2004.

GODDU, G. C. What Exactly is Logical Pluralism? **Australasian Journal of Philosophy**, v. 80, n. 2, p. 218–230, 1 June 2002. Publisher: Routledge.

GOMES, Evandro; D'OTTAVIANO, Itala. **Para além das Colunas de Hércules, uma história da paraconsistência: de Heráclito a Newton da Costa**. [S.l.]: Editora da Unicamp, 2017. Cit. on p. 13.

HAACK, Susan. **Philosophy of logics**. [S.l.]: Cambridge University Press, 1978. Cit. on pp. 29, 34.

HAAPARANTA, Leila. **The development of modern logic**. [S.l.]: OUP USA, 2009.

HJORTLAND, Ole Thomassen. Logical Pluralism, Meaning-Variance, and VerbalDisputes. **Australasian Journal of Philosophy**, v. 91, n. 2, p. 355–373, 1 June 2013. Publisher: Routledge. Cit. on p. 53.

HOFWEBER, Thomas. Logic and Ontology. In: ZALTA, Edward N. (Ed.). **The Stanford Encyclopedia of Philosophy**. Spring 2021. [S.l.]: Metaphysics Research Lab, Stanford University, 2021.

KEEFE, Rosanna. What logical pluralism cannot be. **Synthese**, Springer, v. 191, n. 7, p. 1375–1390, 2014. Cit. on p. 36.

KLEENE, Stephen Cole. Introduction to metamathematics, 1952. Cit. on p. 21.

KNEALE, William; KNEALE, Martha. **The development of logic**. [S.l.]: Oxford University Press, 1962. Cit. on p. 18.

KUHN, Thomas S. **The structure of scientific revolutions**. [S.l.]: Chicago University of Chicago Press, 1970. v. 111. Cit. on p. 72.

KUSCH, Martin. Epistemic relativism, scepticism, pluralism. **Synthese**, Springer, v. 194, n. 12, p. 4687–4703, 2017.

KUSCH, Martin. **Relativism in the Philosophy of Science**. [S.l.]: Cambridge University Press, 2020a. Cit. on pp. 62, 63, 66, 68, 69, 72, 73.

KUSCH, Martin. Relativist stances, virtues and vices. In: OXFORD UNIVERSITY PRESS, 1. ARISTOTELIAN Society Supplementary Volume. [S.l.: s.n.], 2019. v. 93, p. 271–291. Cit. on p. 66.

KUSCH, Martin. **The Routledge handbook of philosophy of relativism**. [S.l.]: Routledge, 2020b. Cit. on pp. 66, 68, 71, 74–76.

LUDWIG, David; RUPHY, Stéphanie. Scientific Pluralism. In: ZALTA, Edward N. (Ed.). **The Stanford Encyclopedia of Philosophy**. Winter 2021. [S.l.]: Metaphysics Research Lab, Stanford University, 2021. Cit. on p. 30.

MACFARLANE, John Gordon. **What does it mean to say that logic is formal?** [S.l.]: University of Pittsburgh, 2000. Cit. on p. 18.

MANIN, IU I; ZILBER, Boris. A course in mathematical logic for mathematicians. Springer, 2010. Cit. on p. 29.

MARIN, Sonia; PEREIRA, Luiz Carlos; PIMENTEL, Elaine; SALES, Emerson. Ecumenical modal logic. In: SPRINGER. INTERNATIONAL Workshop on Dynamic Logic. [S.l.: s.n.], 2020. p. 187–204. Cit. on p. 54.

MENDELSON, Elliott. **Introduction to mathematical logic**. [S.l.]: Chapman and Hall/CRC, 2009.

NASCIMENTO, Victor Luis Barroso. **Logical Ecumenism**. 2018. MA thesis – Pontifícia Universidade Católica do Rio de Janeiro.

PEÑA, Lorenzo. Critical Study of da Costa's foundations of logic. **Logique et analyse**, JSTOR, v. 25, n. 100, p. 447–466, 1982. Cit. on p. 67.

PRAWITZ, Dag. Classical versus intuitionistic logic. **Why is this a Proof**, p. 15–32, 2015. Cit. on p. 54.

PRIEST, Graham. A note on mathematical pluralism and logical pluralism. **Synthese**, Springer, v. 198, n. 20, p. 4937–4946, 2021. Cit. on p. 52.

PRIEST, Graham. **An introduction to non-classical logic: From if to is**. [S.l.]: Cambridge University Press, 2008.

PRIEST, Graham. **Doubt Truth to be a Liar**. [S.l.]: Clarendon Press, 2005. Cit. on pp. 29, 32, 37, 51, 52, 56, 75.

PRIEST, Graham. Logic: One or many. **Logical consequences: Rival approaches**, Hermes Scientific Publishers Oxford, p. 23–38, 2001.

PRIEST, Graham. The logic of paradox. **Journal of Philosophical logic**, JSTOR, p. 219–241, 1979. Cit. on pp. 21, 51.

QUINE, Willard V. **Philosophy of logic**. [S.l.]: Harvard University Press, 1986. Cit. on pp. 18, 34.

READ, Stephen. Monism: The one true logic. In: A logical approach to philosophy. [S.l.]: Springer, 2006. p. 193–209. Cit. on pp. 37, 56.

RESTALL, Greg. Carnap's tolerance, meaning, and logical pluralism. **The Journal of Philosophy**, v. 99, n. 8, p. 426–443, 2002. Publisher: JSTOR.

RUSSELL, Gillian. Logical nihilism: Could there be no logic? **Philosophical Issues**, v. 28, n. 1, 2018.

RUSSELL, Gillian. Logical Pluralism. In: ZALTA, Edward N. (Ed.). **The Stanford Encyclopedia of Philosophy**. Summer 2021. [S.l.]: Metaphysics Research Lab, Stanford University, 2021.

RUSSELL, Gillian. One true logic? **Journal of philosophical logic**, Springer, v. 37, n. 6, p. 593–611, 2008. Cit. on pp. 36, 46, 47.

RUSSELL, Gillian. The justification of the basic laws of logic. **Journal of Philosophical Logic**, Springer, v. 44, n. 6, p. 793–803, 2015.

SHAPIRO, Stewart. **Foundations without foundationalism: A case for second-order logic**. [S.l.]: Clarendon Press, 1991. v. 17. Cit. on p. 18.