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PERSPECTIVISM AND UNDERSTANDING IN SET THEORY

Perspectiva e entendimento em teoria de conjuntos

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Resumo

Perspectivismo na ciência pode ser resumido como a posição que diz que todo conhecimento sobre o mundo é perspectival. Meu trabalho aqui foi tentar adaptar essa posição para a teoria de conjuntos, motivado pelo modo como teóricos de conjuntos contemporâneos trabalham, em sua maioria construindo novos modelos de teoria de conjuntos por meio de técnicas como forcing e axiomas de grandes cardinais. Eu inicio discutindo o que é perspectivismo na Filosofia da Ciência, assim como algumas de suas peculiaridades importantes para a adaptação feita adiante e quando confrontadas com o realismo tradicional. Então, eu passo a incorporar um novo componente na discussão, ao dizer que pretendemos considerar uma versão epistêmica de perspectivismo, que possui foco em como é possível ganharmos conhecimento sobre o mundo, ao invés de determinar o que há no mundo. No entanto, para satisfazer nossa adaptação para a teoria de conjuntos, o critério de conhecimento parece ser forte demais. Portanto, mais uma vez, eu adapto esta versão epistêmica de perspectivismo, substituindo entendimento por conhecimento como conceito epistemológico fundamental para nossa visão. Em poucas palavras, um agente A entende um tópico quando A capta um corpo coerente e compreensivo de informações e, além disso, tem a habilidade de utilizar estas informações para expandir seus fins cognitivos. Por fim, eu aglutino ambos os desenvolvimentos teóricos acima descritos dentro de um arcabouço da teoria de conjuntos e mostro que esta proposta difere de outras que temos desenvolvidas até o momento e, além disso, pode ser utilizada para justificar a prática na teoria de conjuntos. Além disso, eu também aplico as ideias desenvolvidas ao Multiverso desenvolvido por Hamkins (2012), argumentando contra a tese de que esta formulação de Multiverso pode ser considerada um tipo de perspectivismo.

Palavras-chave: Perspectivismo; Teoria dos Conjuntos; Epistemologia; Multiverso.

Abstract

Perspectivism in science can be summarized as the view that claims that every knowledge about the world is perspectival. My job was to give a first attempt of adapting this view to set theory, motivated by the contemporary work of set theorists, which consists of constructing new models of set theory by techniques such as forcing and large cardinal axioms. The work begins by discussing what is perspectivism, in the sense of Philosophy of Science, and discussing a few of its peculiarities when confronted with standard realism. Then, I move to incorporate a new component into the discussion, by saying that we are after an epistemic account of perspectivism, that focuses not so much on determining what there is in the world but how we come to gain knowledge about what there is in the world. However, for the adaptation to set theory, it seems that the factive condition of knowledge is way too strong. So I, once more, adapt this epistemic account of perspectivism to substitute understanding for knowledge as the primary epistemic concept we are going to accept. In short, an agent understands a topic when they grasp a coherent, comprehensive body of information about that topic and is able to use that information to further one's cognitive ends. My last job was to put these two developments together inside a set theoretical background and to show that his approach is different than we already have developed and, moreover, serves as a theoretical background that supports set theoretical practice as it is done. Besides, I also apply these developments to Hamkins' Multiverse (2012), arguing against the claim that the Multiverse as Hamkins describes it can be understood perspectivally.

Keywords: Perspectivism; set theory; Epistemology; Multiverse.

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Introduction

This dissertation has the objective of being a first work in the view named perspectivism in set theory. The original idea came to me when studying forcing and realizing that the technique has many points where *decisions* are made by the working mathematician. The set theorist must choose, first, an adequate axiomatization that allows them to build set theory. Then, they must choose a model of that system which is adequate for the application of forcing, usually a countable transitive model. Then, they must define a generic filter inside that model. Further, they need to pick names for the objects given that filter they chose and then expand the original model so as to assign a truth value to the proposition they are interested in working with.

Not only this convoluted process caught my attention, but also the fact that, inside the original model, nothing can be said about the assertion we are trying to reach. It is as if, during the process, the mathematician grabs a telescope and looks further into their set theoretical universe, so they can observe set theoretical stars that were not visible to their set theoretical eye. This fact sprung my creativity and gave me an idea: set theory is perspective-dependent. Set theoretical constructions and assertions not only depend on the models being built, but on the decisions made by the set theorist who, when constructing a model, picks parameters with which they can say something about the assertion they are after.

Luckily for me, there was a similar view in progress since the early 2000s in the Philosophy of Science, named Perspectivism, whose roots sprung from Nancy Cartwright (1983), Ronald Giere (1988) and, later, Ernest Sosa (1991), but gained robustness after Giere's work (2006) when he coined the term. In short, perspectivism is the claim that says that any human knowledge is possible only perspectivally. Moreover, Giere defends a sort of perspectival realism, within which realist claims can be justified based on his perspectivism. Others agree in parts with Giere, either by defending realism with perspectivism in different ways (CHAKRAVARTTY, 2010; CHIRIMUUTA, 2015; TELLER, 2020) or by saying that perspectivism gives an explanation on *how* we gain knowledge of nature (MASSIMI, 2012a; SAATSI, 2020). I tend to agree mostly with the latter position, hence this is the perspectivist formulation I try to adapt when shifting from science to set theory.

However, the factivity requirement of knowledge seems like a problem for us. When dealing with set theory, and specially recent set theoretical practice, the truth value of a set theoretical assertion is not absolute, neither it can be, since we are usually dealing with assertions that are undecidable. Hence, when asking for *knowledge*, factivity seems too strong for our purposes. I decided thus to follow Christine Elgin (2017a,2017b) in saying that our primary epistemic concept should be *understanding* instead of knowledge. This is in accordance with what Juha Saatsi (2020) thinks, although their account of understanding differ. Saatsi defends an explanatory account of understanding that to understand is to be able to answer *what-if* questions in a way that preserves the explanatory relation between explanandum and explanans, whereas Elgin defends a form of objectual understanding, which says that to understand is to grasp not only those information which are relevant for an account and that give epistemic support for other assertions in the same account, but how these assertions relate to each other inside an account. Both views are applicable to our case, so I consider both of them when trying to argue for a perspectivism in the way I conceive it.

Lastly, another motivation of this whole work was for me to better understand the Universism versus Multiversism debate in Philosophy of Set Theory. The debate began with the work of Hamkins (2012), further developed by Woodin (2011) and Steel (2014), and being further developed by others¹. On one side we have Universists who believe that all of set theory is done inside an all-encompassing huge Universe, whereas Multiversists think that a number of different Universes exist. We focus our attention in Hamkins' work, since I believe it is the easier one to see the results of perspectivism. Hamkins believe that each model of set theory determines a universe, and that each universe determines its own concept of set, together with truth value for set theoretical assertion. I discuss in the last section of the text if Hamkins' position can be considered a form of perspectivism.

The text is organized as it follows:

chapter 1 begins with an intuitive presentation of perspectives, followed by a presentation of scientific perspectivism. I then discuss whether scientific perspetivism can

¹Recent works on different multiversist conceptions include the use of the notion of *core* by set theoretical geologists and the construction of the Ultimate-L (USUBA, 2019; BAGARIA; TERNULLO, 2021), or even novel conceptions of the cumulative hierarchy as is the chase with Level Theory presented recently by Tim Button (2021a, 2021b, 2022).

be thought of as a form of realism. Then I move to answering a few problems that might come with thinking about a simple application of scientific perspectivism to set theory, namely the agency problem and the bounded knowledge. Finally I bring a brief discussion about how perspectivism deals with practical science.

In chapter 2 I make the case for substituting knowledge for understanding in the epistemic version of perspectivism. The first part of the chapter is dedicated to discuss the definition of understanding, mainly motivated by Elgin (2017b), whereas the second part is dedicated to arguing for the incorporation of understanding in perspectivism.

In chapter 3 I take the results derived from both the previous chapters and give a formulation of perspectivism that I find suitable for set theory. Then I answer the question of why is perspectivism interesting and how it differs from other positions already formulated. Then I explain the relation perspectivism, and specially perspectivism in set theory, has with relativism. I end the chapter briefly discussing whether Hamkins' position can be considered a form of perspectivism.

Chapter 1

Perspectives

We begin our discussion by talking about perspectives and how we should understand perspectives in a scientific context. Our first impulse is to think of *perpectives* as a synonym for a point-of-view and, despite being useful in some situations, this commonsense view of perspectives does not helps us move far into a scientific debate. Here we need a formulation that actually brings sense to scientific practice inside the robust theoretical framework that science is done. In this vein, Ronald Giere (2006) will be our first author to discuss, given he was the first to coin the term and the notion of perspectivism in science in the way we are going to discuss here. Since his formulation, though, many other authors have tried to perfect the notion by refining Giere's formulations.

One of the first mentions to perspectivism was done by Bas van Fraassen (2008), who discussed the role of representations in science and brought a notion of perspectives as being a kind of purpose-driven representation. This view is in accordance with Giere's, however for different reasons. In van Fraassen's work (2008), representations and thus perspectives are taken to be the results of measurement techniques, with the intent of passing a message when being used to represent an object or phenomenon. Perspectives in this work are taken to be *cluster concepts*, that is, concepts that do not have necessary nor sufficient conditions which characterize them, but that can be given a multitude of conditions to be satisfied so that the notion purported can be grasped and used. Later, other authors such as Anjan Chakravartty (2010), Michela Massimi (2012a, 2018), Margaret Morrison (2011) and Paul Teller (2020) came with refinements of the notion of perspectivism, that either accept the realist condition given by Giere or reject it on different accounts.

In the sections that follow, I begin by characterizing perspectives together with the idea of representation and ask ourselves if that idea fits the scientific world. Then I give the notion of scientific perspectivism as pushed forward by Giere and others, which is the claim that human knowledge of nature is always perspectival with models being the main representative tool used in science. Then I discuss the points of approximation between perspectivism and realism as shown in the literature, with one sole exception, that being of Morrison (2011), who thinks perspectivism might be a form of antirealism. I end the chapter with discussion of three points I take to be relevant to my position towards a set theoretical perspectivism. First, the inclusion of agency in the perspectival account of science. Second, the explicit commitment to a bounded possibility of knowledge. The third, more akin to the scientific practice than to the rest of my text, but nonetheless interesting, talk about the possibility of reinforcement of biased results by a perspectival account of science. I believe that by discussing these topics we can give a good panorama of how perspectivism can fit into Philosophy of Set Theory and how it can deal with mathematical practice, two topics I am interested in.

1.1 What are perspectives?

Suppose you are at Rio de Janeiro, the sun is blazing high and the gentle breeze brushes your face as you admire the beautiful Sugarloaf Mountain. Depending on where you stand on the city, it could be the case that the famous Cable Car (If you do not know what it is, just watch The Simpsons' episode in which they come to Brazil¹) is not visible to you. You know, however, that it exists, and it is very natural to accept that some variables prevent you from seeing it: it may be the case that the mountain covers the Cable Car from your point-of-view; there might be other buildings in the way, so you are not able to see it; It might be the case that you suffer from a severe case of myopia – as it's mine –, so seeing things from a distance is quite difficult for you.

One can understand a *perspective* as it is applied to visual arts in the way Panofsky (1993) describes it. We say that we are talking about a perspectival view of the space when our view of that space can be compared to the scene we perceive when looking through a window. Anytime we are dealing with a perspective in a visual artwork, we are talking about a fixed a point-of-view with the direction of our gaze, how much we are able to see, which aspects are seen from that vantage point and others aspects which are decided by the artist. In fact, Panofsky's approach makes sense in arts, where perspectives are taken to be a technique of measurement, which one can apply to their drawings, paintings, sculptures and other visual arts products and the work of the artist

¹This refers to episode 15, season 13, named "Blame it on Lisa".

is to decide from which "window" that artwork is intended to be seen. The example of the Cable Car allows us to have a naive comprehension of *perspective*: a point-ofview. Meshing this with Panofsky's description, however, does not entail a satisfactory definition which is sufficiently comprehensive and rigorously defined as to encompass what a perspective is. It at most give us an initial step towards defining it, but restrained by either the ingenuity of the first comprehension of the term or the technical bounds of visual arts. We need something more metaphysical.

Missing from this first attempt is the fact that a *perspective* involves things such as *occlusion* (some objects depicted may partially or totally occlude others in the background, for example), *grain* (how fine-grained are the textures and details in the picture), *angles* (together with occlusion, this gives us only a limited number of angles from which a particular object can be seen), and *marginal distortion* (how distorted are representations that are not central to the picture). Panofsky's defense goes even further in saying that perspective *per se* is a way of giving meaning to the pictures painted on a canvas², by means of those characteristics. By occluding and distorting represented objects on a canvas, the painter says something about that object in that context, so the decision putting an object at the center of the picture passes a message at least about the importance of the object being depicted, while putting it at its margin makes it so that another message is passed on.

This first characterization of *perspectives* puts us closer to some sort of representation. A perspectival view of something is a representation of that something which is partial and includes things like distortion, occlusion, angle, grain, etc. So by saying that we are taking a perspectival view of a topic means that we are actually representing what we are intending to or just making some kind of idealization of the object that does not coincide with its real features?

The question can be raised from the way van Frassen (2008) discuss perspectives in his work. For him, a perspective is just a cluster concept³ with a series of criteria hallmarks. The hallmarks are just the characteristics above exposed, together with *ex*-

 $^{^{2}}$ A more detailed notion of art as a Symbolic form is discussed in ((BUNDGAARD, 2011)), which takes both Panofsky's and Cassirer's work and develop on them by arguing how art can be seen as symbolic form.

 $^{^{3}}$ A cluster concept can be taken as a concept with no definition in particular, but that can be given conditions to be satisfied, without these conditions being exhaustive. More shortly, no set of conditions is sufficient and necessary for the concept to be used.

plicit non-commitment⁴ and presenting a horizon of alternatives⁵. But still, a perspective is a representation. i.e., a depiction made by someone of something being depicted as having (or being, or instantiating) a characteristic, in order to fulfill some determined purpose. It does not seem, however, totally accurate to say that this is the notion of perspective we are after. This also does not answer fully the question I proposed earlier, so we need something more.

Despite the red herring, thinking about Van Fraassen's characterization might light the way towards a better characterization of *perspective* for our purposes in the future. First, we have to concede that even if, strictly speaking, we are not talking about representations, the perspectival approach to science uses representations as its trademark, mainly by saying that scientific models⁶ are the main object of study in science. Second, even if we are not talking directly about representing, we can use this as an analogy to what we do in our scientific practice. Normally we study – at least in undergraduate classes – idealized general schemes of a fact, and we start refining those schemes as we advances in our studies. But even these refinements, as long as one pertains to the academy and does not try to go berserk and study *every* aspect of a fact, remains circumscribed in an area or domain: biology, physics, chemistry, philosophy, etc. This means that our take is not only idealized models, but studied according to some interests in particular. In this sense, Van Fraassen gives us a close insight of what is done in science, by saying that the models science construct are, in fact, a perspectival representation of the world.

In almost the same spirit as the idea proposed by van Fraassen, Giere (2006) gives us a characterization of how perspectives could be understood when he introduces his map analogy⁷⁸. It is widely known that when representing Earth in a flat surface it is necessary to add some distortion to the representation. According to Giere, the aspects being depicted in each map projection depends on the purpose of each map being depicted:

⁴According to van Fraassen (2008), given a property F and a subject S being depicted by a representation, a representation is *commital* with respect to F if it represents S as either having F or not having F. It is *explicitly commital* with respect to F if depicts S as having a property or properties that preclude the representations of being commital with respect to F.

⁵A horizon of different points-of-view that can be used to represent an object or phenomenon as thus or so.

⁶From now on and for the length of chapters 1 and 2, I will use the words *model* and *scientific model* interchangeably. On chapter 3, however, most of uses of the word *model* will refer to the model theoretic definition. So model, for us here and in the next few uses is a conjunction of assumptions that are supported by a theory. I will signal when changing the meaning of the word, so it does not get so confusing.

⁷This analogy is present in other texts as well, such as (FRAASSEN, 2008) and (CHAKRAVARTTY, 2010), but with different relevant aspects being highlighted in each of them.

⁸There are other ways that Giere characterizes perspectivism, one of them being by relating it to color vision, that is, how our perception of color occurs. A good summary of Giere's view and development of scientific perspectivism is made in the first section of (TELLER, 2020).

azimutal projection might be useful for military radars, but the more common Mercator projection is useful for navigation, but might not be for geopolitical representations, given its non realistic distortions of the regions close to the poles. Even my suboptimal sketches that I unashamedly present to my players during our RPG sessions are useful according to *some* purpose (I am glad they never complained about my drawing skills; thanks, guys!). What is on the line here, is that *perspectives* are always partial, but that is not a problem, because in doing so, the scientist, philosopher, game master, artist is just selecting what they think is relevant to be known in order to better represent that aspect of reality (or of some fictional D&D world).

Another commitment the perspectivist makes is regarding the impossibility of non-perspectival views of reality⁹, i.e., we always depict reality by representing some aspects of it according to some parameters. Actually, this commitment is even stronger: for the perspectivist reality can *only* be depicted perspectivally, so any attempt to extract information of reality is bounded to incur in some kind of preemptive selection of the aspects which are considered relevant to fulfill some designated purpose. The purpose at hand might be studying some phenomenon in particular, saying something about a domain or topic or even mundane things like talking about your vacations by the beach. Here, the analogy with maps is useful, since in a scenario where any depiction distorts the reality to some degree, given some purposes, it is important to choose that representation of reality which depicts specifically those aspects that are considered relevant to be analyzed as close to reality as possible, despite the occurrence of distortions in some of the chosen parameters. Another aspect of perspectives, however less important for what is going to be discussed here, is that whenever we talk about *laws* in a determinate theory, we are talking about generalization of claims, and not some deterministic axiom or principle set in stone. This is very important for some accounts, mainly in scientific perspectivism, since this view is consonant with the idea that models only *represent* reality in some way, so *laws* are those generalizations which can be applied, under some perspective, to all kinds of phenomena which can be represented by those parameters. Canonical examples can be Newton's laws, First and Second thermodynamic laws and Maxwell's equations. Each of these are sufficiently general, but Newton's laws are not applicable to magnetic fields or to thermodynamic systems.

 $^{^{9}}$ In fact, the original formulation is that there cannot be non-perspectival *knowledge* of the world, as described in (CHAKRAVARTTY, 2010). However, my approach leaves knowledge aside for the notion of *understanding*, so I chose to present the perspectivist's commitment with the word *views* instead of knowledge.

1.2 Scientific Perspectivism

We have seen that using the term *perspective* closer to a common sense may be subject to it having different meanings and uses, depending on the area it is being applied to, the theoretical background given to the term, and the intention with which the term is being used. Hence, if we want to use it in a scientific context, we have to give a strict definition of the term used in these situations. Luckily for me, the work was already done by Ronald Giere (2006), and a few who followed his work and the usage of the term in a scientific context¹⁰.

We may start by saying, following (GIERE, 2006; TELLER, 2020) that perspectivism is, broadly speaking, the view that human knowledge is always perspectival. In other words, human knowledge is only possible through some particular perspective or point of view. In Giere's case there is a commitment to saying that, besides this being the case, science occupies itself in building inexact models of nature, rather than search for true laws or principles. Moreover, he believes that scientific models are the primary tools of representation used in science, as evidenced by the following quote:

Scientists use models to represent aspects of the world for various purposes. On this view, it is models that are the primary (though not the only) representational tools in science. (...) They are designed for use in representing aspects of the world. The abstract objects defined by scientific principles are, on my view, not intended directly to represent the world. (GIERE, 2006, p. 63)

There are a few points to be made about this quote. The first being to reinforce that scientific perspectivism claims that models are the primary tools scientists use to represent the world. This is clearly in conflict with the standard realist position, which claims that scientific claims about the world must be taken at face value. The reason for such a strong claim by the perspectivist is justified when we object the realist with questions about discarding data or possibly inconsistent measures taken during an experiment. How then can science make claims that assert anything about the world if what science takes into its own realm of discussion are rough approximations to reality?

¹⁰I will refer to them in the next pages, but the reference can also be done here. I will focus on the following works: (CHAKRAVARTTY, 2010), (TELLER, 2020), (MASSIMI, 2012a), (MORRISON, 2011), (SAATSI, 2020) and (CHIRIMUUTA, 2015).

Despite going against standard realism, Giere opts to defend the idea that perspectivism is a realist position. In his text, he tries to push forward the idea that perspectivism should be a position in-between realism and antirealism, but, by the end, he defends it as a form of realism. Since his ideas focus on models being the primary tools of science, the models constructed by scientists tell us what the world is like from that particular standpoint. Since these models tell us something about the world and, moreover, are constructed within reliable bases, that is, are constructed using parameters, theories and other models that gives us a satisfactory amount of information of how the world¹² – or at least a *good enough* approximation of the world – functions. any realist claim made using those models are justified and conditional.

Trying to bring models forward in a scientific context seems more natural and closer to scientific practice than using a direct reference to the world. Still, one must notice that models say something about the world. That is, even though the activity of the scientist is to build models and to work with them, these models are representations of the world, and if they *successfully* represent the world, then the claims being made about models are actually claims about the world itself. Here enters a notion of *successful representation*, which is brought forward by Bas van Fraassen (2008), who tries to draw a line in order to determine when a representation successfully represents something as being (or having) something. According to his account, a representation is uccessful when it trades its success on *selective resemblance*¹³. What that means is that even if we end up with a model that does not resemble in every aspect the phenomenon we are trying to represent, what matters is that it resembles the phenomenon (at least) in

¹¹There are attempts to answer these kinds of problems by realists as is the case with David Lewis' On the Plurality of Worlds (1986), using the notion of 'approximation to truth'. However, as it will be further discussed here, this is not sufficient for the matter in hand: Perspectivism also takes into account cases where incompatible models model the same phenomenon with success, as evidenced by (MORRISON, 2011) and (RUPHY, 2016), which undermines the standard realist position. Moreover, there is an insurgence of big data sciences, which I happen to have had contact with when beginning my scientific career, which seem to not be contemplated by the realist position. A perspectivist approach to such cases can be seen in (CALLEBAUT, 2012).

¹²The idea of *good enough* approximations can be found on the works of van Fraassen (2008) and Elgin (2017a, 2017b). In short – and doing some sort of meshing together of the two presentations –, an approximation of something is *good enough* when it represents that something in a suitable way so as to highlight the relevant aspects of the thing represented in such a way that it makes it possible for us to grasp how the thing behaves.

¹³Van Fraassen defines *selective resemblance* as resemblance that trades on selecting certain aspects of an object for its success. Thus, it is a representation of something because it sufficiently resembles that something or because it highlights aspects of the object being represented in such ways that contribute for achieving the goal the representation intends to. For a more thorough discussion on how selective resemblance works, check (FRAASSEN, 2008, section 1.2)

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the relevant aspects in a suitable way. That is to say that the representation needs to bear "a *sufficiently* adequate resemblance in all *relevant* aspects for the *purpose* at hand" (FRAASSEN, 2008, p.49) to its represented¹⁴.

By introducing *representations* into the mix, we might observe that both Giere and van Fraassen conjecture that science uses *representations* (in the form of models) in order to say something about the world. For Giere, scientists talk about those models, which in turn say something about the world. So in the last few lines of Giere's quote, when saying that models are not intended to directly represent the world, what he wants to say is that they represent the world, but the scientist can only say something about the models they formulate, tested, built and worked with. These models will bear a suitable resemblance to the object being represented, but they are not the object itself. This reinforces the thesis that human knowledge is perspectival. It may be true that we might gain knowledge about how the world is, but *how* we gain knowledge of it can only be perspectivally (MASSIMI, 2012a).

Noteworthy in both Giere's quote and van Fraassen's idea of a suitable approximation, is the presence of *purpose* when deciding which aspects of the phenomenon to highlight when making a representation of it. Since each perspective is partial and requires the agent to know which aspects of the representation to highlight in order for the representation to be successful, then a model always presupposes that there is a choice of perspective made by an agent when being constructed. We can further bring together the idea that representing requires agency to be successful (FRAASSEN, 2008). We will see in chapter 3 that agency might be a problem to deal with in the Philosophy of Mathematics since mathematical platonism presupposes the existence of mind-independent mathematical objects, which I am clearly rejecting here.

Before moving on, we have still to tackle the main purpose of this section. I focused on defining and discussing the general aspects of perspectivism, but I did not restrict the discussion to *scientific* perspectivism. It seems that by saying that scientific models are the representations being used to represent the world, and the people building researching, modifying, proposing those models are scientists, the requirements are satisfied. One can still question if there is an intrinsic characteristic that makes it *scientific*, since inserting some view of science in a scientific context does not make it scientific. For example, we can discuss how astrology may predict some of the behaviors of the planets

 $^{^{14}}$ In his text, van Fraassen uses the term *scaling* to refer to approximations that satisfy these conditions, as he is giving examples of these kinds of representations. Since I think that *scaling* has a restrict use and my purposes involve talking about a science that does not scale anything, I will use *suitable approximation* instead.

and their movements, although nobody in their right mind would claim that the view of the world that astrology purports is scientific. To that question, I answer that the name *scietific perspectivism* might be misleading. We should call it perspectivism applied to science, but the name was created by Giere and it stuck ever since. The crucial difference in both cases is that perspectivism can be faced as a very general thesis about how we, humans, perceive the world and how we extract information from the world¹⁵. This claim is close to the idea I want to propose. In the same vein, it should be noted that what I call "mathematical perspectivism" should also be understood as perspectivism applied to mathematics, however, a few tweaks must be made in order for that application to work. This might make the comprehension of what this work is about a little easier to grasp: not only defining and discussing perspectivism, but trying to apply such a general view of the world to mathematics and say that it grounds mathematical practice as an activity whose ends are cognitive: we want to better understand how mathematics work and what we can do with it.

I now turn to discussing a few points about perspectivist that I believe are weaknesses of the view. I hope this discussion improves the understanding of perspectivism in general – at least how I conceive it – which will be an useful step when trying to fit it into mathematics. First, I will briefly present the discussion whether perspectivism should be counted as a relist or antirealist position. There are arguments from both sides, although I will defend some kind of realism; the next step is to evaluate the perspectivist exigence of agency and if this can be detrimental to the view as a whole, specially because this point should be a concern to the philosopher of mathematics; then I discuss one problem that might haunt us further into the text, that is the impossibility of knowing everything, another point which should concern the philosopher of mathematics;

1.2.1 Perspectivism: realism or antirealism?

The view purported by Giere (2006) should be understood as a form of realism. Obviously, not the standard realism, but some form of it. Giere tries to push forward the idea that, despite model-building having a social aspect and models, in general, being intrinsically partial, scientific models, built upon a scientific background, following fairly successful and reliable methods, using reliable instruments and being done honestly would be enough to justify a realist stance towards any realist claim that follows from the results generated this way. It drifts away from standard realism when it leaves space for

 $^{^{15}\}mathrm{This}$ assertion is close to what Michela Massimi (2012a) defends.

more than one model being sufficiently adequate to model the same phenomenon, from different perspectives, that is, there is no *best* model to describe a single phenomenon, but differently adequate models to represent a phenomenon according to the purpose at hand.

Perspectivism can also be viewed as entailing a form of relativism, according to (CHAKRAVARTTY, 2010) and (KUSCH, 2021). Chakravartty, for example, gives two formulations of perspectivism, by saying that human knowledge is perspectival because either (1) perspectival facts are the only ones we can grasp; or (2) there are only perspectival facts to be known. The first is considered by Chakravartty to be a sort of conceptual relativism, while the second, stronger, derives a conceptual relativism from an ontological relativism. At the end of his article, Chakravartty makes a defense of a dispositional realism where the perspectivist approach should be taken as a form of gaining knowledge about the world in practice. More rigorously, he argues that perspectivism entails dispositional realism: the dispositional properties of the objects in reality are differentially revealed by perspectival detection methods. Scientists then gather together the information given by those distinct readings, which allows for a reading of the world as a whole through those perspectives¹⁶. It strikes me as a curious position, because despite his criticisms towards perspectivism, he concedes at the very end that science happens perspectivally and that this form of doing science is valuable in gaining information about the world. In a sense, it brings his position closer to Massimi's (2012a).

At the other end of the spectrum, we have a defense of perspectivism being an antirealist position done by Morrison (2011). She points out problems in the possibility of constructing models cumulatively, improving upon already established reliable information. For her, models in science are improved upon in this way, but the perspectivist stance undermines such a position. The possibility of having incompatible models leaves the door open for the perspectivist to ascribe inconsistent "fundamental properties" to the target of inquiry, which renders impossible the construction and the extension of models in a cumulative way. For her, this would be equivalent to an antirealist position towards the target of inquiry.

A third defense of realism comes from Mazviita Chirimuuta (2015), who defends a form of perspectival realism which incorporates interestedness, partiality and interaction. For her, the analogy of color vision used by Giere (??pp. 31-36]giere2006) is not

¹⁶As far as I understand Chakravartty's idea, it would be like building a puzzle in which each perspective would give us a fair amount of overlapping information, but some new information that other persepctives would not. The picture of reality would be given by adjoining all of those views together.

adequate to capture the perspectival nature of science. She argues that tactile perception is closer to what perspectivists want to define. Tactile perception involves engagement and interaction with the world in an interested way: there is intentional manipulation of the aspects of the world and that is analogous to what happens when a scientist builds a model. Scientists build models trying to attain an objective and manipulates the data they have in order to do so.

Another proponent of the thesis that realism is an adequate label to describe perspectivism is Paul Teller (2020). According to Teller, perspectivism is in consonance with at least two of the general premises of realism. As already mentioned, perspectivism accepts the ontological aspect of realism, that is, the existence of a mind-independent world which is the target for scientific knowledge and understanding, as well as the semantic aspect of realism, that scientific claims should be taken at face value, for the reasons explained in the first paragraph of this section. It remains to explain why it supports mature theories being successful in explaining true (or approximately true) statements about the world or objects in it. Teller argues that an statement being 'approximately true' needs further development and proposes to understand it as an statement which is sufficiently similar to what it describes in the relevant context according to some purposes. According to him, what forces the scientist to content with approximations to truth is the contingent fact that the world is way too complex for our limited intellect to grasp every aspect of it at once. Hence, the acceptance of contextuality involved in doing science in practice implies perspectivism.¹⁷

The list of defenders of perspectivism as a form of realism continues with Juha Saatsi (2020). He proposes that what is actually involved in perspectivism are explanations. For Saatsi, this is done via explanatory perspectives, that is to say, ways of thinking or representing an object or phenomenon in an explanatory context which functions to increase our understanding of the phenomena we are theorizing about. This is specially appealing since explanations do not have a factivity requirement. However, explanatory perspectives in the scientific context requires that there is some *factivity* involved, by admitting that an explanation is successful when explanations latch on to reality in appropriate ways. It is, thus, a factive approach towards explanations.

There is still Michela Massimi (2018), who proposes another way of realism, distinct from Giere's, in which a scientific claim satisfies a criterion of "success from

¹⁷There is another criterion to be satisfied, but I judge it to be of minor importance for my discussion. Teller also argues that referential tools used in science must be considered to make reference to objects as if they existed as supposed by the idealized models they occur in.

within" by performing adequately according to criteria of its own, expressing a proposition that is in fact true and by meeting standards of performance adequacy appropriate to its original context as assessed from another perspective. The last claim says that a scientist can judge the success of a claim by the standards of its originating perspective, as these standards are interpreted in their own perspective. They are, however, blocked by crossperspectival distance. That is to say that a perspectives that is too 'far away' from another where the original claim occurs is not adequate to evaluate success from such a claim.

Given so many defenses of a realist perspectivism, I tend to believe that perspectivism, at least in science, is best understood as a realist position. The different arguments and stances can be brought together as a robust defense of perspectivism being a realist position. Giere's proposition seems to me more like a proto-perspectivism which has since been worked upon and improved, so that a clear cut of what it entails and how it should be understood arises. Being a realist position, it shall remain for us to discuss what are the implications it has when dealing with set theory, if it is at all compatible with the work done there.

1.2.2 The problem of agency

We briefly discussed that one of the consequences of perspectivism is that the view asks for agency to be part of our account. The first question to be asked is if we can get rid of this exigence and formulate perspectivism without agency. If we are not able to do so, we discuss if agency is really such a big of a problem to have, or if it is actually an asset this view engenders.

Now let us think about taking agency out of perspectivism. As it stands, perspectives have a close relation to representations, and since our take on representations follow closely to van Frassen's, as presented in (FRAASSEN, 2008), we have to concede the point that van Fraassen, in his original formulation, does not leave out the agency component of a representation. As he says, a representation is something "(...) that is used, taken or made to represent something as thus or so"(FRAASSEN, 2008, p. 23). Noticeably is the use of passive voice on the verbs "use", "take", "make", so as to suggest that someone or something uses, takes or makes the representation to represent as thus or so¹⁸.

Suppose we were in a situation where no agent is the sole responsible for making such a representation. We can think of a scenario where humans were only entitled to

¹⁸The discussion of agency is also present in the same book.



Figure 1.1: The classical duck-rabbit example.

discover correct representations of aspects of the world, being them objects or phenomena. Something like a disposition of things to be representations of other things. But then, how can one explain the fact that different people might use one thing to represent different objects? One famous case is the duck-rabbit picture (see fig. 1.1). What does it represent? A duck? A rabbit? Another thing entirely? If agency is taken out of the account of representations, then we should provide an objective answer to such cases and even other more mundane ones, which are not fabricated only to instigate bar-discussions among undergraduate philosophy students¹⁹.

Accepting agency, then, seems part of a bigger goal of perspectivism. Not only we better deal with problems like the aforementioned, but we can also better ground perspectivism itself on agency. Recall that perspectivism says that we view the world through partial representations and that this is the *only* way humans can have knowledge about the world. If we take agency out of our desiderata when dealing with perspectives, we weaken our own view. Since agency may be the element that provides the grounds upon which we can argue that deciding which perspective is sufficiently adequate for each purpose at hand, given the chosen parameters. Analogously to the duck-rabbit picture, if there was no agency, then the question of which representation better represents a thing as thus or so for some determinate purpose would be objective, and no decision, usage, take would be involved in the process.

This problem is also addressed in (GIERE, 2006). For him, being obligated to assume that there is an agent constructing the models which represent aspects of the world is not, in the long run, a problem, because science can only be done inside a human society and in a perspectival way. This is not what gives science its perspectival aspect, however. As Teller puts it (TELLER, 2020), it is a contingency that we have

¹⁹Specially recurrent during my undergrad years.

only perspectival knowledge about the world. The way things are makes it so that our best attempts to grasp information about a world whose complexity is beyond our limited cognition end up being perspectival. Hence, despite our best efforts, agency still seems to lurk in the perspectivist background, as a constitutive part of the view. Teller does not commit himself to the thesis that the perspectival approach to knowledge is a definite prescriptive thesis. He retains himself in saying that it is just a description of how our scientific knowledge is gained at this point in time.

The obligation of assuming an agent is thus not that problematic and we can stretch our argument further and provide it a better justification by saying that perspectives make conceptualization possible. We take, for example, a God's-like view of the Universe and may imagine that there were some absolute representation of reality, given by some very general model, so that *every* observable phenomenon could be explained by that model (even if not 100% accurately). One could quickly architecture some example that would be really awkward to explain by this model without appealing to agency.

Think about the case where there is one event ϵ , and that this event can be explained by a comprehensive absolute account in two distinct ways which are incompatible with one another in at least one parameter. Suppose that these incompatible parameters are named p_1, p_2 respectively and that each of these phenomena are fundamental to explain ϵ in their respective accounts. Now, since p_1 is distinct from p_2 , it is plausible to assume that p_1 depends on parameters that p_2 does not and vice-versa. But then the bigger model should address these dependencies accordingly and as such, explaining event ϵ passes through explaining phenomena p_1 and p_2 respectively. But we assumed that these phenomena were incompatible, hence, either our account is inconsistent, or we should stand beside one explanation over another and there goes the possibility of having incompatible models. Hence, if we decide to accept perspectivism, we cannot accept a God's-eye view of the world, since we admit the possibility of existence of incompatible models.

Of course one could object by saying that this example is way too artificial to be actually tenable in practice. But then I regress to the duck-rabbit case and we can deal with it in the same way. Perspectivism has no space to allow an absolute view of the universe because it has its roots in the conception that human perception of the world is always perspectival and how we gain information about it and not only that, but we, as humans, select which aspects of the world to highlight in order to understand it, interpret it and also to make it so that we can represent it in such a way that we can operate the world scientifically.

1.2.3 Bounded knowledge

Another case can be made about how perspectivism deal with knowledge and if we are ever entitled to know everything there is about a certain topic, object or phenomenon. Again, here the best strategy is to understand how the pieces fit together and if it is possible for perspectivism to accommodate such a premise. We might as well investigate what happens when we assume that our knowledge is always limited and how this might be another justification for assuming a perspectival approach to science.

Beginning by assuming that we might end up with an absolute knowledge of an aspect of the world. We have to analyze the problem into its components so we divide between the 'knowledge' and the 'limitation' parts. Recall that a knowledge requires factivity, so knowing a fact p presupposes that p has to be true. This might already be a problem for the defenders of perspectivism. Recall that perspectivists claim that any scientific claim is a claim about scientific models which represents aspects of the world in some way. Also recall that when discussing representations, I stressed that representations trade on selective resemblance in order to fulfill a purpose, in general, representing the represented as thus or so. If any representation is as such and perspectivism claims that the primary tool of science is some kind of partial representation of the world, then it cannot be the case that there is an absolute representation of the world. In this sense, saying that a fact p is true through the lens of science might be hindered because science only gets information about the world from partial representations. According to van Fraassen (2008, chapters 2,3), these representations are not even *exact* representations of those highlighted aspects, but rather good enough approximations²⁰. Approximations that are sufficiently close to the real object or phenomenon being represented, so as the difference between what a scientist calculates and the world is negligible. The point to be made here is thus not that the perspectival view assumes that the only way we may have knowledge or understanding of the world is through partial representations of it, so that any effort in trying to build an absolute model of a phenomenon or object falls short of its objective.

Hence, we cannot strictly call *knowledge* of p what is drawn from perspectivism. For the purposes in hand in this chapter we can relax our criteria for knowledge and we can call knowledge of p what would be knowledge of sufficiently good approximations of p. But if we weaken our criteria, then we face another problem that is the limitation of knowledge. For let us say that knowledge of sufficiently good approximations of p

 $^{^{20}\}mathrm{Recall}$ the definition of $good\ enough$ approximations given by both van Fraassen and Elgin, shown in footnote 12.

grants knowledge of p, so that we can make a case for knowing everything we can about a fact. Now, if it is possible to know everything we can about p, how can we claim that our perception, understanding or comprehension of p is perspectival? For it is absolute, whence it does not depend on any parameters we judge pertinent to grasp the relevant aspects of p. Since we know everything about p, we know every aspect of p that is relevant and also every aspect that is negligible, so we can point out which ones are to be considered and the ones to be rejected. Moreover, there would be only one way to correctly know p, analogous as the case with the duck-rabbit picture, rendering the perspectival approach obsolete.

Once again, our initial presupposition is thrown out of the window, but we can make the case that what remains is exactly what renders our approach successful. Perspectivism affords a view of knowledge that is essentially limited: humans in general, cannot know, at the same time, and with accuracy and correctness all the aspects of the world, not even all the aspects of some part of the world²¹. Hence, perspectivism is a way to explain the scientific practice which highlights and chooses which aspects of the world the scientist wants to talk about. In doing so, the knowledge resulting from such a practice cannot help but be partial and, somewhat, limited. As in the agency case, this is far from a bad situation to be in. On the contrary, this is on par with our assumptions and also reinforces the reasons why one would opt to choose a perspectival view of science: to bring it closer to practical science.

Thus, by saying that all we may have is limited knowledge we are actually on a par with what perspectivism presupposes. We are far from saying that his knowledge is minimal, or that almost nothing can be known by these procedures, since, as discussed in section 1.2.1, perspectivist supports a realist stance towards scientific claims. We are just stating that even though we may know a lot about a fact or a phenomenon through its models, and by considering that approximations of p can grant us knowledge about p, there is always something beyond the scope of our human comprehension that is not attained by these methods of detection. We must hence pick which aspects of the world we want to grasp at each time as well as decide which ones to leave out of our account.

 $^{^{21}}$ Paul Teller (2020) makes his case on similar grounds, although his approach is not strictly epistemic. For him, it is a contingency that our world is too complex for our limited human cognition to grasp all aspects of it at once. Hence, we are able to grasp approximate truths about the world, which implies a perspectivist view of science. This was already discussed in section 1.2.1.

1.2.4 The faulty scientist objection

The third problem we may face when proposing a perspectival view of science is that since science is done by humans, and humans are faulty, then it is not enough to assume that the results we are getting are exactly free of bias or even particular interests²². In general, perspectivism would grant that people choose perspectives that correspond to their own interests, and we could end up in a situation where each person has a particular view of the whole, and, in a limit case, these views end up disjoint, so no two people can talk about one topic, because their knowledge does not agree in none of its claims.

This is a very challenging objection to be made, since many real life examples can be used to sweep any rational argument I give. However, we can idealize the situation – and I guess this is how the system of blind-peer reviews was envisioned to be – in which any choice of model chosen by a particular agent will, down the line, be independently and honestly checked by other agents which can find faulty models or badly chosen fits, or even sub-optimal choices of models, so that, in the long run, these mistakes tend to be corrected.

I am also obliged to better explain what I consider to be a bad fitting or a bad model in this case. Since we are talking about science models, given the tools and resources available to the agent, a good model gives sufficient and adequate enough information so as to answer the questions presented by the hypotheses assumed. We can grant that a good representation will fulfill the goals it was envisioned to. That is to say that when evaluating the subject, the model chosen does not give a huge discrepancy with what is being observed in the world, but even more than that, it is backed up by other statements which are more fundamental and that we already know about the world, inside a given area of knowledge. A claim is not to be evaluated on its own, which makes the decision in the community of accepting or denying a claim a job backed up by reliable information. It is obvious that this is not the case for *every* situation, and it may happen that revisions become necessary on our previous theory in order to accommodate for new results. Still, this does not render neither our initial theory faulty, nor our new discovery. The discussion also bears on a variety of topics, that might escape the scope of this section.

We are allowed to do that since we can say that bad fitting, bad data manipulation, and even bad theories tend to not last long after published and also tend to not

 $^{^{22}}$ I do not believe this section to be essential to the work done here, however, it is a start towards discussing perspectivism and its consequences inside the everyday practice of science, hencewhy I decided to keep it in this dissertation. This part of the discussion reflects mostly what I have thought when objected with this question by a friend of mine, Siiri Kivimäki, when she read my first drafts on the topic. Any solid results should be better studied in future works.

agree with many of the claims we judge to be basic or even very hard to change. In short, I assume that despite perspectivism leaving open a window for faultiness in principle, in a community it seems harder to happen, because science is a collective activity and scientists use bits of knowledge from one another to build their own models, representation, conceptions and comprehensions of the world. So it would be a plausible scenario had scientists be locked down in their basements as mad scientists in movies do. As science is made in real life, though, the scenario, although theoretically possible, is practically implausible.

Chapter 2

Understanding in perspectivism

Perspectivism, as discussed on chapter 1, is the view that claims that human knowledge is always perspectival. The implications of such a view are enormous in science. Despite the initial shock, I have shown that a realist stance is compatible with perspectivism, as do many of the perspectivists authors in the literature. We must endeavor through, though, the epistemological implications of perspectivism. I claim that my version of perspectivism in set theory is an epistemic version of perspectivism, but if that is the case, then we have to deal with epistemic gain. Not only that but epistemic gain concerning truth values of mathematical propositions and perhaps mathematical explanation. To my account, that is close to what perspectivism entails.

In the formulation of an epistemic version of perspectivism, I follow Massimi (2012a) in saying that what a perspectivist account of science gives us is to provide justification on how we have epistemic gain about the topic. I, however, drift away from her thesis when deciding that epistemology in grounded on understanding instead of knowledge¹², where we get closer to the work of Elgin (2017b). The work of the latter is based on the concept of objectual understanding, which refers to the grasping of a range of phenomena in an account³. How understanding, grasping and other correlated terms shall determine will be discussed further in the next section.

What is important to have in mind right now is to why this shift occurs. The main goal of this thesis is to argue for an epistemological view of mathematics.

¹We here say 'knowledge' for 'scientific knowledge'. since our goal is to deal with sciences and mathematics, so our scope is restricted to those areas.

²It was noted during my thesis' defense that knowledge could still be the ground concept of epistemology, without an exigence for strict factivity. I, however, have not endeavoured through this path, but should be a way to pursue in the nearby future.

³Elgin's notion of account is defined as "(...) a constellation of mutually supportive commitments that bar on a topic." (ELGIN, 2017b, p. 12).

Perspectivism gives us the big scheme of things, by saying that whatever we can grasp of the world, we do so perspectivally, that is, partially, and making decisions on what aspects of the world we will be focusing our attention on. In science, we normally do so trying to reach an objective and, as will be argued in chapter 3, mathematics has an analogous way of doing the same kind of thing. This partial, often idealized, sometimes modeled version of reality, has one aspects that refrain it from allowing to provide knowledge. Our grasping of reality being partial does not allow that, for a fact p, we know something of p. Simply because when analyzing models, idealized versions of p, or even approximations to p, we end knowing about those models, idealizations or approximations, not p itself. But to have knowledge of p, p must be true, but our best efforts only give us truth about things that are almost p, but not p. Hence, saying that perspectivism gives a form of having knowledge about p seems wrong⁴. Instead, we can argue for perspectivism being a way of providing understanding about p under some conditions. And in giving understanding about p under different evaluations, perspectivism gives us a way of having understanding about the whole topic in which p resides.

Before we move on to argue that perspectivism does, in fact, provide understanding of a determinate topic, we must endeavour through what understanding actually is and what does it mean to say that someone understands a fact or proposition p. Then, we move on to applying the notion of understanding to mathematics and arguing that perspectivism is the best background we have upon which we can develop set theoretical understanding.

2.1 Understanding

We usually think about epistemology in terms of *knowledge*. The problem we have is that this conception of epistemology entails that for someone to know that proposition φ is the case, it is required that φ is true. If we join this with perspectivism, we end up in a troublesome situation. To know a fact p, p must be true, but we cannot assert p's truth, because everything we grasp is perspectival. Hence, the factive approach to epistemology, that is, that to know a fact, that fact must be true and the knowledge of the fact implies its truth, by applying deduction rules does not seem to cope well with a perspectivist background.

 $^{{}^{4}}$ I again stress the fact that I am considering a traditional factive account of knowledge here. Knowledge which accounts for truth approximations is not in the scope of this study hitherto, but shall be in the near future.

Not only there is a problem with perspectivism, but the factive approach to epistemology leaves out of the conversation many cases which are cognitively valuable to us as epistemic agents, but in which the propositions being used are not strictly true. For example, when examining a tissue through a microscope, the scientist has many parameters previously chosen by or for them. The sample has to be prepared in accordance with what the microscope is able to take images of, the tissue, cell or organelle being observed has also been chosen, and the situation observed is controlled in such a way that conclusions drawn usually refer to at most a few aspects of the phenomenon in question. The whole process occurs in controlled environments, with controlled parameters chosen beforehand and the conclusion drawn is not about any situation in which that phenomenon occurs, but only in those particular situations that were chosen for the purposes of that study in particular⁵. Not only that, but usually even the samples studied vary a little bit when compared one to another, which we correct by mathematical numerical approximations. So in the end, the conclusion drawn is not about the phenomenon, but about the phenomenon observed in controlled circumstances, under some parameters, and moreover, the data being read is approximated and the calculations are the results obtained from these approximations.

The standard factive approach to epistemology also does not accommodate for cases where we gain understanding of a topic by construing initial broad and false idealizations of the facts, but start growing our understanding of a topic by refining our models ending up with ones that are more accurate and better explain the fact we are studying. An example is how we learn topics with different depths as we grow older, having contact with them at different stages in our academic lives. We start by learning very general, sometimes even borderline wrong, models of some topics, but as we grow older, we start seeing the same topic, but with models that gather more information and that have more refinement of details, more intricacies, so that we can say that we have a *better* understanding of that topic than we had before when we grasp the latter idealizations. Notice that, different than factive knowledge, understanding allows for *degrees*. We can understand something we are told for the first time, but if we revisit a topic we might realize we hadn't grasped all of the details, so we may say that, given the new information grasped, we have a *better* understanding of the topic. With knowledge, either one has it or don't. Simply because knowledge, and specially the factive approach, requires that we only know about things that are true. According to Elgin (2017b), epistemology

⁵It is not uncommon to see presentation titles that indicate that an experiment is extremely specific and the results drawn are only about an experiment done within those parameters and nothing else.

should thus be the area of philosophy responsible for saying why there are such degrees of understanding and what they consist of.

All these cases motivate an epistemic turn towards *understanding* being the central concept in epistemology, rather than standard factive knowledge. For us, this is yet more significant since I believe understanding fits perfectly with perspectivism in building grounds for arguing that set theoretical practice intends primarily to provide epistemological gain, in our newly coined terms, better understanding of the topic being studied. Understanding presupposes *grasping* as its primary cognitive competence. We also follow Kvanvig (2003) in saying that there are two kinds of understanding, propositional and objectual. While propositional understanding involves grasping a fact, objectual understanding involves grasping a range of phenomena. Where they differ is that objectual understanding, according to Elgin, is not a simple conjunction of many propositions, but rather has something else that makes everything gel together. More precisely, for someone to have objectual understanding it is also important to grasp the relations between the different facts which happen in an account and how they support one another inside that account. There is still another conception of understanding (GRIMM, 2006) which ties it to explanation, called explanatory understanding, which is based on saying that to understand is to be able to answer *why* something is so or to be able to comprehend an explanation of why something is so. Another account of explanatory understanding, purported by Juha Saatsi (2020), defines it in being able to answer what-if questions about a topic so that the relevant explanatory dependencies between explanandum and explanans are preserved even when we vary the explananda by means of counterfactual assertions. These two options bring understanding closer to a notion of knowledge of approximations of truth, but again, this shall be better discussed in future works.

Going back to Elgins's account, though, we are still to explain what grasping means. Elgin (2017b, p. 33) suggests that "to grasp a proposition or an account is at least in part to know how to wield it to further one's epistemic ends". We now have understanding tied to *know-how*. In particular, Elgin ties understanding to a sort of coherentism, which involves grasping how the different truths relate to each other and to other elements of a particular account where the phenomenon we are trying to understand lies. It is also an important condition the fact that whoever understands something is willing to use the information obtained, for example, as a working hypothesis, applying to a situation, or any other means to furthers their cognitive ends. This goal-oriented aspect of understanding is what makes it a perfect match to fit inside a perspectival view.

The connection understanding has with explanation can be said to happen

because of many "fundamental" concepts. Few authors (STREVENS, 2008; GRIMM, 2006) believe that understanding why something is so involves being able to explain that something or to comprehend explanations about that something. This position, however requires understanding to be factive. Instead of knowledge that p, we are now focusing on knowledge why p, but still, a knowledge. Others, like (LIPTON, 2004), say that understanding is a knowledge of causes. Again, it seems very narrow, because some topics are not causally dependent, such as is the case with mathematics. There are proofs in mathematics that do not provide *explanation*, but rather just tells us that some statement is true or not⁶. This only corroborates the claim that perhaps mathematical *explanation* and *understanding* are not so tightly correlated. As is current in set theory, we can axiomatize a theory of sets in a number of ways, by, for example, choosing different propositions, different notations, if our theory of sets admits or not *Urelemente*. In some of these axiomatizations a determinate proposition is well explained by a proof, but others might not have the tools necessary to give such an explanatory proof. One of the examples I can mention is when talking about cardinality and defining ω_1 as the least ordinal to not be in bijection with ω . There are other ways to define ω_1 – such as the cardinality of $\mathscr{P}(\mathbb{R})$, for example - but, for me in particular, this is the one that better *explains* what kind of object we are dealing with – and even give *some* explanation on Cantor's theorem, which, for the record, does not need the definition of ω_1 to be proved. Nonetheless, it is also notable that other ways of defining such objects might even further one's understanding of set theory in general. By defining the same objects in different manners and by showing that in each of these variations the same object appears, in various ways, we actually end up gaining understanding on the topic itself, not about ω_1 in particular.

Given those attempts to solve what is the connection between understanding and explanation, perhaps it is time to weaken our requirements. There are instances in which someone understands something without having grasped the necessary tools to formulate an explanation of the phenomenon. One good example is that fluent speakers of a language may not be able to tell what makes a sentence a well-formed sentence⁷. They usually claim it comprehensible by means of being exposed to such sentences during their

⁶I am aware that we can call such proofs "inelegant", but, as a logician, I would argue that a proof is a proof, no matter how 'beautiful' it looks like. Regardless, this seems to be the case with proofs using proof assistants or even computer proven theorems. We only know that the propositions being proved or disproved are true or false, but, mainly, in the proof, there is no explanation of *why* it is so. Still, we cannot say that these methods of proof do not provide *understanding* to the topic being studied.

⁷I am not, for obvious reasons, using "well-formed" as we do in logics. Here, a well-formed sentence in natural language just means a sufficiently grammatically correct according to rules of usage that happens in everyday life, so that fluent speakers can comprehend correctly what is being said by someone.

whole lifetime, but this has nothing to do with their ability to explain why it is so. Other instances where an explanation is not available, such as in subway systems⁸ could be useful as well, since one can explain some aspects of the system and how it works according to their account, but not explain the whole system, the operation behind departure of trains so that they do not collide, maintenance, what powers a train and so on. The point is that a person grasps *enough* of the system so as to *understand* those aspects which are relevant for them in such a way that they can ride the system in an optimal way and to access every place possible and take detours to get back home if necessary without major problems.

Thus, to sum up, understanding as a matter of explaining why something is so or to comprehend such an explanation seems off because it presupposes that the explanans is independent of and prior to the explanandum, at least epistemically. However, it looks like understanding calls for a more holistic approach, which instead takes the explanation of the fact together with the elements that provide such an explanation and everything that supports them. As Elgin (ELGIN, 2017b, p.44) puts it, "(...) the understanding of a particular matter of fact derives from the understanding of a suitably unified, integrated, tenable body of information that bears on that fact.". So objectual understanding is taken to be the main concept of understanding because it is what makes such an approach to understanding possible, and in fact, it *is* what Elgin describes in the above quote. Moreover, it provides a criterion of discrimination between saying that someone understands a topic and saying that someone knows particular truths about that topic, and how theese particular truths relate to each other under some account.

What is missing for us to deal with is the need for grasping to be tied with know-how. That grasping a topic involves understanding of the elements of that topic, how they mesh together, and also knowing how to wield that information so that we can further our cognitive ends is clear. What is not clear is what *knowing how to wield such information* refers to.

A few aspects of knowing how might be important to note. We may say that knowing how to do something might be a matter of multitrack dispositions⁹ (RYLE,

⁸Both examples are presented in (ELGIN, 2017b, pp. 37,42,52), although the language one is presented to talk about understanding as rule-following, which I do not delve into here.

⁹We say multitrack dispositions because knowing how to do something usually involves knowing how to do isolated more specific things in a certain order and with some degree of proficiency. As the example of goal scoring in ice hockey goes, one might say that it involves dispositions to skate, to hold a stick correctly, to hit the puck with the blade of the stick, to angle the stick correctly so that it hits the area of the goal and to do so in a fast and precise way so as to score. One might even refine those dispositions in order to get a better track of each of these dispositions. Hence, knowing how to do something involves multitrack dispositions.

1949), but better than that, we take disposition to be some sort of readiness, propensity or reluctance to do some things accordingly in various circumstances. Moreover, knowing how to do something does not require to do that something reliably – or at least in absolute terms. One good example is scoring in ice hockey. Any fan of the sport will agree that players such as Alex Ovechkin, Mario Lemieux, Pavel Bure are great goal scorers, but none of them has a shooting percentage greater than 20% ¹⁰. Which means that they only score once every five shots they take. One could say that this is not reliable. It may be the case that the defending team does a good job blocking shots, so that these players are held to a maximum of three shots in a game for a few games and these players end up not scoring during that span. Still, this fact does not undermine their disposition to score given the opportunity and conditions. No goaltender would like to be in net while Bure takes a penalty shot, for example – maybe in the 90's this would be case, since he has retired in mid-2000's. We can say for sure that no goalie likes to stand for a slapshot taken by Ovechkin from the top of the left face-off circle nowadays, simply because he *knows how* to score from that position ¹¹.

What we mean by saying that Ovechkin knows how to score from that position is that he does this well, rightly, correctly. There is some notion of virtue behind the idea of *doing something well*, which I will not delve into here, for lack of a greater understanding of the Aristotelian theory involved in this topic. What I can say, though, is that this rather sensitive-to-circumstances account seems to better fit our purposes here with understanding. If understanding is to be able to grasp and know how to use information, this *knowing how* needs also to be adjustable, given suitable circumstances. If we change our hypotheses, we might be able to adjust our behavior towards an analysis, based upon what are the parameters we are trying to read from the data, for example.

The relation to virtues give birth to a normative kind of knowing how, which says that knowing how to do something is to follow the rules of a practice (ABEL, 2012).

 $^{^{10}}$ Shooting percentage refers to the number of goals per shots on goal taken by the player. It already discounts the number of shots not registered on goal, either blocked by a defender or that misses the mark, so if we consider shot attempts, in general, this number would be even smaller. The best all-time shooting percentage in the National Hockey League pertains to Craig Simpson, with an impressive 23.7%, according to QuantHockey<https://www.quanthockey.com/nhl/records/nhl-players-all-time-shot-percentage-leaders.html> , who played from 1985 to 1995, splitting his carreer between the Pittsburgh Penguins, the Edmonton Oilers and Buffalo Sabres. The stats for the players referred to can be found on the NHL official website: https://www.nhl.com/player/alex-ovechkin-8471214>, https://www.nhl.com/player/pavel-bure-8455738>, https://www.nhl.com/player/craig-simpson-8451376>.

¹¹The left face-off circle in hockey is also called "Ovechkin's office", much like the area behind the goal was called "Gretzky's office". The term refers to an area where the referred player has great proficiency in making plays or scoring.

Ginsborg (2010) adds that some norms are primitive, that is, there are norms which do not depend on some rule that existed beforehand. We come to know these rules of practice by learning from people who already know how to master that practice, in general. That is how students, for example, come to learn a subject or topic. Where it comes closer to the Aristotelian view is when we say that someone learns to do something well by doing it. Moreover, when someone knows how to do something well, then that something turns second nature to that person. They do it without having to think in the whole process. One might say that Ovechkin's slapshot from the top of the left face-off circle is second nature to him and I would add that, after almost 20 years in the NHL, he has adapted his game many times, besides scoring from that position being somewhat second nature to him. Why? Because he *knows how* to score from that position, He does it well and he knows how to adapt given the circumstances. Are there periods when he does not score that much? Of course, but that's when learn by doing comes in hand, so he can reshuffle his arsenal and relearn his game (and still be relevant nowadays).

It remains to be discussed whether objectual understanding is factive or not, because, if it is, then we should be able to distinguish it from knowledge and from being just a case of knowledge of a conjunction of propositions. If objectual understanding is factive, then everything it relates and everything it takes into account must be true. The biggest problem with assuming that objectual understanding is factive is that we lose the track of topics which are not themselves, true. Let me explain: when shifting from knowledge to understanding, we should allow work with propositions that are not strictly true, but rather say something about a topic of study that is sufficiently close to being true. In this sense, understanding should require only that our body of information is coherent and sufficiently true. However, we should also allow for degrees of understanding, as already discussed above. We can say that a university professor on Ethics has a better understanding of Ethics than a first-year undergraduate student. This better understanding does not come simply by grasping more true propositions, but also from recognizing which of the information the professor has had contact with during his learning years are significant to the topic at hand. So the propositions need not only be true, but also significant to the topic being learned. The professor may also know how these significant information are tied together and how they support important claims inside a topic.

This makes a divide in what might be said about understanding being factive. If we concede that not every aspect of an account must be true, but at least most of them, and at least the significant ones, it still seems that there is something missing. So, what to do when idealizations come into play? Idealizations and models do not seem to satisfy the truth condition, although one might say that they understand a topic, simply by grasping enough significant information about idealizations and models. This undermines the claim that objectual understanding might be factive. Were it factive, then this scenario of idealizations could not happen. We know, however, that science is done in this way, and we can certainly say that a professor of any science in a university has a good understanding of their topic of work.

So how everything fits inside of mathematics, if they do? We gave a few hints on the answer where we discussed above the case of definitions in mathematics, that the same object can be defined in different axiomatizations, using different tools, names and approaches. Some will give a better explanation of the object in particular, but taking them all into consideration give us a better general explanation on the topic being studied. For example, in talking about cardinals, we might be able to better understand what is necessary for such a cardinal as ω_1 to exist inside a set theoretical axiomatization. We might understand how such cardinals relate to other cardinals, such as $\omega, \omega_2, \omega_n, \omega_\omega$ and even larger cardinals such as when we assume at least an inaccessible κ to exist. This happens because when dealing with different axiomatizations, we change the initial assumptions, our initial endorsements regarding one topic, and with that, there is a movement towards acceptance of different bodies of a coherent set of informations.

2.2 Bringing perspectivism and understanding together

Our last task of the chapter is to bring perspectivism and understanding together and show how they supposedly form a suitable background ready to be applied to set theoretical practice. One should be able to already see how these two views can be drawn together. On one hand we have perspectivism that allows for idealizations and models to be integrated in our Philosophy of Science. On the other hand, we have objectual understanding that says that to understand a topic is to grasp how the elements of an account relate to each other, what are the elements that support these claims and to be willing to use it to further one's cognitive ends.

The claim of this section is simply to say that perspectivism permits a defense that scientific practice, as it is done in practice, provides epistemic gain in terms of understanding. So what one gains when studying a topic is that they better understand the topic, they gather more information to better judge what information is significant, and also what is cognitively valuable between the information they have access to together with the new information being incorporated to the account they assume. So it seems that perspectivism has a good ally when incorporating understanding into its team.

We should then be able to incorporate understand into perspectivism in some way so that their relation is not a simple matter of speculation, but we have solid grounds upon which our view is built. As already mentioned, I am akin to Massimi's (2012a) approach to perspectivism, in saying that perspectivism is a way to ascertain how we gain scientific knowledge of nature. My thesis goes away from hers in saying that perspectivism, more specifically, perspectivism in set theory, in fact, is a way to explain how we get epistemic gain on the topic. However, the great shift occurs when I assume objectual understanding as epistemology's main concept instead of knowledge.

So why do I choose this approach? Why saying that perspectivism is not a view on philosophy of science that intends to support for truth, but for epistemic gain and how we gain understanding? Reasons vary from both the scientific and the philosophical side. And I shall try to explain the main motivations on both sides, without being exhaustive¹².

From the scientific side, the reason is quite simple, actually. I concur to the main idea of scientific perspectivism in saying that the accounts we have for science forget to deal with the actual scientific practice, whereas other accounts read too much into scientific practice disregarding the also existing scientific objectivity and commitment to saying something objective about the world. In this sense, perspectivism should be a middle ground between a realist position and an anti-realist position towards science, as endorsed in the works of (CHAKRAVARTTY, 2010; GIERE, 2006; MASSIMI, 2012a). Understanding enters the game right here to give us better odds at defending such an hypothesis. To draw her thesis, Massimi uses Sosa's epistemological reading on perspectivism (SOSA, 1991) what closely resembles what we are trying to argue for in this dissertation. For Sosa, the justification of our beliefs takes place within an epistemological perspective. Since our epistemology draws upon understanding, what we can say about our perspectivism is that, in the first place, our epistemic practice gives us information about models. Besides, we must be able, of course, to compare new gained information to the reliable information we already have, which is gained from, for example, years of reliable scientific investigation. Having done this, we know if the new information fits right into our body of information, furthering it in some aspect, or if it clashes it some other information. If there is a clash, then we should be able to tell if any of the significant truths were affected. If so, then we might want to rebuild that part of our body of

¹²Since this is the first major work I do on the topic, I will try to explore this non-exhaustive list. It might be interesting to give a more comprehensive and detailed list of reasons why one should adopt this kind of perspectivism, but this would result in a whole work by itself.

information on that topic. If not, then the new information is brought together with the pre-accepted ones and furthers our understanding of the topic.

This whole process is what I claim to be the advantage of putting perspectivism and understanding together. By saying that we gain understanding when assuming that science is perspectival, we actually have grounds upon which perspectivism can be built. There is no commitment, as I said before, to ontological theses, because this is not what perspectivism is dealing with. Instead, perspectivism is a way to explain how we gain understanding, particularly in science. This is the philosophical advantage I think we have when adopting perspectivism. Again, there might be some commitment to truth and to saying what things are true and which are not, as, for example, differentiating astronomy and astrology. The thing is that perspectivism would not delve into that debate. Rather, perspectivism should be able to deal with both bodies of information in a similar way. This does not mean that perspectivism is not *useful* or that it does not give us philosophical advantages. On the contrary, we can say that, on other grounds, astronomy claims better approximate phenomena that occurs in our world, so it is a better scientific theory. This aspect however, is not tied into perspectivism as its roots, but can be attached to it, in order to apply, for example, a perspectival view to science. When dealing with a science in particular, we might want to have extra criteria to distinguish between reliable and unreliable information. Remember, however, that recognizing significant information is one of the conditions that we took for saying that someone better understands a topic than others. Taking this condition together with another that says that understanding a topic comes together with grasping enough of the already existent information on that topic, we are somewhat reliably vaccinated against some sort of deviation in our scientific purposes.

Chapter 3

Perspectivism in Set Theory

Set theory is a well-established area of mathematics, both from the philosophical and the mathematical standpoints. Many of the great philosophers and mathematicians have worked in the area throughout the XX century and still nowadays many great thinkers are occupying themselves with the problems of the area¹. One of them is Joel David Hamkins (2012) whose contribution was to challenge the hegemony of the view that set theory happens inside an all-encompassing universe within which every set-theoretical assertion has a definite truth-value. Hamkins (2012) argues in his article for another conception of set theoretical construction, which determines a multiverse of sets, which accommodates for different concepts of sets each one of them instantiated in a universe, and these universes show particular truth-values for each set-theoretical assertion.

The shift towards a multiversist conception comes after many years of attempts to give a definite truth-value to independent propositions, such as **CH**. For many years², set-theorists have worked with tools that modify the initial assumptions, by adding new axioms to the **ZFC** axioms or building models which would allow for an answer to be achieved regarding such propositions. One of the early examples is Gödel's constructible universe **L** (GöDEL, 1938). Hamkins' shift comes not only from accepting different universes, but to indentifying "(...) set concepts with the models of set theory to which it gives rise"(HAMKINS, 2012, p.2) This view also maintains the ontological foundation of mathematics given by set theory, since when working in a determinate universe all of the

¹We can list as mathematicians David Hilbert, Gödel, Poincaré, Cohen, Russell and others. From the philosophical side we have people such as Quine, Frege, Putnam, Benacerraf, just to name a few.

²We can give a starting point of these kinds of works by saying that they began in the late 1960s, after the works of Paul Cohen (1963, 1964). However, these kinds of different formulations, axiomatizations and even a few set-theoretic models were already developed before the forcing technique was developed by Cohen. Examples of these can be seen already in Cantor (1932a, 1932b), Zermelo (2010a, 2010b, 2010c), Gödel (1938, 1947), Fraenkel (1922) and others.

objects inside of it are existent.

This new possibility of comprehending set theory through a multiversist view has not only opened up the debate about what kind of mathematical objects exist and how they exist, but also whether one can be a monist or a pluralist concerning universes of set theory. The ontological debates also have to include debates about what kind of objects universes themselves are and how they fit into the mathematics of set theory. One of the attempts to deal with objects different than sets is done by Steel (2014), who poses a two-sorted logic which admits of worlds besides sets³.

One question may be raised when dealing with the multiversist view proposed by Hamkins that shall motivate our change into perspectivism: if each universe instantiates a concept of set, is there a real unique truth value for each set theoretical statement? Answering this question might open space for us to bring the discussion of whether or not Hamkins' view can be seen as a form of perspectivism.

One can argue from a pluralist standpoint that the same sentence may have multiple truth values, depending on the model it is being evaluated. Although this might be true, it does not seem a satisfactory answer. Simply because the background upon which this claim lies is not very robust. It is clear that, if one believes that the sentences have a unique truth value, by having partial information about it from distinct universes initially independent one from another, then this commitment to a pluralist acceptance does not help our case. It simply says that in each of those universes the sentence has a determinate truth value, but it says nothing about the real value of the sentence.

Two problems are at stake here: first, deciding whether any set theoretical sentence has a unique real truth value; Second, there is the problem of, if not, then what justifies our belief that it does not, and why are we going after one thing that has no definite truth value. In order to answer those questions I am going to first define what perspectivism in set theory shall be understood as, then discuss what justifies having a perspectival view of set theory. Since our second question might raise objections reagarding a relativist position, I then turn to deal with that. Finally, we answer the question of whether Hamkins' Multiversism can be considered a form of Perspectivism.

³Steel's multiverse is a form of generic multiverse, which is developed with the intention of taking bias out of the language we define our set theory in, namely \mathcal{L}_{\in} . So, for Steel (2014) (also check (MADDY; MEADOWS, 2020)), the idea is to think of the language of set theory as being comprised of symbols for sets and symbols for universes, the worlds.

3.1 Perspectivism in set theory

My point in this work is to defend that *perspectivism* gives satisfactory motivations for both problems raised in the last paragraph of the previous section. For the first, the perspectivist philosopher shall say that set theoretic statements have determined truth values which are dependent on the axioms, model and notation we are working with. For the second, since we gave a negative answer to the first question, we will back up our answer by saying that what mathematicians gain when doing this is better *understanding* of set theory⁴⁵.

Given the above motivations and the developments from the previous chapters, we are ready to define perspectivism in set theory:

We call *perspectivism* in set theory the view that claims that the different possibilities of axiomatization of set theory reflect epistemological decisions, made by agents, who intend to reach a certain objective, i.e., to say something about a statement, and do so by picking from the large amount of possible frameworks epistemologically available to them those ones that permit the agent to say something about the statement they are working with. Moreover, we commit ourselves in saying that set theory is done only perspectivally.

Perspectivism is born in the following way: we take a fact, that there are many ways in which one can axiomatize what should be called "set theory"⁶, and try to justify it by epistemological means. There is also a compromise with saying that set theory can be done only in that way. One can ask if the objects being talked about by set theory change when we change our theory of sets. Given the realist account of perspectivism discussed in section 1.2.1, we would say that what changes are the aspects of such objects which we have epistemic access to.

For the perspectivist, axiomatizing a theory of sets in different ways is analogous to the activity of describing objects in a shelf in different ways. A perspective is chosen when one highlights certain aspects of the objects being described and that is

⁴Understanding here should be *objectual understading* as already discussed in chapter 2. For a thorough explanation of what are the differences between objectual and propositional understanding then I give here, see (ELGIN, 2017b; KVANVIG, 2003).

⁵The epistemological shift I follow here is is accordance, at least in its motivations, with the epistemological shift perpetrated by Massimi (2012a) as already discussed in chapter 2. There, she argues that perspectivism is a thesis on *how we form knowledge*. My personal shift goes from knowledge to understanding, but the general idea is the same.

 $^{^{6}}$ A cautious evaluation of what can be considered a *theory of sets* is done by Luca Incurvati (2020). There, Incurvati argues that for a theory to be a *theory of sets*, it is mandatory that it is extensional, satisfies comprehension, be consistent and derive enough results so regular mathematics can be done within it.

exactly what an axiomatization does. Not only that, but it is current in set theoretical practice to deduce analogous results starting from different vantage points: the difference might be the work needed to get there, but in general, if our theory is strong enough, the relevant common-between-axiomatizations results should be achievable.

Another relevant aspect of perspectivism is that it might be the case that one chooses an axiomatization that does not say anything about the objects they are interested in or the relation between those objects. It is not sufficient to undermine our view: what the perspectivist would say is that the perspective chosen is not a good one to deal with the problems at hand⁷, so another perspective should be chosen in its place. This is where the *purpose* comes in. Not only we are entitled to choose an axiomatization, but we do so in order to fulfill an objective. If that goal is not achievable by the initial conditions we picked, then we made a bad decision from the beginning, and we need a fitter model that highlights those aspects we want from the concept of sets we are working with.

Still, one can challenge the notion of "epistemological decision" I used in the definition. The 'decision' part should be clear enough: it amounts to the decision regarding which axioms, models or notation are assumed to hold or to be used. It is a decision because a person who works with set theory knows that, although it is not finitely axiomatizable⁸, it may be described finitely by using axiom schemas. The matter turns into what kind of objects and what behavior of those objects will the theory be able to describe: sets, classes, Urelemente, worlds, and so on. It can also be done both in an active way, by explicitly declaring which axioms, models, notation are assumed to hold by the mathematician, or passively, by simply assuming and using **ZFC**, for example, as most mathematicians not working with set theory usually do. In any case, this might not be the first part of the process of work, but it is an important one. Without a background theory, a determinate set of axioms well chosen, a result might not be achievable, simply by lack of certain tools. Hence, this step, despite sometimes implicitly contained in the mind of the mathematician, is done somewhere along the line of work and, moreover, dictates what is possible to be proven in certain cases. This adequacy of background theory might also be challenged when trying to deal with independent statements, or even when realizing that one of the previously axioms assumed might not be important for some result and can be achieved without it.

⁷I discussed in chapter 1, following van Fraassen (2008, p.49) that a representation is approximately true of something when it depicts that something with sufficient resemblance in the relevant aspects for the purposes at hand, so the case here fails to satisfy the last condition.

⁸We are working with first-order **ZFC** here, hence why this statement is written in the way. Higher order set theories, such as **NBG** or even **ZFC**₂, are examples of set theories that are finitely axiomatizable.

What about the 'epistemological' part of 'epistemological decision'? The short answer is that perspectivism says that any epistemic access to information about the objects that exist is perspectival, so any decision taken by the agent is made in the realm of epistemology, since it changes only what information they will be able to grasp from the object. It is thus a decision with a cognitive objective, not an ontological one. In this sense, perspectivism is an epistemological thesis. This is simply to say that we are not choosing what objects are there in reality, but we are simply deciding which aspects of those objects we are going to concentrate our attention and cognitive power on, in order to understand something about them⁹. A good analogy would be to think of an astronomer deciding to use either an X-ray or an infrared telescope to observe a nebula. The object is there and emits or reflects a vast spectrum of electromagnetic waves, but the astronomer must filter which aspects of the nebula are going to be relevant for their research and each of the telescopes used will give them different information about the same object¹⁰.

I also think that it is obvious that since there is a decision involved, someone or something must be the agent behind such a decision. An *agent* is not necessarily a sole human being, but might be a group of humans, a whole community of academics in a certain area, a renowned author in a subject, or a computer program. By saying that an agent makes such a decision should not be a weird side addition to the definition for one who sees it for the first time. However, since we are dealing with a subject about which platonists¹¹ have a lot to say, this could be a problem. Perspectivism in general heavily depends on the existence of agents as we discussed in section 1.2.2. Remember that one of my attempts is to apply the more general formulation of perspectivism to mathematics, with the appropriate modifications. But then, since agency is such an integral part of the thesis, should we try to abandon it, because it clashes with more traditional mathematical views? What is at stakes here is that for a full platonist, mathematical objects exists by themselves and should not depend in any way on agents' mind states or their

⁹I say 'understand' instead of 'know' because I take perspectivism to justify the thesis that mathematical practice grants understanding of mathematics, not necessarily knowledge. Again, this follows my previous discussion in chapter 2.

¹⁰The examples using astronomy are fun to give because they illustrate very well the case when we have reliable information that there is a such-and-such object somewhere in the night sky so that we can point measurement devices towards them. However, our tools are so limited that we must, at each time, choose which aspects of these objects we are going to have more information about. It is also very illustrative because the sum of a great number of studies using different measurement devices and techniques provide us with more understanding of the object. This is a great analogy for perspectivism. Giere (2006) and Ruphy (2016) use astronomy examples throughout their work, and I myself find them very useful.

¹¹A reference on platonism in mathematics is Balaguer's Platonism and Anti-platonism in Mathematics (BALAGUER, 1998).

comprehension or how they view the world. The existence of those objects is not a matter of how or if we are able to talk about them, and the same can be said about their truth values.

In putting agency together into the mix for the justification of set theoretical practice we are siding ourselves against platonists. One way out of this issue is to dodge the problem entirely by saying that since our approach is epistemological, there is no explicit commitment for or against the existence or not of independent mathematical entities. The only possible commitment a perspectivist might make is to say that the set theorist may *talk about* what presumably could be objects by means of partial models and axiomatizations. They can be fictional entities, for as long as the perspectivist cares, or they may exist in a realist sense, but the perspectivist, although buying the thesis that the models and axiomatizations give information about something, does not delve further into what kind of objects these formulations talk about. Neither of these assumptions may be attached to the decision made by the perspectivist nor to the compromise they assume when making a choice. What can, however, be said in this regard is that the perspectivist is interested in having a deeper *understanding* about the topic they are investigating. Statements, assumptions, perspectives, axiomatizations and notation may, in the long run, be wrong, but the strength contained within this practice comes from the fact that it generates better understanding of the topic and may help the community in better grasping new notions and new possibilities within the domain by, as goes the analogy with astronomy, extracting more information from objects.

There is still another problem perspectivist might face, regarding the *fulfilling* a purpose criterion. It seems that in assuming that a goal must be established before committing oneself to a perspective, the perspectivist cannot help but work from a *petitio* principii scenario. However there are many ways out of this accusation. First, we may weaken our objective in order no to determine what we want to say about an statement, but rather to say something about it. When discussing the status of **CH** regarding its truth value, Gödel (1947) does not commit himself with saying that **CH** is false, but that he believes **CH** to be false, given the evidence gathered so far in his studies. Perspectivally this should configure enough of a motivation to be a purpose: the set theorist wants to better understand what the absolute truth value of **CH** is. Hence, it is mandatory for their theory to be able to determine, in some way or another, what the truth value of **CH** is. Granting that this evaluation might be partial – and indeed time and set theoretical development has shown it to be – the sole motivation to say something about **CH**, and doing so by choosing an axiomatization that is strong enough, does not seem to fall under the *petitio principii* definition,

The second reason might be a bit more speculative from my part, however it meshes well with mathematical practice¹². The mathematician when working with a problem usually gathers information from definitions, axioms and already proven theorems, propositions and lemmas, that is, from all the reliable information they have at their disposal. When attacking a problem, it is natural to have built already some sort of *intuition*¹³, so a mathematician might never work fully blinded when trying to solve a problem. Joining this assumption together with the condition to establish a framework which says *something* about the statement the working set theorist is interested in results in a scenario where fulfilling an objective does not seem that problematic. Now, backed up by reliable understanding built from working with true statements under some assumptions, the mathematician has the way paved to commit themselves to a stronger thesis, such as " φ should be true under the assumptions I chose".

Having explained what the terms in the definition refer to and how they should be understood, I turn now to a question that was asked to me since I first began to develop the idea of a set theoretical perspectivism: what we gain by accepting this view that is not already described or contained in other traditional views of set theory? I already gave a hint of what the answer might be by saying that it provides *understanding*, but I think there is more to it than that.

3.2 What justifies perspectivism in set theory?

The idea of looking at set theory perspectivally was motivated by the seemingly incompatible models that can be achieved by now standard set theoretical techniques, such as forcing and large cardinal axioms. The original idea was to build a way of looking into set theory and accommodate for these scenarios and addressing them, not only as a fiction, or maybe some thing happening inside some hitherto unknown big universe that encompasses all of these constructions, but rather to address these happenings as *actual* happenings and to incorporate them into our studies.

With the early development of the work, I found a few other reasons we should look forward into adopting a perspectival approach to set theory – at least as its practice goes –, the main one being to give a *juistification* for the techniques being used beyond the

 $^{^{12}}$ This goes hand-in-hand with the motivations that Giere (2006) presents when defending that perspectivism is a realist position. I briefly discussed these motivations in section 1.2.1.

¹³I can make an argument for a kind of *trained* intuition, developed inside an area of knowledge. This should take a long detour from our purposes here, however, so it shall be developed in future works.

pragmatical reasons. One can better *understand* forcing, large cardinal constructions, and even generic multiverses by trying to give a reason why we use them and, even stronger, why we *should* use them as we do. Another reason I can think of at the moment is that by assuming a perspectival approach to set theory, we can shift our epistemological focus from knowledge to understanding, what makes the justifications above to fit perfectly inside this view. Hence, by selecting a primarily epistemological approach and by saying that the primary aspect of our epistemology is *understanding*, we can give an *explanation* of the use of the techniques mentioned above as well as why there may be incompatible models in set theory.

Before I move on, I would like to address the subject of *explanation*. As I briefly discussed in section 1.2.1 some accounts of understanding allow for understanding to be explanatory. Either by saying that understanding is the ability to explain why or something is as thus or so (GRIMM, 2006), or by saying that understanding is the ability to answer a range of *what-if* questions about an object that does not break the explanatory relation between explanandum and explanans (TELLER, 2020). What I will briefly say here is that, mathematical explanation makes things a little muddler because it seems that mathematics itself bears on explanation, at some level, to develop itself. The means with which explanation is achieved bears on proofs which might also be regarded, for example, by intuitionists, as a means to determine not only what is valid about a determinate object, but of asserting the existence of such object. It seems, though, that our account better fits a platonist account of mathematics, which asserts the existence of mind-independent mathematical objects. In this account, model constructions, axiomatizations, proofs, etc. would be means of gathering information about the objects of study¹⁴. Explanatory power would come, in the perspectivist view, by gathering as much information as possible about an object, by means of as many constructions that tell something about that object as possible. According to Saatsi (2020), explaining would be a matter of providing information about systematic patterns of counterfactual dependence. These explanatory counterfactuals correctly capture objective, mind-independent modal connections between explanandum and explanans, in such a way that it is extremely useful for cases where there is no causal explanation, such as in set theory.

Bringing the last idea of the above paragraph back to our discussion, the shift from knowledge to understanding justifies the idea of maximizing understanding, by trying

¹⁴W.W. Tait (1986) brings the discussion of proofs being or not the assertion of existence in mathematics. The article discusses the issue from both the platonist and the intuitionist stances and also the role proofs play in explaining mathematics.

to search for different models that give more information about a determinate object. So let us focus on **ZFC** for now. In building models, in proposing solutions, in trying to grasp information about sentences that are independent from **ZFC** we end up gathering more information on how **ZFC** behaves what are its limits and what it can and cannot express. This is analogous when we do that for an extension of **ZFC** with some large cardinal axiom. We explore its possibilities of giving us information about a particular sentence, a particular object or a group of objects contained within it. One good example in set theory that uses forcing is the Solovay Model which entails that all sets of reals are Lebesgue measurable. The construction of these models assumes the Axiom of Dependent Choice $(\mathbf{DC})^{15}$ and the existence of an inaccessible cardinal¹⁶, so it is a great example for us ¹⁷. As I said before, constructions like the Solovay Model might be faced as being working hypotheses that not necessarily reflect the actual status of the world – in this case, the set theoretical world, whatever that might refer to -. No commitment to the existence of the objects being mentioned is made at any point in the construction of the models and axiomatizations. However, were those objects existent, they would behave or have the properties as shown in the results studied.

Hence, it is possible to conjecture that perspectivism allows for model building, different axiomatizations, usage of set theoretical techniques to be treated as working hypotheses, without any commitment to their actual existence. When saying that a perspective is the result of a decision made by an agent, no ontological commitment is made at this point. Perspectivism in this sense is a view that might be attached to both realist and anti-realist views of mathematics. This might be a drawback to the position, since it cannot be used to decide the question of what kinds of mathematical objects are there. However, also recall that perspectivism can, and has shown to be, not only compatible with realism, but as a good fit for such a label (TELLER, 2020). I also claim that what is at stakes here is not so much what kind of objects there are, but the epistemological factor: how do we grasp these objects, concepts, idealizations, entities inside a fiction or what so ever they might be, and what information can we deduce from

 $x_{n+1}Rx_n$

¹⁵Given R a binary relation on a set $X \neq \emptyset$, and for every $x \in X$ there is a $y \in X$ such that yRx, then there is a sequence $x_1, \ldots, x_n, \cdots \in A$ such that

for all $n \in \mathbb{N}$. (JECH, 2006, p. 50)

¹⁶An infinite cardinal κ is *inaccessible* if $\kappa > \omega$, κ is regular and for all $\lambda < \kappa$, $2^{\lambda} < \kappa$.

¹⁷An intuitive explanation of this construction is presented in section 11.2 of (??). For the full technical presentation, see chapter 28 of (KANAMORI, 2000).

the ones we already have, should the (mathematical) world agree with our hypotheses ¹⁸. The drawback in this enterprise is that, in not committing ourselves to any ontological position from the start, we are able to defend both Universism and Multiversism using perspectivism. Again, this should not be a concern to the person strictly interested in drawing the epistemological argument, but might be a factor of discussion between philosophers of mathematics and mathematicians.

Despite not picking sides in the ontological discussion¹⁹, perspectivism is a very useful tool when the discussion is giving a reason why we have and are allowed to continue to use, build, and propose models of set theory that are incompatible one with another in the way set-theorists currently do. Even in a scenario where we have strict extensions of a model, perspectivism is sufficient to give us a *reason* why it is so. For example, let us take **ZFC** and compare it to **ZFC** + "there is an inaccessible cardinal". Up to κ , where κ is the first inaccessible cardinal, these system behave the same way. Without mentioning κ , everything happens exactly the same in both scenarios. So, what changes when we add the large cardinal axiom to **ZFC**? Ontologically, one might say that we add such an object. But we might go further and start conjecturing different hypotheses, how many of these objects there are? By saying that there is one, do I make a commitment towards a scenario where there is only one or more than one that are equivalent to κ ? How far can I commit myself to the existence of such objects? And how far my hierarchy goes, if I only commit to the existence of the lowest object in the hierarchy? It seems like these questions are looked upon ontologically by the working mathematician, but we can also analyze them from an epistemological stance.

A perspectivist can dodge all of these questions justifiably, without losing the motivation to search for them. We are not, in principle, committing ourselves to the existence of any kind of objects in particular, but we are proposing a working hypothesis. The hypothesis can be summed up as "What information can I deduce from this hypothesis should the world behave in the way I propose?". My argument goes in the direction that this should be enough motivation for a mathematician to pursue an answer to the questions posed, without committing themselves to any form of ontological thesis. This is so because the mathematician should gain a better *understanding* of set theory itself, so there is an intrinsic value for the all the work to be done. It is not as the mathematician should look further than a good puzzle to solve to be motivated to know its answer.

 $^{^{18}{\}rm The}$ counterfactual requirement seems a big part of Saatsi's presentation of perspectivism (SAATSI, 2020)

¹⁹I want to reinforce that not saying what kind of objects exist does not refrain us from our commitment to the existence of objects. We are committed, for now, with the existence of something besides universes.

Many of the mathematical theorems have no foreseeable applicable reasons which make mathematicians want to solve them and yet mathematicians all around the world are eager to give them satisfactory answers.

In defending and giving reasons for such an approach, it seems that perspectivism give us a new way of looking at set theory, that other approaches do not. In accepting that incompatible models may coexist, because they reflect different possible scenarios, with different possible working hypotheses, perspectivism grants the person working with them reasons to continue to operate as they do, but now backed up by a *reason* and not only for pragmatical interest. It could be a pragmatical factor as well, since when we choose our objectives to be achieved, we end up tracing a route on what aspects we consider relevant for reaching such an objective. The point in favor of perspectivism is that it *explains* why such pragmatical stance is achieved, by giving epistemological background to the working mathematician.

3.3 Perspectivism and Relativism

Speaking of drawbacks, one might raise the problem that perspectivism might be a view that brings with it some sort of relativism²⁰. Instead of trying to deny such accusations, I embrace them and try to argue for a position that accepts relativism, but also not any kind of relativism. What I argue for is that, despite relativism being a side effect of perspectivism, it is not harmful to our purposes. First, let us understand how perspectivism might lead to a relativism and then I will try to make an argument for the inoffensiveness of such relativism.

It should be clear that although perspectivism can be viewed as some sort of relativism, it is not an *anything goes* kind of relativism. What I mean by this is that it cannot be asserted that a certain set theoretical proposition is true or false simply by looking at different systems. In general an independent statement is true (or false) *under* some assumptions qua pertaining to that system in particular. Having changed the axioms or models we are using and even a different choice of notation we are working with, the criteria for asserting the truth or falsity of the statement in question is also altered.

²⁰One relativistic view of perspectivism is brought up by Graham Priest, in his recent preprint "Perspectivism on the Universe" (PRIEST, 2020), in which he proposes that truth values of assertions only make sense inside a perspective. Priest does not, however, goes much further than that in the draft much of what I discussed at length in this work is given in two pages by Priest.

The great argument for perspectivism in this regard is that it is exactly in this way that truth or falsity is approached by mathematicians when dealing with different models of set theory and when trying to give an answer to the Continuum problem. Usually, one says that **CH** is true or false under some assumptions, or they build models within which the truth value of **CH** can be ascertained. Since the statement is independent of **ZFC** and so are the assumptions that permit us to build the models which say something about **CH**, we end up in no man's land, where we may only ascertain so much about it, that is, that it is true *under those assumptions*.

Among the authors that mention relativism, we have Martin Kusch²¹ (2021). who agrees that perspectivism entails a form of relativism. For him, Giere's scientific perspectivism accepts dependence and plurality, and there is some doubt whether or not it accepts conversion. Regarding symmetry, Kusch deems Giere's perspectivism accepting only nonneutrality, that perspectives are impossibles to rank without reporting to some other set of standards, and locality, that perspectives are based on contingent, local sources of credibility. Another author that deems perspectivism entailing relativism is Chakravartty (2010), who says that the strongest of his formulations entails a form of conceptual relativism which is derived from a form of ontological relativism ²². This is so because the second of his formulations entails that perspectivism is the view that every knowledge is perspectival because that is all there is to know. So in his formulation, what the world offers to us is perspectival knowledge, that is how the world is, hence, a form of ontological relativism: what there is in the world depends on the observer's perspective. I think that, at most, perspectivism as I formulated entails a conceptual form of relativism, which says that what we can grasp of the world - and here, of the set theoretical world is relative to the perspectives we assume.

More than that, and following the argument I drew in the previous section, I think we advance the debate regarding set theory not only by accommodating for the actual mathematical practice, but we also improve upon it by actually *explaining* why it is done in this way and giving an epistemological cushion upon which our methods might rest. Perspectivism thus in fact give birth to some sort of relativism, but not a

²¹Kusch draws criteria that a theory has to satisfy in order to be considered a relativistic theory: dependence, a belief has an epistemic status relative to epistemic standards; plurality, there are more than one standards to each domain; conflict, epistemic verdicts based on different sets of standards may exclude one another; conversion, changing the set of standards is a leap-of-faith move; and symmetry, which itself is comprised of four other criteria, but in sum, says that sets of standards are equally valid, cannot be rankeable without recurring to another set of standards, there is no transference of evaluative terms between sets of standards and they are based on local, contingent causes of credibility.

 $^{^{22}}$ I discussed both positions and formulations in section 1.2.1.

pathological one. Despite this being a drawback to the position, this might be the best we can hope for in the scenario we find ourselves in and it seems like an improvement on what we already have, as far as explanations for set theoretical methods go.

3.4 Is Hamkins' Multiversism a form of Perspectivism?

So we have come this far to answer the question: Can Hamkins' position be a perspectivist one? After all, his position entails not only the existence of multiple universes, but that they are equally valid in their assertions and that each of them determines its own concept of set, and the truth values are valid inside those universes.

We could think about each of these universes as a *perspective*. It is on a par with our definition of perspectives that they act in this way. By picking some particular model construction, one ends up constructing one particular universe in Hamkins' view, hence determining a particular concept of set that is instantiated in this universe and set theoretical assertions will have some definite truth value in this universe. For Hamkins, this is all one can ascertain the existence of. That is, what we construct as set theorists are universes and each set theoretical assertion has a definite truth value in that universe. Problem is that we cannot ascertain whether or not there are other objects besides the universes in our ontology. Hence, any attempt to defend that Hamkins' view is perspectival in the way I defined it falls short, because a perspective is a perspective of *something*. We cannot ascertain what kind of *something* that is, but we can say that perspectives talk about these objects. And they give us information about those objects, through models and assignment of truth value according to the assumptions we made. It seems that if we take Hamkins' view to be perspectival, there is something missing, because no commitment to the existence of anything else than the universes is made. In this reading, universes would be innocuous, going only so far as saying that they determine set conceptions and truth values for set theoretical assertions. When we formulated models in scientific perspectivism, there was a commitment to the existence of something, of which the model is a representation.

We could, instead, try to go for an antirealist position of perspectives, that says that all we have are our models and that is all we can ascertain that exist. For scientific perspectivism, this seem too weak of a position to hold up against very contundent objections. Hence, perspectivism is, there, better off as a realist position. It strikes me as curious whether or not one can forge a strong antirealist perspectivism for set theory. As it stands, I believe that we are better off being realists. Not only it encompasses different views on existence of mathematical objects which are not the universes, such as sets, as allows for readings such as Steel's, who admits both sets and worlds. Moreover, it seems easier to assume that there are objects of which our models, axiomatizations talk about, so that our position commits itself *only* on the justification of mathematical practice, and does not delve into ontological debates.

Conclusion

I believe this text has a few purposes that are far away from setting in stone what should be the definite version of perspectivism in set theory and how should we comprehend its intricacies. Instead I believe this is a work that should push forward the discussion of what justifies set theoretical practice in the way it is done and the idea of perspectives inside mathematics in general. Perspectivists in science have come a long way in better defining the notion so I hope we, as philosophers of mathematics, can do that as well.

As a first work on the subject, however, I believe we accomplished a few things. First, it was possible to present perspectivism in science, by defining it as the view that claim that every human knowledge is perspectival. The implications of this thesis made us make a stance towards perspectivism being a realist position, by saying that what science works with, i.e, models, idealizations, measurement data and so on, although being partial and being chosen accordingly in order to fulfill a purpose, actually say something about the world. That what they say is partial is not a hindrance to the perspectivist who defends a realist position, as shown in section 1.2.1, because realist claims can be supported by perspectivism. Not only that, but we have authors (TELLER, 2020) that argue towards realism being a label that *better suits* perspectivism.

Having argued for this realist stance, we also came to the conclusion that perspectivism, although compatible with realism, should be understood as a view that is committed to giving an explanation of *how* we gain knowledge about the world (MASSIMI, 2012b). I retorted that, despite this way of arguing being very enticing, it does not serve our purposes when working within set theory. The problem is that, by assuming that we have *knowledge* of a determinate set theoretic assertion, we commit ourselves to that assertion being true or false. However, as we know, there are independent and undecidable propositions, whose truth value are determined only relative to some axiomatization or model, hence, the absolute notion of *knowledge* seems off here. I thus proposed that we changed our epistemic primary concept to *understanding*, following Elgin (2017b), which admits of non-factivity, so one is able to use the information gathered by their investigations in order to further one's cognitve ends.

This shift from knowledge to understanding seems to be the move that makes our initial presupposition stronger. By saying that adopting a perspectivist stance justifies mathematical practice seems to have more than one good point. First, it allows for a justification of set theoretical practice, not taking into account what kind of objects there are; second, it allows for the discussion to drift away from the universist vs multiversist debate not solving it directly, but giving a third option that gives the set theoretical practice amounts to their activity; third, it gives an *explanation* of what set theoretical practice amounts to without compromising any of the ontological positions regarding universes. Although we compromise ourselves with the existence of mathematical objects, neither we make a stance towards what kinds of objects there are, nor do we say whether universes really exist. Understanding allows for us to conjecture, use these formulations simply as hypotheses, by creating *what-if* scenarios, that even then are epistemically valuable to us. Since we cannot have full knowledge of the set theoretical world, we can conjecture, from our limited cognitive position, what that part of the set theoretical world would be should the things behave as we suppose them to.

Finally, I believe that my brief application of my ideas to Hamkins' Multiversism is a hint of what perspectivism might bring us: new ways of interpreting and evaluating ontological stances that have been formulated throughout the years. I hope that in the future I can expand these evaluations to both Woodin's (2011) and Steel's (2014) generic multiverses, and even try to deal with universism. As it stands, I rest my case for now.

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