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ARTICLE

Gravitational induction with Weber's force

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Abstract: According to Faraday's law of induction, when we change the current intensity in a primary electric circuit we can induce a current in a secondary circuit under appropriate conditions. An electric current means charges in motion. Microscopically we can express Faraday's law by saying that when we accelerate charges in the primary circuit, a force is exerted on charges of the secondary circuit, which can accelerate them. A similar effect also exists in gravity with accelerated masses, but of course with much less intensity. The phenomenon is called frame dragging and can be derived from general relativity theory. Here we present an alternative way to calculate such gravitational induction forces based on Weber's law that only involves simple mathematics and incorporates other fundamental concepts, such as Newton's third law, and Mach's principle as the origin of inertia. It therefore summarizes all low-velocity gravitationally relevant effects into a single equation.

Key words: electromagnetic induction, gravitational induction, relational mechanics.

Résumé : Selon la loi d'induction de Faraday, en changeant l'intensité du courant dans un circuit primaire, nous pouvons induire un courant dans un circuit secondaire sous des conditions appropriées. Un courant électrique signifie des charges en mouvement. Microscopiquement, nous pouvons exprimer la loi de Faraday en disant que suite à une accélération des charges dans le circuit primaire, s'exerce alors sur les charges du circuit secondaire une force capable de les accélérer. Un effet similaire existe aussi en gravité pour des masses accélérées, mais évidemment avec une intensité beaucoup plus faible. Le phénomène s'appelle l'effet Lense–Thirring et peut être obtenu de la relativité générale. Nous présentons ici une façon alternative de calculer de telles forces gravitationnelles induites basée sur la loi de Weber qui n'implique que des mathématiques simples et incorpore d'autres concepts fondamentaux comme la troisième loi de Newton et le principe de Mach comme origine de l'inertie. Nous résumons ainsi sous la forme d'une seule équation tous les effets de basse vitesse pertinents en gravitation. [Traduit par la Rédaction]

Mots-clés : induction électromagnétique, induction gravitationnelle, mécanique relationnelle.

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1. Introduction

Frame dragging (also called the Lense–Thirring effect) is a well-known phenomenon in general relativity. It affects, for example, the trajectory of satellites in polar orbit and generates precession forces on spinning gyroscopes. This effect has been experimentally verified most notably with the LAGEOS and Gravity-Probe B missions [1, 2]. General relativity involves advanced mathematics that is not easily accessible to undergraduate students or engineers, therefore scientists tried to derive a simple framework to calculate these effects, which is called gravitomagnetism [3–7]. Forward was one of the first scientists to derive a set of Maxwell-like equations [8] that allow calculation of frame-dragging effects similar to electromagnetic effects in the weak-field and low-speed limits of general relativity.

We will show that the same effects can be derived out of Weber's force law [9–11] for gravitation that includes all well-known mechanical forces (such as Newton's third law) and the so-called fictive forces (such as the centrifugal or Coriolis force). Weber's law also incorporates Mach's principle (the origin of inertia is due to gravitational interaction with the distant galaxies) and therefore summarizes gravitational phenomena in a single equation that is easily accessible to undergraduate students. Further, as we will show in this paper, Weber's force also includes the frame-dragging predictions of general relativity.

Weber's law, therefore, seems to be an excellent starting point to teach the concepts of mechanics and even higher-order gravitational effects with simple mathematics. As an example, we will show how frame dragging or gravitational induction works with Weber's law and calculate the difference in the orders of magnitude between electromagnetic and gravitational induction.

2. Electromagnetic induction

According to Faraday's law of induction (1831), the electromotive force (emf) induced in a closed circuit C is given by

$$\text{emf} = \oint_C \frac{\boldsymbol{F}}{q}\cdot d\boldsymbol{l} = -\frac{d\Phi_B}{dt} \tag{1}$$

where $\boldsymbol{F}$ represents the force acting on an electric charge q belonging to the circuit, $d\boldsymbol{l}$ is an oriented infinitesimal length pointing along the circuit at each point, $\Phi_B = \iint_A \boldsymbol{B}\cdot d\boldsymbol{a}$ is the flux of the magnetic field $\boldsymbol{B}$ through the loop of the circuit having a total area A, an infinitesimal area orthogonal to the loop in each point of its surface is represented by $d\boldsymbol{a}$ (with the directions of $d\boldsymbol{l}$ and $d\boldsymbol{a}$ connected by the right-hand rule), while t represents the time.

As a specific example we consider a spherical shell of radius R uniformly electrified with a total charge Q spinning around the z axis passing through the center of the sphere with an angular

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velocity $\boldsymbol{\Omega}$ relative to an inertial frame of reference S. The center O of the coordinate system will be considered the center of the shell. This situation produces a vector potential $\boldsymbol{A}$ and a magnetic field $\boldsymbol{B}$ at a point $\boldsymbol{r}$ inside the shell given by, respectively [12–14]:

$$\boldsymbol{A} = \frac{\mu_o Q}{12\pi R}\boldsymbol{\Omega} \times \boldsymbol{r} \tag{2}$$

and

$$\boldsymbol{B} = \frac{\mu_o Q \boldsymbol{\Omega}}{6\pi R} \tag{3}$$

where $\mu_o = 4\pi \times 10^{-7}$ kg m/(A^2s^2) is the permeability of free space.

It should be observed that the magnetic field inside this spherical shell is uniform, having the same direction and intensity at all internal points.

Consider now a metal ring of smaller radius r_o and larger radius r, with $r_o \ll r < R$, centered at O and located in the xy plane orthogonal to this magnetic field. The magnetic flux through the area $A = \pi r^2$ of the ring is given by $\Phi_B = \pi r^2 B = \mu_o Q r^2 \Omega / 6R$. According to (1), the emf induced in the ring is given by

$$\text{emf} = -\pi r^2 \frac{dB}{dt} = -\frac{\mu_o Q r^2}{6R}\frac{d\Omega}{dt} \tag{4}$$

According to Ohm's law, neglecting the self-inductance of the ring and considering the time interval in which the angular acceleration $d\Omega/dt$ remains constant, the current I induced in the ring, having a resistance R_o, is given by $I = \text{emf}/R_o$.

A similar effect is known to exist in general relativity, although the calculations leading to this prediction are reasonably complex [8, 15, 16]. In this work we present an alternative formulation of gravitational theory that also predicts a similar effect, namely, Weber's force.

3. Gravitational induction with Weber's force

In the sequence of the paper we consider how it may be possible to produce a similar effect with gravitation. We will be dealing only with neutral bodies here, so that the electromagnetic forces can be neglected.

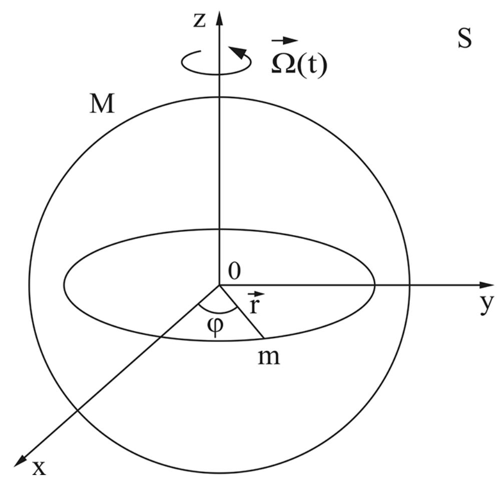
Consider a spherical shell of mass M and radius R that is spinning around an axis passing through the center of the shell with an angular velocity $\boldsymbol{\Omega}$ relative to an inertial frame of reference, S. An internal test body of mass m is located at a position vector $\boldsymbol{r}$ relative to the center O of the shell, moving with velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ relative to S. According to Weber's law applied to gravitation, the force exerted by this spinning spherical shell and acting on the test body is given by [9, 11]

$$\boldsymbol{F} = -\frac{2GmM}{Rc^2}\left[\boldsymbol{a} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}) + 2\boldsymbol{v} \times \boldsymbol{\Omega} + \boldsymbol{r} \times \frac{d\boldsymbol{\Omega}}{dt}\right] \tag{5}$$

where $G = 6.67 \times 10^{-11}$ m^3/kg s^2 is the constant of universal gravitation and $c = 2.998 \times 10^8$ m/s is the velocity of light in vacuum. The second term in the square bracket yields in non-inertial frames of reference the centrifugal force, while the third term yields the Coriolis' force, yielding a mathematical implementation of Mach's principle [9–11].

In what follows we will consider that the test body of mass m belongs to a ring of radius r centered at the center O of the spherical shell and located in the xy plane, orthogonal to the direction z of the rotation of the spherical shell, $\boldsymbol{\Omega} = |\boldsymbol{\Omega}|\hat{z} = \Omega\hat{z}$. The ring will be considered initially at rest and the position vector of one of its elements can be represented in polar coordinates by (r, φ), where φ is the azimuthal angle, Fig. 1.

Fig. 1. Spherical shell spinning around the z axis with an internal ring at the equatorial plane.



We will be interested only in the azimuthal motion of the ring and how it is influenced by a variable rate of rotation of the surrounding spherical shell. Therefore the position, velocity, and azimuthal acceleration of the test particle can be represented by $\boldsymbol{r} = r\hat{r}$, $\boldsymbol{v} = r\dot{\varphi}\hat{\varphi}$, and $\boldsymbol{a} = r\ddot{\varphi}\hat{\varphi}$, respectively, where $\hat{\varphi}$ is the unit azimuthal vector at the location of the particle. The component of the force $\boldsymbol{F}$ proportional to $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})$ points along the radial $\hat{r}$ direction, as does the component proportional to $2\boldsymbol{v} \times \boldsymbol{\Omega}$, so they will not be considered in the sequence.

Combining (5) with Newton's second law of motion, $\boldsymbol{F} = m\boldsymbol{a}$, yields the following equation describing the motion of the test particle in the azimuthal direction:

$$-m_W r\ddot{\varphi} + m_W r\frac{d\Omega}{dt} = mr\ddot{\varphi} \tag{6}$$

where $m_W = 2GmM/Rc^2$ is what we call Weber's mass for this geometry.

By considering a spherical shell with $R = 1$ m and $M = 100$ kg, it is easy to see that $m \gg m_W$. We are then led to

$$\ddot{\varphi} = \frac{m_W}{m}\frac{d\Omega}{dt} = \frac{2GM}{Rc^2}\frac{d\Omega}{dt} \tag{7}$$

This equation represents gravitational induction. That is, whenever the rate of rotation of the surrounding spherical shell changes as a function of time, there will be an azimuthal force acting on each element of the internal ring. If it is free to rotate around its axis and friction can be neglected, the ring will undergo an induced azimuthal acceleration given by (7).

An interesting question that may be asked related to (7) is the following gedanken experiment: Is the angular acceleration field propagating inside the shell core? The answer to this question, according to Weber's law applied to gravitation, would require calculations that are beyond the scope of this paper. In any event, it is relevant to remember here the deduction by Kirchhoff and Weber, in the period 1849–1864, of the telegraphy equation based on Weber's electrodynamics [17–24]. When the resistance of the wire was negligible, the telegraphy equation simplifies to the wave equation. The wave equation obtained by Kirchhoff and Weber then predicted the propagation of signals along the wire with light velocity. As Kirchhoff put it [18 p. 210, 21 p. 406], the velocity of propagation of an electric wave "is independent of the cross-section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41950 German miles in a second, hence very nearly equal to the velocity of light in vacuo." These works of Weber and Kirchhoff, based on Weber's electrodynamics, were obtained prior to Maxwell's electromagnetic theory of light in the period 1864–1873 [25–27]. One of the authors (AKTA) discussed these Weber and Kirchhoff papers in more detail in other works [28–34]. The calculations of Weber and Kirchhoff utilized the interaction of two types of charges, namely, positive and negative ones. As we have only one kind of mass, it is not straightforward to extend their calculations to the case of purely gravitational interactions, as is the case considered in the present paper. But this is an interesting topic to be analyzed in the future.

To know if the induced angular accelerations predicted by (7) would be relevant to the case of Earth internal geophysics, the Earth should not be considered as a rigid body. After all, in that case there would be no effects, according to Weber's law applied to gravitation, which may be due to a relative acceleration between interacting masses. In any event, it would be valuable to highlight this possibility in the case of differential rotations of the Earth shell (as the rotating sphere) and the Earth core (as the probing mass) rotating at different angular velocities relative to an inertial frame of reference. Additional research may be relevant in this case.

The effect described in this paper represented by (7) is analogous to electromagnetic induction with three main differences.

- The first one is that the tangential acceleration of the ring will happen in the same direction as the tangential acceleration of the surrounding shell. In electromagnetism, on the other hand, the induced current will always flow in a direction such that the flux it produces tends to cancel the change of the external flux, according to Lenz's law.
- The second difference is that the induced angular acceleration $\ddot{\varphi}$ will always have the same value for all test masses, no matter the value of m. This is similar to the linear acceleration of free fall in the gravitational field of the Earth. The free fall acceleration does not depend on the mass of the test body. This effect also demonstrates the validity of the equivalence principle.
- The third difference happens in the order of magnitude of the effect. Electromagnetic induction has been known to exist since 1831 with Faraday's experiments. It has a reasonably strong effect that is easily detected in the laboratory. By calculating the dimensionless magnitude $2GM/Rc^2$ of (7), on the other hand, we obtain with $R = 1$ m and $M = 100$ kg the extremely small value of $2GM/Rc^2 \approx 1.5 \times 10^{-25}$. Therefore, by changing the rate of rotation of the surrounding spherical shell such that $d\Omega/dt$ = 1 rad/s^2 we obtain $\ddot{\varphi} = 1.5 \times 10^{-25}$ rad/s^2. Such an extremely small angular acceleration cannot be measured in the laboratory. Moreover, it would not even be produced in practice, due to the inevitable presence of friction. In any event, as stressed before, this effect may be relevant in the case of the differential rotation of the Earth. If the spinning spherical shell is considered as the Earth's mantle, then $2GM/Rc^2 \approx \times 10^{-11}$. The test body may be considered the Earth's core. Therefore, this effect may be relevant for the Earth's internal geophysics, as stated before.

Weber's law complies with the principle of action and reaction. Therefore, the ring will exert an opposite torque in the spherical shell, which will try to decrease its changing rate of rotation. This effect is analogous to the mutual inductance of classical electromagnetic theory.

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