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Total hadronic cross section and the elastic slope: An almost model-independent connection

D.A. Fagundes *, M.J. Menon

Universidade Estadual de Campinas – UNICAMP, Instituto de Física Gleb Wataghin, 13083-859 Campinas, SP, Brazil

Abstract

An almost model-independent parametrization for the ratio of the total cross section to the elastic slope, as function of the center of mass energy, is introduced. The analytical result is based on the approximate relation of this quantity with the ratio R of the elastic to total cross section and empirical fits to the R data from proton–proton scattering above 10 GeV, under the conditions of asymptotic unitarity and the black-disk limit. This parametrization may be useful in studies of extensive air showers and the determination of the proton–proton total cross section from proton–air production cross section in cosmic-ray experiments. © 2012 Elsevier B.V. All rights reserved.

1. Introduction

In addition to their intrinsic astrophysical importance, cosmic-ray experiments constitute a valuable tool for the investigation of particle and nuclear physics at energies far beyond those obtained in accelerator machines. However, at the highest energies a direct approach to particles properties and their interactions is difficult due to the decreasing flux with the increase of the energy. Presently, an indirect method, based on extensive air shower (EAS) studies, is the usual way to treat the subject [1]. In these events, the distribution of the first interaction point allows, in principle, the determination of the proton–air production cross section [2] and in a second step, the estimation of the most fundamental quantity in hadronic interactions: the proton–proton total cross section [3,4].

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* Corresponding author.

E-mail addresses: fagundes@ifi.unicamp.br (D.A. Fagundes), menon@ifi.unicamp.br (M.J. Menon).

However, in practice, the interpretation of the EAS development depends on extrapolations from phenomenological models that have been tested only in the accelerator energy region, resulting therefore in systematic theoretical uncertainties. That represents a crucial point because different models with distinct physical pictures and at the same time consistent with the experimental data up to c.m. energies ~ 2 TeV, present, in general, contrasting extrapolations at the cosmic-ray region (above ~ 50 TeV). Therefore the theoretical uncertainties (bands) involved are relatively large and very difficult to be estimated. As a particular consequence, the estimations of the proton–proton total cross section from different cosmic-ray experiments and analyses are characterized by large error bars and even discrepant results, as discussed in [5] and references therein.

EAS studies are essentially based on Glauber's multiple diffraction theory [6,7] and its extensions and/or corrections, including Gribov–Regge screening effects and other ingredients. In this context the evaluation of the hadron–nucleus elastic and quasi-elastic cross sections, which contribute to the nucleon–air production cross section, depends on the ratio of two physical quantities, the total cross section σ_{tot} and the elastic slope B. That ratio just represents one of the main sources of uncertainties in model extrapolations.

In this work an almost model-independent parametrization for the ratio σ_{tot}/B is proposed, which may avoid uncertainties from models tested only at lower energies. The parametrization is based on the approximate, but experimentally justified, connection of σ_{tot}/B with the ratio R of elastic to total cross section. By means of unitarity arguments and empirical fits to R data from pp scattering above 10 GeV and up to 7 TeV, an analytical parametrization for $R(s)$ is introduced and then extended to the ratio σ_{tot}/B.

The manuscript is organized as follows. In Section 2 we introduce the physical quantities of interest with explicit reference to the importance of the ratio σ_{tot}/B in cosmic-ray studies. In Section 3 we recall some formal (rigorous) results from axiomatic QFT and how some inequalities can be connected with experimental results. In Section 4 we introduce the analytical parametrization and present the fit results. The conclusions and some final remarks are the contents of Section 5.

2. Physical quantities and the Glauber formalism

Let us first recall the main physical quantities related to high-energy elastic hadron scattering, defining the notation and normalizations [8]. Neglecting spin effects and denoting $F(s, t)$ the invariant elastic amplitude in terms of the Mandelstam variables, s and t, the differential and total cross sections at high energies ($s \gg 1$ GeV^2) are expressed, respectively, by

$$\frac{d\sigma}{dt}(s, t) = \frac{16\pi}{s^2} |F(s, t)|^2, \tag{1}$$

and

$$\sigma_{tot}(s) = \frac{16\pi}{s} \, \text{Im}\, F(s, t = 0) \quad \text{(Optical Theorem)}. \tag{2}$$

The parameter ρ, the ratio between the real and imaginary parts of the forward amplitude, is given by

$$\rho(s) = \frac{\text{Re}\, F(s, t = 0)}{\text{Im}\, F(s, t = 0)}, \tag{3}$$

and the slope of the elastic differential cross section in the forward direction is defined as

$$B(s, t = 0) = \left[\frac{d}{dt} \left(\ln \frac{d\sigma}{dt} \right) \right]_{t=0}. \tag{4}$$

From (1) to (3), the optical point is expressed by

$$\frac{d\sigma}{dt}\bigg|_{t=0} = \frac{\sigma_{tot}^2 (1 + \rho^2)}{16\pi}. \tag{5}$$

The integrated elastic cross section reads

$$\sigma_{el}(s) = \int_{t_0}^{0} \frac{d\sigma}{dt}(s, t)\, dt, \tag{6}$$

where t_0 defines the physical (kinematic) region and, from unitarity, the inelastic cross section is obtained:

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s). \tag{7}$$

For our purposes let us recall two formulas in the Glauber formalism that play a central role in EAS studies [1]. The first one is the expression for the sum of the elastic and quasi-elastic cross section for hadron–nucleus (hA) scattering,

$$\sigma_{el}^{hA}(s) + \sigma_{qel}^{hA}(s) = \int d^2b \left| 1 - \prod_{j=1}^{A} [1 - a_j(s, \vec{b} - \vec{b}_j)] \right|^2 \left[\prod_{j=1}^{A} \tau(\vec{r}_j)\, d^3 r_j \right], \tag{8}$$

where $\vec{r}_j$ and $\vec{b}_j$ are the coordinate and impact parameter of the individual nucleons, $\tau(\vec{r}_j)$ the single nucleon density, $\vec{b}$ the impact parameter of the cosmic-ray hadron and $a_j(s, \vec{b} - \vec{b}_j)$ the nucleon–nucleon impact parameter amplitude (profile function). In addition to possible configurations for the nucleus, the profile function constitutes the main ingredient for the connection between hadron–hadron and hadron–nucleus scattering. Typically this profile is parametrized by [1]

$$a_j(s, \vec{b}_j) = \frac{[1 + \rho(s)]}{4\pi} \frac{\sigma_{tot}(s)}{B(s)} e^{-\vec{b}_j^2 / [2B(s)]}, \tag{9}$$

where ρ, σ_{tot} and B are the quantities defined above, demanding inputs from models to complete the connection. As clearly illustrated by Ulrich et al. [1], the uncertainty bands for these three quantities resulting from high-energy extrapolations based on representative phenomenological models, are larger than the range covered by all available MC interaction model results, as QGSJET01c, EPOS1.61, SIBYLL2.1 and QGSJETII.3, leading the authors to the conclusion that presently, "the extrapolation of hadronic cross sections to cosmic-ray energies might be underestimated" [1].

In what concerns the above three fundamental quantities, we recall that forward amplitude analyses connect $\sigma_{tot}(s)$ and $\rho(s)$ through dispersion relations. Detailed tests on different parametrizations have been developed by the COMPETE Collaboration, with the selection of the highest rank result [9,10], which also appears in the Review of Particle Physics by the Particle Data Group [11]. Recent results by the TOTEM Collaboration on σ_{tot} at 7 TeV [12] and the expected estimation of ρ at this energy, will certainly shed light on novel analytical parametrizations and therefore more reliable extrapolations with less model dependency.

However, that is not the case for the elastic slope $B(s)$ since the available data from pp and $\bar{p}p$ elastic scattering can be extrapolated in a very large band of possibilities and moreover, any

result is strongly model dependent. As a consequence, from Eq. (9), despite the uncertainty in the effective radius of the nucleon–nucleon amplitude, any extrapolation is strongly dependent on the ratio

$$\frac{\sigma_{tot}}{B}(s), \tag{10}$$

namely the unknown correlation between σ_{tot} and B in terms of energy. Although some analytical connections have already been investigated from fits to the experimental data [13], the statistical and systematic errors in both quantities and the model dependencies involved put limits on these results.

Based on the above comments, we understand that even under some reasonable approximate conditions, an almost model-independent parametrization for the above ratio may reduce the uncertainty band in the extrapolations from accelerator to cosmic-ray-energy regions. That is the point we are interested in here.

3. Formal and experimental results

In this section we first recall some rigorous (formal) results related to $\sigma_{tot}(s)$ and $B(s)$ and their connections with the experimental data presently available. Based on these considerations in the next section we introduce our proposed parametrization and present the results.

3.1. Rigorous results

General principles and high-energy theorems have always been a fruitful source of model-independent results for physical quantities in the asymptotic regime [14]. In this context, two well-know inequalities have been derived for the total cross section and the elastic slope. The first one is the Froissart–Lukaszuk–Martin upper bound, stating that asymptotically ($s \to \infty$)

$$\sigma_{tot}(s) \leqslant \frac{\pi}{m_\pi^2} \ln^2 \frac{s}{s_0}, \tag{11}$$

for some s_0 [15–17]. The second one, playing here an important role, is the lower bound of MacDowell and Martin, obtained from unitarity together with properties of the Legendre polynomials and involving forward quantities [18],

$$2\left[\frac{d}{dt} \ln \operatorname{Im} F(s,t)\right]_{t=0} \geqslant \frac{1}{18\pi} \frac{\sigma_{tot}^2(s)}{\sigma_{el}(s)}. \tag{12}$$

From the definition of the forward slope, Eq. (4), together with Eq. (1) and under the assumption

$$\operatorname{Re} F(s, t=0) \ll \operatorname{Im} F(s, t=0),$$

it follows that

$$\frac{d}{dt} \ln \operatorname{Im} F(s,t)\bigg|_{t=0} \approx \frac{1}{2} B$$

and from (12) an upper bound is obtained for our ratio of interest,

$$\frac{\sigma_{tot}(s)}{B(s)} \leqslant 18\pi \frac{\sigma_{el}(s)}{\sigma_{tot}(s)}. \tag{13}$$

Although rigorous, relations involving inequalities have a limited practical applicability, except for bounds imposed on the construction of phenomenological models. To go further in the

search for almost empirical or model-independent results, experimental data and formal inequalities must be checked, as follows.

3.2. Experimental results

The highest energies reached in accelerator machines for particle and antiparticles reactions concern pp and $\bar{p}p$ scattering, covering the region up to ~ 2 TeV ($\bar{p}p$) and, presently, up to 7 TeV (pp). These data indicate that at the highest energies

$$\rho(s) \lesssim 0.14,$$

which means that the above assumption, $\rho \ll 1$, constitutes a considerable approximation. On the other hand, at the optical point, Eq. (5), an assumption like

$$1 + \rho^2 \approx 1 \tag{14}$$

certainly represents a reasonable approximation. We shall return to this point in what follows. It may be interesting to note that these information allow us to derive bound (13) from (12) under different assumptions, as shown in Appendix A.

Concerning the differential cross section, experimental data indicate a sharp forward peak, followed by a dip-bump or dip-shoulder structure above ~ 0.5 GeV^2 (Tevatron, LHC). Typically, these structures are located more than 5 decades below the optical point, Eq. (5). These experimental facts are important in the determination of the integrated elastic cross section, since in this case the differential cross section can effectively be represented by an exponential fall off, simulated by a model-independent parametrization [12],

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dq^2}\bigg|_{q^2=0} e^{Bt}, \tag{15}$$

with B the (constant) forward slope. In that case, with the reasonable approximation (14) at the optical point (5) and assuming $t_0 \to -\infty$ in Eq. (6), the integrated elastic cross section reads

$$\sigma_{el}(s) = \frac{1}{B(s)} \frac{\sigma_{tot}^2(s)}{16\pi}$$

and therefore,

$$\frac{\sigma_{tot}(s)}{B(s)} = 16\pi \frac{\sigma_{el}(s)}{\sigma_{tot}(s)}, \tag{16}$$

which is very close to the approximate bound (13). However, the main ingredient in this result is the possibility to investigate the behavior of $\sigma_{tot}(s)/B(s)$ from formal and experimental information on the ratio $\sigma_{el}(s)/\sigma_{tot}(s)$, as discussed in what follows.

4. Analytical parametrization and fit results

In Fig. 1 we display the experimental information presently available on the ratio σ_{el}/σ_{tot} from pp scattering above 10 GeV [11], including the recent TOTEM result at 7 TeV [12] (highest energy reached in accelerators). From a strictly empirical point of view, the data in the linear-log scale may suggest a parabolic parametrization in terms of $\ln s$, with *positive* curvature. However, unitarity demands an *obvious bound*,
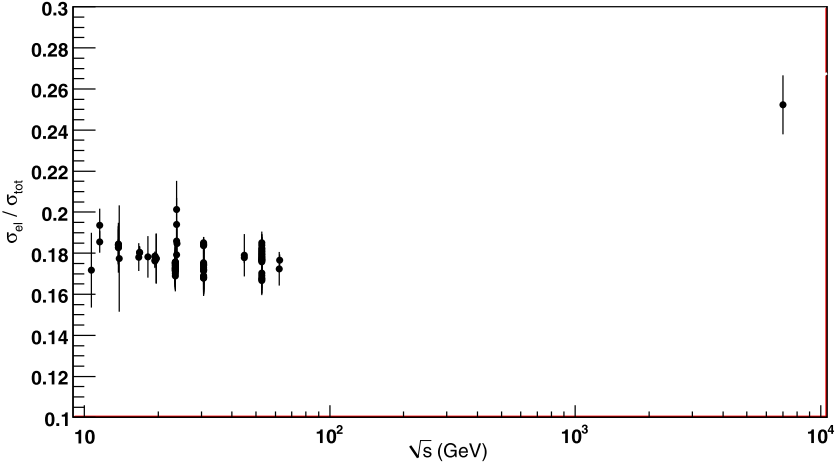
Fig. 1. Experimental data on the ratio between the elastic and total cross sections from pp scattering above 10 GeV [11,12].

$$\frac{\sigma_{el}}{\sigma_{tot}} \leqslant 1.$$

In addition, naive models, as for a Gaussian profile or the gray-disk, predict [19] $\sigma_{el}/\sigma_{tot} = C/2$, where C is a constant (absorption coefficient) and in the black-disk limit $C = 1$. These results indicate a constant asymptotic limit for the ratio

$$\lim_{s \to \infty} \frac{\sigma_{el}}{\sigma_{tot}} = A \quad \text{(constant)}$$

and therefore a *change of sign* in the curvature, at some finite value of the energy, is expected. Moreover, since from Fig. 1 the data at low energies indicate $\sigma_{el}/\sigma_{tot} \sim \text{constant} \approx 0.18$, a general behavior related to a logistic or sigmoid function can be conjectured, at least above 10 GeV (the high-energy region). Several functions with this property can be considered. However, for tests on goodness of fit some quantitative information on the value of the asymptotic limit A is necessary.

Looking for a wide range of possibilities in the phenomenological context, we shall consider two contrasting pictures that have been discussed in the literature. On the one hand, the amplitude analysis by Block and Halzen favour the asymptotic black-disk [20], which is also predicted, for example, in the models by Chou and Yang [21] and by Bourrely, Soffer and Wu [22]. On the other hand, the U-matrix unitarization scheme by Troshin and Tyurin predicts $\sigma_{tot}(s) \sim \sigma_{el}(s) \sim \ln^2 s$ and $\sigma_{inel}(s) \sim \ln s$ [23], which is beyond the black-disk limit and in agreement with the above obvious unitarity bound. Therefore these two contrasting pictures suggest

$$A = \frac{1}{2} \quad \text{(black-disk limit)} \quad \text{and} \quad \frac{1}{2} < A \leqslant 1 \quad \text{(beyond the black-disk limit)}.$$

Although, in principle, it might be possible to explore all the real interval for A beyond the black-disk, we consider here only its maximum value. As we shall show, that is adequate and sufficient for our purpose to infer wider uncertainty bands from the above mentioned phenomenological context.
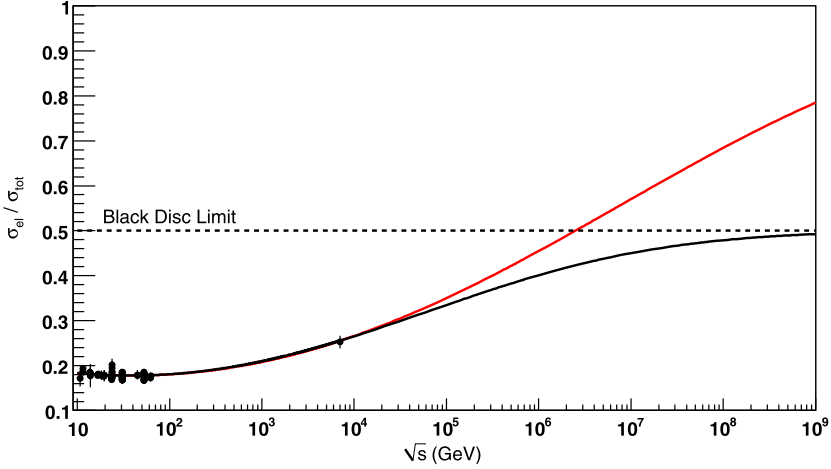
Fig. 2. Ratio between the elastic and total cross section and fit results through parametrization (17), including uncertainty from error propagation, for $A = 1$ (upper curve) and $A = 1/2$ (lower curve).

Table 1
Fit results with parametrization (17) for the ratio σ_{el}/σ_{tot} from pp scattering above 10 GeV. In both cases the degrees of freedom (*DOF*) are 87.

	$A = 1/2$	$A = 1$
$\gamma_1 (\times 10^{-1})$	4.66 ± 0.18	2.204 ± 0.078
$\gamma_2 (\times 10^{-2})$	-2.59 ± 0.49	-1.11 ± 0.20
$\gamma_3 (\times 10^{-3})$	1.77 ± 0.33	0.76 ± 0.13
χ^2/DOF	1.167	1.168

Based on these considerations and inspired in different physical phenomena, we have tested several functional forms to fit the σ_{el}/σ_{tot} data. The best statistical result has been obtained with the following novel model-independent parametrization:

$$\frac{\sigma_{el}}{\sigma_{tot}}(s) = A \tanh\left(\gamma_1 + \gamma_2 \ln s + \gamma_3 \ln^2 s\right), \tag{17}$$

where γ_i, $i = 1, 2, 3$, are free fit parameters and A represents the asymptotic limit, for which we consider only the two extreme cases $A = 1/2$ and $A = 1$.

The data reductions have been performed with the objects of the class TMinuit of ROOT Framework [24]. We have adopted a Confidence Level of $\approx 68\%$ (one standard deviation), which means that the projection of the χ^2 distribution in $(N + 1)$-dimensional space ($N =$ number of free fit parameters) contains 68% of probability [25]. The fit results for $A = 1/2$ and $A = 1$ are displayed in Fig. 2 and Table 1 together with the statistical information. In both cases, the error propagation from the fit parameters has been evaluated and are also displayed in the figure; the bands however are indistinguishable.

From the approximate result (16), the ratio σ_{tot}/B can be predicted as function of the energy and in an almost model-independent way. The results together with the experimental data [11,12,26] are displayed in Fig. 3 and show that, in fact, Eq. (16) is very close to the approxi-
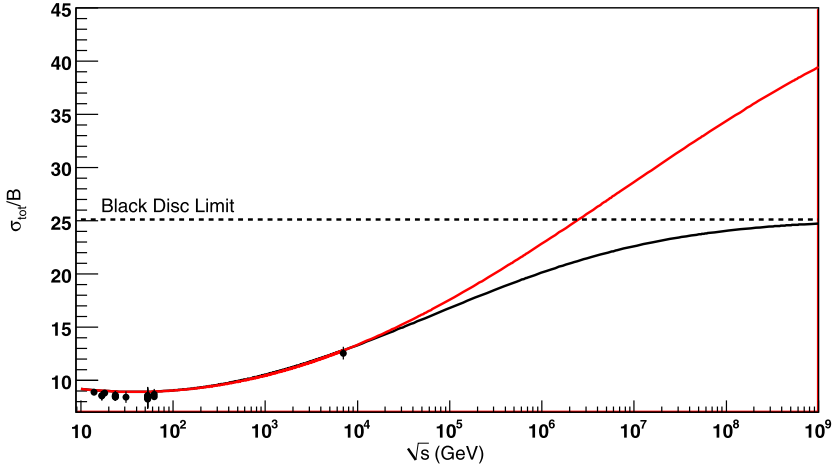
Fig. 3. Experimental data on the ratio between the total cross section and the elastic slope [11,12,26] and predictions from Eqs. (16)–(17), including uncertainty from error propagation, for $A = 1$ (upper curve) and $A = 1/2$ (lower curve).

Table 2
Predictions from Eqs. (16)–(17) for the ratio σ_{tot}/B at the LHC energy region and the TOTEM result at 7 TeV [12].

$\sqrt{s}$	$A = 1/2$	$A = 1$	TOTEM
7.0 TeV	12.827 ± 0.047	12.821 ± 0.024	12.56 ± 0.59
14 TeV	13.811 ± 0.068	13.903 ± 0.033	–

mate bound (13). Up to our knowledge, the only rigorous result indicating a constant asymptotic value for this ratio appears in the recent formal analysis by Azimov, on boundary values for the physical cross section and slope [27]. The numerical predictions with uncertainties for the ratio σ_{tot}/B at the LHC energy region are displayed in Table 2, together with the experimental value at 7 TeV.

5. Conclusions and final remarks

Based on unitarity arguments and fits to the experimental data on the ratio $R = \sigma_{el}/\sigma_{tot}$ from pp scattering above 10 GeV, a novel empirical parametrization for $R(s)$ has been introduced, Eq. (17). The approximate connection between this quantity and the ratio σ_{tot}/B, Eq. (16), allows us to infer the corresponding energy dependence for this ratio in an almost model-independent way. All the results are in agreement with rigorous inequalities derived from the axiomatic formulation.

Although depending on the unknown asymptotic limit represented by the constant A, the results here presented lead to, at least, four main conclusions:

1. If the black-disk represents a reliable physical limit [20], its saturation is very far from presently available energies: $\sqrt{s} \gtrsim 10^9$ GeV, from Fig. 2;
2. If the Froissart–Lukaszuk–Martin bound is saturated then in this region $B(s) \sim \ln^2 s$;
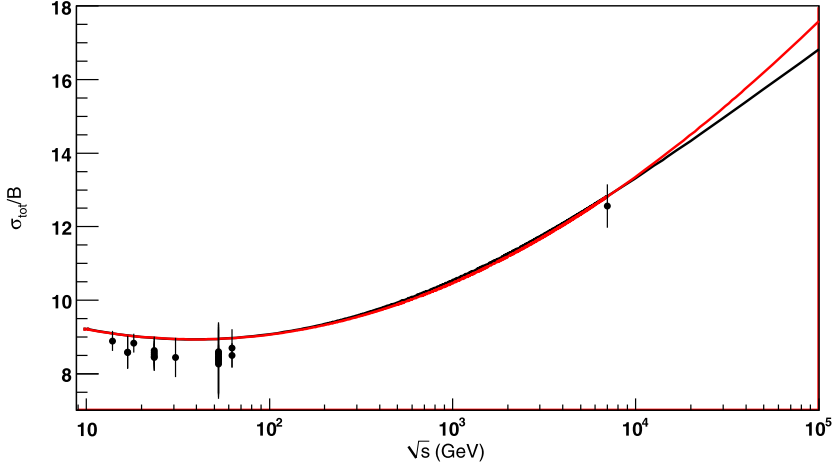
Fig. 4. Detail of Fig. 3 up to the Auger energy region.

3. Either for $A = 1/2$ or $A = 1$, the uncertainty bands evaluated by error propagation from the fit parameters in Eq. (17) are negligible: upper, central and lower curves in Figs. 2, 3 and 4 overlap;

4. Even with $A = 1/2$ and $A = 1$ as lower and upper bounds, extrapolation of the ratio σ_{tot}/B to the Auger energy region, $\sqrt{s} \sim 50$–60 TeV, indicates a reasonably small error band, over-estimated from Fig. 4 to be in the interval 15.5–16.3.

At last, we understand that the applicability of our results in the context of the Glauber formalism can be further developed and improved along the following lines:

- The recent TOTEM result for σ_{el} at 7 TeV has been obtained through the steps outlined in Section 3.2 [12] and therefore has been evaluated from the results for σ_{tot} and B. The forthcoming measurement of σ_{tot}, by means of a luminosity-independent method, and the corresponding σ_{el} determination may improve our fit result, since this region is just associated with the change of curvature in parametrization (17), as shown in Fig. 2. Moreover, further results at 14 TeV will certainly contribute with additional improvements in the fit results.

- Here we have limited the discussion to the extreme values indicated from unitarity and the black-disk limit, together with the almost model-independent result for the ratio σ_{tot}/B. However, as commented in Section 3, analytical parametrizations for the total cross section and the ρ parameter, as those obtained by the COMPETE Collaboration (or possible deviation from this result, if confirmed by the experimental data [28]), may be combined with our results in the Glauber connection, Eqs. (8)–(9), reducing the uncertainties bands in the extrapolations to cosmic-ray energies.

Note added

After this paper was submitted for publication, we have noticed the results from a gray-disk-model analysis, in which a different transition on $\sigma_{el}/\sigma_{tot}(s)$, from low to high energies, is proposed [29] (see also references therein).

Acknowledgements

Appendix A

Beyond the forward direction we can define

$$B(s, t) = \frac{d}{dt}\left[\ln \frac{d\sigma}{dt}(s, t)\right] \tag{A.1}$$

and

$$\rho(s, t) = \frac{\operatorname{Re} F(s, t)}{\operatorname{Im} F(s, t)},$$

so that from Eq. (1),

$$B(s, t) = 2\frac{d}{dt}\ln \operatorname{Im} F(s, t) + \frac{d}{dt}\ln\left[1 + \rho^2(s, t)\right]. \tag{A.2}$$

Under the reasonable assumption that at least in the *neighborhood of* $t = 0$

$$\operatorname{Im} F(s,\ t) \geqslant \operatorname{Re} F(s, t)$$

and by expanding the second term in the r.h.s of (A.2) we obtain at $t = 0$

$$\frac{d}{dt}\ln\left[1 + \rho^2(s, t)\right]\Big|_{t=0} = 2\rho(s)\frac{d}{dt}\rho(s, t)\Big|_{t=0} + \mathcal{O}\left(\rho^3(s)\right).$$

Since from the experimental data $\rho(s) \lesssim 0.14$ and under the assumption

$$\lim_{t \to 0} \frac{d}{dt}\rho(s, t) = 0,$$

Eq. (A.1) at $t = 0$ reads

$$B(s) \approx 2\frac{d}{dt}\ln \operatorname{Im} F(s, t)\Big|_{t=0},$$

leading through Eq. (12) to the upper bound (13).

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