



UNIVERSIDADE ESTADUAL DE CAMPINAS  
SISTEMA DE BIBLIOTECAS DA UNICAMP  
REPOSITÓRIO DA PRODUÇÃO CIENTÍFICA E INTELLECTUAL DA UNICAMP

**Versão do arquivo anexado / Version of attached file:**

Versão do Editor / Published Version

**Mais informações no site da editora / Further information on publisher's website:**

<https://www.osapublishing.org/oe/abstract.cfm?uri=oe-16-17-13267>

**DOI: 10.1364/OE.16.013267**

**Direitos autorais / Publisher's copyright statement:**

©2008 by Optical Society of America. All rights reserved.

DIRETORIA DE TRATAMENTO DA INFORMAÇÃO

Cidade Universitária Zeferino Vaz Barão Geraldo

CEP 13083-970 – Campinas SP

Fone: (19) 3521-6493

<http://www.repositorio.unicamp.br>

# Optical frequency combs generated by four-wave mixing in optical fibers for astrophysical spectrometer calibration and metrology

Flavio C. Cruz

*Gleb Wataghin Physics Institute, University of Campinas - UNICAMP  
13083-970, Campinas, SP, Brazil  
[flavio@ifi.unicamp.br](mailto:flavio@ifi.unicamp.br)*

**Abstract:** Optical frequency combs generated by multiple four-wave mixing in short and highly nonlinear optical fibers are proposed for use as high precision frequency markers, calibration of astrophysical spectrometers, broadband spectroscopy and metrology. Implementations can involve two optical frequency standards as input lasers, or one standard and a second laser phase-locked to it using a stable microwave reference oscillator. Energy and momentum conservation required by the parametric generation assures phase coherence among comb frequencies, while fibers with short lengths can avoid linewidth broadening and stimulated Brillouin scattering. In contrast to combs from mode-locked lasers or microcavities, the absence of a resonator allows large tuning of the frequency spacing from tens of gigahertz to beyond teraHertz.

©2008 Optical Society of America

**OCIS codes:** (190.4223) Nonlinear wave mixing; (190.4370) Nonlinear optics, fibers; (190.4380) Nonlinear optics, four-wave mixing; (120.6200) Spectrometers and spectroscopic instrumentation.

---

## References and links

1. Th. Udem, J. Reichert, R. Holzwarth and T.W. Hänsch, "Absolute Optical Frequency Measurement of the Cesium D1 Line with a Mode-Locked Laser," *Phys. Rev. Lett.* **82**, 3568-3571 (1999).
2. Th. Udem, R. Holzwarth, and T. W. Hänsch, "Optical frequency metrology," *Nature* **416**, 233-237 (2002).
3. W. H. Oskay, S. A. Diddams, E. A. Donley, T. M. Fortier, T. P. Heavner, L. Hollberg, W. M. Itano, S. R. Jefferts, M. J. Delaney, K. Kim, F. Levi, T. E. Parker, and J. C. Bergquist, "Single-Atom Optical Clock with High Accuracy," *Phys. Rev. Lett.* **97**, 020801 (2006).
4. D. Ludlow, T. Zelevinsky, G. K. Campbell, S. Blatt, M. M. Boyd, M. H. G. de Miranda, M. J. Martin, J. W. Thomsen, S. M. Foreman, Jun Ye, T. M. Fortier, J. E. Stalnaker, S. A. Diddams, Y. Le Coq, Z. W. Barber, N. Poli, N. D. Lemke, K. M. Beck, C. W. Oates, "Sr Lattice Clock at  $1 \times 10^{-16}$  Fractional Uncertainty by Remote Optical Evaluation with a Ca Clock," *Science* **319**, 1805-1808 (2008).
5. S.N. Lea, "Limits to time variation of fundamental constants from comparisons of atomic frequency standards," *Rep. Prog. Phys.* **70**, 1473-1523 (2007).
6. A. Baltuska, Th. Udem, M. Uiberacker, M. Hentschel, E. Goulielmakis, Ch. Gohle, R. Holzwarth, V. S. Yakovlev, A. Scrinzi, T. W. Hänsch, F. Krausz, "Attosecond control of electronic processes by intense light fields," *Nature* **421**, 611-615 (2003).
7. M. J. Thorpe, K. D. Moll, R. J. Jones, B. Safdi, and J. Ye, "Broadband Cavity Ringdown Spectroscopy for Sensitive and Rapid Molecular Detection," *Science* **311**, 1595 (2006).
8. S. A. Diddams, L. Hollberg, and V. Mbele, "Molecular fingerprinting with the resolved modes of a femtosecond laser frequency comb," *Nature* **445**, 627-630 (2007).
9. A. Marian, M. C. Stowe, J. R. Lawall, D. Felinto, J. Ye, "United Time-Frequency Spectroscopy for Dynamics and Global Structure," *Science* **306**, 2063-2068 (2004).
10. E. Goulielmakis, V. S. Yakovlev, A. L. Cavalieri, M. Uiberacker, V. Pervak, A. Apolonski, R. Kienberger, U. Kleineberg, and F. Krausz, "Attosecond Control and Measurement: Lightwave Electronics," *Science* **317**, 769-775 (2007).
11. J. C. Knight, "Photonic Crystal fibers," *Nature* **424**, 847-851 (2003).
12. H. R. Telle, G. Steinmayer, A. E. Dunlop, J. Stenger, D. H. Sutter, and U. Keller, "Carrier-envelope offset phase control: A novel concept for absolute optical frequency measurement and ultrashort pulse generation," *Appl. Phys. B* **69**, 327-332 (1999).

13. T. M. Ramond, S. A. Diddams, L. Hollberg, and A. Bartels, "Phase-coherent link from optical to microwave frequencies by means of the broadband continuum from a 1-GHz Ti:sapphire femtosecond oscillator," *Opt. Lett.* **27**, 1842-1844 (2002).
14. P. Del'Haye, A. Schliesser, O. Arcizet, T. Wilken, R. Holzwarth, and T. J. Kippenberg, "Optical frequency comb generation from a monolithic microresonator," *Nature* **450**, 1214-1217 (2007)
15. P. Del'Haye, O. Arcizet, A. Schliesser, R. Holzwarth, and T. J. Kippenberg, "Full stabilization of a microresonator frequency comb," *Phys. Rev. Lett.* **101**, 053903 (2008).
16. M. T. Murphy, Th. Udem, R. Holzwarth, A. Sizmann, L. Pasquini, C. Araujo-Hauck, H. Dekker, S. D'Odorico, M. Fischer, T. W. Hänsch, A. Manescau, "High-precision wavelength calibration with laser frequency combs," *Mon. Not. R. Astron. Soc.* **380**, 839-847 (2007).
17. C.-Hao Li, A. J. Benedick, P. Fendel, A. G. Glenday, F. X. Kärtner, D. F. Phillips., D. Sasselov, A. Szentgyorgyi, and R. L. Walsworth, "A laser frequency comb that enables radial velocity measurements with a precision of 1 cms-1," *Nature* **452**, 610-612 (2008).
18. D. A. Braje, M. S. Kirchner, S. Osterman, T. Fortier, and S. A. Diddams, "Astronomical spectrograph calibration with broad-spectrum frequency combs," *Eur. Phys. J. D* **48**, 57-66 (2008).
19. G. P. Agrawal, *Nonlinear Optical Fiber Optics*, Academic Press, New York, 2001.
20. H. G. Weber, R. Ludwig, S. Ferber, C. S. Langhorst, M. Kroh, V. Marembert, C. Boerner, and C. Schubert, "Ultrahigh-Speed OTDM-Transmission Technology," *J. Lightwave Technol.* **24**, 4616-4627 (2006).
21. S. Pitois, J. Fatome, and G. Millot, "Generation of 160-GHz transform limited pedestal-free pulse train through multiwave mixing compression of a dual-frequency beat signal," *Opt. Lett.* **27**, 1729-1731 (2008).
22. S. Pitois, C. Finot, J. Fatome, B. Sinardet, and Guy Millot, "Generation of 20-GHz picosecond pulse trains in the normal and anomalous dispersion regimes of optical fibers," *Opt. Commun.* **260**, 301-306 (2006).
23. J. Fatome, S. Pitois, and G. Millot, "320/640 GHz high-quality pulse sources based on multiple four-wave mixing in highly nonlinear optical fibre," *Electron. Lett.* **41** (2005)
24. J. Fatome, S. Pitois, and G. Millot, "20-GHz-to-1 GHz repetition rate pulse sources based on multiple four-wave mixing in optical fibers," *IEEE J. Quantum Electron.* **42**, 1038-1046 (2006).
25. A. Cerqueira S. Jr, J. M. Chavez Boggio, A. A. Rieznik, H. E. Hernandez-Figueroa, H.L. Fragnito, and J. C. Knight, "Highly efficient generation of broadband cascaded four-wave mixing products," *Opt. Express* **16**, 2816-2828 (2008).
26. L. S. Ma, P. A. Jungner, J. Ye, and J. L. Hall, "Delivering the same optical frequency at two places: accurate cancellation of phase noise introduced by an optical fiber or other time-varying path," *Opt. Lett.* **19**, 1777-1779 (1994).
27. B. C. Young, F. C. Cruz, W. M. Itano, and J. C. Bergquist, "Visible lasers with subhertz linewidths," *Phys. Rev. Lett.* **82**, 3799-3802 (1999).
28. Jun Ye, Jin-Long Peng, R. Jason Jones, Kevin W. Holman, John L. Hall, and David J. Jones, Scott A. Diddams, John Kitching, Sebastien Bize, James C. Bergquist, and Leo W. Hollberg, Lennart Robertsson and Long-Sheng Ma, "Delivery of high-stability optical and microwave frequency standards over an optical fiber network," *J. Opt. Soc. Am. B* **20**, 1459-1467 (2003).
29. N. R. Newbury, P. A. Williams, and W. C. Swann, "Coherent transfer of an optical carrier over 251 km," *Opt. Lett.* **32**, 3056-3058 (2007)
30. K. O. Hill, D. C. Johnson, B. S. Kawasaki, and R. I. MacDonald, "cw three-wave mixing in single-mode fibers," *J. Appl. Phys.* **49**, 5098-5106 (1978).
31. J. Zhou, R. Hiu, and N. Caponio, "Spectral linewidth and frequency chirp four-wave mixing components in optical fibers," *IEEE Photon. Technol. Lett.* **6**, 434-436 (1994).
32. M. Rodwell, J. E. Bowers, R. Pallela, K. Gilboney, J. Puhl, and D. Nguyen, "Electric and optoelectronic components for fiber transmission at bandwidths approaching 100 GHz," in *LEOS Summer Topical Meetings*, (Institute of Electrical and Electronics Engineers, Piscataway, N.J., 1995), pp. 21-22.
33. P. Chen, G. A. Blake, M. C. Gaidis, E. R. Brown, K. A. McIntosh, S. Y. Chou, M. I. Nathan, and F. Williamson, "Spectroscopic applications and frequency locking of THz photomixing with distributed-Bragg-reflector diode lasers in low-temperature-grown GaAs," *Appl. Phys. Lett.* **71**, 1601-1603 (1997).
34. X.-C. Zhang, Y. Jin, and X. F. Ma, "Coherent measurement of THz optical rectification from electro-optic crystals," *Appl. Phys. Lett.* **61**, 2764-2766 (1992).
35. Q. Quraishi, M. Griebel, T. Kleine-Ostmann, and R. Bratschitsch, "Generation of phase-locked and tunable continuous-wave radiation in the terahertz regime," *Opt. Lett.* **30**, 3231-3233 (2005).
36. A. Dreyhaupt, S. Winnerl, T. Dekorsy, and M. Helm, "High-intensity terahertz radiation from a microstructured large-area photoconductor", *Appl. Phys. Lett.* **86**, 121114 (2005).
37. D. Pereira, J. C. S. Moraes, E. M. Telles, A. Scalabrin, F. Strumia, A. Moretti, G. Carelli and C. A. Massa, "A review of optically pumped far-infrared laser lines from methanol isotopes," *Int. J. Infr. Mill. Waves* **15**, 1-44 (1994).
38. C. Champenois, G. Hagel, M. Houssin, M. Knoop, C. Zumsteg, and F. Vedel "Terahertz Frequency Standard Based on Three-Photon Coherent Population Trapping," *Phys. Rev. Lett.* **99**, 013001 (2007).
39. D. J. Berkeland, J. D. Miller, J. C. Bergquist, W. M. Itano, and D. J. Wineland, "Laser-Cooled Mercury Ion Frequency Standard," *Phys. Rev. Lett.* **80**, 2089-2092 (1998).

40. S.A. Diddams, Th. Udem, J.C. Bergquist, E.A. Curtis, R.E. Drullinger, L. Hollberg, W. M. Itano, W. D. Lee, C.W. Oates, K.R. Vogel, D.J. Wineland, "An optical clock based on a single trapped Hg-199(+) ion," *Science* **293**, 825-828 (2001).
  41. Bingwei Xu, Yves Coello, Vadim V. Lozovoy, D. Ahmasi Harris, and Marcos Dantus, "Pulse shaping of octave spanning femtosecond laser pulses", *Opt. Express* **14**, 10939-10944 (2006).
  42. M. C. Stowe, F. C. Cruz, A. Marian, and J. Ye, "Coherent population transfer dynamics controlled by pulse accumulation and spectral phase manipulation," *Phys. Rev. Lett.* **96**, 153001 (2006).
  43. L. E. E. de Araujo, "Selective and efficient excitation of diatomic molecules by an ultrashort pulse train," *Phys. Rev. A* **77**, 033419 (2008).
- 

## 1. Introduction

Direct measurement of optical frequencies was a long standing problem in science and technology, which was solved with the advent of optical frequency combs (OFC) based on femtosecond mode-locked lasers [1], [2]. Since then, these combs allowed for example the development of optical atomic clocks [3],[4], tests of possible variations of fundamental constants [5], extreme nonlinear optics [6], sensitive, high resolution and broadband spectroscopy [7],[8],[9], and high-harmonic generation of attosecond pulses [10]. The development of OFC was based on the availability of two technological ingredients: femtosecond lasers in the near infrared, and the development of photonic crystal fibers [11]. These fibers allowed a phase-coherent spectral expansion of the comb (supercontinuum generation), required for stabilizing the carrier-to-envelope offset frequency [12]. Later, ultrabroadband femtosecond lasers [13] were developed that dispensed the use of these fibers, and improved long-term operation. Recently, OFC at 1.5  $\mu\text{m}$  with frequency spacing higher than 88 GHz were generated by four wave mixing (FWM) and Kerr nonlinearity in monolithic high-Q silicon microcavities [14], [15]. These combs require a single input low-power cw laser, and generate new frequencies over demonstrated bandwidths of 500 nm. OFC with line spacing of a few tens of gigahertz have also been demonstrated by spectral filtering of mode-locked lasers with  $\approx 1$  GHz repetition rates, producing frequency spacing between 10 and 40 GHz. Such combs have been proposed for high precision calibration of astrophysical spectrographs in the near infrared [16], [17], [18]. High-precision and long-term stability of such "astro-combs" can increase the accuracy in velocity determination, allowing the search for terrestrial mass planets orbiting distant stars. All of the above implementations of OFC require a resonator, whose physical size fixes the line spacing and allows very limited tuning.

In this paper, it is proposed that OFC generated by multiple four-wave mixing (FWM) in short and highly nonlinear optical fibers can overcome this limitation, providing a broadband comb of stable and precise optical frequencies with adjustable spacing. They can be well suited for astrophysical spectrometer calibration, direct frequency comb spectroscopy, and frequency metrology. It is discussed a particular implementation that involves two pump waves, in which one is an optical frequency standard, whose frequency is known to high precision, and the other laser is phase-locked to it using a stable microwave oscillator. Another possible implementation involves two optical frequency standards. "FWM-fiber-combs" will benefit from integration with telecom technology, and have prospect for compactness, reduced cost, and long-term operation. Section 2 addresses what is needed for a stable FWM-fiber-comb and how short fibers can avoid noise propagation. Section 3 discusses the analogies with OFC from mode-locked lasers and how the comb frequencies can be linked to a microwave frequency standard. Section 4 proposes an implementation for an "astro-comb", and is followed by the conclusion in section 5.

## 2. Precision and stability of four-wave mixing fiber combs

FWM has been extensively studied, particularly in optical fibers [19] since it can be a basic limitation to the operation of fiber-based optical telecommunication systems. In a usual configuration, two cw, single-frequency pump fields with frequencies  $f_1$  and  $f_2$  (say  $f_2 > f_1$ ), generate Stokes and anti-Stokes waves at frequencies  $f_3 = f_1 - \Delta f$  and  $f_4 = f_2 + \Delta f$ , with  $\Delta f = f_2 - f_1$ .

These new fields undergo parametric gain, which depend on the pump powers ( $P_{1,2}$ ), the nonlinear coefficient of the fiber ( $\gamma = n_2 2\pi f / c A_{\text{eff}}$ , where  $f = (f_1 + f_2) / 2$ ,  $n_2$  is the nonlinear refractive index and  $A_{\text{eff}}$  is the effective core area of the fiber) and the wavevector mismatch between the four waves ( $\Delta k = k_3 + k_4 - k_1 - k_2$ ) [19]. These new amplified frequencies can in turn generate new ones, in a cascade process known as multiple FWM. A broadband optical comb can be generated in this way provided there is sufficient parametric gain over a large range, and energy and momentum are conserved. Energy conservation assures that the new frequencies are exactly spaced by  $\Delta f$ . Momentum conservation (phase-matching) can be achieved in single-mode fibers when the wavevector mismatch, due to material dispersion, is cancelled by the nonlinear mismatch  $2\gamma(P_1 + P_2)$  (induced by self-phase modulation, SPM, and cross-phase modulation, XPM). This requires fibers with anomalous dispersion. The parametric gain extends over a range of  $\Delta k$  between 0 and  $-4\gamma(P_1 + P_2)$  and is maximum when  $\Delta k = -2\gamma(P_1 + P_2)$  [19]. The generation of broadband optical combs by multiple FWM therefore involves a management of the fiber dispersion, the frequency spacing  $\Delta f$ , fiber nonlinearity and the pump powers. Several groups have been investigating multiple FWM with a primary interest on the generation of short and high-repetition rate pulses for optical time division multiplexing (OTDM) in optical telecommunications [20]. Short pulses were obtained by compression of the original beat note in fibers with anomalous dispersion [21], or compression of pulses produced first in normal dispersion fibers [22]. Well separated pulses with no pedestal have been demonstrated, at rates extending from 20 GHz to up to 1 THz, and with durations from 280 fs to a few ps [21, 22, 23, 24]. Pulse trains generated by FWM can have special features such as near transform-limited pulses (e.g. with near zero spectral phase) and fixed pulse-to-pulse temporal phase [21-24]. Recently FWM-fiber-combs spanning 300 nm, with 740 GHz spacing, were generated by injecting two lasers into highly nonlinear fibers (HNLF) with lengths of only 2 m [25]. In ref. [25], two independent single-frequency cw lasers at 1555 and 1561 nm were injected into the HNLF after amplification into Erbium-doped fiber amplifiers (EDFAs) to powers of several Watts. To avoid saturation of the EDFAs, both lasers were switched into 40 ns pulses with low duty cycle. The power in the comb lines varied from -30 dBm (1  $\mu$ Watt) for frequencies in the wings, to -10 dBm (100  $\mu$ W) for central frequencies.

The generation of an optical comb of stable frequencies by multiple FWM in a fiber requires at least two conditions: 1) the frequency and phase stability of the input lasers must be preserved at the output of the fiber. Their linewidths must not be degraded by technical noise added by the fiber or by EDFAs; 2) the new generated frequencies should be phase-coherent with the input lasers. Condition 1 can be satisfied either by using a short length of optical fiber, well isolated from environmental noise, or by employing well established active techniques of fiber noise cancellation [26,27,28]. In this case a low power, frequency shifted portion of one input laser, sent through the fiber, is reflected back and then heterodyned with the input beam in order to produce an error signal for cancellation of fiber noise. In this technique, the servo bandwidth is limited by propagation delays, which will set the maximum fiber length whose noise can be corrected. The scheme has been recently employed for coherent transfer of a laser with 1 Hz linewidth over a fiber link of 251 km [29]. This link also included four EDFAs with combined gain of more than 124 dB. Condition 2 is intrinsically met by FWM, as consequence of the required momentum conservation (phase-matching) [19], [14]. However, the frequency (phase) fluctuations of the two independent free-running input lasers will propagate to the new frequencies as they are generated as linear combination of the input frequencies  $f_1$  and  $f_2$ . This effect, which for input lasers with a few Hertz linewidths could still be acceptable, can quickly increase the linewidth of the successive comb frequencies, and has been reported in the literature of FWM in fibers [30],[31].

Assuming a fiber length much bigger than the coherence length of the laser, the microwave (or THz) separation  $\Delta f = f_2 - f_1$  between free-running fields with independent

fluctuations  $\delta f_1$  and  $\delta f_2$ , will have fluctuations  $\delta(\Delta f) = [(\delta f_1)^2 + (\delta f_2)^2]^{1/2}$ , which are directly responsible for the jitter in the generated pulse train. This is a limiting factor for their use in optical communications, where low jitter is required [20]. Using the notation in which the Stokes frequencies on the red side of  $f_1$  are  $f_{2k+1} = (k+1)f_1 - kf_2$  (e.g.,  $f_3 = 2f_1 - f_2$ ,  $f_5 = 3f_1 - 2f_2$ , ...) and the anti-Stokes ones on the blue side of  $f_2$  are  $f_{2k+2} = (k+1)f_2 - kf_1$  (e.g.,  $f_4 = 2f_2 - f_1$ ,  $f_6 = 3f_2 - 2f_1$ , ...), the fluctuations in the individual optical comb frequencies, due only to propagation caused by multiple FWM, would be  $\delta f_{2k+1} = [(k+1)^2(\delta f_1)^2 + k^2(\delta f_2)^2]^{1/2}$  and  $\delta f_{2k+2} = [k^2(\delta f_1)^2 + (k+1)^2(\delta f_2)^2]^{1/2}$ , thus scaling as  $\approx (2)^{1/2}k$ . For example, if  $\delta f_1 = \delta f_2 = 100$  kHz, typical of short-term linewidths of free-running extended cavity telecom diode lasers, the  $k = 20$  sidebands would have  $\delta f_{41} \approx \delta f_{42} = 1.41(20) \delta f_1 = 2.8$  MHz. Two input lasers with a few Hz linewidths ( $\delta f/f = 10^{-14}$ ), would still produce  $k \approx 100$  comb lines with sub-kiloHertz linewidths. However, if FWM-fiber-combs can be generated in fibers much shorter than the coherence length of the input lasers, the fiber can work as a high-pass filter, with a cutoff frequency given by  $f_c = (\text{transit time})^{-1} = 100$  MHz for a 2-m fiber. Such "short" fibers can then avoid noise propagation, since for most lasers the frequency fluctuations have technical origin (electrical, acoustic, seismic, thermal) and are dominated by low frequency contributions, scaling roughly as  $1/f$  and seldom with components over a few MHz. For free-running telecom lasers with typical short-term linewidths of 100 kHz, the fastest frequency fluctuations occur in a time  $t \approx 1/\delta f = 1/100\text{kHz} = 10$   $\mu\text{s}$ , which is much longer than the transit time through the fiber,  $t_t = 2\text{ m}/(c/n) = 10$  ns, with  $n=1.5$ . As a consequence, the fluctuations of the input lasers will not propagate to the new comb frequencies due to the FWM process, as long as parametric generation takes place in a short fiber length. Stabilized lasers or optical standards with very small linewidths, would allow use of longer fibers. Short lengths of fibers are also advantageous since the influence of dispersion fluctuations resulting from imperfections during the drawing process or sensitivity of the material to external conditions, are greatly reduced. Another major factor that can negatively affect the stability of FWM-combs is the stimulated Brillouin scattering (SBS) [19]. SBS can generate light propagating in the backward direction, shifted typically by about 10 GHz from the pump lasers, which in turn can generate new FWM products. Its threshold depends on the fiber effective core area and length [19], so that its influence can be largely reduced also by using fibers just a few meters long. This is the case in ref. [25], where SBS was minimal and a phase modulator typically used to prevent it (which of course would degrade the spectral purity of the pump lasers) was not required.

### 3. Analogy with combs from mode-locked lasers

In mode-locked lasers, a short pulse is formed, for example in a Ti:sapphire laser crystal, by combination of self-phase modulation (which creates new phase-locked frequencies) and Kerr lens mode-locking (which favors pulse over cw oscillation). In a time-domain picture, as this pulse propagates inside the cavity, dispersion shifts the carrier frequency with respect to the pulse envelope. After a round trip time  $T$ , a phase  $\delta\phi$  is acquired between the carrier and the envelope. Because this pulse circulates in the cavity (being damped by the output coupler and amplified by the gain medium), this phase-shift  $\delta\phi$  is acquired between successive output pulses, giving rise to a frequency shift  $f_{\text{ceo}} = \delta\phi/(2\pi T)$  in the comb spectrum with respect to the cavity resonances. In FWM-fiber combs, however, the absence of a cavity implies that any phase-shift that might be present in the output pulse (arising from dispersion, nonlinear effects, etc) will be the same for each one. Therefore there will be no frequency shift in the comb spectrum. There is however a phase shift between successive pulses which comes from the original beatnote and is equal to  $\pi$  [23, 24]. The beating between the two injected frequencies  $f_1$  and  $f_2$  consists of their sum frequency modulated by the difference frequency. This modulation causes a  $\pi$  phase difference from one beatnote to the next. New frequencies, generated in the fiber as linear combinations of the two input frequencies, have the effect of

compressing the beatnote, while still keeping the  $\pi$  phase difference [21-24]. This phase has been measured [21-24], although not yet with the precision required for metrology. The analogy with frequency combs generated by mode-locked lasers can be explored further. In a mode-locked laser, the  $n^{\text{th}}$  tooth of the comb has a frequency  $f_n$  given by:

$$f_n = n f_{\text{rep}} + f_{\text{ceo}} \quad (1)$$

where  $f_{\text{rep}}$  is the repetition rate of the laser,  $n$  is an integer and  $f_{\text{ceo}} = \delta\phi / (2\pi T) = (\delta\phi f_{\text{rep}}) / 2\pi$ . Both  $f_{\text{rep}}$  (which is set by the laser cavity length) and  $f_{\text{ceo}}$  are typically RF or microwave frequencies, ranging from tens of MHz to about 1 GHz. For optical frequency measurements, both  $f_{\text{rep}}$  and  $f_{\text{ceo}}$  are actively stabilized to a RF or microwave frequency standard, so that  $f_n$  can be known once  $n$  is determined by a wavelength measurement, using for example a wavemeter. Furthermore, an optical clock is realized if  $f_n$  is locked to an optical frequency standard. By setting for example  $f_{\text{ceo}}=0$ ,  $f_{\text{rep}}$  will be the RF or microwave output frequency of such clock, having stability and accuracy superior to microwave clocks. In FWM-fiber-combs (Fig.1), the  $m^{\text{th}}$  comb line can be written as:

$$f_m = m \Delta f + f_{\text{Offset}} \quad (2)$$

where  $m$  is integer. Of course, since  $\Delta f$  (which will be set by a microwave reference oscillator) is independent of the input lasers, eq. (2) does not imply a phase-coherent link. However, as discussed below, the optical comb frequencies  $f_m$  can be actively phase-locked to the microwave oscillator which sets  $\Delta f$ . Equation 2 is formally identical to eq. (1), but its quite different physical interpretation simply expresses the fact that any frequency can be written as a multiple of a lower frequency plus an offset. This offset frequency  $f_{\text{Offset}}$  is simply another microwave frequency (or THz, depending on  $\Delta f$ ), by definition smaller than  $\Delta f$ . It turns out that  $f_{\text{Offset}}$  can be measured with the same techniques used to measure  $f_{\text{ceo}}$  (Eq. 1) in mode-locked lasers [12]. A convenient way is when the comb spectrum covers one octave.

Figure 1 depicts the case in which laser 1 (master) is a narrow linewidth laser (optical standard) and laser 2 (slave) is phase-locked to it. For detection of the beatnote  $\Delta f$ , a distinction can be made between  $\Delta f$  in the microwave ( $< 100$  GHz) or terahertz region ( $\approx 100$  GHz). In the first case, fast photodiodes are available [32], even commercially for  $\Delta f \approx 40$  GHz. In the second case, photomixers, photoconductive antennas [33] or nonlinear crystals [34] can be employed. They have been used to generate TeraHertz frequencies from cw [35] and mode-locked lasers [36]. Ref. [35] demonstrates phase-coherent generation of THz frequencies at the  $10^{-15}$  level by mixing two cw lasers at 800 nm. The THz field generated by these antennas will be synchronous with the repetition rate of the optical pulse train, and can be directly heterodyned with a THz frequency standard using for example MIM or Schotky diodes. The THz standard can be either a far-infrared gas laser [37], synthesized directly from optical sources by photomixing [38], obtained from frequency multiplication of stable microwave frequencies, such as the cesium standard at 9.2 GHz or the mercury ion standard at 40 GHz [39] or even the microwave output of an optical clock [40].

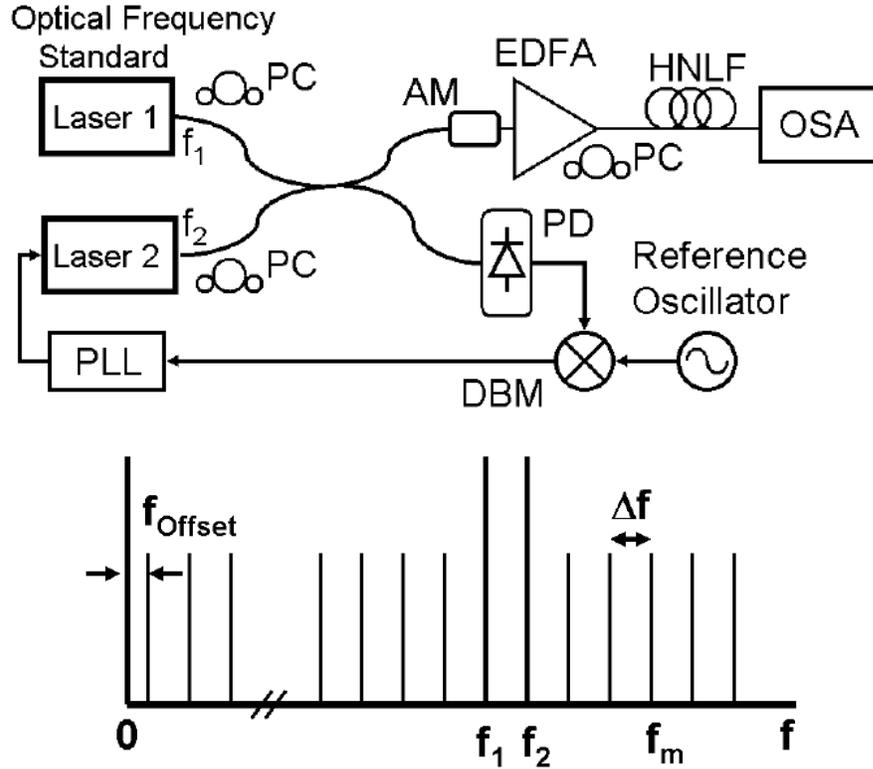


Fig. 1. Schematic diagram of the experimental setup used to generate an OFC by FWM in a highly nonlinear fiber (HNLF). Laser 1 (frequency  $f_1$ ), is an optical frequency standard and laser 2 (at  $f_2$ ) is phase-locked to it using a stable reference oscillator. PC: polarization controller, AM: amplitude modulator, EDFA: Erbium doped fiber amplifier, PD: photodetector; DBM: double-balanced mixer, PLL: phase-locked loop, OSA: optical spectrum analyzer.

The stabilization of the beatnote  $\Delta f$  with respect to the microwave (THz) oscillator, with frequency  $f_{\text{osc}}$ , sets  $\delta(\Delta f) = \delta f_{\text{osc}}$  and  $\delta f_2 = \delta f_1 + \delta f_{\text{osc}}$ , where  $\delta f_{\text{osc}}$  is the frequency fluctuation of the microwave (THz) oscillator, given by its Allan variance. Assuming that  $\delta f_{\text{osc}}$  can be as low as  $10^{-14}$  in 1 s [38], we find that upon stabilization of  $\Delta f$ , the noise of the individual comb frequencies would be  $\delta f_{2k+1} = (\delta f_1) + (k+1)(\delta f_{\text{osc}})$  and  $\delta f_{2k+2} = (\delta f_1) + k(\delta f_{\text{osc}})$ , if we again consider a fiber length much longer than the laser coherence length. For short fibers, it will be assumed that  $\delta f_{2k+1} = \delta f_{2k+2} \approx \delta f_1 + \delta f_{\text{osc}}$  (except  $\delta f_1$  for the master laser), and if  $\delta f_1 \ll \delta f_{\text{osc}}$ , all comb frequencies will have fluctuations  $\delta f_m \approx \delta f_{\text{osc}}$ . In this case, the fluctuations of  $f_{\text{Offset}}$  will also be  $\approx \delta f_{\text{osc}}$ , and eq. (2) can be used to know any comb frequency  $f_m$  if  $\Delta f$  and  $f_{\text{Offset}}$  are known. It is interesting to note that the contribution from the microwave (THz) oscillator to the fluctuations of the individual comb frequencies should be in anti-phase ( $180^\circ$ ) for the Stokes frequencies on the red side of  $f_1$  (e.g.,  $f_{2k+1}$ ) and the anti-Stokes frequencies on the blue side of  $f_2$  (e.g.,  $f_{2k+2}$ ). This is because the FWM process leads to phase conjugated beams.

If, on the other hand, we assume a short fiber and free-running input lasers, with  $\delta f_1, \delta f_2 \gg \delta f_{\text{osc}}$ , after stabilizing  $\Delta f$  with respect to  $f_{\text{osc}}$  by phase-locking  $f_2$  to  $f_1$ , all comb frequencies will have fluctuations  $\approx \delta f_1$ , and can be measured and phase-locked to  $f_{\text{osc}}$  using eq. (2). For illustration, let's consider that the comb covers one octave, and that  $f_1$  is the comb line selected for a  $f$ - $2f$  scheme [12]. Its second harmonic (with fluctuations  $\approx 2\delta f_1$ ), generated in a nonlinear crystal, is heterodyned with the nearest comb tooth (with fluctuations  $\approx \delta f_1$ ). The

resulting beatnote  $f_{\text{Offset}}$  (with  $\delta f_{\text{Offset}} \approx \delta f_1$ ) is a microwave (THz) frequency that contains all information about the noise of the optical frequency  $f_1$ . This would be a new feature of the present scheme: the frequency fluctuations of an optical source are directly transferred to a microwave (THz) frequency, without recurring to any other stable optical reference. Once  $f_{\text{Offset}}$  is detected, it can also be measured with respect to the microwave (THz) oscillator, and phase-locked to it. This is done by feeding back corrections to laser 1, whose frequency fluctuations will also be reduced to the level of  $\delta f_{\text{osc}}$ . In this way all individual comb frequencies will be known within the uncertainty given by the microwave (THz) frequency standard, according to Eq. (2). It is worth mentioning that the derivation of a phase-coherent microwave frequency from the optical input fields, such as realized in optical clocks based on femtosecond frequency combs, is not possible in FWM-combs considered here. This is simply because the two input lasers are independent and already require a reference oscillator to stabilize their difference frequency.

It is likely that a considerable extension of the spectral coverage in FWM-combs can be achieved either by optimizing the fiber and lasers setup, by cascading more stages of nonlinear mixing in different fibers, or by phase-locking separate combs with different spectra. Ref. [12] discusses several possibilities for measuring  $f_{\text{ceo}}$  (Eq.1) in mode-locked lasers, depending on the spectral bandwidth of the comb. The demonstrated scheme with minimum bandwidth requires the ratio (BW/f) of comb bandwidth, BW, to the central frequency f, to be at least 0.29. Presently the largest bandwidth reported for multiple-FWM combs seems to be BW/f = 0.2 of ref. [25].

#### 4. Implementation of “astro-combs” of precise optical frequencies

The particular implementation of a FWM-fiber-comb depicted in Fig. 1 can be used to generate a comb of precise optical frequencies, particularly with the required spacing for astrophysical applications.

The stable master laser whose frequency  $f_1$  is known with high precision can be a diode laser stabilized to acetylene or its second harmonic stabilized to Rubidium. The slave laser with frequency  $f_2$  is phase-locked to it using a reference microwave oscillator. The precision and stability of the optical standard can be much higher than for the microwave standard. In this case,  $f_1$  will be a stable frequency at the fiber output, with fluctuations  $\delta f_1$ , while all other comb frequencies will have fluctuations approximately given by those of the microwave oscillator. For astrophysical spectrometer calibration, fractional stability and accuracy near  $10^{-11}$  would be sufficient to measure velocity variations of 1 cm/s [17]. Another obvious implementation of a FWM-fiber-comb involves two optical frequency standards, in which the frequency spacing instability will be set by their combined instabilities. In this case, even for long fibers, with full noise propagation into the generated comb lines, fractional instabilities  $<10^{-11}$  would be attainable.

Finally, it is worth noting that near zero spectral phase can be obtained for FWM-fiber combs, allowing optical pulses which are nearly Fourier transform-limited [24]. This should allow the generation of broadband second harmonic spectra [41], thus extending the spectral coverage into the near-infrared which might be required for astrophysical applications [18]. Applications of FWM-fiber-combs may include direct link between optical and THz frequencies, combined fiber transmission of THz and optical standards, secure communications, optical frequency synthesis, and frequency comb spectroscopy. In this case it can be possible to explore the large tuning of the frequency spacing or the coherent accumulation effect [42] at high repetition rates (possibly using alternated pulses) for transitions with short-lived states. A recent proposal for selective excitation of cold molecules demands pulses at 2 THz rates [43].

## 5. Conclusion

Optical frequency combs based on multiple four wave mixing in short and highly nonlinear fibers are proposed for use as precise frequency markers and metrology. A distinct feature of such combs is the absence of an optical resonator and largely adjustable frequency spacing, which can be set from tens of GHz to beyond one THz. In particular, spacings of a few tens of GHz are suited for calibration of astrophysical spectrometers. A broadband comb of stable and precise optical frequencies can be generated by pumping with an optical frequency standard and a second laser phase-locked to it, or with two independent frequency standards. Fiber technical noise can be actively cancelled or minimized with use of short fiber lengths, isolated from environmental noise. Short fibers should avoid linewidth broadening of the comb lines, and minimize the influence of stimulated Brillouin scattering. The expected performance of FWM fiber combs needs to be demonstrated, but additional work will hopefully be stimulated by their potential for compactness, reduced cost, long-term operation, and integration with telecom technology.

## Acknowledgments

The author thanks Arismar Cerqueira Sodre Junior, Hugo L. Fragnito and Christiano de Matos for discussions on multiple FWM and highly nonlinear optical fibers. Financial support from FAPESP, CEPOF, CNPq is gratefully acknowledged.