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SPECIAL ISSUE

SCIENCE IN THE FOREST, SCIENCE IN THE PAST

Is there mathematics in the forest?

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Scholars from different fields and epistemological orientations —including anthropologists, science historians, and mathematicians—have argued that technical and social practices of indigenous peoples, exemplified by ornamentation in textiles and kinship taxonomies, embody the mathematical capacity of illiterate people. I take as my main example the kinship calculi of the Cashinahua at the Brazil-Peru frontier, bringing into focus the complex mathematical operations and structures embedded in this domain and inextricably embedded in their ontology. Does that mean I am imposing modern mathematical concepts on indigenous ontologies? Against this charge of epistemic colonialism, I argue in favor of the existence of universal mathematical capabilities (evidenced by the recursive rules used to produce consistent patterns that are transportable across distinct domains of thought and action) across ontological boundaries.

Keywords: Amazonia, mathematics, ontologies, kinship, Cashinahua Indians

Is there mathematics in the forest?

Is it possible to translate forest mathematics into modern language? The immediate answer is yes, because otherwise there could be no ethnography—not to mention history—of mathematics, a bleak conclusion that would deprive of meaning many works on counting systems among illiterate people and on their worldviews.

This argument, of course, begs the point, which is precisely whether or not there *is* mathematics among nonliterate, indigenous cultures in the first place—that is to say, whether we are talking about the same thing when we include finger-counting among indigenous societies and theorem-proving in axiomatic style as comparable instances of mathematics. Are we not, in so doing, committing another act of charitable translation, by redressing other people's acts and assertions so as to make them look better in our modern garb? And, granted that there is, so to speak, mathematics in the forest, is it the same as Western mathematics, and can it be translated without distorting the peculiarities of indigenous ontologies in which it is embedded?

Ethnographies as well as histories of mathematics that deal with different cultures suggest strongly that we can actually engage in meaningful conversations with people in other cultures, in the sense of talking significantly to each other, and not merely just misunderstanding each other. Thus it is that a contemporary introduction to the Theory of Numbers invokes the "sophisticated means" employed by Babylonian clerks to generate Pythagorean triples, uses the "Chinese remainder theorem" in proofs, and places Euclid's "division algorithm" as the foundation stone of the whole subject.[1] Anachronistic as these references may seem to the eyes of the modern scholar, and notwithstanding the deep differences between the worldviews of ancients and moderns, there remains the fact that Euclid's division algorithm survives a variety of translations from bad to excellent. The reason for this fact is what is at stake. I am aware that while mathematical agreement in the pragmatic and structural sense may be consistent with ontological pluralism, it can also be the case that this sense of familiarity of moderns when reading ancient mathematical texts lies in the fact that our own mathematics belongs to the Greek tradition, and has developed along successive rewritings of Euclid's as well as of Archimedes's and

1. On Babylonian mathematics, the Chinese Remainder Theorem, and Euclid's Division Algorithm, see Stillwell (2003: 12–13, 66, 158, 171–76); see also Lloyd (1996, 2004) and Cuomo (2001).

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Diophantus's works.[2] But there is more to it than that, because we also understand intuitively the use of Roman and Greek calculating boards as well as of Chinese and Japanese sorobans and African counting systems (Cuomo 2001, 2007; Zaslavsky 1973; Lloyd 1990, 1996; Lockhart 2017). When it comes to nonliterate Amerindian societies, these issues are the subject of controversy. Anthropologists argue that there is indeed mathematics among indigenous people, involving counting with the body, with actions, with beads, and embodied in social life. They look for mathematics embedded in social practices and institutions as well as in kinship, cosmology, and religion (Mimica 1988; Crump 1990; Urton 1997; Verran 2001; Passes 2006; Ferreira 2015), and also "in the stones" (Hugh-Jones 2016) as well as "in everyday life" and "in the street" (Lave 1988; Nunes, Schliemann, and Carraher 1993; Mesquita, Restivo, and D'Ambrosio 2011).

Ethnographic studies tend to conclude that "forest mathematics" is incommensurable with modern mathematics, and oppose the ontological content of indigenous numeral systems—for instance, with the supposedly abstract, disembodied, ontology-free arithmetic of modern times. Against this stance, mathematicians such as Ubiratan D'Ambrosio, Marcia Ascher, and Hervé Bazin have argued that there are common mathematical ideas expressed by different means in different cultures and times, and look for indigenous "mathematical ideas" that overlap modern mathematical themes (Ascher and Ascher 1981; Ascher 1991, 2002a; D'Ambrosio 2001; Bazin and Tamez 2002).[3] Concurrent with the second interpretation, mathematicians have in the last century illustrated abstruse areas of pure mathematics, such as crystallographic groups, knot theory, and permutation groups, by means of such concrete subjects as Egyptian decorative patterns (Tietze 1942), Polynesian navigation charts and quipus (Speiser [1922] 1937) and Australian kinship systems (Weil [1949] 1967). These examples suggest that the "unreasonable effectiveness of mathematics" in the natural sciences (Wigner 1960) may have a counterpart in the human sciences, where mathematics appears to play a role similar to that of music as a means of communication between different cultures, although with different meanings. The latter view can of course be dismissed as charitable at best, and Eurocentric at worst, or, in other words, as yet another variant of a Whig view in which all previous modes of knowledge converge toward contemporary science. Should then anthropologists counter this supposed scientific ethnocentrism with the thesis of radical noncommensurability? Against this dismal epistemic posture, I think that a defense of mathematical translatability across time and space is compatible with the acknowledgement of the unlimited varieties of mathematical activity in different cultures and epochs.[4] A plurality of mathematical ontologies and the consequent ambiguity and indeterminacy of mathematical translation are not an impediment to transcultural mathematical understanding. More specifically, I argue that the pragmatic effects of mathematics, as well as its relational and iconic character, account for its interculturality, despite the multiplicity of ontologies associated with mathematical activities in the same or in different cultures.[5] The thesis, of course, is far from new. It is a reinstatement of a view pioneered by Wilhelm von Humboldt early in the nineteenth century, the point being that all languages are capable of expressing any human thoughts, although with different grammatical means and carrying, accordingly, distinct connotations.[6] To use an analogy employed by Edward Sapir, translating between languages is like changing the coordinate system when representing a geometrical figure. The representation will come across in both frames of reference,

2. An authoritative author argues that Gauss "not only did see that Euclid was right, . . . he also saw that [the parallel axiom] implied the existence of a geometry different from that of Euclid" (Kelly and Matthews 1981: 12). Archimedes is described as the forerunner of the Integral Calculus (Pólya 1973: 155).

3. Cf. Sahlins's proposal that there is a common core of "kinship ideas" recognizable across all known cultural forms (Sahlins 2013).

4. This point may perhaps be taken as a special case of Lloyd's argument against the incommensurability thesis and the homogeneity of mentalities (Lloyd 1990).

5. For these points I am indebted to Da Costa's pluralistic philosophy of science (Da Costa, Bueno, and French 1998; Da Costa and French 2003), and from his reading of Peirce's pragmatism (Peirce 1932, 1965).

6. "No language has ever been found that lies outside the boundaries of complete grammatical organization . . . even the so-called rude and barbaric language families already possess everything that is needed to a complete usage." Humboldt's most noteworthy example is "a literature flourishing since millennia in a language nearly devoid of any grammar in the usual sense of the word," that is, Chinese (Humboldt [1820] 1994:1ff.; [1822] 1994).

but some reference systems allow an elegant representation, while others lead to a cumbrous formulation, as becomes evident when we look for an equation representing a circumference in the Cartesian plane (Sapir [1924] 1949).[7] The important point in Sapir's analogy is that the completeness of all human languages is not the same as semantical equivalence.[8]

One could advance the argument a bit further to suggest that modern mathematics is well equipped for conveying the ontological variety of non-Western cultures, sharing somehow the similar claim of anthropological and historical disciplines. For contemporary mathematics is a multicultural continent where Platonists, formalists, and constructivists live together while disagreeing on basic issues of existence and method (Connes, Lichnerowicz, and Schützenberger 2001; D'Espagnat and Zwirn 2017). Mathematics is ultimately "what mathematicians do." Incidentally, this is not a unique feature of modern culture, for a similar plurality of views and methods flourished in ancient Greece and China (Lloyd 1996). Platonic realism, represented by some of the most eminent modern mathematicians, maintains that sets exist in a realm of their own that is independent of human thought and inaccessible to senses,[9] and that mathematicians have an intuitive perception of such suprasensible beings.[10] Kurt Gödel's ontological and epistemological position is diametrically opposed to the views that mathematics is the result of human activity. But in this camp, there is no consensus either, because there are those who say that only mathematical objects exist that can be constructed by well-defined rules (Bridges and Richman 1987), and those for whom mathematics is the free creation of the human mind—not to mention the naturalistic attitude that sees mathematics as an empirical science dealing with properties of the physical world (Maddy 1997).[11] In short, just as indigenous and ancient mathematics are laden with multiple metaphysical worlds, contemporary mathematics overbrims with ontological and epistemological varieties ranging from idealism to constructivism and formalism,[12] just as anthropology has its own corresponding epistemic strategies—namely, looking for metaphysical systems, describing how things are actually constructed, searching for rules and algorithms.

Do Indians have numbers?

The thesis of the universal existence of mathematics across cultures would seem to have been refuted by Daniel Everett's thesis, according to which the alleged absence of "recursiveness" in the Pirahã language explains why the Pirahã Indians lack "numbers of any kind or a concept of counting" (Everett 2005: 621). Everett empirically supported his argument with the absence of words for numbers among the Pirahã (Everett 2005:

7. Jerrold Katz stated the point as the "principle of effability" (Katz 1978). The creator of the "Sapir-Whorf" hypothesis was neither Sapir nor Whorf, but the editor of the posthumous works of Whorf (Whorf [1941] 1956: 134ff.).

8. While asserting that the "Eskimo" have linguistic means to express the notion of causality and to translate Kant's work, Sapir calls attention to how grammatical schemata have ontological implications: "Stone falls is good enough for Lenin, as it was good enough for Cicero . . . [the] Chinese . . . content themselves with a frugal 'stone fall,' and in Nootka no stone is assumed at all, and 'the stone falls' may be reassembled into something like 'It stones down' " (Sapir [1924] 1949: 124, 158–59, 160–66).

9. "The objects of transfinite set theory . . . clearly do not belong to the physical world, and even their indirect connection with physical experience is very loose (owing primarily to the fact that set-theoretical concepts play only a minor role in the physical theories of today)" (Gödel (1964) 1990: 267–68).

10. "I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them" (Gödel (1964) 1990: 268). Meinong's ontology, which allows for the existence of "impossible objects" such as square circles (Meinong [1904] 1999) and Da Costa's "paraconsistent logic," which allows inconsistent propositions (Da Costa 1974) are examples of contemporary ontological anarchism (Almeida 2013).

11. It is perhaps of interest to anthropologists who struggle with "neopositivism" to mention that Quine deconstructed a long time ago the "two dogmas of empiricism"—that is to say, the separation between "logical truths" (independent from experience) and "synthetical truths" (relying on experience), and between theory and observation (Quine 1953).

12. These three broad branches are not the whole story. There are radical constructivism (cf. Bridges and Richman 1987), naturalism (Maddy 1997), and structuralist mathematics (Bourbaki 1994), among other varieties.

621).[13] In his 2005 article, Everett relied heavily on Peter Gordon's counting experiments among the Pirahã, from which Gordon concluded that Pirahã were unable to count large "numerosities" with exactness, attributing this failure to the lack of number-words (2004).[14] The same issue of *Science* that features Gordon's report includes another experiment on indigenous counting abilities, this time with the Munduruсu, whose language lacks words for "numbers beyond 5," but who "are able to compare and add large approximate numbers that are far beyond their naming range," although "failing in exact arithmetic with numbers larger than 4 or 5" (Pica et al. 2004); the point being that there is a distinction between a system of approximate counting without numerals and a language-based system for counting that consists in a routine for pairing in a one-to-one way objects with numerals (Pica et al. 2004: 499, 503). We can conclude from the latter statement that the "no number, no counting" thesis is based on the mistaken identification of the number concept with the use of numerals, and of the counting concept with "counting with numerals" (Gelman and Butterworth 2005). In fact, that is not all there is to it. In a later paper, contra Gordon (2004) and contra Everett (2005), Michael Frank (2008) and collaborators (including Everett) recognize after new experiments—this time using a more culturally friendly setting—that Pirahã speakers, although they have "no linguistic method of expressing any exact quantity, even 'one,'" are, after all, able "to perform exact matching tasks with large numbers of objects when these tasks do not require memory," the conclusion being now that the Pirahã lack "words for numbers," which are a "technology" indispensable for memorizing and comparing "large quantities" (Frank et al. 2008: 820). Notwithstanding, the Pirahã proved in these experiments to be able to pair quantities one-to-one and thus compare quantities as larger and smaller: "performance on the one-to-one matching task was nearly perfect, and performance on the uneven match task was close to ceiling as well" (Frank et al. 2008: 822). The authors conclude: "a total lack of exact quantity *language* did not prevent the Pirahã from accurately performing a task which relied on the exact numerical equivalence of large sets" (Frank et al. 2008: 823; my emphasis). Facing this evidence, the remaining argument is that, although the Pirahã can check the "numerical equivalence of large sets," they lack "memory" devices for numbers, which are supposedly dependent on *words*.

How about the "absence of a number concept" and of a "counting concept"? Let us recall the main empirical facts revealed by a second counting experiment, which differed from the one performed by Gordon in that "matching" was done with objects familiar to Pirahã: first, Pirahã can distinguish a collection with n from another having $n+1$ objects, and can compare cardinally two collections, as larger and smaller.

In fact, in Peano's axioms, natural numbers are constructed from a sign for one, and by the act of adding one to a number already constructed—that is, from 1 and from the operation designated as $n+1$, or, even more basically: starting from $|$, juxtaposing $|$ successively, so as to obtain $|$, $||$, $|||$, and so on.[15] Therefore, the Pirahã, having the ability to make these distinctions, already have all that is needed for doing Peano's arithmetic—without the use of numerals. Also, in Cantor's set theory, *infinitely large* numbers are *compared* by means of one-to-one matching of two collections, an act that can be performed with bundles of sticks.[16] And this is precisely the method used by another Amazonian indigenous group. As for memorizing quantities, the Palikur of northwestern Amazonia, when inviting a neighbor for a party, used a "day-counting" device consisting of "a number of finger-sized sticks," "richly

13. A large section of Everett's 2005 essay is dedicated to the "absence of a number concept," giving as corroborating evidence the absence of numerals, or number-words, among the Pirahã (Everett 2005: 623–24, 626). We are also told that the Pirahã do not have "ordinal numbers" either, although they order generations as above and below Ego (Everett 2005: 633).

14. Everett's central thesis is that, pace Chomsky and collaborators (Hauser, Chomsky, and Fitch 2002), recursiveness is not a universal feature of human languages, the Pirahã being a counterexample (Everett 2005). Everett gives this thesis as the explanation for the "the absence of numbers of any kind or a concept of counting" (Everett 2005: 621).

15. This is Hilbert's basic characterization of the number system (Hilbert [1904] 1967).

16. In the manual of arithmetic in Tukuya language (Bazin and Tamez 2002; Cabalzar 2012), Tukuya's finger-based counting system is represented as bundles of sticks, with the addition of a Mayan symbol for positional zero (to allow the construction of big numbers). Calculation with an abacus or a soroban is essentially another way of "counting with fingers," without using number-words at all.

decorated with cotton and feathers." Curt Nimuendajú, the German ethnographer, continues: "After receiving the Iyen-ti, the invited person kinks off daily the ends of two sticks. If at the end there is still one stick left, then the party starts at noon of this day, but if there is none left, then the party starts at night" (Nimuendajú 1926: 94, quoted in Vidal 2007: 23, my translation).

There seems to be no doubt left about the presence of a modern *concept* of counting and of number even among the Pirahã, not to mention of the actual ability to *count large numbers* by means of the matching method. If there is any conceptual shortcoming here it is not on the Pirahã's side. As for the "memory" role of numerals, one should recall, besides the *Iyen-ti* technique quoted above, the method of *quipus* and of the Christian rosaries as efficient techniques for storing large numbers without words (Almeida 2015).

Mathematics in the forest

This is our cue to go back to the comparative ethnography of mathematics, which was the starting point of my argument. Studies of indigenous mathematics have focused on number systems (Zaslavsky 1973; Closs 1986; Gilsdorf 2012; Ferreira 2015; Lockhart 2017) and on related pedagogical issues (Verran 2001; Bazin and Tamez 2002; Cabalzar and Bazin 2004). How to go beyond the focus on the metaphysics of numbers in the Tylorian tradition ([1871] 1920), toward a wider view of mathematics?

I go back to Gary Urton's thesis: that Quechua number ontology has a relevant contribution to make to contemporary philosophy of numbers (Urton 1997). While agreeing with the point, I suggest that contemporary mathematics has also a relevant contribution to make to anthropology, by offering a wider view of what mathematics is about. This wider perspective is illustrated by the cooperative work of mathematicians and anthropologists, which has thrown light on nonnumerical, nonmeasure-oriented "mathematical ideas" embedded in human life. One of the best examples is Marcia and Robert Ascher's *Mathematics of the Incas: Code of the quipu* (Ascher and Ascher 1981), a deep analysis of the many uses and possible meanings of *quipu*. In subsequent books, Marcia Ascher drew attention to the interesting and nontrivial mathematics implied in "sand drawings" by Angolan children and in "tracing graphs around rice grains" by Tamil Nadu women (Ascher 1991: 30; 2002a: 162; 2002b; on children's drawing, see also Gerdes 2007). Other areas where Ascher revealed subtle "mathematical ideas" are "the logic of kin relations" and other "systems of relationships" (1991: 67–83; 2002a: 128–59), a point to which I will return. The "symmetric patterns" (1991: 154ff.) and "models and maps" (2002a: 89; 2002b: 122) are other domains where "mathematical ideas" are found. Other examples of cooperative work by anthropologists and mathematicians include the catalogue of plane patterns found in indigenous designs by Dorothy Washburn and Donald Crowe (1988, 2004), based on the theory of groups. This is the approach that I will use in the next section, as a tentative example of how mathematics can be found in social systems.

The incommensurability point: "My father is my son"

I now turn to an example of translation from another modern mathematical theory into a native idiom of kinship, emphasizing the point that the translation does make ontologies commensurable, for what is being translated are ideas about *relations*, not about *things* related by them. I take as an instance the Cashinahua kinship language.[17] First, I argue that ontological translation is unavoidably ambiguous in this case.

Epan is the Cashinahua vocative translated as *pai* in Portuguese and as "father" in English. This is clearly a case of equivocation both in the extensional sense and in the intensional sense, since in standard usage, Brazilian *pai* refers to a single individual at the next ascending generation, while a Cashinahua can address as *epan* not only his "father" in the English sense but also all his "father's brothers" and also his "sons" together with his "brother's sons"—keeping in mind that English kinship terms within quotation marks are not meant as translations of Cashinahua terms. Thus, terminologically, "father" and "son" could refer in different contexts to individuals that a Cashinahua speaker would address as *epan*, as well as all his "father's brothers" and his "brother's sons."[18] As an example, Sian, a Cashinahua of

17. The Cashinahua are an indigenous people inhabiting the course of the upper Jurua River. They belong to the Panoan linguistic family, which encompasses several indigenous groups distributed along the Jurua River and the Ucayali River. My main sources are the monographs by Kensinger (1995) and McCallum (2001), and also Capistrano de Abreu (1941) and Camargo (2002), in addition to conversations with Sian Caxinauá.

18. Anthropologists will be familiar with the terminological identification of "father" with "father's brothers," and of

Jordão River, explained, in the Portuguese language, to an undergraduate class: *"Eu respeito meu filho porque meu filho é meu pai"* ("I respect my son because my son is my father").

Sian's "son" and "father" can be addressed as *epan*: as "grandsons" and "grandparents" on the male line can be called *huchi* (elder *brother*) and *ichu* (younger *brother*), respectively.[19] To put it in other terms: *epa* (vocative *epan*) is used reciprocally among male members of alternating generations in the same moiety, and *huchi* or *ichu* are used among members of the same generation and the same moiety, and between people of the same moiety removed by two generations. A mirror image of this system works for women.[20]

The consequence of this system is that, in each of the two moieties in which people are divided, all generations are collapsed into two alternating sets of "brothers" (who are described as *huchi/ichu* according to relative age, or as *betsa/betsa* when ignoring the relative age distinction), these sets being related to each other as *epan/epan* (Kensinger 1995; McCallum 2001). People in each set are *chutabaibu*, or namesakes. Since there are two moieties, men can therefore belong to four different *xutabaibu*, two alternating namesake classes in each moiety. As there are also four *xutabaibu* for women, the entire system comprises eight *xutabaibu*, which are named categories of same-sex siblings and namesakes.

It is already clear, from the awkward way of expressing Cashinahua's kinship terms in English, that Cashinahua kinship terms combine with one another to produce other kinship terms with their own rules. Thus, my *epan's epan* is my *huchi/ichu* (or just *betsa*)—in the usual mistranslation, "my father's father is my elder/younger brother." In the case of women, *ewa's ewa* ("mother's mother") is *chipi/ichu* (or *betsa*), or, in the English mistranslation, "my mother's mother is my *elder/younger sister*."

A famous case of mistranslation from Cashinahua to Portuguese is *chai*, a reciprocal term that links same-generation males in opposite moieties, being ambiguously translatable as:

(i) a bilateral cross-cousin (sons of a "mother's brother" or of a "father's sister"),
(ii) a wife's brother (an actual or possible wife being addressed as *chanu*),
(iii) persons addressed by a *chai* as *huchi* or as *ichu* (according to relative-age), that is to say, persons who are *betsa* to a *chai* (that is, persons who are addressed by a brother-in-law as "brother")

Chai has been incorporated to regional Portuguese with the generic meaning of "friend," being used by Brazilians of both sexes when addressing any Indian of the region (cf. Viveiros de Castro 2004).

A mathematical translation

I now proceed to a representation of the above fragment of the Cashinahua's rules for combining kinship terms. These rules express the way the Cashinahua relationship words are combined to produce relationship words.[21]

I. There is a *neutral* term, which, when combined with any other term, produces the other term. Here, for convenience, I'll use a male speaker and use *epa* (the descriptive form):

$$\textit{epa} * \textit{betsa} = \textit{epa}$$

The term for "same-sex parent" is *epa*, and the term for "same-sex sibling" is *betsa*. We can also translate the above equation as:

$$\text{same-sex parent} * \text{same-sex sibling} = \text{same-sex parent},$$

which can be mistranslated as:

$$\text{a father's brother} = \text{a father},$$

"sons" with "brothers' sons"; on the other hand, the terminological identification of "father" with "son" is a rare feature of systems with "alternating generations."

19. Sian's father and Sian's son belong to the same namesake class, distinct from Sian's *brother-in-law* class (those he calls "chai"), which is also Sian's grandfather's and grandson's namesake class. Sian's grandson is Sian again, a "little Sian" as he calls him.

20. This rough sketch is not intended as a precise description, being a simplification of actual linguistic usage. For ethnographic details, see Kensinger (1995) or McCallum (2001).

21. For reasons of space and of simplicity, the full set of relational words and the full table of their possible combinations will not be shown here.

and formally, as:

$$f * e = f.$$

The above reasoning holds good when the order of terms is reversed, which means that (in the English mistranslation) "a brother's father is a father."

By putting all this together, we obtain the following formal representation:

$$f * e = f = e * f$$

And this is familiar algebraic property, with which we are familiar in the form

$$a \times 1 = a = 1 \times a,$$

where a is any rational number (this holds also for integers). This holds true not only for *epan* but also for all Cashinahua kinship terms. This means that the formal word e, which here translates formally *betsa* (or the pair *huchi*/*ichu*) behaves syntactically as the number 1 in multiplication. This is the algebraic version of Lewis Morgan's diagnostic trait for "classificatory systems of relationships," by which he meant systems that mix linear and collateral relatives (Almeida 2018).

II. Every term has an inverse. Always assuming a male speaker, and using the descriptive form *epa* as an example, without loss of generality:

$$epan * epan = betsa,$$

or "a father's father is a brother," and, in the algebraic translation,

$$f * f = e$$

This rule means that the inverse of f is f itself, that is to say:

$$f = f^{-1}$$

Rule II says that there is an inverse for every kinship term. This parallels the fact that, for any rational number a, there is a multiplicative inverse a^{-1} such that $a \times a^{-1} = 1$. Recall also that in Boolean algebra, $1 + 1 = 0$, where 0 is the neutral element, so that here 1 plays the role of its own additive inverse. It should be noted that $ff^{-1} = e$, or in the additive version, $1 + 1 = 0$, is the algebraic version of Sian's assertion that "my father is my son," since it is equivalent to say that "my father's father is my brother" in one of many possible mistranslations.

III. *Closure*: Any combination of two kinship terms is equivalent to a kinship term in the Cashinahua kinship vocabulary. We can think of this property as asserting that there is a multiplication table for Cashinahua kinship terms—considering the set of eight kinship terms that are enough to express all Cashinahua kinship relationships (ignoring relative age distinctions).

Finally, we add that the following constraints characterize fully the *structure* of this set of eight relationships.

IV. *Structure*: One constraint says that two terms are enough to generate all eight terms when combined in all possible ways. One possible choice for these two terms are f and s, that can be read as "same-sex parent" and "opposite-sex sibling." These terms share the following properties: $ff = e$ and $ss = e$ ("a father's father is a brother," from the standpoint of a male speaker, and "a mother's mother is a sister" from the standpoint of a female speaker).

V. The relation terms f and s do not commute, that is to say, $fs \neq sf$. This feature expresses the fact that (for a male speaker) "a father's sister is not a sister's mother" (a male's "same-sex genitor's opposite-sex sibling" is *not* a man's "opposite-sex sibling's same-sex genitor"). In other words, a father's sister is not a father's wife.

Extending this analysis would demand a separate essay, so I will stop here, having already probably abused the patience of the reader with what Bronisław Malinowski called depreciatively the "mock-algebra of kinship." The point is that there is here a legitimate isomorphism between the Cashinahua calculus of kinship relationships and a particular mathematical structure, called the "dihedral group of order eight," which is characterized particularly by Rules III, IV, and V above. This structure occurs in many contexts.[22]

22. The specific structure is not trivial at all. For we face here the task of generating *eight* relations (corresponding to the eight *xutabaibu* classes, divided in moieties, generations, and gender) by means of just *two* relations corresponding to filiation and gender-change. The key to this effect is combination of alternating generations and noncommutativity (diagrams in Almeida 2014: 4–6).

Let me return to the analogy traced above between Cashinahua's kinship relational calculus and a mathematical structure. My point is that there is more than an analogy: for the Cashinahua kinship rules are a way of calculating with words, just as the abstract symbols of algebra are a way of calculating with another class of words, and to say that these calculi have a common structure—or, in other words, that there is an isomorphism connecting one system into the other, where by "isomorphism" is meant a dictionary that translates the symbols of one system into symbols of the other system so as to preserve the structure.[23]

The existence of such an isomorphism has the following pragmatic consequence. Suppose one wishes to calculate a product of S and T in a mathematical system having the structure described (although described above in a loose way, the structure can be precisely specified). One way to perform the calculation is as follows: first translate the S and T into Cashinahua kinship words, and ask a Cashinahua speaker to calculate the resulting kinship word; finally, translate back the resulting kinship word to a formal symbol, say U. Or one could obtain the kinship "product" of a string of Cashinahua kinship words by first translating them into symbols of the mathematical system, performing the calculation according to the mathematical rules, and translating the result back into Cashinahua. This intriguing idea was suggested to me by the Tamil anthropologist Ruth Manimekalai Vaz (2014).[24]

The issue of reverse translation

The above example is a particular case of a more general fact. I think that, as a matter of fact, every translation of ancient/forest mathematics in the language of modern mathematics is automatically a candidate for a reverse translation of modern mathematics in indigenous terms.

Keeping the focus on kinship issues, I will suggest a case of a concept in modern mathematics that was originally expressed in the language of kinship. The theory of relations, created independently by Richard Dedekind and by Gottlob Frege as a foundation for mathematical induction, was expressed by Alfred Whitehead and Bertrand Russell in *Principia mathematica* (following the lead of Frege) in the idiom of descent, ancestrality, heredity, succession, and generation (Whitehead and Russell 1910: 570). This is how the principle of mathematical induction works: given that number 1 has the property *P*, and granted that, if a number *n* has a property *P*, its *successor* n + 1 *inherits* the property *P*, then all *descendants* of 1 have the property *P*, and the property *P* is shared by all *ancestors* of *n*. This means that the property is *hereditary*, as Whitehead and Russell put in a nice way: "If *m* is the Peerage, *m* is hereditary with respect to the relation of father to surviving eldest son" (Whitehead and Russell 1910: 570).

The authors of *Principia mathematica* were using—in the first really mathematical chapter of the book—the fact that the ordering of positive integers is isomorphic to the ordering of *peers*, a fact that justifies the use of the language of British peerage to define the concept of an inductive relation as equivalent to that of a hereditary relation.

Could Euclid prove $\sqrt{2} \times \sqrt{3} = \sqrt{6}$?

Dedekind claimed that his construction of irrational numbers afforded for the first time a proof that $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (Dedekind 1963: 40). The mathematician and historian of mathematics John Stillwell countered Dedekind's claim with a proof that $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ in purely Euclidean terms (Stillwell 2016: 156–57). The mechanism of the proof is essentially the same as Euclid's proof of Pythagoras's theorem, since in both cases the point is to show that successive transformations of an initial figure preserve their areas. The beautiful geometrical proof requires, however, as an initial step the translation of the product $\sqrt{2} \times \sqrt{3}$ of irrational numbers (as constructed by Dedekind) into a geometrical figure—namely a rectangle having irrational sides that are the geometrical translations of $\sqrt{2}$ and $\sqrt{3}$. This geometrical object is then successively transformed by means of purely Euclidean constructions, all of them justified in Book I of the *Elements*, resulting in a final rectangle equal to the

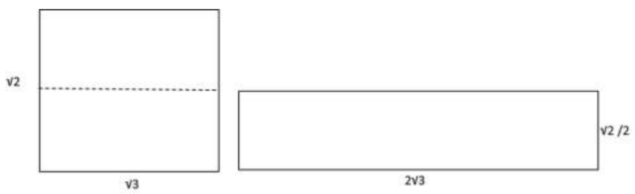
23. Lévi-Strauss once asked the eminent Henri Hadammard for help with a complicated problem in "Australian kinship," being told that "mathematics deals with the four operations and kinship could not be assimilated to any one of them"; he then met younger André Weil, who told him that "only the relations among marriages mattered" (Lévi-Strauss and Eribon 1988: 79). Hadammard was a renowned but aging mathematician, while the young Weil was one of the founders of the Bourbaki structuralist reconstruction of mathematics.

24. I gave a precise formulation for Vaz's conjecture on Dravidian kinship calculation by means of a calculating method borrowed from quantum physics—that is, Pauli matrices (Almeida 2014).

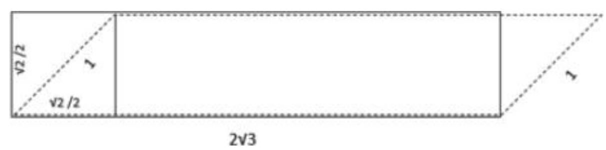
initial rectangle—in modern language, having the same area. And the final rectangle has sides that are the unit and the Euclidean version of √6. Translating the Euclidean result into Dedekind's language, one obtains $\sqrt{2} \times \sqrt{3} = \sqrt{6}$.

Step 0. Translate the product $\sqrt{2} \times \sqrt{3}$ (Dedekind's irrational numbers) into a Euclidean object—namely, an object constructed by means of unmarked rule and compass. This is a rectangle with irrational sides that we can interpret as √2 and √3 (Stillwell 2016: 156).

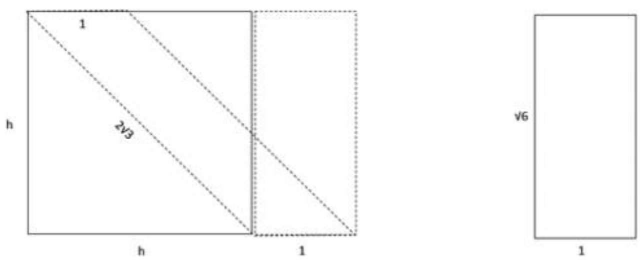
Step 1. Transform the initial rectangle into a rectangle with sides that we interpret as $2\sqrt{3}$ and $\sqrt{2}/2$. This transformation conserves the area.



Step 2. Transform the resulting rectangle into a parallelogram, cutting at left and adding at right half a square. This again conserves the area (*Elements* I). The diagonal of the square is 1 by Pythagoras's theorem.



Step 3. Left: Rotate the resulting parallelogram, transforming it into a parallelogram with base equal to the unit and height equal to the side *h* of a square with diagonal equal to $2\sqrt{3}$. Right: Transform the resulting parallelogram into a rectangle with unit base and height equal to *h*. As it happens, $h^2 + h^2 = (2\sqrt{3})^2 = 4 \times 3 = 12$ by Pythagoras's theorem, so $h^2 = 6$ and $h = \sqrt{6}$. The area of the last rectangle is therefore $1 \times \sqrt{6} = \sqrt{6}$.



And since all the figures are equal in area, the initial rectangle with sides √2 and √3 is equivalent in area to the final rectangle with sides equal to the unit and to √6. Translating this back in Dedekind's irrational numbers, this can be interpreted as meaning $\sqrt{2} \times \sqrt{3} = \sqrt{6}$.

One has again an isomorphism between two proofs. However, there is a catch. While Euclid's geometrical √2 is constructed by means of rule and compass—being the diagonal of a square with unit side—Dedekind's √2 is defined as a couple of infinite sets of rational numbers: those the square of which is less than 2, and those the square of which is bigger than 2. Dedekind's "cut," composed by two *infinite sets* of rational numbers—in the actual sense, not the potential sense—looks very much like two Zeno tortoises approaching one another without ever meeting because there is no rational number for them to meet at—the whole point being that the races themselves define a new kind of number.[25]

And notwithstanding this ontological chasm separating Euclid's and Dedekind's mathematical universes, there is a bridge connecting them. For not only does the translation between the two languages preserve the structure of the proof, but Archimedes and Dedekind would agree on the following: given an arbitrarily small quantity, it is possible to produce a rational number that, when squared, differs from 2 by less than this quantity by excess or by default. Another case in point, and more relevant, is Euclid's proof that, given any list of prime numbers, one can show that there is a prime number not in it (*Elements* IX: 20). Not a single word needs to be changed in Euclid's proof by today's standards, but modern versions of it are often phrased as stating that "the set of primes is infinite," while Euclid's subtle statement avoids any reference to "infinite" altogether. (The Cambridge mathematician Godfrey Hardy, who praised Euclid's proof as an example of immortal beauty in mathematics [Hardy 1940: 12], did not participate in this ontological mistranslation.)

Mathematical translation and ontological bridgeheads

As a final note, I am aware that mathematical agreement in the pragmatic and structural sense may be consistent with ontological pluralism, but it can also be a means for ontological cleansing and active evangelization (Vilaça 2018), a point also exemplified by Gottfried Leibniz's proposal of using his binary mathematics as a bridge-

25. "I still regard the statement . . . that the theorem $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ has nowhere yet been strictly demonstrated" (Dedekind 1963: 40).

head for religious conversion in Chinese (Leibniz 2006: 305–16). Against these ontological invasions and under the disguise of mathematical pedagogy, there remains the alternative of struggles for ontological autonomy also in the domain of mathematics (Viveiros de Castro 2003).

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