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Unitary neutron matter in the on-shell limit *

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Abstract. We compute the Bertsch parameter for neutron matter by using nucleon-nucleon interactions that are fully diagonal in momentum space. We analyze the on-shell limit with the similarity renormalization group and compare the results for a simple separable toy model to realistic calculations with high precision NN potentials.

1. Introduction

About fifteen years ago George Bertsch proposed the following problem [1]: what would be the ground state properties of a many-body system composed of spin-$\frac{1}{2}$ fermions interacting via a short-range contact interaction with an infinitely large scattering length?

The two-body scattering amplitude can be written as

$$\mathcal{T}_2(k,k) \propto \frac{1}{[k\cot\delta - ik]} \, , \tag{1}$$

and at low energies, $k\cot\delta$ can be described by the effective range expansion

$$k\cot\delta = -\frac{1}{\alpha_0} + \frac{1}{2}r_0 k^2 + \cdots \, , \tag{2}$$

where α_0 is the scattering length and r_0 is the effective range. The unitary limit corresponds to $\alpha_0 \to \infty$ and $r_0 \to 0$. In this limit, $k\cot\delta \to 0$ and the scattering amplitude is then reduced to

$$\mathcal{T}_2(k,k) \propto \frac{i}{k} \, . \tag{3}$$

and thus the cross section in the S-wave saturates the unitarity bound $\sigma = 4\pi/k^2$. Therefore the two-body scattering amplitude becomes completely scale invariant and as a consequence the energy per particle for the ground state of such a system would be given by

$$\varepsilon = \xi \times \varepsilon_{FG} \, , \tag{4}$$

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where

$$\varepsilon_{FG} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} \tag{5}$$

is the energy per particle of the free (non-interacting) Fermi Gas and ξ has been called Bertsch parameter. In fact, Bertsch wanted to know the sign of ξ. If $\xi > 0$, the system is a gas with positive pressure, and if $\xi < 0$ the system collapses since the pressure is negative.

Interesting many-body physics emerges when the interaction is close to the unitarity limit. In particular, many-body fermionic systems behave almost like a perfect fluid and may exhibit both BCS crossover and Bose-Einstein condensation which are, in fact, distinct limits of a common phenomenon. If the scattering length is small and negative, the interaction is weakly attractive and the system is in the BCS limit and there are overlapping loosely bound pairs. If the scattering length is large and negative, the interaction is strongly attractive and the system is in the BEC limit where the fermions form deeply bound pairs [2] (see Fig. 1).

Both neutron matter at low densities and ultra-cold atoms close to a Feshbach resonance are strongly interacting fermionic systems and present large pairing gaps when measured in units of the Fermi energy [3]. Monte Carlo simulations of a superfluid Fermi gas with an attractive short-range two-body interaction in the unitary limit (infinite scattering length and zero effective range) estimate the energy per particle of neutron matter in 44% of the energy of the free Fermi gas and the pairing gap to be about twice the energy per particle [4].

The S-wave interaction between two neutrons is very attractive (almost enough to produce a nn bound state) and has a significantly large (and negative) scattering length, $-\alpha_0^{nn} = 18.5\text{fm} \gg r_0^{nn} = 2.7\text{fm}$. Hence, neutron matter at low densities is a system with features close to the unitary limit. Another reason to study neutron matter at low densities is that both superconductivity and superfluidity in fermonic systems are manifestations of quantum coherence at a macroscopic level. An *ab initio* calculation of a Fermi gas in the unitary limit shows that, at $T = 0.2\,E_F$, the viscosity is close to the lower limit for a perfect fluid [5].
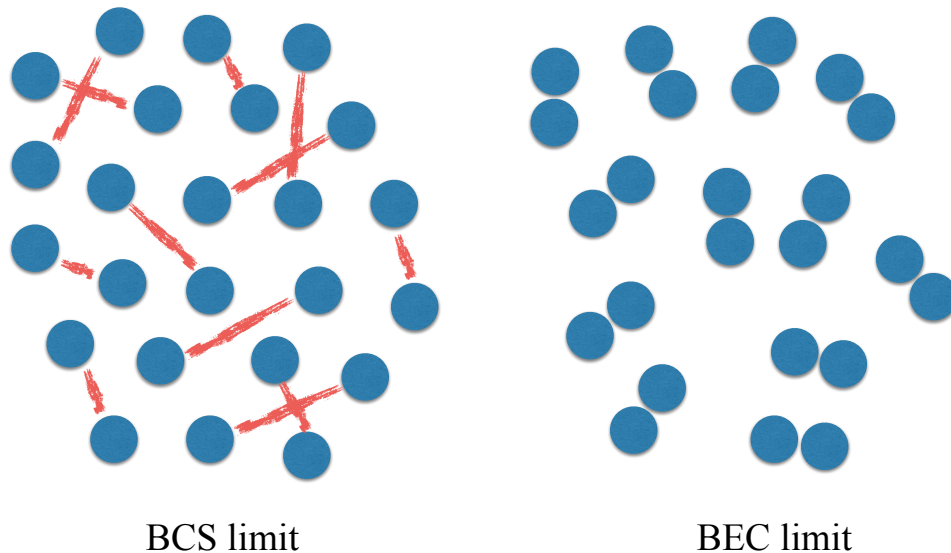

BCS limit   BEC limit

Figure 1. Pictorical representation of the BCS and BEC limits of a many-fermion system. The blurred red lines indicate the weak pairing. Adapted from Living Rev. Relativity **11**, 10 (2008).

2. On-shell interactions from the SRG and the Bertsch parameter

In this work we want to study neutron matter at the unitary limit with on-shell interactions obtained by evolving the nuclear force with the similarity renormalization group (SRG) towards the infrared region of the similarity cutoff λ. The SRG has been widely applied to nuclear structure calculations and the nuclear many-body problem [10, 11]. The technique is based on a flow equation which for the nucleon-nucleon interaction and the Wilson generator reads

$$\frac{dV_s(p, p')}{ds} = -(p^2 - p'^2)^2 \, V_s(p, p') + \int_0^\infty dq \; q^2(p^2 + p'^2 - 2q^2)V_s(p, q) \, V_s(q, p') \,, \tag{6}$$

where the flow parameter s is usually written in terms of the so-called similarity cutoff λ (which has dimension of momentum) as $s = \lambda^{-4}$. The potential before evolution (initial) corresponds to $s = 0$ or $\lambda = \infty$ and the matrix elements of the evolved potential are denoted as $V_\lambda(p, p')$. The unitarity of the transformation ensures that all observables computed with $V_\lambda(p, p')$ are exactly the same as the observables computed with the non-evolved initial potential $V_\infty(p, p')$. This applies in particular to phase-shifts, which do not depend on λ.

Recently, we have developed techniques in nuclear physics in order to study the infrared fixed-point of the SRG by pushing the evolution towards the on-shell limit $\lambda \to 0$ [6, 7, 8] and have found an elegant an simple way to determine phase shifts from fully diagonal interactions in momentum space complying with isospectrality and Levinson's theorem [9].

A simple S-wave gaussian separable potential toy model allows to carry studies with a moderate numerical effort

$$V(p, p') = C \; \exp\left[-\frac{1}{L^2} \left(p^2 + p'^2 \right) \right] \,, \tag{7}$$

where the parameters C and L are obtained by fitting the scattering length and the effective range. The nn interaction cannot be measured directly, but since the nn and np interaction in the $^1S_0$ channel have similar (and large) scattering lengths, $\alpha_0^{nn} = -18.5$ fm and $\alpha_0^{np} = -23.7$ fm, we use the np phase-shifts to access how well the toy model describes the nuclear force in the S-waves. This gives $C = -1.916$ fm and $L = 1.2$ fm^{-1}.

At similarity cutoffs close to $\lambda \sim 1$ fm^{-1} the flow equation becomes extremely stiff so that it is nearly impossible to study the infrared limit of the similarity cutoff, $\lambda \to 0$, if the potential has a long tail in momentum space, which is the case for high precision nucleon-nucleon potentials. This is the reason why we have constructed the toy model since it gives good qualitative results for the nucleon-nucleon S-waves but has a short tail in momentum space, allowing the SRG evolution towards the infrared region of the similarity cutoff with a moderate numerical effort. The fully diagonal on-shell interaction at $\lambda = 0$ is obtained by using the energy shift prescription of Ref. [9].

3. Bertsch parameter

Here we apply the toy model to compute the Bertsch parameter in the infrared region of the similarity cutoff with different grid sizes. We also compute the Bertsch parameter with high precision nucleon-nucleon potentials to compare them to the results from the toy potential.

The Bertsch parameter is the ratio between the total energy of a system of interacting fermions in the unitary limit and the energy of a free Fermi gas:

$$\xi_\lambda(k_F) = \frac{T(k_F) + V_\lambda(k_F)}{T(k_F)} = 1 + \frac{V_\lambda(k_F)}{T(k_F)} \,. \tag{8}$$

The kinetic energy in neutron matter is given by

$$T(k_F) = \frac{3k_F^2}{10m_n} \,, \tag{9}$$

where m_n is the neutron mass and k_F is the Fermi momentum. The potential energy can be obtained in the Hartree-Fock approximation and is due to the toy interaction in the $^1S_0$ channel:

$$V_\lambda(k_F) = \frac{4}{m_n}\frac{2}{\pi}\int_0^{k_F} dk\, k^2 \left(1 - \frac{3k}{2k_F} + \frac{k^3}{2k_F^3}\right) V_\lambda^{^1S_0}(k,k)\,. \tag{10}$$

For realistic high precision interactions, there are also contributions from higher partial waves and the energy per particle can be written as

$$\varepsilon(k_F) = \frac{3k_F^2}{10m_n} + \frac{4}{m_n}\frac{2}{\pi}\int_0^{k_F} dk\, k^2 \left(1 - \frac{3k}{2k_F} + \frac{k^3}{2k_F^3}\right)$$
$$\times \left[V_\lambda^{^1S_0}(k,k) + 9V_\lambda^{^3P_c}(k,k) + 5V_\lambda^{^1D_2}(k,k) + 21V_\lambda^{^3F_c}(k,k) + 9V_\lambda^{^1G_4}(k,k)\right]\,, \tag{11}$$

where ^3P_c and ^3F_c are linear combinations of the P and F waves, which are given explicitly in Ref. [12].

4. Numerical results

In Fig. 2 we show the Bertsch parameter as a function of the Fermi momentum computed with the toy model for several values of the similarity cutoff, mostly in the infrared region, for different number of grid points. For similarity cutoffs from infinity down to 0.5 fm^{-1} the number of grid points almost do not affect the results. However, for smaller values of λ the number of grid points start to change the results and in the limit $\lambda = 0$ the difference is huge as can be seen in the last panel of Fig. 2.

At $\lambda = 1$ fm^{-1} the results obtained with the toy model are very close to the results that come out if we consider only S-waves from high precision nucleon-nucleon potentials. This can be observed in Fig. 3 where we display the Bertsch parameter computed with Argonne v18 [13], Nijmegen II [14], N3LO (2003) [15] and N3LO (2005) [16] nucleon-nucleon interactions. The minimum value of the Bertsch parameter lies in between $\xi = 0.42$ and $\xi = 0.45$ for both the toy model and high precision potentials at k_F between 1.1 fm^{-1} and 1.3 fm^{-1}. Sophisticated quantum Monte Carlo calculations give $\xi = 0.44$ [4], so it is quite impressive that a simple separable potential can provide results that are so close to more accurate approaches.

While the contribution from the S-waves is rather independent of the nucleon-nucleon potential, when higher partial waves are included the results depend on whether one uses phenomenological potentials (Av18 or NijII) or chiral potentials (2003 N3LO or 2005 N3LO). This can be seen in Fig. 4 where the Bertsch parameter is computed summing up to G-waves. Also, the minimum of ξ gets much smaller and is displaced towards larger k_F.

When the similarity cutoff reaches the limit $\lambda = 0$ the interaction becomes fully diagonal and all off-shell ambiguities are then eliminated. In Fig. 5 we show a comparison of the Bertsch parameter at $\lambda = 0$ for the toy model and the high precision potentials with only S-waves (left panel) and summing up to G-waves (right panel). In the region $0 < k_F < 0.5$ fm^{-1}, the toy model matches the calculation with high precision potentials with only S-waves. For larger k_F the result is also very reasonable if one considers the extreme simplicity of the separable potential.

The λ-dependence of the Bertsch parameter is rather strong and shows that it is not determined just by two-body scattering information unless the extreme on-shell limit $\lambda \to 0$ is taken. Of course, the real limit $\alpha_0 \to -\infty$ and $\lambda \to 0$ remains an interesting challenge.
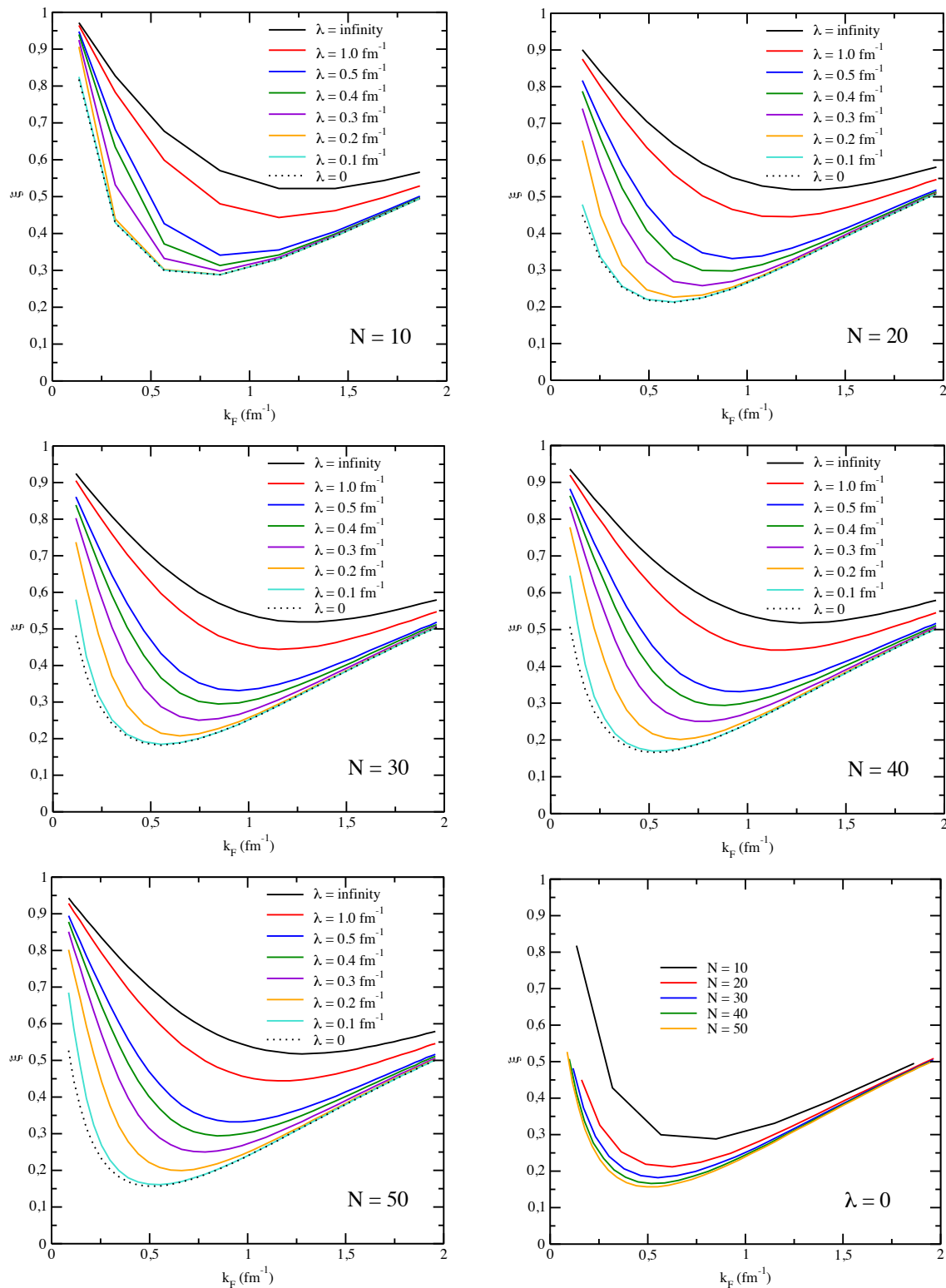
Figure 2. Bertsch parameter from the S-wave toy model for several values of the similarity cutoff λ (Wilson generator) and different number of grid points N. The right panel in the last row shows the grid dependence in the limit $\lambda = 0$. The on-shell interaction $V_{\lambda=0}(k)$ was obtained by applying the eigenvalue method of Ref. [9].
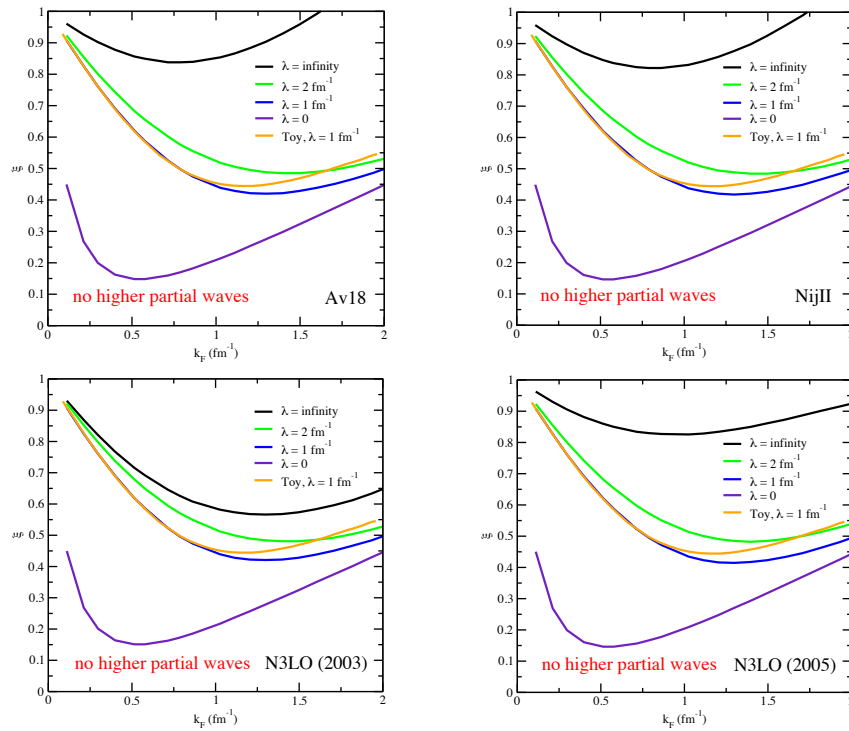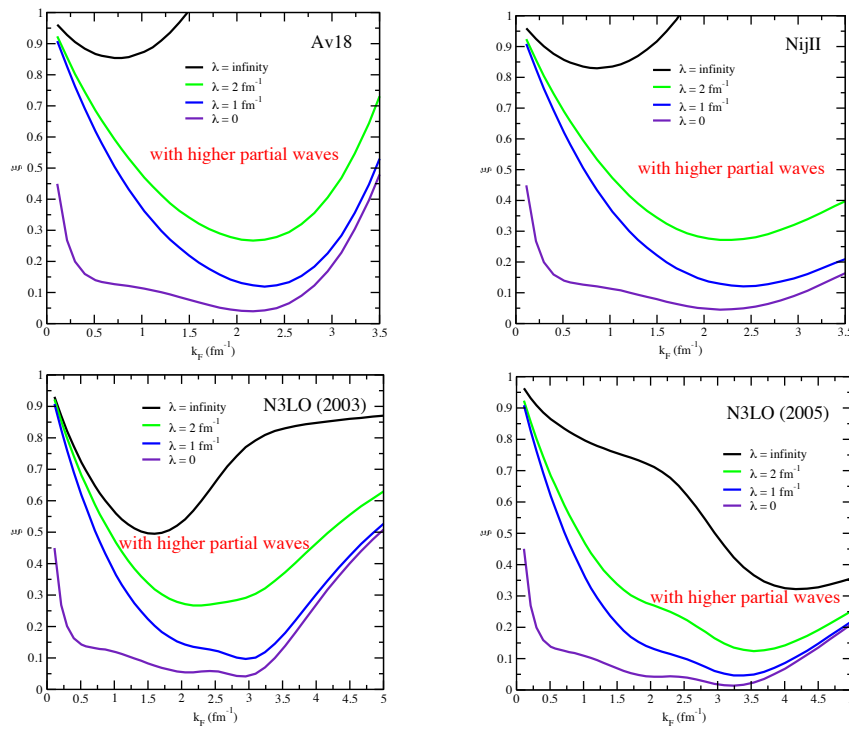
Figure 3. Bertsch parameter from high precision potentials for some values of the similarity cutoff λ (Wilson generator) with $N = 200$ grid points, taking only S-waves into account.

Figure 4. Bertsch parameter from high precision potentials for some values of the similarity cutoff λ (Wilson generator) with $N = 200$ grid points, but summing up to G-waves.
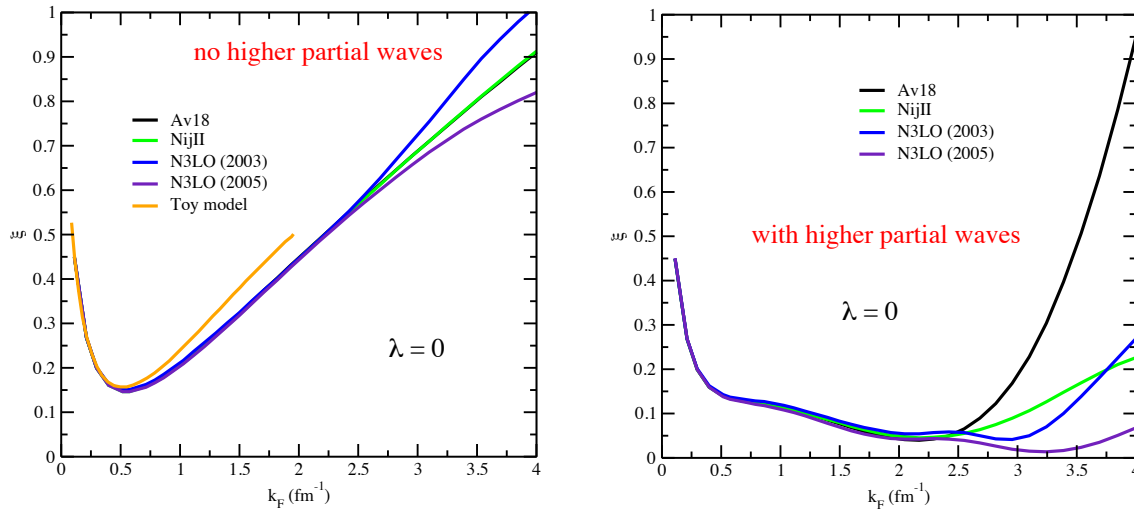
Figure 5. Bertsch parameter in the limit $\lambda = 0$ for some high precision potentials with $N = 200$ grid points with only S-waves (left) and summing up to G-waves (right).

5. Conclusions

So far we have studied the ground state of neutron matter in the unitary limit with a two-nucleon interaction given by a simple separable potential and compared the results to what is obtained with high precision nucleon-nucleon interactions. We also extended the study to the infrared region of the similarity cutoff by evolving the interactions with the SRG flow equation for the Wilson generator. The limit $\lambda = 0$ was obtained by using our energy shift prescription. Our results for the toy model provide a good estimate for the Bertsch parameter at $\lambda = 1$ fm^{-1}. In the limit $\lambda = 0$, since the two-body force gets small, ξ lowers dramatically at intermediate k_F.

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