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COMPARISON OF VOID FRACTION CORRELATIONS FOR DRIFT-FLUX MODEL IN VERTICAL UPWARD FLOW

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Abstract. *Multiphase flow is a common phenomenon that occurs in a variety of industries, such as petrochemical, refrigeration and chemical. The gas-liquid flows have a spatial and temporal phase distributions in a pipe section, which are related to the phases velocities, thermophysical properties and pipe geometry. Bhagwat and Ghajar (2014) developed a flow pattern independent void fraction correlation using a wide range of literature database in order to estimate the parameters of the drift-flux model, i.e., distribution parameter and local drift velocity. The main objective of this study is to compare the results of total pressure drop applying their set equation with the results using the classical parameters of drift-flux model dependent flow pattern simulated by Lima (2011) and experimental data collected by this author, Bueno (2010), Rosa and Mastelari (2008) and Owen (1986). A steady, one-dimensional and isothermal air-water flow through pipes with 0.026 m and 0.032 m internal diameters were assumed. The frictional pressure drop was estimated using the homogenous approach. The expected result with this analysis is to verify the possibility of despising the use of flow pattern maps in the description of the air-water hydrodynamic behavior.*

Keywords: *Drift-flux model, Flow pattern dependence, Isothermal flow*

1 INTRODUCTION

Multiphase flow is found in nature (raindrops, sediment transport in rivers and estuaries, fog formation) and in industrial activities, such as producing oil and gas wells, chemical and nuclear reactors, thermal power plants, food production, aerospace and automotive industries. The term multiphase flow is used to refer to any fluid flow consisted of more than one phase or component, for example, a two phase flow can be structured by two immiscible compounds or phases: liquid-liquid, solid-gas, solid-liquid or gas-liquid.

In gas-liquid flow the phases can assume different spatial configurations inside the duct, named flow patterns, which are influenced by phase velocities, pressure, temperature, densities, viscosities, surface tension, pipe diameter and inclination. At low gas flow rates, the gas phase tends to rise through the continuous liquid medium as discrete bubbles (bubbly flow) and the pressure drop is less affected by the gas presence. An increase in the gas velocity causes a coalescence of the small bubbles and form larger bubbles. At sufficiently high gas flow rates, the agglomerated bubbles become large enough to occupy almost the entire pipe cross section (slug flow) and both phases significantly interfere in pressure gradient. Higher flow rates increase the shear stress between the Taylor bubble and the liquid film inducing a breakdown of the liquid film and the bubbles and some liquid can be entrained in the continuous gas phase as droplets, while the rest of the liquid flows up the wall through the annulus formed by the pipe wall and the gas core (annular flow) and the pressure drop is mainly caused by the presence of gas phase (Ishii and Hibiki, 2010).

The flow pattern, relative velocity, void fraction and phases head loss are important parameters to determine the heat and mass transfer processes in order to analyze the economic viability, environmental risks as well as process optimization. Those data are generally determined by experimental procedures but it is limited to the physical laboratory conditions, which are why commercial softwares have been developed to gas-liquid flow, such as OLGA®, TACITE® and LedaFlow® (Bai and Bai, 2005). However, the correlations and physical models are not thoroughly known by the users, who need to explore different cases of practical situations.

Lima (2011) developed a computational code to describe air-water, isothermal flow in permanent regime based on drift-flux model, which is extensively used to describe flow behavior of gas-liquid systems and considered the primitive variables like a pseudo-homogeneous mixture (França and Lahey, 1992). The relative motion between the phases was determined using specific set equation for each flow pattern. These parameters and pattern flow dependence were disestablished by Bhagwat and Ghajar (2014) through a flow pattern independent void fraction correlation.

The objective of this work is to compare experimental and Lima (2011) simulation data with the results obtained by applying Bhagwat and Ghajar (2014) approach in a steady, one-dimensional, no mass transfer and an isothermal air-water flow. This comparative analysis may reveal some advantages of using the approach adopted.

2 PROBLEM FORMULATION

This section introduces the main equations used to the development of the computational code which was employed to solve the momentum and continuity equations for two-phase air and water flow in a pipe.

Some simplifying assumptions were considered, such as: (i) steady-state, (ii) one-dimensional, (iii) isothermal flow, (iv) vertical pipe, (v) single pipe of constant cross-sectional area, (vi) flow without mass change and (vii) negligible surface tension effects, i.e., there is no pressure gradient between the phases in a section.

The required equations to calculate the pressure drop through gas-liquid drift-flux model are presented.

2.1 Mass and momentum equations

The mass conservation equations in axial direction z for gas and liquid phase, according to the hypothesis above, are Eqs. (1) and (2), respectively.

$$\frac{d}{dz}\left[\alpha\rho_G U_G\right]=0 \tag{1}$$

$$\frac{d}{dz}\left[(1-\alpha)\rho_L U_L\right]=0 \tag{2}$$

Where the sub-indexes G and L are used to represent gas and liquid phase, respectively. The density for the incompressible liquid (water) is ρ_L = 997 kg/m³ and for gas phase (air) is ρ_G, calculated by the ideal gas behavior at 293 K.

The drift flux model assumes that the momentum equation is obtained by considering the two-phase flow as a gas-liquid mixture governed by gravitational force due to the inclination angle θ and wall shear force according to Eq. (3).

$$\frac{d}{dz}\left[\alpha\rho_G U_G^2+(1-\alpha)\rho_L U_L^2+P\right]=-\frac{\tau_w S}{A}-\rho g\sin\theta \tag{3}$$

Where S is the pipe perimeter, A is the cross sectional area and g is de standard gravitational acceleration. The mixture density ρ and viscosity μ can be estimated by Dukler et al. (1964) expression. Equation (4) generalizes these properties by ψ as an average of phases densities or viscosities weighted by the void fraction α (gas fraction).

$$\psi=\alpha\psi_G+(1-\alpha)\psi_L \tag{4}$$

Noting that the drift-flux model has five unknown variables: pressure P, gas phase velocity U_G, liquid phase velocity U_L, void fraction α and wall shear stress τ_w.

The gas phase velocity can be estimated by the drift kinematic law proposed by Zuber and Findlay (1965) according to Eq. (5), which has two parameters: distribution parameter C_0 and local drift velocity V_G.

$$\frac{J_G}{\alpha}\equiv U_G=C_0 J+V_G \tag{5}$$

In the open literature, there are several correlations to calculate the distribution parameter and the local drift velocity through an independent or dependent flow pattern equation. Bhagwat and Ghajar (2014) developed a general void fraction equation using a wide range of fluid combination, pipe hydraulic diameter and pipe orientation, in terms of the drift-flux parameters defined by Eqs. (6) and (7) for vertical flow.

$$C_0=\frac{2-\left(\rho_G/\rho_L\right)^2}{1+(\mathrm{Re}/1000)^2}+\frac{C_1}{1+(1000/\mathrm{Re})^2} \tag{6}$$

$$V_G = 0.35\sqrt{\frac{gD\Delta\rho}{\rho_L}}(1-\alpha)^{1/2}C_2C_3 \tag{7}$$

The parameters C_1, C_2 and C_3 are function of fluid themophysical properties: phases densities, ρ_G and ρ_L, liquid viscosity, μ_L, and surface tension, σ, and quality, x, phases superficial velocities, J_G and J_L, and hydraulic pipe diameter, D, according to Bhagwat and Ghajar (2014) definitions given by Eqs. (8), (9) and (10) for circular pipe.

$$C_1 = 0.2\left(1-\sqrt{\rho_G/\rho_L}\right)\left[\left(2.6-J_G/J\right)^{0.15}-\sqrt{C_f}\right]\left(1-x\right)^{3/2} \tag{8}$$

$$C_2 = \begin{cases} \left[0.434/\log\left(10^3\mu_L\right)\right]^{0.15} & \text{if} \quad \mu_L > 10^{-2} \\ 1 & \text{if} \quad \mu_L \le 10^{-2} \end{cases} \tag{9}$$

$$C_3 = \begin{cases} \left(40\mathrm{La}\right)^{0.9} & \text{if} \quad \mathrm{La} < 0.025 \\ 1 & \text{if} \quad \mathrm{La} \ge 0.025 \end{cases} \tag{10}$$

Where $\mathrm{La} = [\sigma/(g\Delta\rho)]^{1/2}/D$ is the Laplace variable, that is essentially the inverse of the non-dimensional hydraulic pipe diameter defined by Kataoka and Ishii (1987), and $\Delta\rho = \rho_L - \rho_G$ is the density difference.

The mixture Reynolds number introduced in Eq. (6) was defined by Eq. (11) in terms of mixture superficial velocity, $J = J_G + J_L$, pipe hydraulic diameter, D, and liquid phase density and viscosity, ρ_L and μ_L.

$$\mathrm{Re} = \frac{JD\rho_L}{\mu_L} \tag{11}$$

The frictional pressure gradient of two-phase flow was determined from the homogeneous mixture hypothesis, two-phase multiplier model proposed by Friedel (1979) and Beggs and Brill (1973) expression. For all cases, the Fanning friction factor C_f for turbulent flow regime was estimated using the Haaland (1983) correlation for a pipe with roughness ε, according to Eq. (12).

$$C_f = \left\{-3.6\log\left[\left(\frac{\varepsilon}{3.7D}\right)^{1.11}+\frac{6.9}{\mathrm{Re}}\right]\right\}^{-2} \tag{12}$$

For calculation of the frictional pressure gradient using the homogeneous mixture hypothesis, the wall shear stress τ_w is defined according to the Eq. (13).

$$\tau_w = \frac{1}{2}C_f\rho J\left|J\right| \tag{13}$$

3 SOLUTION METHODOLOGY

The solution algorithm was implemented in Fortran 90 programming language. The equation set was solved applying an implicit in pressure fourth order Runge-Kutta (RK) algorithm. It was used a fixed integration step, which corresponds to one pipe diameter for each tested case.

As the phase velocities, void fraction and gas density are dependent with the pressure the momentum equation were solved iteratively for each RK step through the implicit equation $f(P)$, Eq. (14), which solution was obtained by the secant method.

$$f(P) = P + G_G U_G + G_L U_L - \varpi = 0 \tag{14}$$

Where ϖ is the dependent variable used in numerical integration by RK algorithm. The solution algorithm was done up to the deviation of $f(P)$ reach in 10^{-1} Pa for each step of RK procedure from the beginning of the duct to its end. Figure 1 shows a simplified flowchart for the solution algorithm with the needed initial conditions and numerical integration arrangement described above. The liquid superficial velocity, J_L, and phases mass flux, G_G and G_L, are constant because there is no mass transfer.

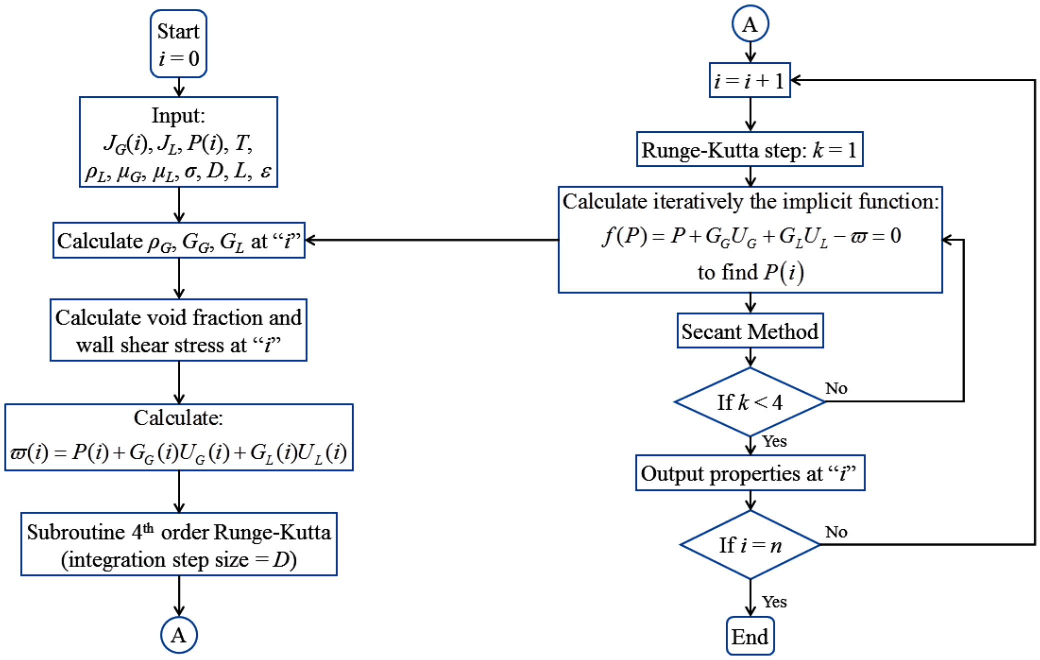

Figure 2. Flowchart for the solution algorithm of the drift-flux model

4 RESULTS AND DISCUSSION

The results of present approach was compared using experimental and drift-flux dependent flow pattern simulated data. Besides the methodolody used in this work does not require previous knowledge of phase distribution in the pipe, three different scenarios were analyzed: dispersed, separated and intermittent pattern.

For the investigated flow patterns, the pressure gradient were calculated using drift-flux model, using Bhagwat and Ghajar (2014) definitions, by three setups: with homogeneous mixture wall friction model (BGH), with Friedel's (1979) correlation (BGF) and with Beggs and Brill (1973) equation (BGB).

Table 1 shows the operational conditions to dispersed upward flow in vertical pipe with 0.026 m internal diameter and 4.68 m length. Those experimental data were collected by Lima (2011), Bueno (2010) and Rosa and Mastelari (2008), and consist of phases superficial velocities, J_G and J_L, pressure gradient, $\Delta P/L$, and absolute pressure at pipe outlet, P_{outlet}.

Table 1. Operational conditions for dispersed upward flow of air and water in a vertical pipe with 0.026 m internal diameter and 4.68 m length

Test [#]	J_G [m/s]	J_L [m/s]	$\Delta P/L$ [kPa/m]	P_{outlet} [kPa]	Test [#]	J_G [m/s]	J_L [m/s]	$\Delta P/L$ [kPa/m]	P_{outlet} [kPa]
1	0.123	0.60	9.6	107.3	12	0.209	1.19	9.7	108.5
2	0.196	0.60	8.9	106.0	13	0.130	1.21	10.0	109.0
3	0.214	1.18	9.8	88.3	14	0.515	2.12	10.6	110.1
4	0.281	2.16	11.0	83.9	15	0.264	2.16	11.1	100.2
5	0.189	2.22	11.3	80.1	16	0.152	2.22	11.4	87.2
6	0.262	0.90	8.1	108.2	17	3.038	2.86	12.0	128.1
7	0.246	1.18	8.8	111.3	18	1.716	2.92	12.0	121.2
8	0.450	1.24	8.6	115.4	19	0.925	2.95	12.3	128.7
9	0.143	0.29	8.3	105.7	20	0.522	2.99	12.3	110.5
10	0.132	0.60	9.1	107.2	21	0.273	3.05	12.6	93.6
11	0.213	0.61	8.5	106.3	22	0.159	3.09	13.2	88.9

A comparison between the results obtained by Lima (2011) simulated data, results of this work (BGF and BGB) and homogeneous flow (HH), which is frequently used in dispersed flow pattern, is presented in Fig. 2. Mostly results obtained by drift-flux model presented pressure gradient percentage error less than ±10%. This low value was expected once this pattern flow is strongly dependent with the gravitational force, which depends on the mixture density and void fraction.

Figure 2 shows that the equations used to determine the frictional pressure gradient and void fraction interfere on predict pressure gradient value, which was underestimated for non-slip flow. Besides, the results of BGB approach are in good agreement with Lima (2011) for 14 cases among the 22 present in the Tab. 1.

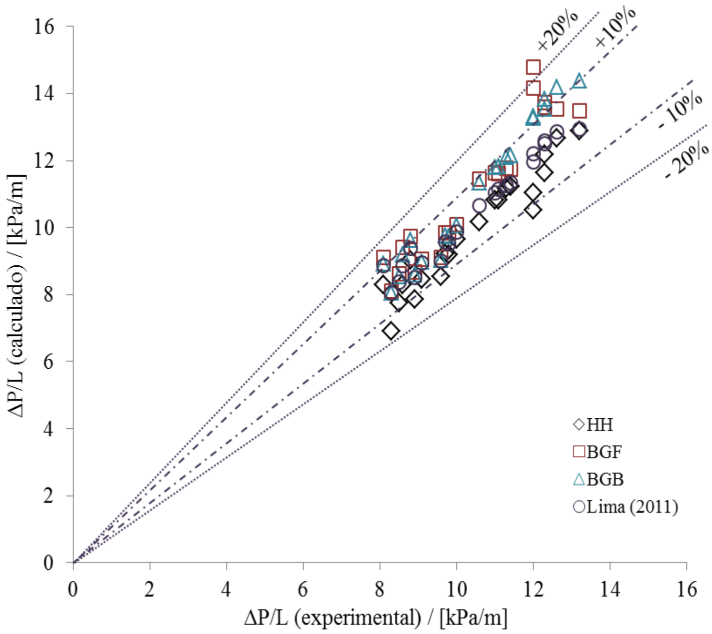

Figure 2. Pressure gradient comparison for dispersed upward flow of air and water in a vertical pipe with 0.026 m internal diameter and 4.68 m length

Table 2 lists from Test 1 to 18 the experimental data obtained by Rosa and Mastelari (2008) and Lima (2011) for annular and semi-annular upward flow in vertical pipe with 0.026 m internal diameter and 4.68 m length. From Test 19 to 36 the experimental data were obtained by Owen (1986) for the same flow pattern and pipe orientation describe above in a pipe with 0.032 m internal diameter and 1.25 m lenght. Those experimental data consist of phases superficial velocities, J_G and J_L, pressure gradient, $\Delta P/L$, and absolute pressure at pipe outlet, P_{outlet}.

Table 2. Operational conditions for separated upward flow of air and water in a vertical pipe with: (Tests 1-18) 0.026 m internal diameter and 4.68 m length; (Tests 19-36) 0.032 m internal diameter and 1.25 m length

Test [#]	J_G [m/s]	J_L [m/s]	$\Delta P/L$ [kPa/m]	P_{outlet} [kPa]	Test [#]	J_G [m/s]	J_L [m/s]	$\Delta P/L$ [kPa/m]	P_{outlet} [kPa]
1	19.09	0.23	3.4	99.0	19	29.11	0.401	5.07	240.0
2	17.33	0.35	4.1	101.5	20	28.79	0.401	5.35	240.0
3	24.61	0.62	7.6	114.8	22	20.96	0.401	4.39	240.0
4	11.09	0.58	4.6	101.7	23	19.47	0.401	4.22	240.0
5	9.76	0.59	4.8	111.3	24	17.84	0.401	4.00	240.0
6	14.36	1.20	9.9	131.6	25	16.53	0.401	3.94	240.0
7	9.09	1.28	7.8	124.8	26	15.82	0.401	3.88	240.0
8	8.11	1.99	10.0	133.4	27	14.68	0.401	3.84	240.0
9	5.39	2.03	9.5	130.7	28	5.83	0.199	2.32	240.0
10	3.06	2.09	9.4	122.5	29	5.98	0.199	2.24	240.0
11	11.02	2.09	13.2	151.7	30	6.40	0.199	2.19	240.0
12	7.71	2.69	12.9	146.8	31	17.54	0.199	2.84	240.0
13	4.76	2.79	12.2	138.0	32	5.53	0.401	2.94	240.0
14	21.06	0.22	3.2	103.9	33	5.92	0.401	3.07	240.0
15	20.19	0.240	3.30	104.4	34	8.51	0.401	3.20	240.0
16	17.58	0.350	4.00	107.5	35	10.11	0.401	3.30	240.0
17	16.54	0.650	6.30	116.0	36	12.38	0.401	3.56	240.0
18	19.78	1.200	11.60	144.0					

Figure 3a shows a similar pressure gradient prediction for BGB and Lima (2011) simulated results, with the root mean square of the relative deviations (RMS) 7.3% and 5.4%, respectivelly, which can be assumed that the results obtained by these models converge to the experimental data for both cases. The BGF results superestimated the literature data for gradient pressure up to 9 kPa/m and high gas velocities. Mostly results obtained by BGF presented pressure gradient percentage error between +10% and +20%.

Figure 3b shows that for highest pressure as well as superficial liquid velocities and bigger pipe diameter, the RMS calculated for BGF, BGB and Lima (2011) were similar and equal to 6.1%, 7.8% and 8.1%, respectivelly. Mostly results obtained by all approachs presented pressure gradient percentage error less than ±10%.

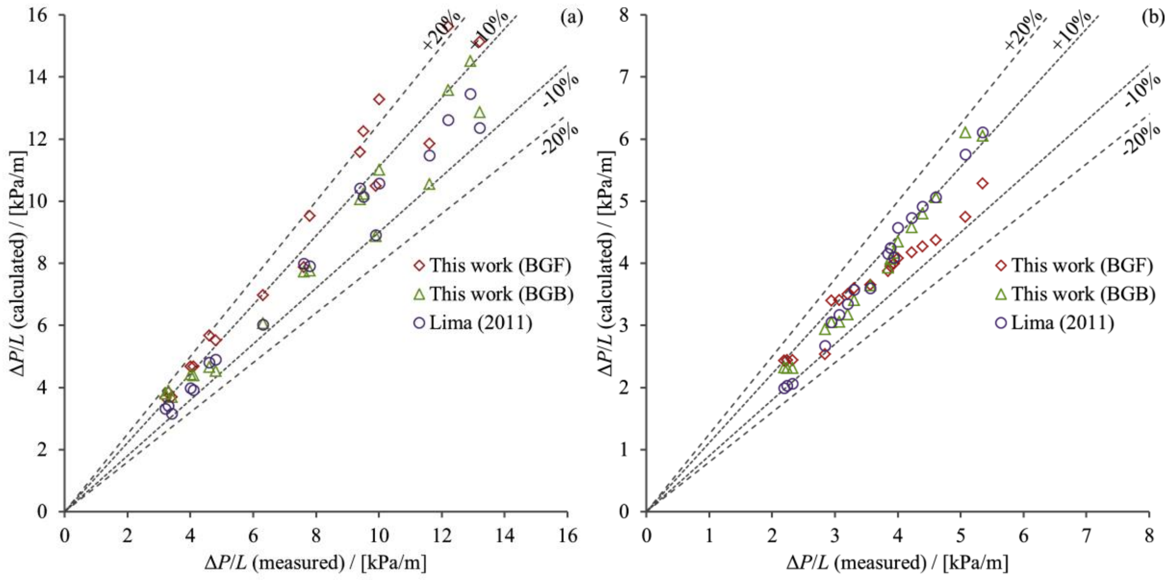

Figure 3. Pressure gradient comparison for separated upward flow of air and water in a vertical pipe with: (a) 0.026 m internal diameter and 4.68 m length (b) 0.032 m internal diameter and 1.25 m length

Table 3 shows the operational conditions to intermittent upward flow in vertical pipe with 0.026 m internal diameter and 4.68 m length. Those experimental data were collected by Lima (2011), Bueno (2010) and Rosa and Mastelari (2008)), and consist of phases superficial velocities, J_G and J_L, pressure gradient, $\Delta P/L$, and absolute pressure at pipe outlet, P_{outlet}.

Table 3. Operational conditions for intermittent upward flow of air and water in a vertical pipe with 0.026 m internal diameter and 4.68 m length

Test [#]	J_G [m/s]	J_L [m/s]	$\Delta P/L$ [kPa/m]	P_{outlet} [kPa]	Test [#]	J_G [m/s]	J_L [m/s]	$\Delta P/L$ [kPa/m]	P_{outlet} [kPa]
1	0.207	0.29	8.3	105.1	13	1.023	0.61	5.3	108.1
2	0.530	0.33	6.1	101.7	14	0.358	0.79	7.8	108.1
3	2.460	0.35	3.5	98.0	15	0.762	0.88	6.9	112.8
4	0.936	0.37	4.9	99.9	16	0.705	1.18	8.0	119.3
5	1.450	0.39	4.2	99.1	17	2.025	0.30	3.1	104.1
6	0.259	0.58	8.7	105.7	18	1.881	0.63	4.7	107.0
7	0.931	0.60	6.0	101.5	19	0.546	1.20	8.7	107.3
8	0.549	0.61	7.2	103.2	20	1.825	1.24	7.1	110.8
9	2.243	0.64	4.8	100.3	21	1.090	1.25	7.7	107.5
10	0.231	0.28	6.9	106.6	22	1.024	2.13	10.1	111.7
11	1.651	0.30	3.2	101.9	23	1.856	2.13	9.6	115.1
12	0.519	0.61	6.6	107.4					

Figure 4 shows that the pair mixture model and homogeneous friction (HF) approach employed by Lima (2011) presents the result for pressure gradient equal than 9.2% and BGH, 6.2%. Besides, Lima's results applying drift-flux model and phenomenological friction (PF) approach is in accordance with BGB, 5.6% and 6.0%, respectively. These mean that for intermittent flow, the equation proposed by Bhagwat and Ghajar (2014) is able to well-predict the pressure drop in the operational conditions described.

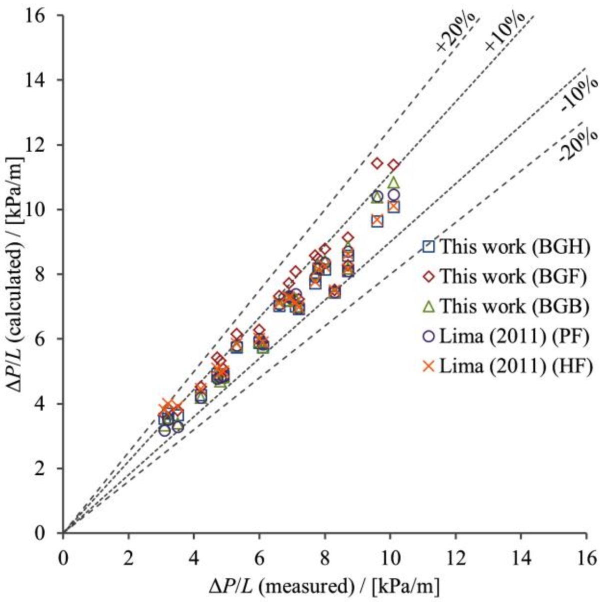

Figure 4. Pressure gradient comparison for intermittent upward flow of air and water in a vertical pipe with 0.026 m internal diameter and 4.68 m length

5 CONCLUSIONS

A flow pattern independent void fraction correlation developed by Bhagwat and Ghajar (2014) with Friedel (1979) and Beggs and Brill (1973) wall shear stress models were used for modeling the pressure gradient of air-water flow. The results showed that this approach predicted well the pressure gradient distribution compared to the existing experimental data available in the literature and results obtained by flow pattern dependent void fraction for the operational conditions analyzed in dispersed (bubbly flow), separated (annular flow) and intermittent (slug flow), which reveals a strong point of this formulation, once the simulated approach goes without regime transition criteria of two-phase flow.

The results obtained also showed that the wall shear stress model by Beggs and Brill (1973) proved to be slightly better than the one proposed by Friedel (1979), both used in this study. For horizontal or quasi-horizontal flow cases, the wall shear force become dominant, and thus further analysis regarding the approach to estimate the frictional pressure gradient become necessary.

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