Universidade Estadual de Campinas
Faculdade de Ciências Aplicadas

André Ignacio Lara Rodriguez

Forecasting Volatility in the FOREX and Bitcoin Market
A model comparison

Limeira
2017

Universidade Estadual de Campinas
Faculdade de Ciências Aplicadas

André Ignacio Lara Rodriguez

Forecasting Volatility in the FOREX and Bitcoin Market
A model comparison

Trabalho de Conclusão de Curso apresentado como resquisito parcial para a obtenção do título de Bacharel em Engenharia de Produção à Faculdade de Ciências Aplicadas da Universidade Estadual de Campinas

Orientador: Prof. Dr. Luiz Eduardo Gaio

Limeira
2017

Agência(s) de fomento e nº(s) de processo(s): Não se aplica.

Ficha catalográfica
Universidade Estadual de Campinas
Biblioteca da Faculdade de Ciências Aplicadas
Renata Eleuterio da Silva - CRB 8/9281

	Lara Rodriguez, André Ignacio, 1994-
R618p	Previsão de volatilidade no mercado FOREX e de bitcoin : uma comparação de modelos / André Ignacio Lara Rodriguez. – Limeira, SP : [s.n.], 2017. Orientador: Luiz Eduardo Gaio. Trabalho de Conclusão de Curso (graduação) – Universidade Estadual de Campinas, Faculdade de Ciências Aplicadas. 1. Volatilidade (Finanças). 2. Câmbio. 3. Modelo GARCH. I. Gaio, Luiz Eduardo,1983-. II. Universidade Estadual de Campinas. Faculdade de Ciências Aplicadas. III. Título.

Informações adicionais, complementares

Título em outro idioma: Forecasting volatility in the FOREX and bitcoin market: a model comparison
Palavras-chave em inglês:
Volatility (Finances)
Exchange
GARCH model
Titulação: Bacharel em Engenharia de Produção
Banca examinadora:
Daniel Henrique Dario Capitani
Data de entrega do trabalho definitivo: 29-11-2017

RODRIGUEZ, A. I. L. **Previsão de volatilidade no mercado de FOREX:Uma comparação de modelos.** 2017. 32 f. Trabalho de Conclusão do Curso (Graduação em Engenharia de Produção) Faculdade de Ciências Aplicadas. Univesidade Estadual de Campinas. Limeira, 2017

Resumo

O cambio de moedas é o maior dos mercados financeiros e, recentemente, novas moedas como e-currencies começam a aparecer e atrair a curiosidade dos investidores, com isso em mente estudos são necessários para entender sua viabilidade. Volatilidade é um dos mais importantes fatores que instituições financeiras devem pesquisar para descobrir o risco nas transições. Esse papel irá testar os modelos ARCH, GARCH, TARCH, EGARCH e APARCH nos mercados de Euro-USDollar, Yen-USDollar, Euro-Yen, Bitcoin-USDollar, Bitcoin-Euro e Bitcoin-Yen. Nós também testamos uma abordagem out-of-sample para checar se os modelos ainda possuíam uma previsão satisfatória. A analise dos resultados mostram que o EGARCH é o modelo que melhor se adapta aos dados da maioria dos mercados, que o modelo APARCH melhor se adapta em relação ao tempo e espaço n maioria dos mercados e ainda provem os melhores resultados na abordagem out-of-sample.

RODRIGUEZ, A. I. L. **Previsão de volatilidade no mercado de FOREX:Uma comparação de modelos.** 2017. 32 f. Trabalho de Conclusão do Curso (Graduação em Engenharia de Produção) Faculdade de Ciências Aplicadas. Univesidade Estadual de Campinas. Limeira, 2017

Abstract

Currency exchange is the largest financial market, recently, new currencies such as e-currencies start to appear and increase the curiosity of investors, in this sense some studies are necessary to understand their viability. Volatility is one of the most important factors financial institutes have to research to discover the risk in transactions. This paper will test the ARCH, GARCH, TARCH, EGARCH and APARCH models in the Euro-USDollar exchange market, the Yen-USDollar exchange market, the Euro-Yen exchange market, the Bitcoin-USDollar exchange market, Bitcoin-Euro exchange market and Bitcoin-Yen exchange market. We also tested with and out-of-sample approach to check if the model would still give satisfactory forecasting. The results show that the EGARCH was the model that best adapted to the data in most markets, and the APARCH model was the one that best adapted to time and space in most markets and still would give the best results in the out-of sample approach.

1- Introduction

The financial market provides ways of gaining earnings from goods, services, tax rates and even currencies. Participants of this sort of transaction can be from up to professionals who studied the subject extensively to people who never dealt with this market and receive help from banks to operate. The investments can be made at random, but the majority tries to rely on models to predict the risk involved in each sector and operation.

The Foreign Exchange Market (also called Forex or FX) deals with currencies and with an estimated value of $1 trillion traded every day, in 2000, is the largest and most liquid of the financial market (YAO; TAN, 2000). Some popular markets and those assessed in this study are the EUR/USD, JAP/USD and EUR/JAP. Since most of the countries work with non-fixed exchange rates, the transactions are dynamic and take into account exchange parities, inflation, production, consumption, interest rate levels, foreign currency reserves, income and others (PIMENTA JUNIOR; LIMA; GAIO, 2014).

A new currency emerging in the globalized world are electronic currencies, or e-currency, and just like other types of currencies they work on transactions of goods and services, but unlike the normal currencies they aren't backed up by governments and should aim for some specific properties: Security, Anonymity, Scalability, Acceptability, Offline Operation, Transferability and Hardware Independence(MEDVINSKY; NEUMAN, 1993).

The BitCoin is one E-currency based on cryptographic proof, instead of needing a "trusted third party" to intermediate between two parties. It uses a "peer-to-peer distributed timestamp server to generate computational proof of the chronological order of transactions" according to Nakamoto, the pseudonym of the creator (or group) of the BitCoin (NAKAMOTO, 2008). Its market value is of US$4175, approximately, and comparing it to its value in 2011 of US$2,5 (GRINBERG, 2012) it had a growth of 1670% in the last 6 years.

According to Kumar (2006) more complex models give better results, like the Exponential weighted moving average and GARCH models, also Yao e Tan (2000) uses neural network to accurately forecast forex. There are a lot of models for forecasting volatility but there is still no consent for the best model in the academia.

Banks and Financial institutions are obliged to provide the volatility and the VAR for investments and by providing the best forecasting this institutions give the best answers to their clients and can more accurately describe the market. Some models are too complicated to rely and simpler models can provide similar results.

So what model gives the most accurate answer for the volatility? The current paper seeks to test existing models on the volatility of currencies in the forex market, American dollars, Japanese yens and Euros, and also check the volatility of the BitCoin when compared to said currencies. For that it will be necessary to:

- Gather data from the period
- Study each model
- Test the models
- Analyze and compare the results
- Decide which the best model is

2- Foreign Exchange Market

The Foreign Exchange Market, also called FOREX, Currency Market and FX, is the financial market from trading of currencies, is the most liquid market, decentralized and over-the-counter. Despite being used by many people its biggest user are banks and financial firms, having 95% of the trading occurring between them, rather than costumers (FRANKEL, FROOT, 1990). It's the biggest of the financial markets with an estimated value of $1 trillion traded every day, in 2000 (YAO; TAN, 2000).

The trades are made based on how one currency compares to another, such as US Dollar and Japanese Yen or Euro and Great British Pound for example. It's necessary to make such comparison on currencies.

The real exchange rate of a country will be decide by its overall competitiveness, so when it starts depreciating it will induce a switch in the demand from foreign goods to domestic ones (FLASSBECK, LA MARCA, 2009). The value of one currency in comparison to others is measured on how competitive is the country, how safe it is and what is the overall sentiment of the world on it, Engle and Victor (1993) show models that measure the impact of news on volatility.

According to the Bank for International Settlements, trading in FX averaged $5.1 trillion per day in April 2016, down from $5.4 trillion in April 2013, for the first time since 2001 spot transactions fell, from $2.0 trillion in 2013 to $1.7 trillion in April 2016. The US Dollar is on one side of 88% of the transactions. Other strong currencies such as the Australian Dollar, the Euro and the Yen lost market share, while emerging countries increased theirs, the renminbi, Chinese currency doubled its share to 4% and has 95% of his trading against the US dollar. The United Kingdom, the United States, Singapore, Hong Kong and Japan intermediated 77% of all foreign exchange trading according to Triennial Central Bank Survey of foreign exchange and OTC derivatives markets in 2016.

.

3- BitCoin

The BitCoin is an electronic currency created by someone or a group under the pseudonym Satoshi Nakamoto (NAKAMOTO, 2008). It was created under the premise that today's online commerce have to rely on financial institutions to act as a trusted third party, this third party will normally do its job, but will charge for it. While this work well for medium to big transactions, small casual fees won’t be possible, because of the fee cost.

What Nakamoto offers is an electronic payment system based on cryptographic proof so that parties can negotiate directly. The transactions are irreversible so the sellers are protected, and routine escrow mechanisms protect the buyers.

Each owner of a digital signature can transfer Bitcoins to the other and all transactions will be publicly announced in a timestamp block, each block will carry a proof of work and the machine that generates this proof of work will be awarded Bitcoins, this is the process also known as mining BitCoins. The mining process increase the security of the system and confirms transactions, each block is generated every 10 minutes and after 210.000 transaction block, the reward will drop 50%, for example in 2009 one transaction block solved awarded 50 bitcoins, in 2013 the same would award 25 bitcoins.

One of the BitCoin's biggest problem is it legal status, although being decentralized stops any country from completely shutting down BitCoin, if one such as the USA would make it illegal, be it from the government monopoly on currencies or security regulations(*U.S. Constitution*), the coin could lose its value due to the untrust of its users(GRINBERG, 2012).

This market is still in its early stage, most address have never participated in any transaction, the majority of the transactions that take place move a fraction of a BitCoin and still there are hundreds of transactions that move more than 50000 BitCoins(RON, SHAMIR, 2013).

4- Models

This paper will apply the following Volatility Forecasting Models:ARCH, GARCH, EGARCH, TARCHA, PARCH, FIGARCH, FIAPARCH and HYGARCH.

All models studied are a variation of the ARCH model, the first one developed and the first to be explained.

4.1 ARCH

The Autoregressive Conditional Heteroscedasticity (ARCH) model was developed in 1982 by Robert F. Engle to calculate the variances of United Kingdom's inflation (ENGLE, 1982). One of the premises of this model is that the inflation in any y_t day will depend on y_t day, in other terms, will depend on the past.

$$y_t = \sqrt{\sigma_t^2}\varepsilon_t \quad (1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 \quad (2)$$

, Where y_t is the return and ε_t is a random sequence of variables that follows i.i.d(0,1).

According to Lamoureux and Lastrapes (1990) ARCH is a manifestation of the daily time dependence in the rate of information arrival to the market and can be used in stock markets and foreign exchange market.

4.2 GARCH

The Generalized Autoregressive Conditional Heteroscedasticity was developed by Bollerslev (1986) as an change to perfect the ARCH model, by proposing that beyond the square of the past days y_{t-1}^2, the values itself(σ_t) would affect the volatility, so the new model became:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + v_t \quad (3)$$

Where α is a constant and v_t is a white noise, and ω is a constant. It's a model for short term memory

4.3 EGARCH

The Exponential GARCH was proposed by Nelson (1991) and changed so that changes in variance would be exponential instead of squared, because of it, it incorporates the asymmetric effects of the market and not imposing artificial parameters to the equation (GAIO, PESSANHA, OLIVEIRA, ÁZARA, 2007).

Its simplified form becomes:

$$ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \left(a_i \left(\left| \frac{y_{t-i}}{\sigma_{t-i}} \right| - E \left| \frac{y_{t-i}}{\sigma_{t-i}} \right| \right) + \gamma_i \frac{y_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^{q} \left(\beta_j ln(\sigma_t^2) \right) \quad (4)$$

Where,γ_i allows for asymmetry and if equals to 0 it means there is no asymmetry.The EGARCH models take in account time-series clustering, negative correlation with returns, log normality and even long memory. The model is useful for range data (BRANT, JONES, 2006).

4.4 TARCH

Zakoian (1994) developed the threshold ARCH, which, different than GARCH models, can handle the asymmetry in models when there are moments of high volatility followed by long periods of relative calm (HADSELL, MARATHE, SHAWKY, 2004).

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \gamma y_{t-i}^2 d_{t-1} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + v_t \quad (5)$$

d_{t-1}Is a dummy variable that equals 1 if $y_{t-i} < 0$(bad news) and equals 0 if $y_{t-i} > 0$ (good news).

4.5 APARCH

The asymmetric power ARCH model of Ding, Granger, and Engle (1993) and is one of the most promising ARCH models, for it combines at least 7 ARCH models (LAURENT, 2003), the model is shown below:

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{p} \alpha_i (|y_{t-i}| - \gamma_i y_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta} \tag{6}$$

Where $\delta \geq 0$ and $-1 < \gamma_i < 1$

This model can become the Arch model, the GARCH model, the GJR model of Glosten. Jagannathan e Runkle (1993), the TARCH model, the NARCH model of Higgins and Bera (1992) and the Log-ARCH model of Geweke(1986) by changing the values of δ, γ_i and β_j.

4.6 Other studies on the models

The topic of which model perform best is on debate in the academia and several studies were made comparing different models,Danielsson (1994) used the S&P 500 index from 1980 to 1987 and compared the ARCH, GARCH, IGARCH and EGARCH and found that the EGARCH model outperformed the rest. Varma (1999) compared the GARCH model with the EMWA using Indian stock market data from 1990 to 1998 and discovered that the GARCH gave better results.

A multi country study with stock markets from over 10 countries from 1989 to 1996 said that the APARCH could be used to analyze most countries volatility (BROOKS et al, 2000). In Guidi's study (2008) with AGARCH, EGARCH, PGARCH and TGARCH models on the stock market of UK from 1999 to 2008, Switzerland and Germany and discovered that each model worked better in a different country. Cheong (2009) made a study of crude oil markets from 1993 to 2008 and discovered that the GARCH model outperformed the APRARCH, FIGARCH and DIAPGARCH models.

A study with the Hong Kong stock market using data from 1987 to 2007 said that the TARCH model performed better than the GARCH model(SABIRUZZAMAN et al 2010). Charles (2017) said that on crude oil markets from 1987 to 2007 the GARCH EGARCH and GJRGARCH models would outperform the GAS and the EGAS models. Katsiampa (2017) used the Bitcoin market from 2010 to 2016 to discover that the CGARCH performed better than the GARCH, EGARCH, TGARCH APARCH and ACGARCH models. Lin (2017) said that the EGARCH model worked better with SSE composite market from 2013 to 2017 than the GARCH and TARCH models.

The table 1 shows a summary for all the research collected on the models.

Table 1: Summary with all research on this paper

Researcher - year of publication	Data - period	Best model
Danielsson - 1994	S&P 500 index 1980-1987	EGARCH
Varma - 1999	NSE-50 Indian market 1990 -1998	GARCH
Brocks et al - 2000	Stock market from 10 countries 1989-1996	APARCH
Guidi - 2008	Stock market from UK, Switzerland and Germany 1999-2008	No best model
Cheong - 2009	Crude oil markets 1993-2008	GARCH
Sabiruzzaman et al - 2010	Hong Kong Stock market 1987-2007	TARCH
Charles - 2017	Crude oil markets 1992- 2014	GARCH EGARCH GJRGARCH
Katsiampa - 2017	Bitcoin 2010-2016	CGARCH
Lin - 2017	SSE Composite 2013-2017	EGARCH

5- Hypotheses

According to the studies mentioned above and the theory that the EGARCH mostly perform better than the other models this paper will test the following hypotheses:

H1: The EGARCH will have the best adjustment in the FOREX market

H2: The EGARCH will have the best adjustment in the Bitcoin market

H3: The EGARCH will give the best forecast in the FOREX market

H4: The EGARCH will give the best forecast in the Bitcoin market

The hypotheses are constructed based on the fact that, from the papers researched, the EGARCH model was, in most cases, the best model.

6- Methodology

This paper will conduct its research by gathering data from both the Forex market and the Bitcoin market, testing it so that the models work. After the data is adjusted its run through the ARCH model, GARCH model, EGARCH model, TARCHA model, APARCH model, FIGARCH model, FIAPARCH model and HYGARCH model, while dealing with the possible outliers, like crisis and prosperity boons.

The markets in this study are the USD/EUR, EUR/USD, JAP/USD, USD/JAP, EUR/JAP, JAP/EUR and Bitcoin historical prices. The data for both the Forex currencies and the Bitcoin will be collected daily, from the bid price. For the Forex market currencies the period of data will be taken from January1, 2009 to October 21, 2017, this will signify a period of almost 9 years. For the Bitcoin Market, the biggest possible sample was analyzed, for the Bitcoin-Dollar exchange market it will be used the time from the August 18, 2011 to October 21, 2017, and for the Bitcoin-Euro and Bitcoin-Yen exchange market the period from December 8, 2014 toOctober 21, 2017. Since all of these markets are currency based they have no closing time, so all days will be analyzed.

The normality test used will be the Jarque-Bera test, with its p-value, or the probability of not rejecting the hypotheses of a normal pattern. The trust level will be at 95%. The stationarity test will be done through the Augmented Dickey and Fuller unit root test with its p-value, the probability of series having one unit root. The linearity is tested by Brooks et al. (1996) test based on the correlation dimension. The nullhypothesis is that the data follow the i.i.d. behavior.

Normality test (Jarque and Bera)

Bera and Jarque(1981) created this test to detect the normal distribution, the nule hypotheses is that the series follow a normal distribution. It's shown next:

$$\boldsymbol{Jarque - Bera} = \frac{\boldsymbol{N-k}}{\boldsymbol{6}}\left(\boldsymbol{S^2} + \frac{\boldsymbol{(K-3)^2}}{\boldsymbol{4}}\right) \qquad (7)$$

Where S is asymmetry, K is the kurtosis and k the number of estimated coefficients used in the series.

Stationarity test (Augmented Dickey and Fuller unit root test)

The Dickey and Fuller test is used for when the series are generated by an auto-regressive process of first order and its random members follow a white noise. In this case of higher order the augmented version is used. The Dickey and Fuller test is:

$$\Delta R = \alpha R_{t-1} + x_t'\delta + \varepsilon_t \qquad (8)$$

Where$\alpha = \rho - 1$, the hypotheses null and alternative are:

$H_0: \alpha = 0$
$H_0: \alpha < 0$

It's usual to use

$$t_\alpha = \frac{\alpha\prime}{se(\alpha\prime)} \qquad (9)$$

Whereα' is given by α, and $se(\alpha')$ is the standard error.

The augmented Dickey and Fuller test is:

$$\Delta R = \alpha R_{t-1} + \sum_{i=1}^{p-1} \omega_t \Delta R_{t-1} + \gamma T + u_t \qquad (10)$$

Where$\omega = -\sum_{j=i+1}^{p} p_j t$ is the deterministic trend and u_t is the white noise.

Linearity test (Brooks)

Brooks test based on the correlation dimension (BDS) is used to test the hypotheses of an i.i.d behavior. It can be defined as follow:

$$C_m(\varepsilon) = \lim_{T\to\infty}\left(\frac{1}{(T-m)(T-m+1)}\right) \sum_{i,j=1}^{T} I\left(\left(x_i^m - x_j^m\right) < \varepsilon\right) \quad i \neq j \qquad (11)$$

T is the size of the series,x_t^m and $I(.)$ is the indicating funtion, if the null hypotheses of the i.i.d. then $C_m(\varepsilon) = (C_1(\varepsilon))^m$. If these are the results then the test becomes:

$$BDS_m(\varepsilon) = \sqrt{T}[C_m(\varepsilon) - (C_1(\varepsilon))^m]/V_m^{1/2} \tag{12}$$

Where V_m is the variance expression, described by Cromwell et al. (1994)

Next the analysis of the autocorrelation function and the partial autocorrelation function is done to ensure the series is auto correlated. If there is so it is necessary to make adjustments by an ARMA model (autoregressive-moving-average) to eliminate it. The equation of the autocorrelation function is according to Eviews(2005):

$$\rho_t = \frac{\sum_{t=\tau+1}^{T}(R_t-\bar{R})(R_{t-\tau}-\bar{R})}{\sum_{t=1}^{T}(R_t-\bar{R})^2} \tag{13}$$

Where ρ_t the correlation on the discrepancy τ

For the partial autocorrelation function considers the correlation of two data in the series instead of considering the correlation with data between the data. The equation according to Eviews (2005) is:

$$\varphi_t = \begin{cases} \rho_1 & for\ k = 1 \\ \frac{\rho_t - \sum_{j=1}^{\tau-1}\varphi_{\tau-1,j}\rho_{\tau-j}}{1-\sum_{j=1}^{\tau-1}\varphi_{\tau-1,j}\rho_{\tau-j}} & for\ k > 1 \end{cases} \tag{14}$$

Where φ_t is the parameter from the autoregressive model and $\varphi_{\tau,j} = \varphi_{\tau-1,j} - \varphi_\tau\varphi_{\tau-1,\tau-j}$

After it the heteroscedasticity test is done so that the ARCH models can be applied, the heteroscedasticity presence is vital for this model. For it is applied the ARCH-LM from Engle (1982) in the returns, with lag of 2, 50 and 100. The equation for the probability is as in the equation (13), and $H_o: \beta_s = 0$.

$$e_t^2 = \beta_0 + (\sum_{s=1}^{q}\beta_s\, e_{t-s}^2) + v_t \tag{15}$$

Where e is a residue and β are the parameters of regression.

The fourth step consists in modeling the data into the Gaussian distribution and into each model. The table 3 shows each functional volatility model is shown:

Table 3: Functional form of volatility models.

Model	Functional form
ARCH(1,1)	$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2$
GARCH(1,1)	$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + v_t$
EGARCH(1,1)	$ln(\sigma_t^2) = \omega + \left(a_1 \left(\left\|\frac{y_{t-1}}{\sigma_{t-1}}\right\| - E\left\|\frac{y_{t-1}}{\sigma_{t-1}}\right\|\right) + \gamma_1 \frac{y_{t-1}}{\sigma_{t-1}}\right) + \beta_1 ln(\sigma_t^2)$
TARCH(1,1)	$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \gamma y_{t-1}^2 d_{t-1} + \beta_1 \sigma_{t-1}^2 + v_t$
APARCH(1,1)	$\sigma_t^\delta = \omega + \alpha_1 (\|y_{t-1}\| - \gamma_1 y_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta$

The adjustment for the Gaussian distribution is given by the log-likelihood funtion:

$$L_{norm} = -\frac{1}{2}\sum_{i=1}^{n}(ln(2\pi) + ln(\sigma_i^2) + \varepsilon_i^2) \quad (16)$$

After all models are run through the selection of the best model will be based on categories of accuracy and operability. This is used because there are models that give a lot of adjustment for its many variables tend to perform better but are much harder to use, and there are other who are easier to use and still give fairly good results. That is why this paper will use both Akaike Information Criterion or AIC (AKAIKE. 1974) and the Bayesian Information Criterion or BIC (SCHAWARZ. 1978)

The AIC model is shown below:

$$AIC = 2k - 2ln(L) \quad (17)$$

Where *k* is the number of parameters and *L* is the maximum value of likelihood

The BIC model is based on the AIC model and is:

$$BIC = kln(n) - 2ln(L) \tag{18}$$

Where n is the number of observations.

The model with the lowest AIC and BIC value will be the best model.

Two other tests will be made, the Normalized Mean Square Error (NMSE) and the Normalized Mean Absolute Error (NMAE) fromSchittenkopf et al., 2000. These tests are run to show the overall deviations between prediction and measured values. The equation for each model is shown next:

$$NMSE = \frac{\sqrt{\sum_{t=1}^{T}(r_t^2 - \sigma_t^2)^2}}{\sqrt{\sum_{t=1}^{T}(r_t^2 - r_{t-1}^2)^2}} \tag{19}$$

$$NMAE = \frac{\sum_{t=1}^{T}|r_t^2 - \sigma_t^2|}{\sum_{t=1}^{T}|r_t^2 - r_{t-1}^2|} \tag{20}$$

Where r is the return and σ are the values from the models tested. A low value, in both tests, indicates that a model is performing well in both space and time.

The next step is to make an out-of-sample forecast with all models and submit to NMSE and NMAE test and after it compare it to the in-sample forecast. The forecast will be made for the last 100 days of the series.

7- Results

The following graphs will show the bid of all exchange markets as well as the histogram of the returns. The figure 1 shows the bid exchange from the Euro to the US Dollar from January 1, 2009 to October 21, 2017 and the histogram of the returns from the period.

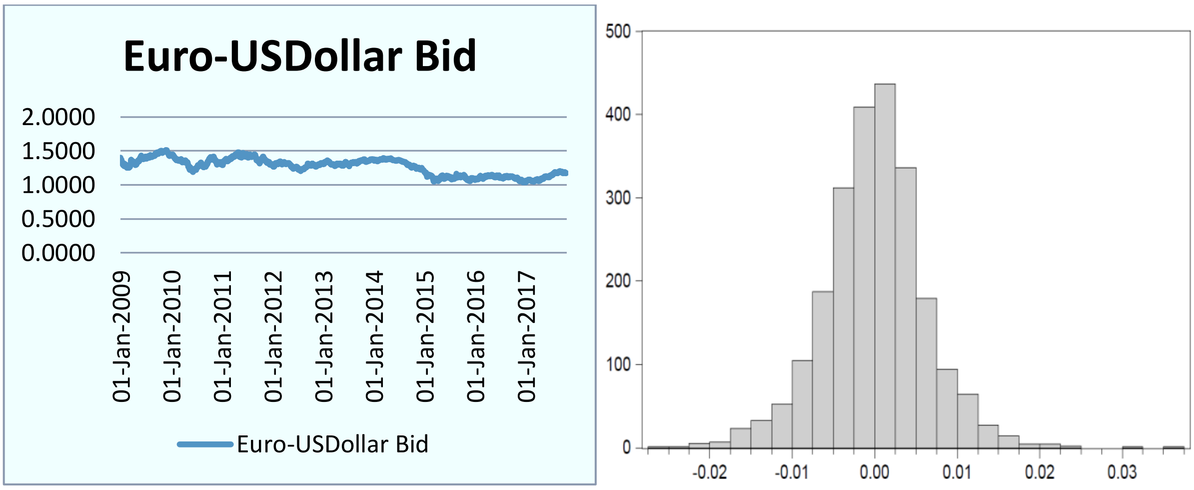

Figure 1: Bid value of the Euro in comparison to the US Dollar from January first 2009, to October 24th 2017.

The graph shows that the Euro since 2009 has lost value overall in comparison to the dollar. It had a minimum value of 1.385 and a maximum of 1.514. The histogram shows an apparent normal curve.

The figure 2 is the bid exchange from the Yen to the US Dollar from January 1, 2009 to October 21, 2017 and the histogram of the returns from the period.

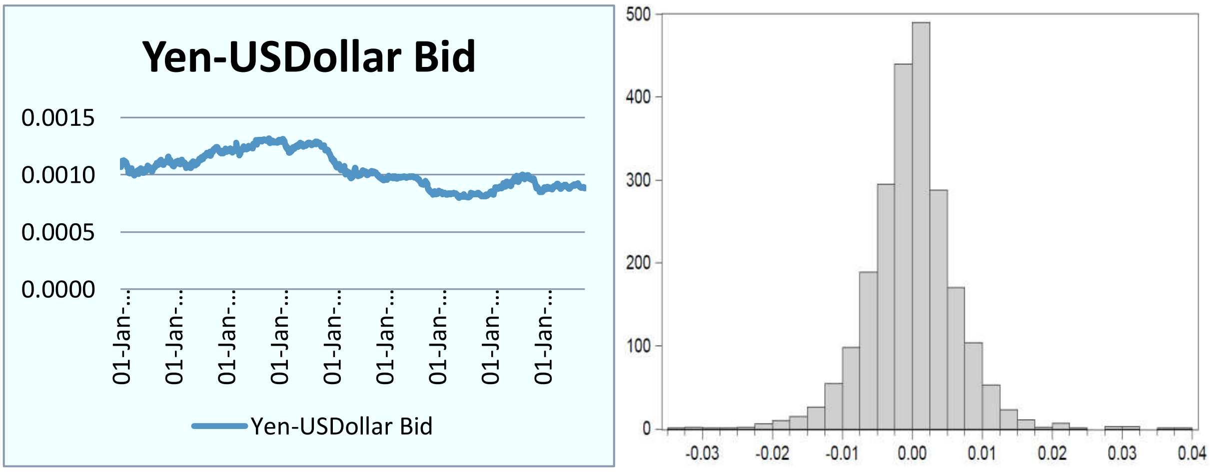

Figure 2: Bid value of the Yen in comparison to the US Dollar from January first 2009, to October 24th 2017.

The first graph shows that the yen lost its value to the dollar since the begging of the period studied. The minimum value of the period was 0.0079 and the maximum was 0.0132. The histogram shows an apparent normal curve.

The figure 3 is the bid exchange from the Euro to the Yen from January 1, 2009 to October 21, 2017 and the histogram of the returns from the period.

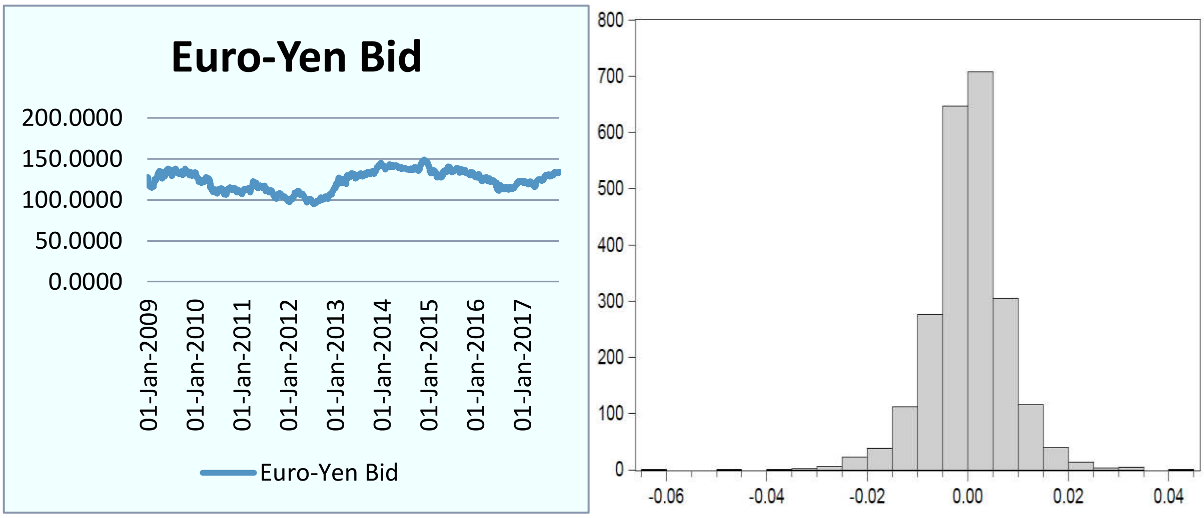

Figure 3: Bid value of the Euro in comparison to the Yen from January first 2009, to October 24th 2017.

The Yen and Euro Exchange Market was stable and their first and last values only are less than 6 values apart. The maximum value was 149.10 and the minimum value was 94.30. The histogram shows an apparent normal curve.

The figure 4 is the bid exchange from the Yen to the US Dollar from January 1, 2009 to October 21, 2017 and the histogram of the returns from the period.

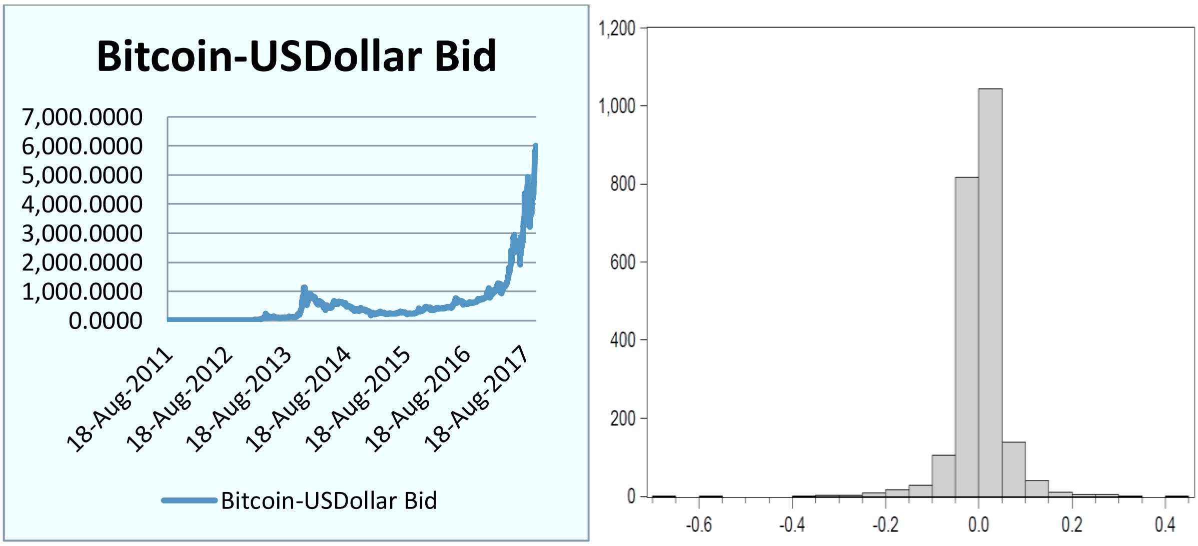

Figure 4: Bid value of the Bitcoin in comparison to the US Dollar from August 18th 2011, to October 24th 2017.

The graph of exchange market for the Bitcoin US Dollar shows a great increase of the value of the Bitcoin currency, it had a low of 2.24 and a high of 6,013.46. The histogram shows a that despite its big increase it doesn't have very big variances in the return and an apparent normal curve.

The figure 5 is the bid exchange from the Bitcoin to the US Dollar August 18, 2011 to October 21, 2017 and the histogram of the returns from the period.

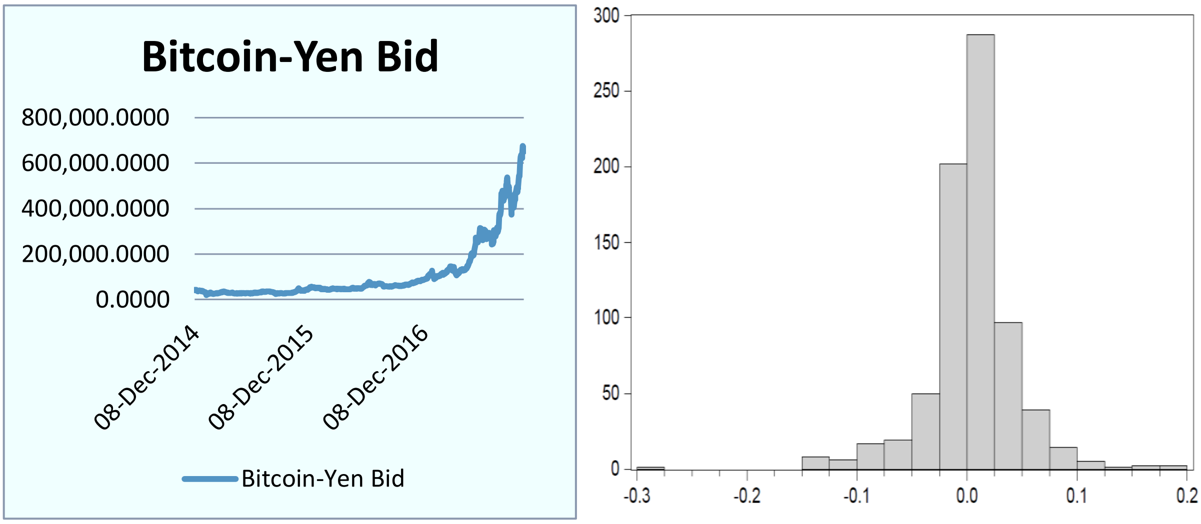

Figure 5: Bid value of the Bitcoin in comparison to the Yen from December 8th 2014, to October 24th 2017.

The graph shows that the Bitcoin since 2014 has increased its value greatly in comparison to the Yen. It had a minimum value of 21,000.28 and a maximum of 675,965.14. The histogram shows an apparent normal curve.

The figure 6 is the bid exchange from the Bitcoin to the Yen from Dezember 8, 2014 to October 21, 2017 and the histogram of the returns from the period.

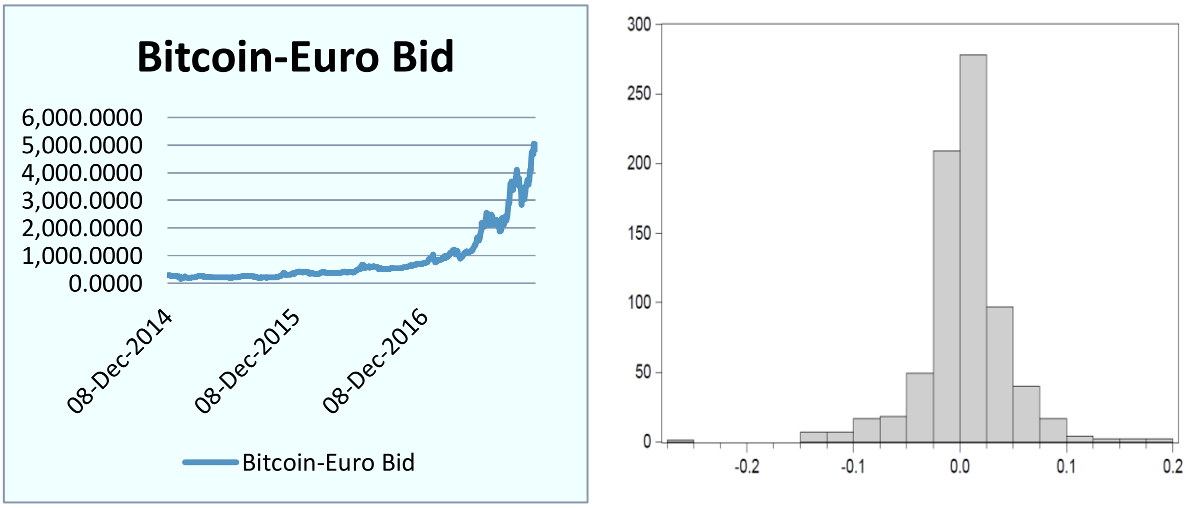

Figure 6: Bid value of the Bitcoin in comparison to the Euro from December 8th 2014, to October 24th 2017.

The first graph shows that the Bitcoin gained value in comparison to the Euro since the begging of the period studied. The minimum value of the period was 298.12 and the maximum was 5,053.15. The histogram shows an apparent normal curve.

All Bitcoin graphs follow the same pattern, they vary in value because each currency has a different value over the others. The US dollar, Euro and Japanese Yen currencies value doesn't gain a big value over each other and tend to go up and down, but isn't on a steady increase like the Bitcoin.

All data collected followed all necessary requirements to use ARCH models as show in table 4.

Table 4: Descriptive statistics, ADF test, ARCH-LM test and BDS test. Varied sample period.

	Euro-US dollar	Yen-US Dollar	Euro- Yen	Bitcoin-US dollar	Bitcoin-Euro	Bitcoin-Yen
Mean	-0.0001	-0.0001	0.0000	0.0028	0.0037	0.0036
Median	0.0001	0.0000	0.0002	0.0022	0.0028	0.0041
Maximum	0.0372	0.0375	0.0418	0.4455	0.1809	0.1905
Minimum	-0.0265	-0.0343	-0.0616	-0.6639	-0.2722	-0.2758
Standard Deviation	0.0062	0.0064	0.0077	0.0529	0.0398	0.0399
Skewness	0.0340	0.1155	-0.3743	-1.4860	-0.4523	-0.5036
Kurtosis	4.7353	6.6542	7.3114	28.7326	9.1538	9.4250
Jarque-Bera	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ADF	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARCH-LM (2)	0.0399	0.0375	0.0046	0.0000	0.0000	0.1720
ARCH-LM (50)	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001
ARCH-LM (100)	0.0000	0.0077	0.0000	0.0000	0.0000	0.0247
BDS(2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
BDS(6)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
BDS(8)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
BDS(10)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: The Jarque-Bera, ADF, ARCH-LM and BDS test are represented by its p-probability values. The ARCH-LM test used tree lags, 2, 50 and 100.

As expected all means of the returns are close to zero, the median of the Bitcoin currencies are bigger, because is a growing market. The p-values of the

Jarque-Bera tests are all zero with indicate that the series follow a non-normality pattern.

The Bitcoin- Dollar exchange has the highest kurtosis, while the Euro-Dollar has the lowest. The unit root test indicates the stationarity of the series, because all p probability areapproximated to 0.

The ARCH-LM test indicates how the market values can oscillate, and the only one that shown big value are the ones tied to the Bitcoin- Yen market since it deals with the biggest value in numbers.

The probability in the BDS test shows that all data doesn't follow an i.i.d. behavior because they approximate to zero. This indicates that there is a temporal dependence between the returns, the future returns depend on the past returns.

The table 5 shows the function of autocorrelation and partial autocorrelation for 5 lags for the returns and squared returns:

Table 5: Autocorrelationand partial autocorrelation for the return and squared return for all data.

Return	Euro- US dollar	Yen- US Dollar	Euro- Yen	Bitcoin- US dollar	Bitcoin- Euro	Bitcoin- Yen
a1(p1)	0.106 (0.106)	0.015 (0.015)	-0.010 (-0.010)	-0.020 (-0.020)	0.024 (0.024)	0.031 (0.031)
a2(p2)	0.075 (0.064)	-0.027 (-0.027)	-0.012 (-0.012)	-0.075 (-0.076)	0.034 (0.034)	0.028 (0.027)
a3(p3)	0.072 (0.059)	-0.029 (-0.028)	-0.047 (-0.047)	0.015 (0.012)	-0.041 (-0.043)	-0.031 (-0.033)
a4(p4)	0.096 (0.080)	0.011 (0.012)	0.024 (0.023)	0.026 (0.021)	-0.021 (-0.02)	-0.028 (-0.027)
a5(p5)	0.089 (0.065)	-0.025 (-0.027)	-0.010 (-0.011)	0.034 (0.037)	0.049 (0.053)	0.047 (0.051)
Squared Return	Euro- US dollar	Yen- US Dollar	Euro- Yen	Bitcoin- US dollar	Bitcoin- Euro	Bitcoin- Yen
a1(p1)	-0.019 (-0.019)	0.094 (0.094)	0.149 (0.149)	0.269 (0.269)	0.213 (0.213)	0.194 (0.194)
a2(p2)	-0.001 (-0.001)	0.080 (0.072)	0.106 (0.086)	0.186 (0.123)	0.080 (0.036)	0.081 (0.045)
a3(p3)	-0.047 (-0.047)	0.054 (0.041)	0.037 (0.010)	0.088 (0.012)	0.071 (0.049)	0.045 (0.022)
a4(p4)	0.008 0.006)	0.030 (0.016)	0.048 (0.034)	0.085 (0.042)	0.079 (0.054)	0.076 (0.063)
a5(p5)	0.017 (0.017)	0.033 (0.023)	0.061 (0.041)	0.186 (0.158)	0.089 (0.060)	0.090 (0.065)

Continue...

Table 5: Continuation

	Euro- US dollar	Yen- US Dollar	Euro-Yen	Bitcoin- US dollar	Bitcoin-Euro	Bitcoin-Yen
$^{2}/_{\sqrt{T}}$*	0.042	0.042	0.042	0.042	0.073	0.073

Note: ai and pi represent the Autocorrelation and partial autocorrelation.
*Asymptotic limit for the autocorrelation function.

Table 5 presents that most of the data are bellow de Asymptotic limit, and therefore doesn't correlate to the past. This is normal according to Fama(1970), since the forecast of markets is expected to be somewhat random, otherwise it wouldn't be an efficient market.

The next tables will show the estimated results and constant values for all models evaluated.The lowest value in BIC and AIC are the best results for this test.As said before the lowest value for NMSE and NMAE will have the lowest overall deviations between prediction and measured values. The best for each test will be in bold. Firstin table 6 it will be presented the estimation results for the Euro-USDollar exchange market.

Table 6: Estimation results for ARCH-type models for the Euro-USDollar exchange market.

	ARCH	GARCH	TARCH	EGARCH	APARCH
ω	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0004)	-0.0818 (0.0000)	0.0000 (0.5293)
α	0.1185 (0.0000)	0.0304 (0.0000)	0.0092 (0.0253)	0.0504 (0.0000)	0.0254 (0.0000)
β		0.9652 (0.0000)	0.9735 (0.0000)	0.9960 (0.0000)	0.9746 (0.0000)
γ			0.0277 (0.0000)	-0.0280 (0.0000)	0.4570 (0.0002)
δ					1.3250 (0.0000)
Log likelihood	8453.864	8590.153	8598.792	**8600.203**	8599.755
AIC	-7.3558	-7.4736	-7.4802	**-7.4815**	-7.4802

Continue...

Table 6: Continuation

	ARCH	GARCH	TARCH	EGARCH	APARCH
BIC	-7.3508	-7.4661	-7.4702	**-7.4715**	-7.4677
NMSE	0.7438	0.7235	**0.7225**	0.7230	0.7234
NMAE	0.8251	**0.7861**	0.7872	0.7872	0.7870

Note: The values in parentheses are the p-probability for direct interpretation of the parameters.

Mostly the models were well adjusted for the data, as notice by the p value being lower than 1%. The model that had the best adjustment was the EGARCH, because of its low AIC and BIC. According to the NMSE test the best model was the TARCH, and according to NMAE it was the GARCH model.

Table 7 shows the estimation results for the models applied in the Yen-USDollar exchange market.

Table 7: Estimation results for ARCH-type models for the Yen-USDollar exchange market.

	ARCH	GARCH	TARCH	EGARCH	APARCH
ω	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	-0.3871 (0.0000)	0.0001 (0.3241)
α	0.1878 (0.0000)	0.0379 (0.0000)	0.0588 (0.0000)	0.1435 (0.0000)	0.0716 (0.0000)
β		0.9512 (0.0000)	0.9373 (0.0000)	0.9724 (0.0000)	0.9242 (0.0000)
γ			-0.0213 (0.0055)	0.0272 (0.0002)	-0.1900 (0.0003)
δ					1.1126 (0.0000)
Log likelihood	8388.798	8457.338	8458.352	**8468.907**	8467.880
AIC	-7.2992	-7.3580	-7.3580	**-7.3672**	-7.3654
BIC	-7.2942	-7.3505	-7.3480	**-7.3572**	-7.3529
NMSE	0.7433	0.7337	0.7348	**0.7334**	0.7340
NMAE	0.8200	0.7939	0.7975	0.7918	**0.7395**

Note: The values in parentheses are the p-probability for direct interpretation of the parameters.

The results for the models indicate that the data was well adjusted, since the p-value was mostly lower than 1%. The EGARCH model had the best test results for the AIC, BIC and NMSE tests and the APARCH model had the best result for the NMAE test.

The estimation results for the Euro-Yen exchange market are shown in table 8.

Table 8: Estimation results for ARCH-type models for the Euro-Yen exchange market.

	ARCH	GARCH	TARCH	EGARCH	APARCH
ω	0 (0.0000)	0 (0.0000)	0 (0.0000)	-0.2565 (0.0000)	0 (0.2906)
α	0.2216 (0.0000)	0.0549 (0.0000)	0.041 (0.0000)	0.1332 (0.0000)	0.0671 (0.0000)
β		0.9378 (0.0000)	0.9355 (0.0000)	0.9841 (0.0000)	0.9358 (0.0000)
γ			0.0287 (0.0017)	-0.03 (0.0001)	0.1989 (0.0004)
δ					1.2667 (0.0000)
Log likelihood	7983.748	8080.236	8083.176	**8090.602**	8088.543
AIC	-6.9467	-7.0298	-7.0315	**-7.0379**	-7.0353
BIC	-6.9417	-7.0223	-7.0215	**-7.0280**	-7.0228
NMSE	0.7601	0.7508	0.7508	0.**7480**	0.7487
NMAE	0.8379	0.8054	0.8045	0.7937	**0.7545**

Note: The values in parentheses are the p-probability for direct interpretation of the parameters.

The p-values for the parameters are mostly 0, which indicates that all models are well adjusted. The model that had the best test results for AIC, BIC and NMSE tests is the EGARCH model. For the NMAE test the APARCH model had the best results.

Table 9 shows the estimation results for the models for the Bitcoin-USDollar exchange Market.

Table 9: Estimation results for ARCH-type models for the Bitcoin-USDollar exchange market.

	ARCH	GARCH	TARCH	EGARCH	APARCH
ω	0.0016 (0.0000)	0 (0.0000)	0 (0.0000)	-0.4027 (0.0000)	0.0005 (0.0010)
α	0.4983 (0.0000)	0.1901 (0.0000)	0.1844 (0.0000)	0.2942 (0.0000)	0.1847 (0.0000)
β		0.8183 (0.0000)	0.8174 (0.0000)	0.9676 (0.0000)	0.848 (0.0000)
γ			0.0138 (0.3685)	-0.0159 (0.0278)	0.0736 (0.0027)

Continue...

Table 9: Continuation

	ARCH	GARCH	TARCH	EGARCH	APARCH
δ					1.2729

					(0.0000)
Log likelihood	3659.896	4098.754	4098.911	4103.102	**4105.595**
AIC	-3.2865	-3.6799	-3.6792	-3.6829	**-3.6843**
BIC	-3.2814	-3.6722	-3.6689	**-3.6727**	-3.6715
NMSE	0.8189	0.8101	0.8113	**0.7918**	0.8005
NMAE	0.9594	0.9486	0.9504	0.8764	0.**7660**

Note: The values in parentheses are the p-probability for direct interpretation of the parameters.

As presented in table 9 most parameters had p-values bellow 1%w, so they are well adjusted. The APARCH model had the best results for the AIC and NMAE test, while the EGARCH model had the best test results for the BIC and NMSE tests.

The estimation results for Models applied to the Bitcoin-Euro exchange market are shown in table 10.

Table 10 Estimation results for ARCH-type models for the Bitcoin-Euro exchange market.

	ARCH	GARCH	TARCH	EGARCH	APARCH
	0.0012	0.0000	0.0000	-0.5021	0.0000
ω	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1977)
	0.2844	0.1604	0.1877	0.3102	0.1544
α	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
		0.8397	0.8449	0.9567	0.8445
β		(0.0000)	(0.0000)	(0.0000)	(0.0000)
			-0.0626	0.0254	-0.1008
γ			(0.0027)	(0.0418)	(0.0014)
					2.0168
δ					(0.0000)
Log likelihood	1379.268	1452.086	1454.073	**1454.830**	1454.060
AIC	-3.6727	-3.8642	-3.8669	**-3.8689**	-3.8642
BIC	-3.6604	**-3.8458**	-3.8422	-3.8442	-3.8334
NMSE	0.7810	0.7921	0.7941	**0.7809**	0.7942
NMAE	0.8653	0.8721	0.8742	0.8424	**0.7660**

Note: The values in parentheses are the p-probability for direct interpretation of the parameters.

In general, the p-value for the parameters arebellow 1%, indicating a well adjustment to the model. The EGARCH model had the best results for the AIC and NMSE tests, GARCH model had the best results for the BIC test and the APARCH model had the best test results for the NMAE test.

Table 11 will show the estimated results for models for the Bitcoin-Yen exchange market.

Table 11: Estimation results for ARCH-type models for the Bitcoin-Yen exchange market.

	ARCH	GARCH	TARCH	EGARCH	APARCH
ω	0.0012 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	-0.4027 (0.0000)	0.0000 (0.1816)
α	0.2701 (0.0000)	0.1649 (0.0000)	0.1948 (0.0000)	0.2942 (0.0000)	0.1544 (0.0000)
β		0.8373 (0.0000)	0.8448 (0.0000)	0.9676 (0.0000)	0.8427 (0.0000)
γ			-0.0702 (0.0010)	-0.0159 (0.0438)	-0.1124 (0.0003)
δ					2.1075 (0.0000)
Log likelihood	1377.109	1445.389	1447.741	1448.591	1447.741
AIC	-3.6670	-3.8464	-3.8500	**-3.8522**	-3.8473
BIC	-3.6546	**-3.8279**	-3.8253	-3.8276	-3.8165
NMSE	**0.7749**	**0.7749**	0.7880	0.7757	0.7894
NMAE	0.8603	0.8596	0.8720	0.8430	**0.7702**

Note: The values in parentheses are the p-probability for direct interpretation of the parameters.

In table 11, is possible to see that the models are well adjusted, since the p-values are mostly below 1%. EGARCH had the best AIC test result, GARCH had the best BIC result and tied with the ARCH model for the best NMSE tests results, while the APARCH model had the best NMAE test results.

The out-of-sample approach used the last 100 data to forecast the results, it was applied to all models and all currencies and tested the NMSE and NMAE for comparison to the in-sample models. Table 12 shows the test results for the NMSE and NMAE test for all markets and all models. The results with the lowest score for both NMSE and NMAE are the ones with less deviation between the prediction and measured values and are in bold.

Table12: NMSE and NMAE test Results for ARCH-type models for the Euro-USDollar exchange market, the Yen-USDollar exchange market, the Euro-Yen exchange market, the Bitcoin-USDollar exchange market, Bitcoin-Euro exchange market and Bitcoin-Yen exchange market.

		ARCH	GARCH	TARCH	EGARCH	APARCH
Euro-USDollar	NMSE	0.7438	0.7236	**0.7227**	0.7233	0.7236
	NMAE	0.8317	**0.7874**	0.7875	0.7877	0.7875
Yen-USDollar	NMSE	0.7433	0.7338	0.7345	**0.7334**	0.7340
	NMAE	0.8271	0.7971	0.7997	0.7958	**0.7472**
Euro-Yen	NMSE	0.7600	0.7509	0.7510	**0.7479**	0.7487
	NMAE	0.8460	0.8080	0.8070	0.7961	**0.7595**
Bitcoin-USDollar	NMSE	0.8269	0.8098	0.8094	**0.7917**	0.8001
	NMAE	0.9677	0.9476	0.9470	0.8767	**0.7639**
Bitcoin-Euro	NMSE	0.7823	0.7906	0.7943	**0.7791**	0.7906
	NMAE	0.8101	0.8552	0.8623	0.8207	**0.7368**
Bitcoin-Yen	NMSE	0.7751	0.7859	0.7893	**0.7744**	0.7957
	NMAE	0.8017	0.8597	0.8639	0.8254	**0.7723**

In table 12 is shown that the TARCH model had the best NMSE test results for the Euro-USDollar market, while for the rest of the markets the EGARCH model had the best NMSE test results. The GARCH model had the best NMAE test results for the Euro-USDollar, while the APARCH model had the best NMAE test results for all other markets.

Table 13 presents all of the models with the best NMSE and NMAE results for in-sample and out-of-sample, also show the best AIC and BIC results.

Table 13: The best models for test results, both in-sample and out-of-sample.for the Euro-USDollar exchange market, the Yen-USDollar exchange market, the Euro-Yen exchange market, the Bitcoin-USDollar exchange market, Bitcoin-Euro exchange market and Bitcoin-Yen exchange market.

	In-Sample	Out-Of-Sample

	AIC	BIC	NMSE	NMAE	NMSE	NMAE
Euro-USDollar	EGARCH	EGARCH	TARCH	GARCH	TARCH	GARCH
Yen-USDollar	EGARCH	EGARCH	EGARCH	APARCH	EGARCH	APARCH
Euro-Yen	EGARCH	EGARCH	EGARCH	APARCH	EGARCH	APARCH
Bitcoin-USDollar	APARCH	EGARCH	EGARCH	APARCH	EGARCH	APARCH
Bitcoin-Euro	EGARCH	GARCH	EGARCH	APARCH	EGARCH	APARCH
Bitcoin-Yen	EGARCH	GARCH	GARCH/ ARCH	APARCH	EGARCH	APARCH

The AIC test shows that the EGARCH model had the best results overall, only not achieving the best score in the Bitcoin-USDollar data. The BIC test showed a similar picture where the EGARCH model had the best results in 4 out of 6 series, the GARCH model had the better results in the Bitcoin-Euro and Bitcoin-Yen market.

In-sample EGARCH had the best NMSE results in the Yen-USDollar market, Euro-Yen market, Bitcoin-USDollar market and Bitcoin-Euro market, the GARCH model had the best NMSE test results for the Euro-USDollar market and tied with the ARCH model for the best results in the Bitcoin-Yen market. The GARCH model had the best NMAE results for the Euro-USDollar market, while the rest of the markets had the APARCH model as the best result.

Out-of-sample the TARCH model had the best NMSE result for the Euro-USDollar market, and the rest of the markets had the EGARCH model as best scored. In the NMAE test the GARCH had the best results for the Euro-USDollar market, while the APARCH model had the best results for all other markets.

The best model for adjustment is the EGARCH test, with 75% of best results on the AIC and BIC tests. The best model for least deviations between predictions and measured values is the APARCH model with 41,5% of best results, also is noted that the EGARCH model had 37,5% of best results indicating that is also a good model for this function, and overall the best model.

This result confirm the hipotheses that the EGARCH model was had the best adjustment for the FOREX market and for the Bitcoin market as well as what Danielsson (1994), Charles (2017) and Lin (2017) published. The hypotheses that the EGARCH gave the best forecast for the FOREX markest wasn’t completely proved as it tied with the APARCH model, and the hypotheses that it was gave the best forecast for the Bitcoin market was wrong, since the APARCH model had the best results.

8- Conclusion

This paper objective is to discover with model gives the most accurate forecast and best adjustment for the series. It creates a comparison between the ARCH, GARCH, TARCH, EGARCH and APARCH models using series from the forex market, specifically American dollars, Japanese Yens and Euros, and series from the Bitcoin market.

This paper describe each model and identify the best one. It is compared the results of each model in the Forex and the Bitcoin market. The Bitcoin market behaves similarly to the Forex market and the same models to study one can be used to study the other one.

This paper is useful for academic purposes, for comparison with other papers and continuation of the study, is useful for bankers, risk managers and financially active people, as explain and provides with which is the best model, between the ones studied, to be used in the Forex and Bitcoin market, besides showing that they work.

The study was limited by the number of models studied, by curve adjustment, it could have been used the t-student curve for example, by the number of currencies and markets studied.

For future works is suggested to increase the number of models and adjustment to curves in the study, also is possible to make a study specialized in only E-currencies, to discover which currency is less volatile and how each works with the models we have today.

9- References

AKAIKE, H. A. **A new look at statistical model identification.** *IEEE Transactions on Automatic Control,* AC-19. p. 716-723. 1974.

Bank for International Settlements .Triennial Central Bank Survey of foreign exchange and OTC derivatives markets in 2016.Foreign exchange turnover in April 2016. September 2016. Available in <http://www.bis.org/publ/rpfx16.htm> Access in September 12, 2017

BOLLERSLEV, T..**Generalized autoregressive conditional heteroskedasticity.***Journal of econometrics*v. 31.3 p.307-327. 1986.

BOUBAKER, H., SGHAIER N..**Modellng Return and Volatility of Oil Price using Dual Long Memory Models**. No. 2014-283. 2014.

BRANDT, M. W.; JONES, C.S..**Volatility forecasting with range-based EGARCH models.***Journal of Business & Economic Statistics*v. 24.4 p. 470-486.2006.

BROOKS, C. **Introductory Economics for Finance.** Cambridge, UK: *Cambridge University Press.* 2002

BROOKS, R.; FAFF, R. W.; MCKENZIE, M. D.; MITCHELL, H.. A Multi-country Study of Power ARCH models and National Stock Market Returns. *Journal of International Money and Finance*. p. 377-397. 2000.

CHARLES, A., DARNÉ, O. **Forecasting crudeoil market volatility: Further evidence with jumps,** *Energy Economics* 2017.

CHEONG, C. W. **Modeling and forecasting crude oil markets using ARCH-type models.** 2009

CONRAD, C.; KARANASOS, M.;ZENG, N..**Multivariate fractionally integrated APARCH modeling of stock market volatility: A multi-country study.***Journal of Empirical Finance*v. 18.1 p.147-159. 2011.

CROMWELL, J. B.; LABYS, W. C.; TERRAZA, W. C. **Univariate test for time series model.***Sage University Papers,* 1994

DANIELSON, J. **Stochastic Volatility in Asset Prices: Estimation with Stimulated Maximum Likehood**. *Econometrics.* p. 375-400. 1994.

DICKEY, D. A.; FULLER, W. A. **Distribution of the Estimators for Autoregressive Time Series with a Unit Root**. *Journal of American Statistical Association.*v.74. p.427-431. 1979.

DING, Z.; GRANGER, C.WJ.; Engle, R.. **A long memory property of stock market returns and a new model.***Journal of empirical finance*v.1.1 p. 83-106. 1993.

ENGLE, R.**Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.**.Econometrica.v.50. p.987-1007. 1982.

ENGLE, R. F.; VICTOR K. N..**Measuring and testing the impact of news on volatility**. *The journal of finance* V. 48.5 p. 1749-1778.1993.

EVIEWS.**User's Guide.** 2005.

FLASSBECK, H.; LA MARCA M..**Global imbalances and destabilizing speculation**. 2009.

FRANKEL, J. A.; FROOT, K..**Chartists, fundamentalists, and trading in the foreign exchange market.** 1990.

GAIO, L. E., et al. **Análise da volatilidade do índice Bovespa: um estudo empírico utilizando modelos da classe ARCH.***Contextus* 5.1 2007.

GEWEKE, J.**Exact inference in the inequality constrained normal linear regression model.** 1986.

GLOSTEN, L.; JAGANNATHAN, R.; RUNKLE, D. **On the relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks.** *The journal of finance,* v. 48. p. 1779-1801. 1993.

GRINBERG, R. **Bitcoin: An innovative alternative digital currency**. *Hastings Sci. & Tech. LJ* 4.2012.

GUIDI, F.**Volatility and Long-Term Relations in Equity Markets: Empirical Evidence from Germany, Switzerland and the UK***.* *The Icfai Journal of Financial Economics.* p.7-39. 2008

HADSELL, L.; MARATHE, A.; SHAWKY, H. A. .**Estimating the volatility of wholesale electricity spot prices in the US.** *The Energy Journal.* p. 23-40. 2004.

HIGGINS, M. L.; BERA, A.**A class of Nonlinear ARCH models.** *International economic review,* v. 33. p. 137-158. 1992

JARQUE, C.; BERA, A. A.**A test for normality of observations and regression residuals.** *Internacional statistical Review,* v. 22. p. 163-172. 1987

KUMAR, S. S. S. **Forecasting Volatility–Evidence from Indian Stock and Forex Markets**. 2006.

LAMOUREUX, C. G.;LASTRAPES, W. D. .**Heteroskedasticity in stock return data: Volume versus GARCH effects.** *The journal of finance*v. 45.1 p. 221-229.1990.

LIN Z. **Modelling and forecasting the stock market volatility of SSE Composite Index using GARCH models.** 2017.

MEDVINSKY, G.; NEUMAN, C..**NetCash: A design for practical electronic currency on the Internet.** *Proceedings of the 1st ACM conference on Computer and communications security.*ACM. 1993.

NELSON, D. B. **Conditional Heteroskedasticity in Asset returns: A New Aproach,** Econometrica. v 59. p.347-370. 1991.

NAKAMOTO, Satoshi. **Bitcoin: A peer-to-peer electronic cash system**. 2008.

KATSIAMPA P.. **Volatility estimation for Bitcoin: A comparison of GARCH models.** 2017.

PIMENTA JUNIOR, T.; LIMA, F. G.; GAIO, L. E. **Volatile behavior of BRIC capital markets in the 2008 international financial crisis.**2014.

RON, D.; SHAMIR, A. **Quantitative analysis of the full bitcoin transaction graph.** International Conference on Financial Cryptography and Data Security. p. 6-24. 2013.

SCHITTENKOPF, C.; DORFFNER, G.; DOCKNER, E. J. **Forecasting time-dependent conditional densities: A semi non-parametric neural network**. Journal of Forecasting. v. 19. p. 355–374. 2000.

SCHWARZ, G. **Estimating the dimension of a model.** *Annals of Statistic.* v. 6. p. 461-464. 1978.

U.S. Constitution. Art./Amend. I, Sec. 8.

VARMA, J. **Value at Risk Models in the Indian Stock Market**. 1999.

YAO, J.; TAN, C.L..**A case study on using neural networks to perform technical forecasting of forex.**2000.

ZAKOIAN, J. Threshold Heteroskedastic Models. Journal of Economic Dynamics and Control. p. 931-955. 1994.