Universidade Estadual de Campinas

Instituto de Filosofia e Ciências Humanas

Pedro Mendes Ferreira Lemos

The Fading Light of Contingency: On Tense, Modals, and Assessment-sensitivity

A Luz Evanescente da Contingência: Operadores temporais, Modalidades, e sensibilidade a Contextos de Avaliação

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A LUZ EVANESCENTE DA CONTINGÊNCIA: OPERADORES TEMPORAIS, MODALIDADES, E SENSIBILIDADE A CONTEXTOS DE AVALIAÇÃO

Tese apresentada ao Instituto de Filosofia e Ciências Humanas da Universidade Estadual de Campinas como parte dos requisitos exigidos para a obtenção do título de Doutor em Filosofia.

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Supervisor/Orientador: Prof. Dr. Walter Alexandre Carnielli

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A Ata de Defesa, assinada pelos membros da Comissão Examinadora, consta no processo de vida acadêmica do aluno.

To Morena

the bitterest truth always comes

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Resumo

O que queremos dizer quando falamos que algo *vai* acontecer amanhã, ou quando falamos que algo *pode* acontecer amanhã? Quais são as condições que precisam ser cumpridas para avaliarmos certas predições sobre nosso futuro, como dizendo (ou tendo dito) algo verdadeiro ou falso? Ou ainda, o que consideramos ter sido expressado por falantes, quando estes fazem afirmações sobre como o futuro *irá* se desdobrar, ou *poderá* se desdobrar? São a estas questões que esta tese se dedica.

Em seu objetivo mais geral, pretendemos familiarizar o leitor com as ferramentas necessárias para se avaliar as diversas teorias envolvendo semânticas para uma linguagem *temporal-modal*, e como elas se saem com respeito ao problema dos futuros contingentes. Assim sendo, o *Capítulo 1* inicia a tese apresentando o problema dos futuros contingentes, sendo então seguido por um levantamento das principais teorias semânticas para uma linguagem *temporal-modal*.

No *Capítulo 2*, fornecemos as definições de teorias $T \times W$, e preparamos o terreno para uma discussão aprofundada (a ser exposta no *Capítulo 3*) sobre a abordagem de MacFarlane a respeito do problema (2003, 2008, 2014), que toma 'futuros contingentes' como expressões cujo estatuto-de-verdade (se são *verdadeiras*, *falsas*, ou *nem verdadeiras, nem falsas*) é determinado não apenas pelo *contexto de uso* da sentença, mas também pelo *contexto de avaliação* de onde se avalia este *uso*.

No capítulo final (*4*), apresentamos finalmente nossa meta particular: o objetivo desta seção é o de investigar o que seria necessário para um *modal* (de *possibilidade*) se comportar como sendo sensível a *contextos de avaliação*. Sugerimos que um modal com este comportamento torna-se saliente no discurso ordinário, especialmente quando ele se encontra em modo *indicativo*: opondo-se assim usos contendo "*pode*", de locuções semelhantes utilizando "*poderia*", quando o modal se encontra em modo subjuntivo. Nós acreditamos que esta sugestão se torna sensivelmente persuasiva através de impasses envolvendo asserções contendo o primeiro modal, porém inexistentes com afirmações semelhantes envolvendo o modal em modo subjuntivo.

Essencialmente, argumentamos que ao se tomar a abordagem de MacFarlane sobre asserções – entendendo que quando alguém assere um conteúdo p, ele implica se comprometer em defender sua afirmação de qualquer *contexto de avaliação* (caso seja contestado) –, e se igualmente supomos que um falante assere um conteúdo cuja forma lógica é '$\neg\phi \wedge \Diamond\phi$', se segue então que este falante acaba se representando como comprometido em defender uma alegação que não pode em princípio ser *avaliada* como verdadeira. Por outro lado, como se mostra, o mesmo não ocorre com afirmações envolvendo o modal subjuntivo (*poderia ser,*), já que ele não é *sensível a contextos de avaliação*, e portanto a conjunção pode em princípio ser *avaliada* como *verdadeira*.

Palavras-chave: Filosofia, lógica, modalidade, tempo, contexto.

Abstract

What do we mean when we say that something *will* happen tomorrow, or when we say that something *can* happen tomorrow? What are the conditions that need to be fulfilled in order to make us evaluate a prediction about our future as saying (or having said) something true or false? Or else, what do we take speakers to have imparted when they make claims involving how the future *will* unfold, or how the future *may* unfold? It is to these questions that this dissertation is devoted.

Within its most general goal, it intends to acquaint the reader with the tools required to assess the many distinct theories involving the semantics of *temporal-modal* languages, and how they square with the problem of future contingents. Thus accordingly, *Chapter 1* sets out by framing the problem of future contingents, which is then followed by a detailed survey of distinct theories regarding the semantics of a *temporal-modal* language: we provide the relevant definitions, and assess their merits and downfalls.

In *Chapter 2*, we provide the definitions of $T \times W$ theories of time, and set the stage for a thorough discussion (to be carried out in *Chapter 3*) about John MacFarlane's distinctive take on the problem (2003, 2008, 2014), which views 'future contingents' as expressions whose truth-status (whether they are *true*, *false*, or *neither*) depends not only on a *context of use* of the sentence, but also on a *context of assessment* from which one evaluates this *use*.

The final chapter (*4*) will then embody our particular goal: the gist of this section is to investigate what it would take for a *possibility-like* modal to be sensitive to *contexts of assessment*. We suggest that a modal behaving as such becomes more salient in ordinary discourse when the modal is phrased in the *indicative* mood: thus opposing uses of '*can*' from similar locutions using '*could*', with the modal phrased in the *subjunctive* mood. The suggestion, we believe, becomes sensibly compelling in light of a puzzle involving defectiveness of assertions embedding such modals, as opposed to non-defectiveness of similar assertions involving '*could*'.

Essentially, it is argued that if we take MacFarlane's view of assertions – by understanding that whenever one flat out asserts a content p, one imparts being committed in defending his claim from any *context of assessment* (if challenged) –, and further suppose that a speaker asserts a content taking the logical form of '$\neg\phi \wedge \Diamond\phi$' – where '$\phi$' is a *future contingent* and '$\Diamond$' is the *assessment-sensitive* modal –, *then* the defectiveness springs from the fact that the speaker is representing himself as committed in defending a claim that cannot be expected to ever have grounds to be *assessed* as true. On the other hand, as we show, the same does not occur with similar assertions involving '*could*', since the modal is not *assessment-sensitive*, and thus the whole conjunction can be expected to be *assessed* as *true*.

Keywords: Philosophy, logic, modality, time, context.

Contents

Introduction

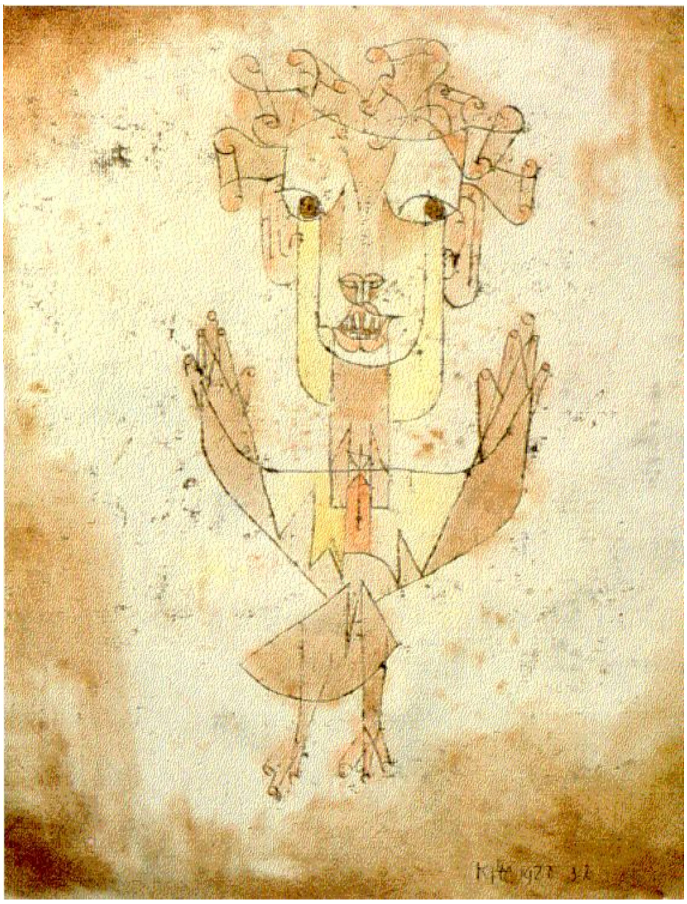


Figure 1: Klee, Paul. *Angelus Novus*. 1920. Israel Musem, Jerusalem.

In the print *Angelus Novus*, shown above, Paul Klee depicts an angel who appears to be – in the words of Walter Benjamin – "[moving] away from something he is fixedly contemplating" (1999 [1940]). As *the Angel of History*, as christened by Benjamin, "his eyes are staring, his mouth is open, his wings are spread... His face is turned toward the past. Where we perceive a chain of events, he sees one single catastrophe which keeps piling wreckage upon wreckage and hurls it in front of his feet. The angel would like to stay, awaken the dead, and make whole what has been smashed. But a storm is blowing from Paradise; it has got caught in his wings with such violence that the angel can no longer close them. This storm irresistibly propels him into the future to which his back is turned, while the pile of debris before him grows skyward. This storm is what we call progress" (Benjamin, 1999, p.257).

This is an allegory employed by Benjamin in the context of his *Theses on the Philosophy of History*. But if you lend some more thought to it, it also conveys a narrative which resembles many of the analogies we use to represent our experience with 'time', and it seems to bear that kind of spark capable of taunting even our most strongly held beliefs. Thus in a way, if you think about it, the storm of time blows and engulfs our wings. Regardless of our actions, we are caught in this incessant motion which pushes us only towards one direction: the future. When we "look back" at our pasts, we cannot "awaken the dead"; we just passively observe and contemplate. Usually, we see a trail of our choices, the decisions we had to make, the outcomes we wouldn't expect, and we amuse ourselves and engage in wishful thinking: what could have happened instead if we had taken different options, and how different our lives would be right now?

"But were them really choices?" – we might ask. After all, could our world have evolved otherwise than it *actually* did? And how then are we justified in saying (or believing) that our world *can* evolve in a number of distinct ways, different from how it will *actually* unfold?

In at least one perspective (one that philosophers have a special regard for), a problem surrounding such questions gets particularly pressing: for instance, we can – and often do – make claims about how the future *will* unfold. To give an example, suppose someone asserts right now the sentence "There *will* be a sea battle tomorrow". Often, we say that such a statement conveys a *future contingent*: a future-oriented declarative sentence, which is neither regarded *necessarily* true, nor *necessarily* false; *viz.* that nothing is presently settled with respect to a sea battle occurring tomorrow. But then we might ask: by asserting this sentence, has someone expressed a *content* which bears a definite truth-value? That is, has someone expressed something which is either now *true* or *false*?

Let us take some stock. What exactly has been imparted? At a first glance, the assertion looks as conveying a content whose 'semantic value' – its extension as a truth-value – is determined by the way things are tomorrow. It doesn't look, to first appearances at least, very different from assertions stating what *has been* the case in the past. Truth-values in the latter case look as being determined by how things were in our past, while in the first case, it just looks as depending on how things are tomorrow. In fact, one has only to hear an assertion about our past – for example, that "Napoleon died in 1822" –, to readily acknowledge that our world already "shouts at us" in giving a definite answer to its truth-status[1]. But the truth is that in cases involving the future, things don't look that clear. After all, if we assume our world to be undetermined with respect to a sea battle raging tomorrow, it looks unintelligible to simultaneously hold that the assertion expresses something already true or false. Indeed, how come our world be undetermined with respect to a sea battle occurring (or not) tomorrow, if it is already true (or false) that it will happen?

At this point, a much better choice is perhaps to think that our world cannot be "shouting at us" in the same way. Granted. But at the same time, there seems to be something incomplete about that story. For instance, just imagine yourself sitting on the dock of the bay, late at night, seeing nothing but calm waters; no ships, no frigates, no thunder of cannons. If you (or anyone else) had asserted yesterday the sentence, "There

[1]In fact it is false; Napoleon died a year before, in 1821.

will be a sea battle tomorrow", what ought you to think about *what was said*? Isn't the world "shouting at us" ? How come we think anything else, except that it expressed something *false*?

As a matter of fact, the problem of future contingents bearing definite truth-values is a well-acknowledged age-old problem. It is headlined by notions familiar to both logicians and philosophers, and in itself, it has motivated revolutions in *Logic* (even if adjuvantly) that moved much beyond its boundaries; *many-valued logics*, as stemming from Łukasiewicz's pioneering works, is perhaps the most well-known example. But most importantly, the problem springs from one of the most scrutinized pieces of text ever written by Aristotle: the relatively short *Chapter 9* of *De Interpretatione*; a challenging and dazzling chapter, almost cryptic in many of its passages, and haunted by concisions that have yielded unending disputes among scholars and logicians alike.

In its birth, and from what Aristotle seems to report, we know the problem is wholly driven by the upshots of a logical argument, being raised against him. But additionally, when we read the chapter, the problem is almost cast as a legal matter. In those times, most of the disputes between different schools would trade on a common strategy. The opponent would take one of your assumptions, he would then couple it with some premises that he would expect to yield no controversies, and he would finally draw on consequences which would directly collide with another assumption you already held. So there, in a chapter filled with obscurity and ambiguities, these elements of the dispute seem at least clear.

First of all, we have clear evidence of what principle of Aristotle was being taken: 'excluded middle'. Further, it also seems clear what other assumption of his was taken to conflict with the consequence that his opponent has drawn: it was Aristotle's view of an open future, and that things happen *as chance has it*. But why exactly, already there in Aristotle, was it a problem for future contingents to bear definite truth-values?

Essentially, the problem amounted to the relations obtaining between the truth-status and the modal-status of statements concerning the future. Thus it looks clear at least, that Aristotle's opponent did exploit both his conceptions of *necessity* and *possibility* – and how they were thought to relate to statements bearing definite truth-values –, in order to advance their *deterministic* conclusion: that nothing happens *as chance has it*. In this respect, we read in A. Iacona:

> "[...] an important relation ties the semantic properties of future-oriented expressions to the modal status of the future itself. The relation may be phrased as follows: if an expression has a semantic value that involves a certain way things might go, it is necessary that things will go that way." (Iacona, forthcoming, p.1)

Much of the *deterministic* [2] arguments will invoke that kind of tenet: if a statement concerning the future bears a definite *truth-value*, then they *cannot fail* to have that truth-value.

For one quite standard interpretation (yet certainly not undisputed), Aristotle's response (in *De Interpretatione's Chapter 9*) is that, albeit sharing with his opponent the view that statements bearing definite truth-values *cannot fail* in having them, *future contingent* statements don't already possess determinate truth-values, and thus things do happen *as chance has it.* It is usually said that Aristotle's position here is to limit the extent of application of the *principle of bivalence*, while showing his opponent that the *principle of excluded middle* remains unharmed, and still holding. The first principle – *bivalence* – states that every proposition is either true or false (or that of every pair p and $\neg p$ – where p is a proposition, and '$\neg$' is negation – one is true and the other false). The second principle, *excluded middle*, says in its turn that every proposition of the form '$p \vee \neg p$' – where $\vee$ is disjunction – is logically true. Thus, according to this interpretation, Aristotle still maintains that instances of excluded middle embedding future contingents are logically true. Therefore, he would maintain that "Either there *will* be a sea battle, or there *won't* be a sea battle" holds the truth-value *true* (in fact, as a consequence of *excluded middle* being logically true). But he would deny that, *in division*, we ought to take either disjunct as already bearing a definite truth-value, since – as we indicated – that would amount to a statement concerning the future *not possibly failing* to possess that truth-value, and hence, that the future couldn't unfold either way, as to make it true, or to make it false.

The problem then is twofold. It is a problem about the truth-status of future contingents, but only insofar it is also a problem of the modal-status of future contingents. The sense then sought, concerning the modal-status, is that it wouldn't make sense for a statement to already bear a truth-value, and still that *it can fail* to possess that truth-value. And it is that kind of tenet, linking a statement's truth-status (of bearing a definite truth-value) to its modal-status (that it *can't* be true [false], if *already* false [true]), that makes Aristotle deny that future contingents could already possess determinate truth-values.

But then we start asking: what is the notion entrenched there, concerning the modal-status of statements involving the future? Of course, many times we don't properly make

[2]Sometimes, a notional difference is drawn between the concepts of *determinism* and that of *fatalism.* The first one is sometimes attached to a notion of *causal determinism* – the view that present state of affairs fully determine subsequent state of affairs. It amounts then, to a notion resorting to causes and effects, given physical laws. The term "fatalism", in its turn, is often attached to the view that no person is able to act freely, in face of the assumption that all statements are either *true* or *false.* We here use the term 'deterministic' not in the *causal* sense, but rather in the sense that all truths (and falsehoods) – be it about our past or our future – are presently determined, and thus nothing happens *as chance has it.* "Fatalism" of course, is a consequence of that view, but it is more directed to the problem of deliberative agents acting freely.

a claim about how the future *will* unfold, but rather resort to something weaker, by making claims about what is *possible*, at a given time. For instance, perhaps we don't assert "There *will* be a sea battle tomorrow", but instead assert something like "It *can* be the case, that tomorrow there *will* be a sea battle". Much different from the first claim, where we may be hesitant in assigning truth or falsehood to the sentence, this last claim sounds plainly and simply true, if we are already assuming the future to be contingent with respect to a sea battle raging tomorrow. But then, what is the sense sought? What are the truth-conditions of such statements, and how it relates to our world?

Here things start to get critical again (and interesting). For one thing, the meanings of the modals would certainly largely deviate from the modern mindset we became familiarized through Kripke's *relational semantics*, where we countenance 'truth' (of propositions) relative to 'possible worlds' – in short, it seems that something more must be added to that story. This is a well-acknowledged problem; for instance, Hintikka writes:

> "An attentive reader of the Aristotelian *corpus* can scarcely fail to notice that in certain respects the Stagirite used the *modal notions* of possibility and necessity in a manner different from our modern ways with them. A case in point is the relation of modality to time" (Hintikka, 1981, p.57).

What then do we mean when we say that something *can* happen tomorrow, and what exact conception of possibility did Aristotle hold, that would relate to the kind of tenet we have alluded (that 'it doesn't make sense to say that something about the future is *already* true, and that it still *can fail* to be true')?

Perhaps we should start here with some notional scrutiny: the sense of modalities employed here is certainly not that of 'logical necessity' or 'logical possibility'. Instead, the concept – either of 'necessity' or 'possibility' – has to do with (or *concerns*) states of affairs that are "within reach" (*attainable*) at a given time. Perhaps a more direct way to grasp the concept, as alluded by Thomason, is to think of "sentences involving the adjective 'possible', such as 'In 1932 it was possible for Great Britain to avoid war with Germany; but in 1937 it was impossible' " (Thomason, 1984, p.137)[3]. This is the primary sense of modalities at play here, and we sometimes call them "historical modals". But even so, when we look at the sense used in the original context of the problem of future contingents, there is hardly agreement on what precise notion these modal conceptions would amount to.

There is in fact another dazzling chapter in this whole story: the question about Aristotle's very conception of 'possibility'. It is a story pervaded with seeming inconsis-

[3] As we will see, when we evaluate a sentence like '*Possibly, I will meet Jake tomorrow*', at a time t, we are inquiring whether *meeting Jake* the "next day", is *then possible* at the time t; that is, if conditions are such, that meeting Jake the next day is "within reach" or attainable at that relevant time, where the sentence is being evaluated.

tencies stemming from a philosopher who, among many traits, was undoubtedly a skilled and brilliant systematist. What was, after all, his conception of possibility?

In fact, the problem gets particularly pressing when we look at the relations holding between *possibility* and *actuality*, in Aristotle. The interpretive problem is so severe, that different scholars will place Aristotle's conception of possibility in radically opposing extremes, all of which are equally well supported (both by textual evidence, and each scholar's line of argument).

For instance: on the one hand, Aristotle seems evidently to subscribe to the view that (1) *not* every possibility has to be fulfilled; that is, *possibilities* need not be 'actualized'. Particularly telling, the oft-cited '*cloak passage*' is commonly invoked to support that position by Aristotle:

> "For example, it is possible for this cloak to be cut up, and yet it will not be cut up but will wear out first" (*De Interpretatione 9*, *19a7* [Ackrill's translation, p.52]).

On the other hand, others have suggested that as far as Aristotle was concerned with *possibilities*, he subscribed to the view that "*all genuine possibilities*, or at least all possibilities of some central and important kind, *are actualized in time...* [and that], [a]ny such possibility thus has been, is, or will be realized; it cannot remain unrealized through an infinite stretch of time; in a sense, everything possible will happen in the long run" (Hintikka, 1981, p.58). This kind of view was labeled the *Principle of Plenitude* by Arthur O. Lovejoy, and Hintikka is perhaps the most well-known supporter for attributing to Aristotle such a position (1964, 1973, 1981). He refers to many textual evidences in support of his view. For example, he writes: "I have argued elsewhere that Aristotle's theory is compatible with the Principle of Plenitude. Indeed, it seems to me that rightly understood, *Met.* Θ *3–4* strongly supports my attribution of the Principle of Plenitude to Aristotle" (Hintikka, 1981, p.63). In the same article, we read a little later: "Aristotle's view is probably motivated by the idea that the only way in which we can think of a possibility to be realized is at some moment of time in our actual 'history of the world' " (Hintikka, 1981, p.68). In effect, the main passage Hintikka elects (to give strength to his interpretation) is the following one we find in Aristotle's *Metaphysics*:

> "If what we have described is identical with the potential or convertible with it, evidently it cannot be true to say "this is possible but will not be", which would imply that things incapable of being would on this showing vanish" (*Metaphysics* Θ *4. 1047b3-6* [Hintikka's translation, 1981, p.67])

In her *Passage and Possibility* (1982), Sarah Waterlow couches this interpretive problem in the form of a puzzle: how are we to understand the fact that Aristotle apparently endorsed the view "that 'possibly', by its very meaning, implies fulfillment at some time" (Waterlow, 1982, p.12), if on the other hand, it seems an "indubitable fact that for him the actual and the possible differ in extension" (Waterlow, 1982, p.4) – as the *cloak passage* and many others suggest?

Interpretive matters aside, perhaps one of the most interesting facts around the problem is how it is generally cast in a same and constant kind of concise formulation, independently of what one has to say about Aristotle's 'right' answer or conceptions. A paramount example, is something in the following lines:

> "Suppose, for the sake of argument, ... [that] it is true (*already* true) that there will be a sea fight tomorrow. But if this is already true today, how can the occurrence of tomorrow's sea fight be contingent? If it is already true that there will be a sea fight tomorrow, the sea fight <u>cannot</u> conceivably <u>fail</u> to come about. By the same token, if it will not take place, then it will be false today to say that it will be fought; and this seems to <u>make it impossible</u> for it to take place" (Hintikka, 1964, p.463 – emphasis added).

It is quite common to phrase the unintelligibility *of a statement failing to possess a truth-value* (supposing it has that value), through locutions such as '*cannot fail*', in the indicative mood. In fact, we face a similar kind of discomfort when we look at a puzzle involving modals phrased in the *indicative* mood. For instance, consider the following claim, as asserted by someone right now:

> (1) There *won't* be a sea battle tomorrow, although it *can* be the case now, that tomorrow there *will* be a sea battle.

The conjunction in sentence (1) strikes us as defective or odd-sounding. In fact, the defectiveness of (1) bears a sense reminiscent of the tenet linking the truth-status and the modal-status of future contingents. After all, in what sense could someone say that something *won't* happen but that it *can* be true now that it *will* happen? If it *really* can happen, then it is because it is not already the case that it won't happen. Agreed. But how ought we to explain the puzzling phenomena?

Some have suggested (notably DeRose: 1991, 1998, 1999), that the defectiveness we hear in sentences such as (1) owes to the fact that the possibility modal, expressed in the second half of the conjunction, must invariably bear an epistemic sense. Thus, the sentence 'It *can* be the case now, that tomorrow there *will* be a sea battle' ought to express – as DeRose argues – that a speaker's *stock of knowledge and beliefs* doesn't rule out a sea battle occurring tomorrow. And since whenever one flat out asserts a content p, one is usually representing oneself as knowing or believing *that* p, *then* by flat out asserting a conjunction such as (1), the speaker is involved "in some kind of pragmatic conflict” (as Seth Yalcin speaks: 2007, p.984); he represents himself as both knowing that there won’t be a sea battle tomorrow, and also that his *stock of knowledge and beliefs* doesn’t rule out a sea battle occurring tomorrow.

This dissertation offers a second option for interpretation: the ‘possibility’ modal in (1) is *assessment-sensitive* (in the sense proposed by MacFarlane: 2003, 2008, 2014). In his *Future Contingents and Relative Truth* (2003), MacFarlane’s main idea concerning the problem of future contingents involved the truth-status problem. It resorted on reconciling two distinct views: that (i) future contingents may lack truth-values (they are *neither* true *nor* false), when a *use* of a sentence is *assessed* from some perspective (a *context of assessment*) where our future is open with respect to *what has been said*; yet on the other hand – he argues –, (ii) future contingents may bear definite truth-values (they are *true*, or *false*), when a *use* of the sentence (think it as an ‘episode of assertion’), is *assessed* from a future privileged context, where one of the outcomes has *come true*, or *false*. Thus, he views a ‘future contingent’ as an expression whose truth-status (whether they are *true*, *false*, or *neither*) depends not only on a *context of use* of the sentence, but also on a *context of assessment* that is evaluating an ‘episode of assertion’[4].

In our view, there is a sense of a (historical) *possibility* modal which behaves in the same way, and which appears (or becomes more salient) when the modal is phrased in the *indicative* mood. That way, when we use expressions that convey a sense that something *can* happen tomorrow (or *may* happen), its truth-value depends not only on the *context of utterance* of such expressions, but also on the *context of assessment* from which one is evaluating the utterance. An explanation for defectiveness of sentences such as (1), is then made intelligible when we conjoin this interpretation (of the possibility modal) with a distinct picture of assertions, whereby one only asserts or imparts a content that is expected to ever have grounds to be assessed *true*. Hence, since both the future contingent (‘There *won’t* be a sea battle tomorrow’) and the modal claim (‘It *can* be the case now, that tomorrow there *will* be a sea battle’) are assessment-sensitive, the conjunction in (1) can never be – presently or retrospectively – expected to be assessed as true.

In a sense, ‘*can*’ is a possibility modal that stands in a middle ground between *actuality* and *counterfactual* possibilities. It is tied to *actuality* in the sense that an assertion embedding this modal cannot persist *true*, if the prejacent proposition (of what was claimed to be possible) doesn’t *in fact* happen. However, it is also tied to *counterfactual* possibilities, in the sense that the claim has its truth secured for as much time while the objective possibilities are still around.

Therefore, the fact that one assesses retrospectively the claim as false (which is the sense that ties ‘possibility’ to ‘actuality’), doesn’t preclude the fact that the same assertion was assessed *true*, for the whole time while what was claimed was still an objective

[4] This is just a glimpse. But don’t worry, our first chapter will spell out in more detail MacFarlane’s view of the problem.

possibility of our world. The stress on the *indicative* mood of the very phrasing of the modal is perhaps crucial; since the mood has the effect of 'directing to facts', the modal claim remains dependent on the present state of the assessor, and the objective possibilities from his perspective – which refers, after all, to "the world" occupied by both him (the *assessor*) and the *utterer* (who has imparted, at a time, that something *can* be true, from the perspective of *his time*, and *in his world*).

This whole last argument is mainly developed and motivated in our last chapter. Thus, our dissertation has first the objective to spell out in detail how one handles the semantics of *tensed* and *modal* sentences, and how each distinct theory (giving truth-conditions for such sentences) squares with the problem of future contingents. This is done mainly in chapters *1* and *2* ('Theories of time', and 'Leading to $T \times W$ Theories'). Having provided all the material we judged essential for understanding the problem (and the formal semantics that are involved), we then expound MacFarlane's distinctive take on the problem of future contingents, and his notion that some expressions are *assessment-sensitive* (*Chapter 3*). Finally, in our *Chapter 4*, we provide our interpretation of an *assessment-sensitive* possibility modal (and the semantics involved), and discuss it in light of the alluded puzzle.

We have also added an Appendix, at the end of this dissertation, containing some updated discussions that involve (in one way or another) MacFarlane's kind of solution to the problem. These texts have been written due to new material appearing in literature during the very writing of this dissertation. It includes some evaluations concerning an additional approach, which appeared in Ciuni & Proietti (2013); a kind of *paraconsistent* cousin of the *supervaluationist* approach (or a *sub-valuationist* kind of solution, as they also term). The approach has some interesting features, which are critically evaluated and compared in light of the other approaches.

The appendix also includes material of perhaps some historical relevance, concerning approaches similar to MacFarlane's, which have appeared before his (2003). This embraces especially a paper published in 1981, by Michael J. White, a scholarly trained philosopher mostly dealing with Aristotle's modal conceptions, at that time. Some passages display startling resemblances to MacFarlane's solution, yet it lacks many of the features of the latter.

Chapter 1

Theories of time

The aim of this chapter is to acquaint the reader with the problem of future contingents, and the task of providing an interpretation for a language containing *tenses* and *modals*. It starts by first outlining the problem, which is then followed by some brief preliminary discussions concerning a temporal language. Finally, it will spell out a host of distinct 'theories of time', and the semantics involved. For each theory, we examine how they may be said to square (or not) with the problem of future contingents.

1.1 Framing the problem

Suppose Thomas is on his way to meet Jake, carrying an 'agreement contract' concerning the sale of his business. Just before knocking on Jake's door, Thomas eavesdrop on a conversation between Jake and his partners. Amidst some altercation, he overhears Jake saying: 'I will meet Thomas today'. He immediately refrains from entering the room and starts reflecting on what has just been said.

While Thomas is making up his mind and reflecting on his choices, contingency of his future starts to shed its light: he knows his reflexes are intact and respond readily to his volitions. He knows he *may* just turn around, go back home, and give some more thought on his decision to sell his long-established family-owned business; on second thought, he also knows he *may* just knock, enter the room, and simply meet Jake.

At this instant, while Thomas is struggling with his decision and before he takes any action, we may start asking ourselves: is Jake's assertion expressing something either true or false? Is what Jake said, 'I will meet Thomas today', expressing a proposition which is either true or false? The problem looks pressing.

This very brief storytelling brings in us some of the common thoughts we share through many of our everyday experiences. We do concede to future some indeterminism: we have a strong intuition that our future is objectively open to a variety of distinct possible outcomes, which would mean that in some of them some events will take place, while in others they won't. As MacFarlane puts the matter, "[i]f there are objective possible futures which would make the prediction true and others which would make it false, symmetry considerations seem to forbid counting it either true or false" (MacFarlane, 2003a, p.321). And what else would we expect? MacFarlane called such intuition the "indeterminacy intuition".

Yet suppose now that Thomas has finally made up his mind. He knocks the door, enters the room and finally shakes Jake's hand. What are our thoughts now concerning Jake's assertion? Should we still assess it as having been neither true nor false? According to MacFarlane, it is just such a situation which compels us for "a strong temptation to say that the assertion does have a definite truth-value, albeit one that must remain unknown until the future 'unfolds' " (2003, p.321). How come we witness both shaking their hands and still claim that Jake's assertion was anything else than *true*? This other kind of intuition, MacFarlane called it the "determinacy intuition".

It seems then that if it is in time where contingency starts to shine its light, it looks as if it is time again which obliterates every potency for objective indeterminism. Time will come, and with it every contingency will see its vindication being impugned. Is this an accurate picture? And if not, why it isn't?

It has been more than a decade now since MacFarlane published his *Future Contingents and Relative Truth* (2003). In its heart, it sought to accommodate both the aforementioned intuitions, which at a first glance, would look unreconcilable. The solution was ingenious: it resorted on breaking the assumption of 'the absoluteness of utterance-truth', as he called it (2003, p.322), and it traded on the idea that the strength of the problem of future contingents rested on resisting giving both views their due. Speaking some years later about this paper, and its driving idea, MacFarlane wrote:

> "I suggested in [*Future Contingents and Relative Truth*] that this odd perspective relativity was both the source of the staying power of the problem of future contingents and the key to its solution. The source of its staying power, because if one focuses on present claims concerning the future, it seems impossible that they could have a determinate truth value, while if one focuses on past claims concerning the present, it seems impossible that they could fail to have one. The key to its solution, because one has only to find a way to give both perspectives their due. The way to do that, I argued, is to relativize the truth of sentences not just to a context of use but to a context of *assessment*." (MacFarlane, 2008, p.90).

But it was also on that article, which MacFarlane first acknowledged that both intuitions were already made sense of, in the very framework of Thomason's *supervaluations* - the main theory relative to which his own approach would show as an improvement. In fact, MacFarlane was always at pains for adjudicating what was that very feature of his truth-relativist theory, which would show itself as an improvement over the supervaluationist solution. For instance, in (2003) he argued that his theory could, while supervaluationists could not, make sense of retrospective judgments of *truth* for past assertions – and thus additionally make sense of the determinacy intuition (in effect, the *indeterminacy* intuition was already covered by *Supervaluationism*). But as we said, he then later acknowledged that in fact, supervaluationists could already make sense of retrospective judgments of truth.

So then he opted, in (2008), for defending that his theory could, while supervaluationists could not, make sense of judgments of truth, for past assertions expressing what would *actually* be the case in the future. And this was a very interesting option, because it dealt with the behavior of a modal-like operator (quantifying over worlds), whose truth would vary along distinct *contexts of assessment*. In fact, the gist of our dissertation is to investigate what it would take for a *possibility modal* to be assessment-sensitive, and what kinds of upshots it would bring.

But before reaching that, we need to show how distinct theories of time are provided, and how we can define temporal structures entrenching some of our pre-theoretical notions concerning time: for instance, what structure best mirrors a *deterministic* view of time, or an *indeterminist* one; furthermore, investigating what a *temporal-modal language* would amount to, and how we provide an *interpretation* for it, based upon each kind of structure; what kinds of *logical validities* and *consequence relations* each theory complies with, etc. We will begin with a discussion about *temporal languages*.

1.2 A first venture into *temporal languages*

First and foremost, we want to evaluate competing semantic theories providing truth-conditions for sentences containing or combining temporal modifiers and modals. In essence, we want to evaluate truth-conditions of sentences such as:

(1) Jake *will* visit Paris.

(2) Jake *has* visited Paris.

(3) Jake *will* visit Paris tomorrow.

(4) It is *possible* that it *will* rain tomorrow in Athens.

(5) It is *necessary/settled* that it *will* rain tomorrow in Athens.

(6) It is *contingent* that it *will* rain tomorrow in Athens.

But before spelling out a semantic theory for a formal language capable of representing or expressing such sentences, let us take some stock. There are different ways one may treat the temporal modifiers we find in such sentences: first of all, once we glimpse at natural languages, we come across a host of entirely distinct features yielding such transformations: we see changes in verb-form, we stumble upon auxiliary verbs expressing future and past tenses - such as *will* and *was* in English; and pretty often we see them complemented by temporal adverbs such as *tomorrow*, *yesterday*, *the day before yesterday*, and the like. How should we then proceed?

There are mainly two options. A first one, pioneered by Prior, following his *Time and Modality* (1957), was to take linguistic items such as 'Jake is visiting Paris', with its main verb in the present progressive tense, as a fully complete sentence expressing "time-neutral" contents, and thus fully entitled to bear distinct truth-values relative to different instants of time[1]. Thus for Prior, we could simply regiment such linguistic items through a set of sentential variables $\{p, q, ...\}$, which under an *assignment*, would take distinct truth-values at different times (more on *assignments* later). So say, that we could assign 'Jake is visiting Paris' to be *true* on 'March 2$^{\text{nd}}$, 1986, 2 p.m.', and the same sentence to be *false* on 'March 3$^{\text{rd}}$, 1986, 5 p.m.'.

And how about sentences such as (1) and (2) above, like 'Jake *will* visit Paris', and 'Jake *has* visited Paris'? Prior sees these sentences, which differ from 'Jake is visiting Paris' in the tense of the main verb and the auxiliary verbs, as sentences containing sentential operators scoping over contents of the first kind.

Based on these assumptions, Prior introduces in a *temporal (or tense) language* two kinds of one-place sentential operators: the future-tense operator F (meaning something in the lines of "It will be the case that"), and a past-tense operator P (in the lines of "It was the case that"). So where $p, q, ...$ are sentential variables, Prior will take the logical forms of sentences such as 'Jake *will* visit Paris', and 'Jake *has* visited Paris', to be respectively Fp and Pp.

Another author who takes this kind of option when building up a *logic of demonstratives* is Kaplan (1978; 1989), who also treats tenses as sentential operators. So in case of any sentences with tenses occurring in it, he also views these tenses as operators scoping over "time-neutral" contents. Kaplan even offers a kind of 'operator argument' to back his position. For him, if we take items such as 'Jake is visiting Paris' as expressing 'contents' already "incorporating reference to a specific time, or state of the world, or whatever" (1989, p.53), or perhaps taking temporal references to be embedded as 'unarticulated constituents' of those sentences, then it would be otiose to ask whether the content expressed by tokenings of the same sentence at different times, would have different truth-values. Yet this doesn't seem an otiose question to be asked.

The second main available option is to treat instead temporal modifiers and ordinary tensed languages as mainly involving *quantification* over instants of time. Thus, this kind of approach will take linguistic items as 'Jake is visiting Paris', "not as a complete

[1]"So, 'time-neutral' contents meaning here 'time-neutral' propositions?", you might ask. Not so fast. In both (1978) and (1989), Kaplan for instance will speak of such 'contents' (expressed by these items) as what "has often been referred to as a 'proposition' " (Kaplan, 1978, p.84), so we might view them as 'propositions'. Yet Lewis for instance (1979a), views propositions as 'sets of possible worlds'; thus he takes 'contents of beliefs' for example, not to be propositions, but as properties having truth values relative worlds, times, and agents (MacFarlane discusses this point in Chapter 4 of his (2014)). But yes, we will take these 'contents' to be something much more akin to propositions than properties.

sentence expressing a proposition and having a truth-value, to be symbolized by a sentential variable $p, q, ...$, but rather as a predicate expressing a property on instants, to be symbolized by a one-place predicate variable $P, Q, ...$" (Burgess, 1984, p.90).

In that sense, this kind of approach will add a symbol to the language corresponding to the present instant (for example: c), and also a symbol $<$ for the *earlier/later* relation. Hence, where P encodes the predicate associated with 'Jake is visiting Paris', the logical form of a sentence such as 'Jake *will* visit Paris' will be $\exists t(c < t \wedge P(t))$.

This kind of approach was one "forcefully advocated in Quine [1960]" for instance, and because it mainly trades on formalizing 'tensed talk' into a *first-order language*, it has the benefit of "[regimenting] ordinary tensed language to make it fit the patterns of classical logic" (as Burgess neatly observes: 1984, p.89).

More recently, this kind of position has been advocated as an approach which outperforms the one favored by Prior, especially by Jeffrey King (2003). He claims that we better ought to understand tenses in natural languages as mainly involving *directly referring terms* and *quantifiers* (either expressions quantifying over instants of time, or expressions directly referring to specific instants, given a context of speech). King's arguments have even forestalled MacFarlane from treating temporal modifiers as sentential operators, in his *Truth in the Garden of Forking Paths* (2008). But in his most recent exposition, (2014), he has shifted back in treating them as operators, while still acknowledging that "[t]here are strong considerations in favor of treating tense using quantifiers instead" (MacFarlane, 2014, p.149).

We will also opt for the 'operator approach', for similar reasons argued by MacFarlane. Thus, it is not because we presume that the problems addressed by King could be avoided. We actually don't. The main reasons summoned by King look quite consistent and forceful[2]. But our choice simply rests on the fact that all the problems here taking place seem orthogonal to the kinds of issues addressed by King.

As a matter of fact, reshaping the framework in order to treat temporal modifiers as referring terms and quantificational expressions, would only take place at a very early stage of our exposition; we would only need to lightly tweak some of the *syntax* and the semantics of tensed formulas, but it would remain untouched the very source of the problems we take, which has to do with consequences following from the assumption that a future-tensed sentence bear definite truth-values. Thus, let us provide a first taste of temporal structures and how they relate to *temporal languages*. Additionally, we will already introduce the reader to some (very brief) notions concerning context-sensitivity.

[2] *See* J. King's defenses, in his 'Tense, Modality, and Semantic Values'. *Philosophical Perspectives* 17. pp.195-245.

1.3 A first taste of temporal structures and context-sensitivity

A thorough account on the semantics and models for formulas of a *temporal-modal language* will follow up the present section, but let us already provide some taste concerning temporal structures, and how they relate to *temporal languages* bearing context-sensitivity. We will even follow our last discussion, and provide some assessment of differences between Prior's and 'Regimentation's' viewpoints concerning a *temporal language* (even though we will leave 'Regimentation' at this section, and only work onwards with the former notion, that of Prior's).

As we saw, Prior's view is to take a language containing, among sentential variables and boolean connectives, the operators F and P. 'Regimentation' instead (let us follow Burgess (1984) on how he calls the view) will take a tensed language to contain, among 'boolean connectives', also a set of one-place predicates expressing a property on instants (each standing intuitively for a present-tensed linguistic item), quantifiers, a set of variables (which will range over *times*), and perhaps 'singular terms' directly referring to 'dates' of time.

These two views will differ on what they take to be a tensed-language, so they will deviate on how to provide an interpretation for the language. Nonetheless, before even getting to an *interpretation* for a temporal language, we need to take stances on how our world is temporally structured. In this thesis we comprise three such ways to temporally structure our world. Let us then provide a glimpse at those structures.

Concerning the first two options, we can start with a set of times T and a binary relation $<$ ordering the members in T (symbolizing the intuitive *earlier/later* relation, if you like). Two different viewpoints on time will impose different constraints on how the members are related. For example, the first view will take all members of T to be *linearly ordered* by the relation $<$, meaning that if you take two non-equal elements t, t' in T, then either t is *earlier* than t' ($t < t'$), or t is *later* than t' ($t' < t$). The second view will order the elements of T as resembling a *tree*, such that it 'forks' or 'branches' only upwards. In any case, a structure of the first or of the second kind can be viewed as a pair $\langle T, < \rangle$, where they differ on how $<$ is relating the members in T.

A third view, which we find in Kaplan (1978; 1989) and Thomason (1984), is to start with a set of times T and a set of possible worlds W, and couple with them a relation $<$ *linearly ordering* the members of T (the same way we mentioned before), and also some kind of binary *accessibility relation* that relates the members of W. Kaplan for instance will simply take all members of W to relate to each other, but Thomason will provide a finer-grained arrangement; he will make the structure resemble something close to a *tree* (yet not quite 'treelike'). To achieve that, he adds a ternary relation $\simeq$, which relates a time t of T, and two *worlds* w, w' of W. Making some abuse of language, instead of saying that 'w_2 is *accessible from* w_1 ', we can start saying that 'w_2 is t-*accessible from* w_1 ', or 'w_2 *is accessible from* w_1*, at* t', and the like. To make it resemble a *tree* (yet not quite!), we just add that for every $t \in T$, if 'w_2 is t-*accessible* from w_1', then for every t' *earlier* than t, w_2 must also be 't'-*accessible* from w_1'. In any case, a structure of this kind can be viewed as a quadruple $\langle T, W, <, \simeq \rangle$. In Kaplan, we would say that for every two worlds w, w' in W, they are related by '$\simeq$' ($\forall w, w' \in W : w \simeq w'$). While in Thomason, it works as we said.

Our dissertation is mainly concerned with indeterministic representations of time, and the structures best suited for such representations are structures of the 'branching' kind $\langle T, < \rangle$ (where times 'branch' upwards), and structures of the kind $\langle T, W, <, \simeq \rangle$. So for now, we restrict attention to these kinds of structures.

So let us now provide a taste of how such structures relate to *temporal languages* bearing context-sensitivity. Let us first enrich our structures with a non-empty set C comprising *contexts*. We will discuss at length much later what contexts are, and what roles they play in the semantics of a *temporal language* and the problem of future contingents, but it suffices now only to say that a 'context' is a possible occasion of use of a sentence: for instance, a possible occasion of use of the sentence '*Jake will visit Paris tomorrow*'. This much will provide us a way to talk about 'tomorrow' and to yield a simple semantics for both *Priorean* and 'Regimentation' languages able to express sentences containing 'tomorrow'.

So we fix that for every member c (a 'context') of the set C, it has at least one parameter informing us what time (of T) counts as being *the time of the context*, and one parameter informing what world (of W) counts as being *the world of the context*[3]. It is very important to notice here that both *the time of the context* and *the world of the context*, must already be a member of T, and of W, for any given structure[4].

So let us work then with a *Priorean language* containing at least, besides sentential variables, the temporal operator F ("It will be the case that"), the (metric) temporal operator *One* ("One day hence, it will be the case that"), and the temporally indexical operator *Tom* (expressing "Tomorrow it will be the case that"). For a 'Regimentation' language, let us consider one containing, at least, besides predicates standing for every present-tensed sentences, also the 'singular terms' *"tomorrow"* and *"one day hence"*.

Now, notice that at this stage, we haven't yet made any considerations on how a *temporal language* is interpreted with respect to any of these structures. But suppose that we first take a structure of the '$T \times W$ kind': $\langle C, T, W, <, \simeq \rangle$ (now enriched with contexts). If our language is that of Prior's, then in order to provide an interpretation for

[3]Much later, we will see how the standard view, which designates a unique *world of the context*, brings trouble for the problem of future contingents.

[4]As in Kaplan (1978), conditions 3.(ii); 3.(iv): "If $c \in \mathscr{C}$, then (ii) $c_T \in \mathscr{T}$ [...] (iv) $c_W \in \mathscr{W}$" (p.88), where $\mathscr{C}$ is the set of contexts, c_T is *the time of context* c, and $\mathscr{T}$ is the set of times; while c_W is *the world of context* c, and $\mathscr{W}$ is the set of worlds.

this language we first provide a function mapping each 'sentential variable' to a subset of $T \times W$ (we can call such a function an *assignment*, and denote it by V – intuitively, V maps which atomic formulas are *true* in each time-slice $\langle t, w \rangle$ of a possible world w)[5]. As a final step, the semantics for arbitrary formulas of the language, embedding connectives and temporal operators, will take the form of a recursive definition of truth; for example, given $\langle C, T, W, <, \simeq \rangle$ and an *assignment function* V, and assuming that $t \in T$, $w \in W$, and $c \in C$:

for atomic formula p:

$\ulcorner p \urcorner$ is true at $\langle c, t, w \rangle$ *iff* V maps p to a subset containing $\langle t, w \rangle$

for temporal operator F:

$\ulcorner F\phi \urcorner$ is true at $\langle c, t, w \rangle$ *iff* ϕ is true at $\langle c, t', w \rangle$,
for some t', such that $t < t'$

for temporal operator One:

$\ulcorner One\ \phi \urcorner$ is true at $\langle c, t, w \rangle$ *iff* ϕ is true at $\langle c, t+1, w \rangle$,

for temporal operator Tom:

$\ulcorner Tom\ \phi \urcorner$ is true at $\langle c, t, w \rangle$ *iff* ϕ is true at $\langle c, t_c + 1, w \rangle$,
where '$t_c + 1$' is *the time corresponding to tomorrow, relative to context c*

Traditionally, we see t_c as labeling *the time of the context c*, and it plays a role when recursively defining truth-conditions for the *Now* operator. For instance, Kaplan symbolizes it in a formula as the operator N scoping over a sub-formula, as in $\ulcorner N\phi \urcorner$. And the formula is *true* at $\langle c, t, w \rangle$, if ϕ is *true* at $\langle c, t_c, w \rangle$ (as we said, with t_c representing the *time of the context c*). And supposing ϕ is an atomic formula, then ϕ will be true if the *assignment function* is mapping ϕ to a subset containing the pair $\langle t_c, w \rangle$. Additionally, we can here assume T to be the set of integers (as Kaplan does in (1978: p.9); and (1989)),

[5]There are also further conditions to impose to an *assignment function* V when '$T \times W$ kinds' of structures are at stake. For instance, it should be imposed that *if* V maps an atomic formula p to a pair $\langle t, w \rangle$, then it should also map p to every other pair $\langle t, w' \rangle$ such that w' *is t-accessible from* w; this is to emulate that two distinct worlds which are accessible at a time t, coincide in their present and pasts, with respect to what atomic formulas are true; and since the structure already organizes worlds in a way that being accessible at t implies being accessible at every $t' \leq t$, this secures that they will coincide in both present and pasts once we add that condition on V. But since we are offering only a taste of *temporal languages*, this fact will come irrelevant, since we are considering only expressions talking about 'tomorrow', 'one day hence', and the future.

and t_c+1 is representing what counts as being *tomorrow, relative to the time of the context* c, while $t+1$ is representing a time *one day hence, relative to* t.

By looking only at the truth-conditions for *One* and *Tom*, notice how they seem to resemble each other. Yet they do not collapse into each other. We will see in a minute some of the interesting features relating both expressions.

Now supposing that *Tom* ϕ is such that ϕ is an atomic formula, then *Tom* ϕ is true if the *assignment* is mapping ϕ to a subset containing the time in T which counts as being 'tomorrow' from the perspective of c. Thus supposing we are evaluating 'John will visit Paris tomorrow', having as its logical form *Tom* p, then it would be *true* at a context c, a time t and a world w, if 'John is visiting Paris' is being mapped to the time concerning tomorrow (relative to the time of c, considered as an occasion of use of the sentence), and the world w[6]. But notice how the time we begun with, t, looks idle here; it simply gets replaced by a time, t_c+1 which doesn't relate to it (albeit it relates to t_c). But with *One* it is different.

If we are evaluating 'One day hence, John will visit Paris', having as its logical form *One* p, then it would be *true* at a context c, a time t and a world w, if 'John is visiting Paris' is being mapped to the time which stands one day ahead of the time t ($t+1$), and the world w.

We could now point to some of the interesting features between *One* and *Tom*, and why we feel that they both resemble each other. But before, we ought to provide the notion of a sentence being *true at a context*.

So far, we have introduced the reader to a definition of truth for sentences, relative to a context, a time, and a world[7]. But we ought to flesh out the richer notion of a sentence ϕ being true 'relative to a possible occasion of its use', or the notion of 'an occurrence of a sentence at a context being true', and the like. We can do this through two main routes.

The first one, that of Kaplan, is to split the process into two stages. We first derive a notion of 'the proposition that would be expressed, had the sentence ϕ been uttered at context c', and then we would account a sentence ϕ as being *true at a context* c[8], just in case, 'the proposition that would be expressed by ϕ at c' is true relative to the *circumstance of evaluation* $\langle t_c, w_c\rangle$, where t_c is *the time of context* c, and w_c is *the world of context* c.

The second route, is to directly appeal to sentence-truth (and not to proposition-truth), and work out the notion of a sentence being true relative to a *context* and an *index*. This is Lewis's style, that we find in his *Index, Context, and Content* (1980), and we will much later be working through this kind of approach. As with many of the things we have been expounding in this section, this is also another matter we will take more

[6]This way to phrase things is just to serve intuition, but we have largely abused language here.

[7]"And when taken under a structure, and an *assignment*...", yes. But we can simply drop these qualifications for now.

[8]Or equivalently, that 'the occurrence of ϕ at c *is true*'.

care much later. But for now, it helps just noticing some of its main features.

First of all, we are relativizing truth (of a sentence) to a triple comprising a *context* and an *index*. For Lewis (1980), the kinds of parameters which get shifted by operators of the language, for example the parameter t of time (which here gets shifted by F, *One*, and *Tom*), or for example the parameter w of world (when we have modal operators), these kinds of parameters assemble an *index*. So here we may consider the *index* we have been working out, to be a pair composed by a time t and a world w[9]. Additionally for Lewis, the kinds of parameters which don't get shifted by any operators in the language are the ones directly provided by a context: for example, the parameter t_c. No operator in the language is capable of shifting this value of a context; what they can do (as we said), is retrieve this parameter from a context and replace one of the coordinates of the *index* with this value (which is what our operator *Tom* has been doing)[10].

But what changes need to be done, if instead our language is that of 'Regimentation'? Actually not much. What happens now is that given a structure $\langle C, T, W, <, \simeq \rangle$, an *interpretation* for the language can be accomplished by coupling to the structure a two-place *interpretation function* f that maps every world in W, and predicate of 'Regimentation's' language, onto a subset of T, and every 'singular term' onto members of T[11].

Concerning well-formed formulas of 'Regimentation', where α is a 'singular term', and Φ is a one-place predicate (standing for a present-tensed sentence), then $\ulcorner \Phi(\alpha) \urcorner$ is an atomic formula. We have now everything needed to run the semantics.

Therefore, given a structure $\mathfrak{A} = \langle C, T, W, <, \simeq \rangle$, and provided an *interpretation function* f, and assuming that $t \in T$, $w \in W$, and $c \in C$, we write: $[\![\alpha]\!]^{\mathfrak{A},f}_{c,t,w}$ as denoting *the extension of* α *at* c, t, w (under the structure $\mathfrak{A}$, and *interpretation* f). The recursive definitions are then as follows:

singular term:

$$[\![\text{tomorrow}]\!]^{\mathfrak{A},f}_{c,t,w} = t_c + 1$$

one-place predicates:

$$[\![\Phi]\!]^{\mathfrak{A},f}_{c,t,w} = f(\Phi, w)$$

atomic formulas:

[9] Worlds don't get shifted here, but they will later (a lot!).

[10] For a thorough analysis concerning both Kaplan's and Lewis's style, *see* MacFarlane, J. (2003b): 'Three Grades of Truth Relativity' (unpubl. manuscript).

[11] This is just one way to do it, that we thought would be simpler and straightforward

$$[\![\Phi(\alpha)]\!]^{\mathfrak{A},f}_{c,t,w} = \begin{cases} \text{True,} & \text{if} \quad [\![\alpha]\!]^{\mathfrak{A},f}_{c,t,w} \in [\![\Phi]\!]^{\mathfrak{A},f}_{c,t,w} \\ \text{False,} & \text{if} \quad \text{otherwise} \end{cases} \tag{1.1}$$

Thus supposing that P is a predicate of the language standing for 'Jake is visiting Paris', then $\ulcorner P(tomorrow) \urcorner$ (when taken in context c, and under structure $\mathfrak{A}$, and *interpretation* f) *is true with respect to time* t *and world* w, just in case $t_c + 1$ is in the extension given by $f(P, w)$. So as far as concerns treating times through 'singular terms', as in 'Regimentation', nothing essential is lost compared to Prior's option. They come in practice to the same. And we will see much later, that the real problem concerns what kinds of consequences there are, if we concede that 'Jake will visit Paris tomorrow' may bear a definite truth-value *already* today. These are the kinds of questions which drive us, so it doesn't matter whether we take Prior's viewpoint or 'Regimentation', as far (and this is an important remark) as we are concerned with simple tensed-sentences of this kind. But of course, we didn't take any trouble into accounting for tenses as quantifying expressions ranging over times, we resorted directly to 'singular terms'. Yet concerning the problem whether 'Jake will visit Paris tomorrow' ought, or not, to have a determinate truth-value, shifting from one approach to the other will just be innocuous. The problem still stands.

One additional thing that we should call into attention, is that someone could ask us why we even take trouble to place time t when we talk about the extension of α through $[\![\alpha]\!]^{\mathfrak{A},f}_{c,t,w}$ if in the end, only either w would take a role in accounting for the extension of 'Regimentation's predicates, or c would play a role when giving the extension of the 'singular term' tomorrow? Well, that is because we could have 'singular terms' such as "One day hence", and for instance the extension $[\![\text{One day hence}]\!]^{\mathfrak{A},f}_{c,t,w}$ would be $t+1$, as we discussed concerning Prior's language. And we could also draw on interesting outcomes if we had presented a definition of *truth at a context*, such as for example, that the sentences 'John will visit Paris tomorrow' and 'One day hence John will visit Paris' would come logically equivalent (but only through a definition of *truth at a context*, which we haven't presented (yet!)). But let us now move on, we have much work to do.

1.4 A Linear-time Theory for Ordinary Tense Logic

Before reaching representations of indeterminist structures, as will be provided in the next section through nonlinear 'branching-time structures', we should start our exposition by showing a first simple and intuitive way to render a semantic theory for a language containing future and past-tensed sentences.

The natural first step is to introduce a *temporal language* and a grammar governing the construction of arbitrarily complex formulas (given this language). Let us then introduce a simple *temporal language* $\mathscr{L}$ as one consisting of: (1) a set of atomic formulas $p, q, ...$ (each one standing for some present-tensed sentence - as we have seen in our previous section); (2) the *booloean connectives* $\neg, \wedge, \vee, \supset$; and finally, (3) the one-place *temporal operators* F ("It will be the case that"), and P ("It was the case that"). We also use the greek letters $\phi, \psi, ...$ to stand for arbitrary formulas (either atomic or molecular). The grammar will be straightforward: if ϕ is a formula, then $\ulcorner \neg\phi \urcorner$, $\ulcorner F\phi \urcorner$, and $\ulcorner P\phi \urcorner$ are formulas (we will sometimes mention formulas like $F\phi$ and $P\phi$ as 'future-tensed formulas' and 'past-tensed formulas', respectively); also if ϕ, ψ are formulas, then $\ulcorner \phi \wedge \psi \urcorner$, $\ulcorner \phi \vee \psi \urcorner$, and $\ulcorner \phi \supset \psi \urcorner$ are formulas.

Our next step is to provide a definition of 'linear-time structures'. Let us do that:

Definition 1 (Linear-time structures)**.** *A* linear-time structure *is a pair* $\langle T, < \rangle$*, where* T *is a non-empty set of 'moments of time'* $t_0, t_1, ...$*, and* $<$ *is a binary ordering relation on* T*, that is (1) irreflexive:* $\forall t \in T : t \nless t$*; (2) transitive:* $\forall t_i, t_j, t_k \in T$*, if* $t_i < t_j$ *and* $t_j < t_k$*, then* $t_i < t_k$*; and also (2) linear:* $\forall t_i, t_j \in T$*, either* $t_i < t_j$*, or* $t_j < t_i$*, or* $t_i = t_j$*.*

We can imagine linear-time structures as a kind of 'chained' succession of nodes, running from left to right, where each node is representing a 'moment of time', and where for each node of the chain, all the nodes standing to the right are seen as later moments, and all standing to the left, are seen as earlier moments of time. As a final step, we may define an *assignment function* mapping every atomic formula to a set of moments, and thereof provide a recursive definition of truth for arbitrary formulas of the language:

Definition 2 (assignment). *Given a 'linear-time structure'* $\langle T, < \rangle$*, an* assignment *is a function* V *mapping every atomic formula of the* temporal language $\mathscr{L}$*, to a subset of* T*.*

Definition 3 (V**-truth value**)**.** *Given a 'linear-time structure'* $\langle T, < \rangle$*, and an* assignment function V *with respect to it, the* V*-truth value* $[\![\phi]\!]_t^V$ *of* ϕ *at the* point t *(where* $t \in T$*), for all sentences* ϕ *of the* temporal language $\mathscr{L}$*, is defined as follows:*

For an atomic formula p:

$$[\![p]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if}\ t \in V(p) \\ \mathit{False}, & \mathit{if}\ t \notin V(p) \end{cases} \tag{1.2}$$

Boolean connectives:

$$[\![\neg\phi]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]_t^V = \mathit{False} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.3}$$

$$[\![\phi \wedge \psi]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]_t^V = [\![\psi]\!]_t^V = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.4}$$

$$[\![\phi \vee \psi]\!]_t^V = \begin{cases} \mathit{False}, & \mathit{if}\ [\![\phi]\!]_t^V = [\![\psi]\!]_t^V = \mathit{False} \\ \mathit{True}, & \mathit{otherwise} \end{cases} \tag{1.5}$$

$$[\![\phi \supset \psi]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]_t^V = \mathit{False},\ \mathit{or}, [\![\psi]\!]_t^V = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.6}$$

Temporal operators F and P:

$$[\![F\phi]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if\ for\ some}\ t' \in T, \mathit{where}\ t < t', [\![\phi]\!]_{t'}^V = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.7}$$

$$[\![P\phi]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if\ for\ some}\ t' \in T, \mathit{where}\ t' < t, [\![\phi]\!]_{t'}^V = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.8}$$

Temporal operators G and H [12]:

$$[\![G\phi]\!]_t^V = \begin{cases} \mathit{True}, & \mathit{if\ for\ every}\ t' \in T, \mathit{where}\ t < t', [\![\phi]\!]_{t'}^V = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.9}$$

[12]Of course, we could just pick either both F, P, or both G, H as primitives in the language, and just define the other temporal operators using negation and the primitive operators. For instance, taking F as primitive and defining $G\phi =_{df} \neg F\neg\phi$. Same with the *boolean connectives*; we pick both negation, and one of either $\wedge$, $\vee$, or $\supset$ (as primitives), and hence define the other connectives. But we choose to simply phrase everything and ease a task of consulting truth-values of complex formulas embedding the connectives and the temporal operators.

$$[\![H\phi]\!]_t^V = \begin{cases} \textit{True}, & \textit{if for every } t' \in T, \textit{where } t' < t, [\![\phi]\!]_{t'}^V = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.10}$$

The following figure depicts a diagram of a 'linear-time structure' $\langle T, < \rangle$, where $t_0, ..., t_4$ are elements of T, with the *earlier/later* relation '<' represented by the arrows (running from left to right), and showing us some of the true formulas at moments of time, according to an *assignment* over the structure:

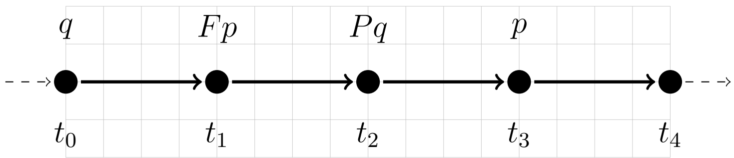


Figure 1.1: a 'linear-time structure', and an *assignment* V where $t_3 \in V(p)$ and $t_0 \in V(q)$.

We see here represented a future-tensed formula Fp that is true at t_1, since there is a *later* moment - namely t_3 - where the V-*truth value* of the atomic p at t_3 is *True*. Similar with Pq being true at t_2; there is an *earlier* moment - namely t_0 - such that q is true at t_0.

But in order to engage in many of the subsequent discussions, relating features of indeterminist representations of time (through 'branching-time structures') with that of 'linear-time', we need to introduce an account of 'linear validity' for formulas; those formulas which stand *True* irrespective of the 'linear-time structure' and *assignment* that is at play:

> **(Linear validity).** A formula ϕ is *linear valid* if, relative to all *linear-time structures* $\langle T, < \rangle$, and *assignments* V over each: the V-*truth value* $[\![\phi]\!]_t^V$ of ϕ at t, is *True* for every $t \in T$.

So just to get started, excluded middle, $\phi \vee \neg\phi$, is here valid, including instances of it embedding future-tensed formulas: $F\phi \vee \neg F\phi$. And this looks in conformity with Aristotle's position in the whole of *De Interpretatione*[13].

[13]It is a controversy whether Aristotle's main escape from the *fatalist's* conclusion, would resort to denying the *Principle of Bivalence* while retaining in full the *Principle of Excluded Middle* (first interpretation), or whether he ain't even denying there *Bivalence*, but rather resorting to something else (second interpretation). Nonetheless, there is some consensus that he is there upholding validity of excluded middle. We will discuss this matter extensively in our section dedicated to the *supervaluationist* approach

Concerning representative *linear validities*, these will include formulas like $\phi \supset PF\phi$, $\phi \supset FP\phi$, $\phi \supset HF\phi$, and $\phi \supset GP\phi$. If ϕ is true at a time, then it *was* the case (at least once) that it would be true that ϕ ($\phi \supset PF\phi$); or, If ϕ is true at a time, then it *was always* the case that it would be true that ϕ.

Especially the first two validities, $\phi \supset PF\phi$ and $\phi \supset HF\phi$, have been taken by many to be the main tenets to which many of the discussions present in Aristotle's *De Interpretatione 9* are being addressed. The very textual evidence therein suggests that Aristotle takes such formulas to be valid. Particularly telling the passage in *18b9-11*, where we find:

> "if it is white now, it was true to say earlier that it would be white; so that it was always true to say of anything that has happened that it would be so" (*De Interpretatione 9*, *18b9-11* [Ackrill's translation, 2002, p.50]).

The notion of *anterior truth* of a prediction, given present truth, looks in the whole of the chapter to be a concession of Aristotle to his opponent. The main trouble thereof for Aristotle, is when his opponent, the fatalist, moves from *anterior truth* (and nothing preventing someone to have made a *true* predicition), to the conclusion that things could not have been otherwise, or *as chance has it*. But here we haven't introduced yet any notion capable of expressing modal contents. But why shouldn't we?

In 1974, Robert McArthur has addressed such matter in an interesting light. In his *Factuality and Modality in the Future Tense* (1974), he begins his paper by observing how "[i]t is commonplace to distinguish between the *factual* future tense, e.g., 'There *will* be a sea fight tomorrow', and the *modal* future tenses, e.g., 'There *must* be a sea fight tomorrow' and 'There *may* be a sea fight tomorrow' " (McArthur, 1974, p.283). And he further claims that it is precisely "the ability to make just such a distinction" (p.238) that is the main requirement for any argument against fatalism, and that wishes to make sense of an indeterministic world.

But on his view, there would simply be no way for grounding such a distinction, because either all three statements are semantically indistinguishable (if one is working within a 'linear-time' framework - thus deterministic); or, if one is working with indeterministic frameworks such as branching structures, then he claims that *factual statements* (the simple future), albeit cast in a factual mood in surface form, they would inevitably have to be understood as being *covertly modal*. That would mean that, whenever indeterministic frameworks are at stake, no precise meaning of the *factual* future tense can be given, besides it either expressing what *must* be the case, or what *may* be the case.

At this stage, it will prove useful to assess the strengths and weaknesses of each proposal, by how they would meet the first kind of challenge introduced by McArthur. Thus, let us phrase it through a question:

(1) Can the framework discriminate between the *factual* future truth, the *possible* future truth, and the *necessary* future truth?

So let us suppose we enrich our *temporal language*, by adding to it (besides F and P) two kinds of operators that would enable us to express the full contents of sentences embedding such modal future tenses. So let us add $\Diamond^F$ as conveying 'future possibility' ("It *may* be the case (sometime in the future) that"), and $\Box^F$ conveying 'future necessity' ("It *must* be the case (sometime in the future) that"). Now let us take a look at the following figure:

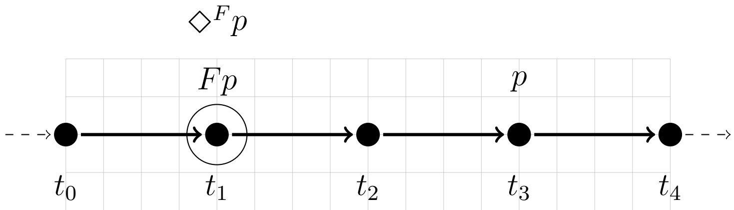


Figure 1.2: a 'linear-time structure'

We already have a good grasp of the semantics for Fp, and it seems that whatever we quarrel about representing time through 'linear-time structures', it surely looks as they do the job for making plain good sense of *factual* future truth. Moreover, we have seen that by providing a definition of *assignment* over these kinds of structures, we accomplish a straightforward notion of validity which will make valid some of the *Tense Logic* formulas for which tenets we find in Aristotle's *DI9* seem to be instances of.

But now taking once more a glimpse at the figure shown above, how about the truth-value of $\Diamond^F p$ at t_1 ("It *may* be the case (sometime in the future) that p")? Is it true just in case what? The problem looks pressing.

If all we have is the kind of representation grounded in 'linear-time structures', then there seems to be simply no room for giving truth-conditions for $\Diamond^F p$, without making it collapse with the future-tense operator. If we instead turn our attention to $\Box^F p$ ("It *must* be the case (sometime in the future) that p"), the same conclusion will look inescapable.

The problem will look even more pressing if we opt to work with sentences containing temporally definite references. Take for instance, sentence like 'It *will* be the case *tomorrow...*', or 'One day hence, it *will be* the case that'[14]. How are we to account for the truth of 'It *may* be the case *tomorrow* that ϕ' that wouldn't make it collapse with the

[14]To account for the semantics of the first sentence, we would have to tweak the definition of a structure, and endow it with a non-empty set of *contexts*, where each of its elements would have at least a parameter informing what time (of T) counts as tomorrow relative to *the time of the context*. And finally, we would have to relativize truth of a sentence to both a context and a time (and not only relative to a time t

factual 'It *will* be the case *tomorrow* that ϕ'? Again, there is simply no room. Neither will there be room for preventing either of these two to collapse with future necessity: 'It *must* be the case *tomorrow* that ϕ'.

As pointed out by McArthur, "due to the restrictions placed on the future alternatives to any given point, whatever is possibly the case is actually the case, and, hence, whatever is actually the case is necessarily the case" (McArthur, 1974, 284). We may thus couch our findings in the form of a verdict: whenever working with a determinist representation of time (such as through 'linear-time structures'), then the *factual* and *modal* future-tenses will become semantically indistinguishable.

Before ending this section, and moving on to indeterminist representations of time, we should provide some clarifications on the concept of *determinism*, and this section seems just the right place to introduce them. First of all, 'linear-time' structures seem just the kind of structure one would expect when representing a deterministic world. Sure, there are many distinct concepts of determinism; for instance, we might take it in the sense of the *Laplacean* definition:

> "We must therefore regard the present state of the universe as an effect of the state preceding it, and as cause of the state which will follow it. An intellect which at a given moment would know all the forces governing Nature and the respective situations of all things of which it is composed . . . would embrace in the same formula the movements of the largest bodies in the universe as well as those of the lightest atom; nothing would be uncertain for it, and future and past alike would be present before his eyes" (Laplace, 1921, p.3 *apud* Sambursky, 1959, pp.57-58)

Or we might even prescind of talk of omniscience; it could simply just be that "the evolution of the world is so chaotic that it could not be predicted, even by an agent with full knowledge of its past states and all the laws of nature." (MacFarlane, 2008, p.81). Nonetheless, it just doesn't matter the stance we take, since we always end up with such a view of a 'chain' of events which 'linear-time structures' seem just well suited to represent. So in essence, when taking representations of temporal succession the way we do, simply *qua* mathematical structures, it won't matter if each temporal state could be predicted by an omniscient being or not, or if each state is an unsurpassable aftermath of previous states, by *physical laws*. For matters of investigating *Tense Logics*, and distinct semantic theories concerning a *temporal language*, it is this kind of approach, of viewing representations of time simply *qua* mathematical structures, that will be enough to do the job.

according to a V, as in $[\![p]\!]_t^V$). To account for the second sentence, we would have to regiment metric indices into the meanings of either F, $\Diamond^F$, or $\Box^F$, and provide some way for the structure to relate two moments by time spans in some unit. We will much later talk a lot about definitions of *truth at a context* and the semantics for such expressions. But for now it is enough to just grasp the intuition.

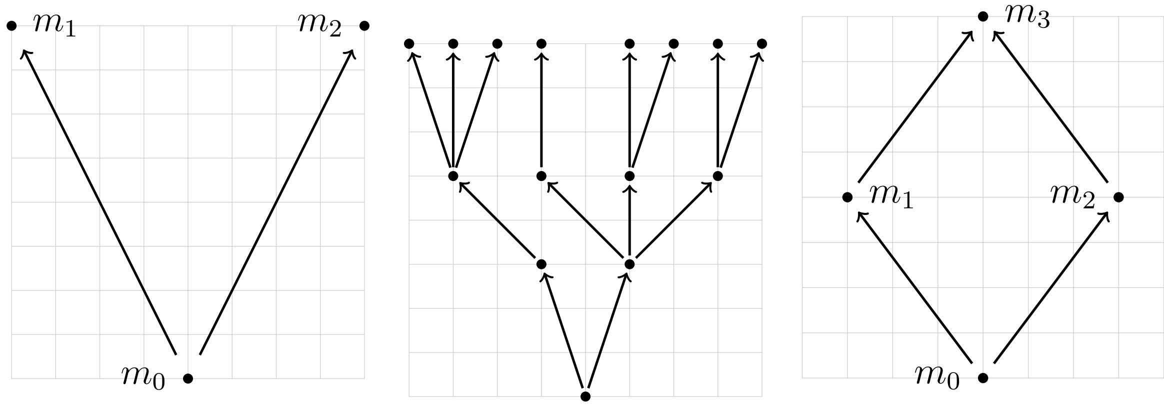
We have thus seen that through such representations, there will be no room for providing good sense of both '*factual* talk about the future' and '*modal* talk about the future'. So let us now carry fresh in mind all such quarrels, and readily move on to the next section where we will introduce indeterminist representations of time, and the main attempts at interpreting the future-tense and the historical modals in terms of these representations. And for a reminder, we will still be guiding our discussions through the kind of challenge brought by McArthur.

1.5 'Branching-time' structures: figuring a way to represent indeterminism.

One way to picture temporal indeterminism is through structures which partially order moments of time, by forking them only in one direction (upwards, if you like), as resembling a tree and depicting how time could evolve indeterministically towards the future. One kind of structure which seems well suited for this kind of representation has been christened in literature with labels such as 'branching time structures' or 'treelike structures'. Here is one way to define these kinds of structures:

Definition 4 (Branching structures)**.** *A* branching structure *is a pair* $\langle M, < \rangle$*, where* M *is a non-empty set of 'moments of time', and* $<$ *is a binary ordering relation on* M*, that is* (1) *transitive: for every* $m_1, m_2, m_3 \in M$*, if* $m_1 < m_2$ *and* $m_2 < m_3$*, then* $m_1 < m_3$*; and also* (2) *such that for every* $m_1, m_2, m \in M$*, if* $m_1 < m$ *and* $m_2 < m$*, then either* $m_1 < m_2$*, or* $m_2 < m_1$*, or* $m_1 = m_2$*.*

Given any such structure, we might imagine each moment (of time) as a 'temporal state of an indeterministic world', which forks upwards towards distinct moments representing its 'future alternatives'. Condition (2) of the above definition guarantees that for every moment of time, there is a unique route going downwards (if you like), such that every moment which stands as being earlier to it will be linearly ordered, and thus comparable by the relation $<$. The following three figures, illustrate two structures of the 'branching' kind (the first two), and one which isn't (the last):



(a) ‘branching’ (b) ‘branching’ (c) ‘non-branching’

Figure 1.3: ‘branching’ and ‘non-branching’ kinds of structures

Each ‘dot’ here is representing a moment of time, and the arrows are representing the earlier/later relation symbolized by <. When focusing on each ‘dot’, every other ‘dot’ following the arrows and being upwards is viewed as *later*, while every ‘dot’ linked downwards by the arrows (contrariwise to its direction) is viewed as being *earlier*. The first figure is representing a structure whose ‘set M’ is $\{m_0, m_1, m_2\}$, and where these moments are ordered in accordance with the above definition; we then have a representation mirroring the relations $m_0 < m_1$, and $m_0 < m_2$. The second figure is also representing a branching structure, only that this structure now comprises more elements (each dot still representing a moment of time), and the arrows are again representing the earlier/later relation. The third figure represents a structure which is not of the ‘branching’ kind. Notice how the elements m_1 and m_2 are earlier than m_3, and yet they are not *comparable*, or related, by <; thus not complying with condition (2).

At this stage, since we are interested in providing a semantic theory for a language containing the future tense, we start asking ourselves again how we ought to interpret such a tense in such indeterministic kinds of structures. The natural first step would be then to summon the *temporal language*, and the grammar governing the construction of arbitrarily complex formulas (given this language), that we have introduced in our previous section.

But for now, let us just introduce the simplest *temporal language* containing only a set of atomic formulas $p, q, ...$ (each one standing for some present-tensed sentence), and the one-place *temporal operator* F (“It will be the case that”). The grammar is again straightforward: where p is an atomic formula, $\ulcorner Fp \urcorner$ is a formula [15].

The final step, we could then think, would be to provide an *assignment function* mapping every atomic formula to a set of moments (sticking to precision, ‘to a subset of M, of a given branching structure $\langle M, < \rangle$’), and thereof provide a recursive definition of

[15]Let us leave for now just a language containing no past-tense and modal operators, nor *boolean connectives* - we will do that later.

truth for formulas with an F operator scoping over a subformula. But let us consider just a very simple case of a branching time structure, as the one represented by figure 1:

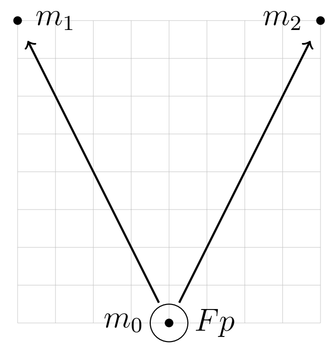


Figure 1.4

How are we to account for the truth-value of Fp at the moment m_0? The problem looks pressing (again!). Is it true, just in case p is true at both m_1 and m_2? Or is it true just in case p is true *at least* in one of either m_1 or m_2?

There are far more lessons to take from these interrogations, than just a technical matter of deciding the right truth-conditions for the future tense. As Thomason puts the matter, when Prior discusses this technical problem and couples his exposition "with bits from figures like Diodorous Cronus, Peter de Rivo, and Jonathan Edwards" (Thomason, 1984, p. 142), he is expounding the main insights lurking behind their logical arguments for determinism. It is precisely for this kind of problem, of finding "[a] definition of satisfaction for a language with tense operators that is suited to such structures" (Thomason, 1984, p.142), that the arguments claim that no meaningful solution can be found. There is simply no good way, they would claim, to escape this.

In the following sections, we will present some of the main approaches for providing a solution to these kinds of problem. For each of them, we will discuss their merits and adversities. As we have already mentioned in the previous section, it will also prove useful here to assess the strengths and weaknesses of each proposal, by how they would meet the kind of challenge introduced by McArthur. But here we yield a more complete account of the matter, through two complementing questions:

> (1) Can an indeterministic framework (like those using 'branching structures' to represent indeterminism) discriminate between the *factual* future truth, the *possible* future truth, and the *necessary* future truth?
>
> (2) After giving an answer for (1), can it still do justice to a conception of real, objective indeterminism?

We think this is enough for setting the stage. So it is now time to query again for proposals.

1.6 *Łukasiewicz's* solution

Łukasiewicz's way out of the problem is to introduce a third truth-value, which he labeled as the value *'possible'* - besides *'true'* and *'false'* - and supply three-valued truth-tables for the logical connectives. Here are the truth-tables that Łukasiewicz provides[16]:

$\neg$		$\wedge$	T	i	F	$\vee$	T	i	F	$\supset$	T	i	F
T	F	T	T	i	F	T	T	T	T	T	T	i	F
i	i	i	i	i	F	i	T	i	i	i	T	T	i
F	T	F	F	F	F	F	T	i	F	F	T	T	T

Figure 1.5: Łukasiewicz's three-valued truth-tables

But this maneuver has often raised suspicion. Thomason (1984, p.143), for instance, takes Łukasiewicz's position to yield outcomes which are at variance with the very intended indeterministic interpretation.

Take a first example. According to the truth-tables for disjunction and negation, as shown above, whenever an atomic formula p is standing for a future contingent, and thus possessing the *'indeterminate'* truth-value (such as *'Tomorrow there will be a sea battle'*), Łukasiewicz's approach will output the instance $\ulcorner p \vee \neg p \urcorner$, of excluded middle, as also having the *'indeterminate'* truth-value. So in terms of truth-values, it puts both on a par p and $p \vee \neg p$; yet the first is standing for a sentence which is properly said a future contingent, while the second formula is not.

Thus at close inspection, Łukasiewicz's solution seems even at variance with Aristotle's approach in *De Interpretatione 9*, and the textual evidence therein. For even if Aristotle would be taking future contingents to have indeterminate truth-values (which is actually a tenable position to impute to Aristotle), it seems he would nonetheless reject any position rendering excluded middle as not valid, and rendering instances of it as not taking the truth-value *true*[17].

A second problem, is that whenever p and q are both standing for future contingents, yet they are not jointly contradictory sentences (for example, let us take the first to be *'Tomorrow there will be a sea battle'*, and the second *'Tomorrow it will rain at Athens'*), Łukasiewicz's semantics would also put on a par both sentences $p \vee q$ and $p \vee \neg p$, in terms

[16] Łukasiewicz, J. (1967 [1920]): On Three-Valued Logic (*In* McCall, S. (1967): *Polish Logic, 1920–1939.* Oxford University Press.)

[17] For more on this respect, *see* N. Kretzmann's (1998) interesting examination connecting Łukasiewicz and Boethius's commentaries on *De Interpretatione 9* (*In*: Ammonius (1998): On Aristotle's On Interpretation 9 with Boethius (1998): On Aristotle's On Interpretation 9. Translated by David Blank and Norman Kretzmann).

of truth-values. Both would possess the same truth-value: *'indeterminate'*. Yet again, the first sounds properly a future contingent, while the second does not.

In conclusion, perhaps Łukasiewicz's gives us some answer to the second question we presented above. After all, if we assume that the future is open, and there is real and objective indeterminism, then taking sentences about the future to possess the 'indeterminate' truth-value looks at least in conformity with this assumption.

Yet surely it give us no answer to question (1). How can it make sense of *factual* future truth, if the very approach is rooted in qualifying future contingents as being indeterminate in truth-value? And besides, blaming the approach for introducing a third truth-value is the least of the charges against it. The real problem is how the approach ends up outputting indeterminate truth-values for constructions we would take as *valid*; most especially, instances of excluded middle. So we better look for other options, as the next one: the *Peircean* approach.

1.7 The *Peircean* solution

Different from Lukasiewicz's solution, the *peircean* approach (which is actually a coinage by Prior (1967), who elaborates the position), will now opt to fully embrace branching structures as representations of indeterminism, and to directly face the problem of giving determinate truth-conditions for future-tensed sentences, by quantifying over the moments of time which stand in the later relation, with respect to the moment of evaluation. But here we arrive at yet another problem: how should we *quantify over the later moments*?

Let us take stock. We are here choosing whether a sentence such as *'John will visit Paris'* should be true, if *'John is visiting Paris'* is true at *some* later moment (relative to at least one way the future might go), or whether it should be true, if *'John is visiting Paris'* is true at *every* some later moment (relative to any possible way the future might go). But now consider the following three figures, where *'John will visit Paris'* is being represented by $F\phi$, and we are asking on what conditions it is true at the moment m_0.

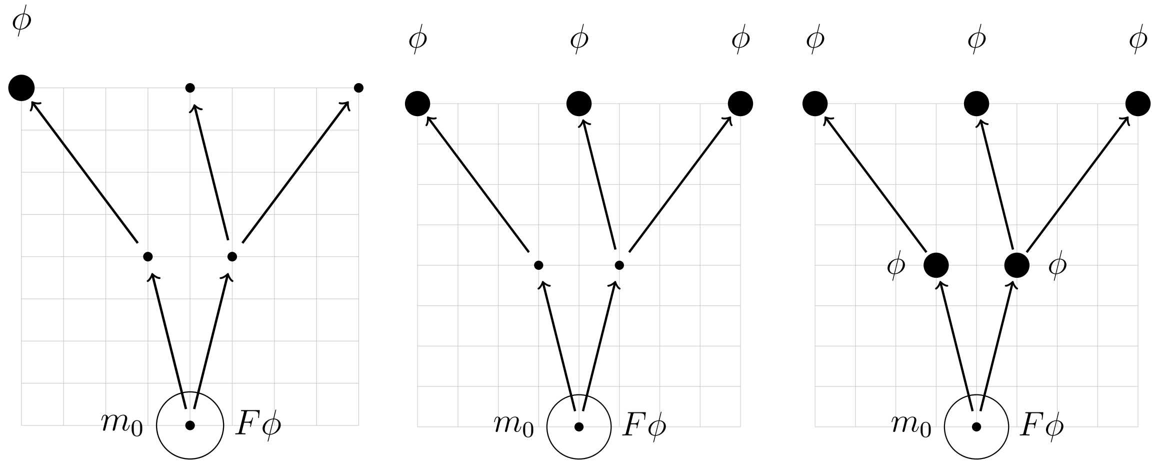


Figure 1.6: Three options for quantifying over *later* moments

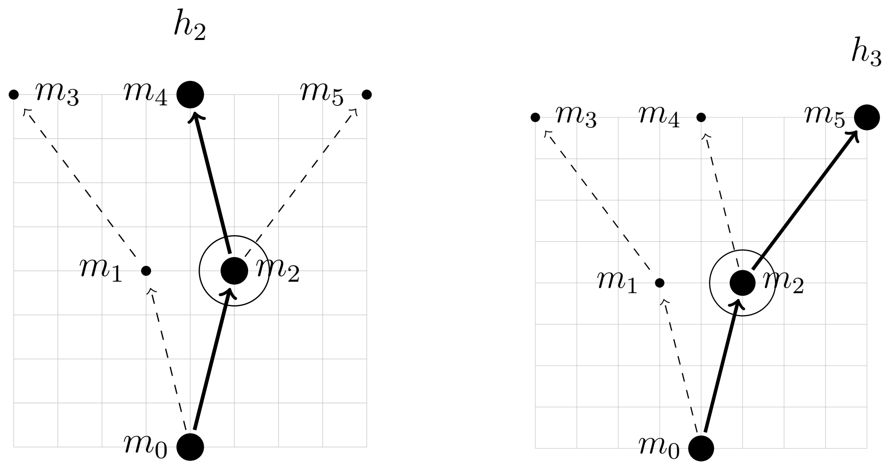
So it looks that we are choosing between whether to interpret $F\phi$ in the sense of the first diagram (Figure 'a'), or the second (Figure 'b'). In the first diagram, *'John will visit Paris'* is true, if *'John is visiting Paris'* is true in at least one later moment, relative to at least one way how the future might go. In the second (Figure 'b'), it is true if *'John is visiting Paris'* is true at every some later moment, whatever the future might go.

But if all we have is the notion of a later moment relative to m_0, then simply quantifying over *all* later moments would be represented by the diagram depicted in Figure 'c'. But we surely don't want to mean that the second option, of interpreting *'John will visit Paris, no matter how the future goes'*, would be true if *'John is visiting Paris'* is true at every single moment from now on; tomorrow, the day after, and for all eternity.

Thus, if we are to choose between the first and the second options (and not the option depicted by Figure 'c'), we will need a richer notion, we will need the notion of a 'history' passing through a moment. Fortunately, the task of defining such a notion is straightforward within branching structures. This is how we define it:

Definition 5 (Histories and Branching sets)**.** *Given a branching structure $\langle M, < \rangle$, and an element $m \in M$, a* history through *m is a maximal linearly ordered subset of M, containing m. The branching set of histories passing through m, is denoted by H_m. And the set of all possible histories, is the set $H(M) = \bigcup_{m \in M} H_m$.*

Intuitively, a branching structure, which is a partial order of moments of time, gives us with respect to a moment m (an element of the given structure), the 'branches' representing the 'alternatives' or 'possible histories' which pass through m. A 'history', which we will denote generally simply by h, is technically a *maximal chain*, which means that every element of a history h is *linearly ordered* by the relation $<$ of the given branching structure. Thus, this means that every two elements of a 'history', must be *comparable* by the relation $<$: $\forall m, m' \in h$: either $(m < m')$, or $(m' < m)$, or $(m = m')$. The following two figures will provide an easy grasp of the concept.



(a) one of the 'histories' through m_2: $h_2 \in H_{m_2}$

(b) the other 'history' through m_2: $h_3 \in H_{m_2}$

Figure 1.7: 'Histories' passing through a moment

Both figures are representing a same branching structure; it is a structure composed by the moments $m_0, ..., m_5$ and a relation partially ordering them. Figure 'a' above is just highlighting one of the 'histories' passing through the moment m_2: it is a subset of all moments, which contains m_2, and such that every two elements of it (m_0, m_2, and m_4) are *comparable* by the relation $<$; we are also calling it here h_2. Figure 'b', in its turn, is representing another 'history' (and actually *the only other* in this case) passing through m_2. Again, it is complying with the above definition of 'histories'. Finally, the *set of all 'histories' passing through* m_2, denoted by H_{m_2}, is exactly the set $\{h_2, h_3\}$.

We now have almost all the tools to make sense of the *peircean* interpretation of the future tense, and thus provide a recursive definition of truth for a language containing the F operator. We just need first to define an *assignment function* mapping each atomic formula to a subset of the set M of moments (of a structure), and then we'll finally provide a recursive definition of *truth* for any constructions embedding operators and connectives of the language.

Definition 6 (assignment). *Given a branching structure $\langle M, < \rangle$, an* assignment *is a function V mapping every atomic formula to a subset of M.*

Definition 7 (V-truth value). *Given a branching structure, and an assignment function V, the V-truth value $[\![\phi]\!]^V_m$ of ϕ at the* point *m, for all sentences ϕ of the language $\mathscr{L}$, is defined as follows:*

For an atomic formula ϕ:

$$[\![\phi]\!]_m^V = \begin{cases} \textit{True}, & \textit{if } m \in V(\phi) \\ \textit{False}, & \textit{if } m \notin V(\phi) \end{cases} \tag{1.11}$$

Boolean connectives:

$$[\![\neg\phi]\!]_m^V = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]_m^V = \textit{False} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.12}$$

$$[\![\phi \wedge \psi]\!]_m^V = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]_m^V = [\![\psi]\!]_m^V = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.13}$$

$$[\![\phi \vee \psi]\!]_m^V = \begin{cases} \textit{False}, & \textit{if } [\![\phi]\!]_m^V = [\![\psi]\!]_m^V = \textit{False} \\ \textit{True}, & \textit{otherwise} \end{cases} \tag{1.14}$$

$$[\![\phi \supset \psi]\!]_m^V = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]_m^V = \textit{False, or,} [\![\psi]\!]_m^V = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.15}$$

Temporal operators:

$$[\![F\phi]\!]_m^V = \begin{cases} \textit{True}, & \textit{if for every } h \in H_m, \textit{there is an } m' \in h : m < m' \ \& \ [\![\phi]\!]_{m'}^V = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.16}$$

$$[\![P\phi]\!]_m^V = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]_{m'}^V = \textit{True for some m' such that m'} < m \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.17}$$

In compliance with what we just have said, notice that the definitions of 'history' and 'branching set' are here needed to recursively define the truth of a future-tensed sentence (as in the clause of $F\phi$ above) relative to a single moment *coordinate*. The sense we sought, as it was claimed, was to interpret a future-tensed sentence as true at a moment m, just in case there was at least one later moment, in each 'history' passing through m, where the untensed sentence is true. Thus, we remarked, we couldn't just simply provide a semantic clause which would *quantify over every later moment*, or over *some later moment*, since this alone wouldn't get the intended interpretation.

Nonetheless, and this is particularly important in discriminating between the *Peircean semantics* and the *Ockham semantics* (which we will expound in the next section), it should be observed that albeit there is *quantification* over histories, they don't play any role as a *coordinate* relative to which truth is defined. The only coordinate here is a *moment*. And finally, observe that no quantification over histories is even needed when accounting for truth of past-tensed sentences; we simply quantify, existentially, over *earlier* moments. And this is because in branching structures, all earlier moments, relative to any moment, are *linearly ordered*.

But before looking for merits and adversities, we should introduce the notion of *peircean* validity:

> **(*Peircean* validity)**. A formula ϕ is *Peircean valid* if, relative to all *branching structures* $\langle M, < \rangle$, and relative to all *assignments* V (with respect to $\langle M, < \rangle$): the V*-truth value* $[\![\phi]\!]^V_m$ of ϕ at m, is *True* for every $m \in M$.

Now for the good parts. Different from Łukasiewicz's solution, the *Peircean* approach is able to make sense of the validity of excluded middle. Every instance of it, including instances embedding the future-tense (as in $F\phi \vee \neg F\phi$), will be true, irrespective of what branching structure or assignment function is at play.

This happens for two reasons: the first is that (1) the *peircean* interpretation translates a sense of future-tensed claims, as expressing something in the lines of *It will be the case, no matter what*. The second reason is that (2) since for every atomic formula ϕ, there is an *assignment* mapping it onto every moment in the structure where it counts true, then for every moment of time, there are only three kinds of situations concerning the future truth of ϕ: either (i) the future is open with respect to it (there are 'histories' where it will (someday) be true, others where it won't (ever) be true); or (ii) the future is settled in its falsehood (there is no 'history' where it ever will be true); or finally, (iii) the future is settled in its truth (for every history there is a later moment where it is true).

In the first two cases, (i) and (ii), $F\phi \vee \neg F\phi$ is true because the truth conditions for $\neg$ and F will then render its second disjunct, $\neg F\phi$, as being true. So if we are at a situation where there is at least one 'history' where ϕ won't ever be true (either open with respect to ϕ being true, or settled that never ϕ will be true), then it is not the case that it *will* be true, in the *peircean* sense. And now with respect to the only remaining case, (iii), where it is settled that ϕ will be (someday) true, at every possible 'history', then now the whole disjunction will be true because its first disjunct, $F\phi$, is now true, in the *peircean* sense.

Now for the bad verdicts. This approach will not make valid $F\phi \vee F\neg\phi$. For instance, it won't be true in situations as described by (i) above, where the future is open with respect to the future truth of ϕ. Nevertheless, this formula is not an instance of excluded middle. But perhaps, we might glimpse at it some sense familiar to what is expressed by the excluded middle instance.

This happens especially when we take ϕ as expressing something which we take that can only happen once. For example, something like *'Jake will die of a heart disease'*. It seems then that if we say something like *Either 'Jake will die of a heart disease'* ***or*** *'it is not the case that Jake will die of a heart disease'* (resembling excluded middle), it looks as expressing the same as: *Either 'Jake will die of a heart disease'* ***or*** *'Jake will not die of a heart disease'*.

In fact, the *peircean* interpretation can't make logically equivalent both $\neg F\phi$ and $F\neg\phi$. And this might sound something of a quirk, if we are agreeable at welcoming sentences such as *'Jake will die of a heart disease'*. And why shouldn't we be?[18]

The reason why they are not logically equivalent, is because the first can be true in a model where there is one 'history' where ϕ won't be ever true, and a second history where ϕ will forever be true (thus, because of the first 'history', it is *not the case* that for every 'history', ϕ will be true at least once; hence $\neg F\phi$ is true). Yet in this same model, $F\neg\phi$ is false, since there is a 'history', the second one, where ϕ is forever true, hence $\neg\phi$ will forever be false, and thus $F\neg\phi$ is false.

Moreover, neither will the *peircean* approach make valid either $\phi \supset PF\phi$, nor $\phi \supset HF\phi$, nor many other formulas which are validities of Linear Time Theories. These two formulas are not valid because of the behavior of F; the fact that ϕ is true doesn't imply that it *was* the case (or it *was always the case*) that ϕ would inevitably be true, since we may construct a model where for some past moment, there was a 'history' passing through it, in which every later moment is such that ϕ is false; thus, the whole material implication $\phi \supset PF\phi$ (and also $\phi \supset HF\phi$) is not a validity in a *peircean* system.

How about our two questions? Well, like Łukasiewicz's approach, the *peircean* solution seems to give a positive answer to the second question: by opting to address the problem of interpreting the future tense directly in terms of giving truth-conditions using branching structures, it fully complies with treating the future as being objectively indeterminate.

[18]MacFarlane (2014, p.216), for instance, suggests that especially when we opt to "regiment" an indexical operator, such as *Tomorrow*, to be a sentential operator, hence yielding a syntactic difference between formulas with distinct interactions of scope between '$\neg$' and '*Tomorrow*', we don't hear a difference between them. We haven't yet talked about indexical operators such as *Tomorrow* (we are saving it for later discussions), but the point is that no difference is heard between '$\neg$ *Tomorrow* ϕ' and '*Tomorrow* $\neg\phi$'; where the first is expressed by a sentence such as *'It is not the case that tomorrow there will be a sea battle'*, and the second, *'Tomorrow there will not be a sea battle'*.

Yet with respect to the first question, it performs as bad as Łukasiewicz's solution. It will just collapse the sense of the future-tense with the sense of settled truth, so it gives no semantic distinction between the *factual* future-tense occurring in 'There *will* be a sea battle tomorrow', and the *modal* future-tense occurring in 'There *must* be a sea battle tomorrow'.

Thus we have reached a forceful dilemma. The only kinds of representations that we have so far, are 'linear-time' and 'branching-time' structures. So either we will be looking at diagrams resembling the first of the following figures, or diagrams resembling the two below:

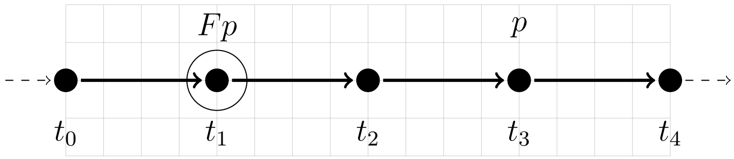
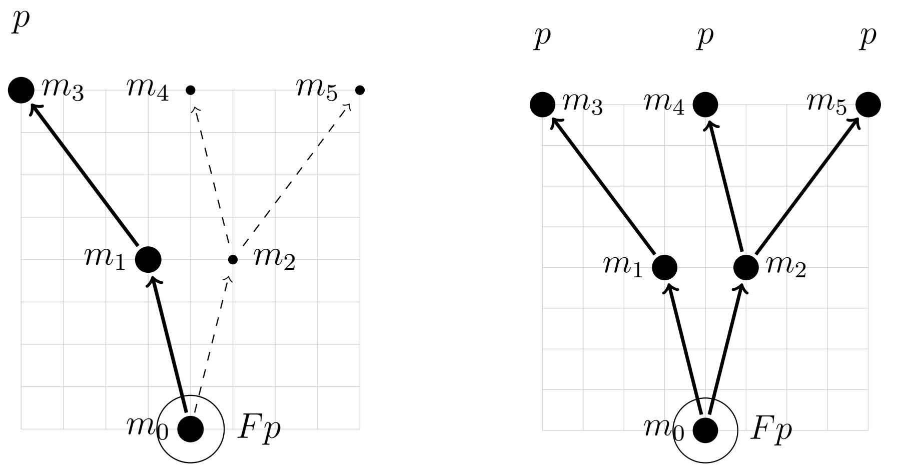


Figure 1.8: a 'linear-time structure', and an *assignment* where $t_3 \in V(p)$.



(a) p is true at some later moment, in *at least one* 'history' through m_0.

(b) p is true at some later moment, in *every* 'history' through m_0.

Figure 1.9: Quantifying over *some* 'history', or over *every* 'history'

Looking at the first case, we perhaps get a good sense of *factual* future truth, but then 'talk of *modal* future-tenses' just become innocuous, since either it is just inexpressible, or if expressible, it just collapses semantically with the future-tense. Looking at the remaining two cases, within 'branching' kinds of representation, we now lose any good sense of the *factual* future-tense, and we end up with the view - advocated by McArthur - "that all future-tense statements should be viewed as either overtly or covertly (when in a factual guise) modal" (McArthur, 1974, p.288).

In conclusion, we have so far three approaches, *Linear-time* theories, Łukasiewicz's and the *peircean* solution. In none of them, a good answer was found with respect to our question (1) - can we discriminate between factual and modal senses of future truth? Perhaps these examples are just showing us the kind of dead end predicted by the arguments for determinism. After all, it seems that as soon as the approach fully embraces objective indeterminism, it loses any ground for making sense of the factual future tense. So why should we postpone the problem? Perhaps there really is no way to give a positive answer to both questions.

But let us not make such hasty a conclusion by looking only at two kinds of solution. Things have just started to get interesting, and the third approach to be introduced, the *Ockhamist* solution, might have enough power to shake things up again.

1.8 The *Ockhamist* solution

According to Thomason, this solution embraces "[t]he most promising of Prior's suggestions for dealing with indeterminist future tense" (1984, p.143). But before moving on, we should first enrich our *temporal language*, by adding to it the one-place *historical modal operators* $\Diamond$ ("It is *possible* that"), and $\Box$ ("It is *necessary/settled* that"). Grammar as usual: we just add that if ϕ is a formula, then $\ulcorner \Diamond\phi \urcorner$ and $\ulcorner \Box\phi \urcorner$ are formulas.

The main idea here is that, given a 'branching structure' and an *assignment*, a formula is now true relative to a pair composed by both a moment (of the structure), and a 'history' containing that moment. In order to make the idea precise, it will require us to slightly tweak the definitions of an *assignment* V and of the V*-truth value* of a formula (at a pair), based upon an *assignment* V. But the definitions are quite straightforward:

Definition 8 (**Ockhamist assignment**). *Given a branching structure* $\langle M, < \rangle$, *an* Ockhamist assignment *is a function* V^{Ock} *mapping every atomic formula into subsets of* $\{\langle m, h \rangle | m \in M \, and \; h \in H_m\}$.

Definition 9 (**Ockhamist** V^{Ock}**-truth value**). *Given a branching structure, and an assignment function* V^{Ock}, *the* V^{Ock}*-truth value* $[\![\phi]\!]^{V^{Ock}}_{\langle m,h \rangle}$ *of* ϕ *at the* pair $\langle m, h \rangle$, *for all sentences* ϕ *of the language* $\mathscr{L}$, *is defined as follows (to ease reading, we will henceforth drop the qualification* V^{Ock}, *and simply write* V *instead):*

For an atomic formula ϕ:

$$[\![\phi]\!]^{V}_{\langle m,h \rangle} = \begin{cases} \textit{True}, & \textit{if } \langle m, h \rangle \in V(\phi) \\ \textit{False}, & \textit{if } \langle m, h \rangle \notin V(\phi) \end{cases} \tag{1.18}$$

Boolean connectives:

$$[\![\neg\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]^V_{\langle m,h\rangle} = \mathit{False} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.19}$$

$$[\![\phi \wedge \psi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]^V_{\langle m,h\rangle} = [\![\psi]\!]^V_{\langle m,h\rangle} = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.20}$$

$$[\![\phi \vee \psi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{False}, & \mathit{if}\ [\![\phi]\!]^V_{\langle m,h\rangle} = [\![\psi]\!]^V_{\langle m,h\rangle} = \mathit{False} \\ \mathit{True}, & \mathit{otherwise} \end{cases} \tag{1.21}$$

$$[\![\phi \supset \psi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]^V_{\langle m,h\rangle} = \mathit{False},\ \mathit{or}, [\![\psi]\!]^V_{\langle m,h\rangle} = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.22}$$

Temporal operators:

$$[\![F\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{True}, & \mathit{if\ for\ some}\ m' \in h, \mathit{such\ that}\ m < m', [\![\phi]\!]^V_{\langle m',h\rangle} = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.23}$$

$$[\![P\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]^V_{\langle m',h\rangle} = \mathit{True\ for\ some\ m'\ such\ that\ m'} < m \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.24}$$

Historical Modal operators[19]*:*

$$[\![\Box\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \mathit{True}, & \mathit{if\ for\ every}\ h' \in H_m, [\![\phi]\!]^V_{\langle m,h'\rangle} = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{1.25}$$

[19] Same as we remarked before about the *boolean connectives* and the *temporal operators*: we could just pick one of either $\Diamond$ or $\Box$ as primitive, and define the other operator (its dual) using negation; but it will prove useful to simply phrase everything, and provide quicker examinations when consulting the truth-values of complex formulas embedding them.

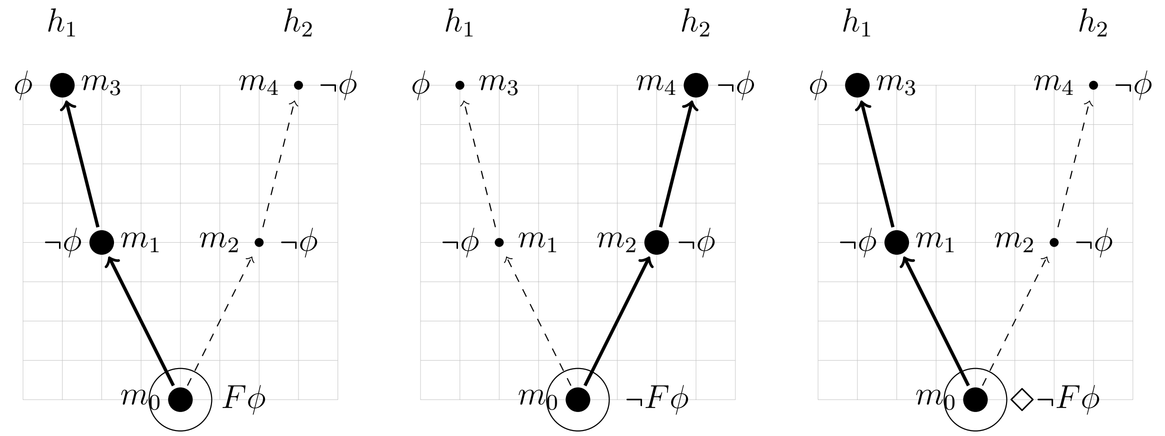
$$
[\![\Diamond\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \textit{True}, & \textit{if for some } h' \in H_m, [\![\phi]\!]^V_{\langle m,h'\rangle} = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{1.26}
$$

And as we have done with respect to the *Peircean* approach, we here also introduce the notion of *Ockhamist validity*:

> **(*Ockhamist* validity).** A formula ϕ is *Ockhamist valid* if, relative to all *branching structures* $\langle M, <\rangle$, and relative to all *Ockhamist assignments* V (with respect to $\langle M, <\rangle$): the V*-truth value* $[\![\phi]\!]^V_{\langle m,h\rangle}$ of ϕ at the pair $\langle m, h\rangle$, is *True* for every $\langle m, h\rangle$, where $m \in M$, and $h \in H_m$.

We can now start assessing the merits of the *Ockhamist* approach. First of all, compared to Łukasiewicz's and *peircean* approaches, the *Ockhamist* solution will simply outperform them in a number of outcomes. Just to get started, we now have a definition of truth for future-tensed sentences which appeals directly to a 'history' coordinate, and only quantifies over later moments of time contained by the 'history'. Thus, a formula like $F\phi$ will be true at a pair $\langle m, h\rangle$ (relative to a branching structure and an *assignment*), when there is at least one later moment contained by h, such that ϕ is true at the pair $\langle m', h\rangle$.

Furthermore, we now have a language able to express modal contents, through the historical modal operators $\Diamond$ and $\Box$. So we are now able to combine tenses with modals, and express things in the lines of *'It is **possible** that there **will** be a sea battle'*, or *'It is **necessary** that there **will** be a sea battle'*. The following three figures show us some of the true formulas at a pair, according to an *assignment* over a given 'branching structure':



(a) $F\phi$ is true at $\langle m_0, h_1\rangle$ (b) $\neg F\phi$ is true at $\langle m_0, h_2\rangle$ (c) $\Diamond\neg F\phi$ is true at $\langle m_0, h_1\rangle$

Figure 1.10: The *Ockhamist* approach: truth at *a moment*, and *a history*

In the first figure ('a' above), a future-tensed formula is true at the pair $\langle m_0, h_1\rangle$, since there is a later moment contained by h_1 - namely m_3 - such that the untensed formula ϕ is true at the pair $\langle m_3, h_1\rangle$. In the second figure ('b'), $\neg F\phi$ is true at the pair $\langle m_0, h_2\rangle$, because there is no later moment contained by h_2 (neither m_2, nor m_4), where ϕ would be true at such pairs. And finally, the last diagram ('c') is showing us how the modalized futured-tensed formula, $\Diamond\neg F\phi$, is true at the pair $\langle m_0, h_1\rangle$, since there is a 'history' passing through m_0 (thus, contained in the *branching set* H_{m0}) - namely, the 'history' h_2-, such that $\neg F\phi$ is true at the pair $\langle m_0, h_2\rangle$.

Compared to the *Ockham* approach, the *peircean language*, which only contains the future and past tenses (besides the *boolean connectives*), has thus less expressive power than *Ockhamist* language. It is, in Prior's words, "a fragment in which contingently true predictions are, perversely, inexpressible. The Peircean can only say 'it will be that p' when p's futurition is necessary; when it is not necessary but will occur all the same, he has to say that 'It will be that p' is false; the sense in which it is true eludes him." (Cf. Prior, 1967, pp.130-131 *apud* MacFarlane, 2014, p.215). Yet the *Ockhamist* approach is here able to express the *peircean* sense of the future-tense, through $\Box F$.

A further important thing, is that different from the *Peircean* approach, the *Ockhamist* Tense Logic can be viewed as a conservative extension of ordinary (Linear) Tense Logic. The reader might remember how the *Peircean* approach wouldn't make valid many of the formulas valid in ordinary Tense Logic. But this is not the case here, because all the *Ockhamist validities* of the tense fragment of the Ockham language (that is, formulas containing no occurrences of $\Diamond$ and $\Box$), are precisely validities of ordinary Tense Logic. So all these ordinary validities, formulas like $\phi \supset PF\phi$, $\phi \supset HF\phi$, are also *Ockham validities.*

Thus speaks Thomason, "indeterminist frames can be accommodated without sacrificing any orthodox validities". And "[t]his is good", he continues, "for those who (like me) are not determinists, but feel that these validities are intuitively plausible" (Thomason, 1984, p.144).

And finally, as a coronation of the approach, the *Ockhamist* seems to finally curb the argument leading to the determinist's conclusion. There are just many ways that this could be illustrated.

So let us put the matter back in terms of the two questions which were guiding the disputes between the indeterminist and the determinist, and which are being requested for the indeterminist to be adequately responded. First question was: (1) can the framework discriminate between the *factual* future truth, the *possible* future truth, and the *necessary* future truth? Second one is: (2) after giving an answer for (1), can it still do justice to a conception of real, objective indeterminism?

So it now seems that the Ockham solution is finally able to work around nicely the first question. It can definitely discriminate between the factual, the possible and the necessary future truths, since now simple truth just won't collapse with necessary truth. And it can do this by allowing a simple future-tensed sentence, $F\phi$, to be true at a pair $\langle m, h\rangle$ (given a branching structure and an *assignment*), while nevertheless, also true that it could never happen; thus, allowing that the formula $\Diamond\neg F\phi$ could be true at the same pair. Our illustration, through Figures 11-13, has shown us precisely that, and it definitely looks as an improvement over its competing theories: Łukasiewicz's and the *Peircean* solutions.

Moreover, it finally seems that the indeterminist can now claim that the *Ockhamist* solution is also able to make good sense of real and objective indeterminism, since it fully works with branching structures. All the notions of an *Ockhamist assignment*, the *V-truth value* of a formula (based upon such an *assignment*) and *Ockhamist validity* are all working with branching structures.

But does the Ockhamist solution really circumvent the determinist's conclusion? Was it that easy? Actually, we will discuss some of the reasons why the *Ockhamist* solution might not exactly work around the determinist argument, and perhaps even worse, why the very appeal to a 'history' might actually be used by the determinist to make his own case.

1.8.1 The determinist's sting against the *Ockhamist*.

Let us take stock once more. At first sight, the Ockhamist solution might suggest a nice work-around for the indeterminist to make his case against the logical argument for determinism. The whole problem for the indeterminist, we should remember, was to render a definition of truth that would make the future tense fit into the kind of indeterminism that branching structures seem able to represent. The determinist, in his turn, is arguing that it is this very indeterminist conception, entrenched in these representations, that blocks any good sense of 'talking about *our* actual future'.

But then we showed how the *Ockhamist* was able to make sense of both $F\phi$ and $\Diamond\neg F\phi$ being true at a pair $\langle m, h\rangle$. But let us suppose that a determinist might resort to a final attempt, by just simply asking how is he able to do this, how is he able to satisfy both sentences.

As we said, the indeterminist takes himself prepared to answer this question, by saying that nothing prevents him from providing a certain branching structure (thus, indeterminist), and an *ockhamist assignment* to do the job.

Yet a determinist might bring some trouble for this kind of solution, in the following way. Certainly, the *Ockhamist* solution was able to work the truth-conditions in a way that $F\phi$ (a future-tensed sentence) might be true, while not necessarily true at the same moment. But for that, he needed to relativize truth to both a moment and a 'history', containing this moment. "But what is this history to stand for?", the determinist might ask.

And thus he argues: if it stands for a historical perspective that takes care of what *will* factually be the case, then it already assigns which history counts as the actual one. And by doing that, it also assigns which histories (passing through a moment) will count as counterfactual ones. So it is not really the case that the other 'histories' (which have already been assumed as counterfactual) could really stand as objective possibilities for how things might *actually* have been different. In some way, they were never really objective options. So this maneuver is actually making the case for the determinist, rather than thwarting his argument. "After all", the determinist might conclude, "the question concerned simply what it would take, for a future tensed sentence, to be true at some moment m in an indeterminist (branching, if you like) framework. But that just means m alone".

As Thomason puts the matter (1970; 1984), the choice of a branch or history has to be entirely *prima facie*. But then, "if a time [m] can have only one "real" future, times located in other alternative futures cannot really bear any temporal relation to [m]. They can bear an *epistemic* relation, being futures for a situation which for all we know is the actual one [m], but strictly speaking this is not a temporal relation. Thus, indeterministic tense logic collapses on this interpretation to deterministic tense logic" (Thomason, 1970, p.271).

Of course, provisionally positing a 'history' might take care of the semantics for the future tense, but then it blocks all the good sense we started with, which was to think that no 'history', representing an objective alternative, is to be privileged. So now we just got back to our dilemma. If we start making sense of 'talk about the factual future', we lose indeterminism; on the other hand, if we fully embrace branching, then coherence about the truth of a future-tensed sentence (of *factual* truth) rests on assuming that a 'history' is privileged among the other merely counterfactual 'histories' passing through a moment.

Should we then concede that there is no good way to behold both 'indeterminism' and 'coherent talk about the future'? Should we give up any way of making good sense of both, if all we have is a single moment of a branching structure?

Actually, perhaps there is one kind of theory which seems to render a nice work-around to circumvent this kind of dilemma. It is to this theory that we now turn our attention, in the next section.

1.9 The *Supervaluationist* solution

1.9.1 A prelude: van Fraassen's method

We saw in our last section how a determinist could object to a solution that would relativize sentence-truth to anything beyond just moments of time, irrespective if one is working with an indeterminist representation of time. But at a first glance, it just looks untenable to make sense of the distinctions between the *factual* and the *modal* senses of the future-tense, if we don't appeal to the notion of a 'history'. So how could we solve this conundrum?

In 1966, Bas van Fraassen published a paper entitled *'Singular Terms, Truth-Value Gaps, and Free Logic'* (1966). His main interest there had nothing to do with *Tense Logic* or the problem of future contingents, but dealt primarily with standoffs involved in assigning truth-values for 'statements' (formulas with no free variables) containing non-referring 'singular terms' (i.e., with non-referring 'definite descriptions' or 'proper names').

B. van Fraassen takes P. Strawson's dispute with Russell, concerning 'definite descriptions', to be a case in point. In his 'On Referring' (1950), Strawson took 'statements' like 'The king of France is wise' to lack a truth-value and to be neither true nor false; thus *contra* Russell, for whom 'statements' containing non-denoting 'definite descriptions' had the logical form of quantifying expressions. For Russell, these 'statements' would not only bear truth-values, but would be false according to the theory of definite descriptions he proposed.

But where Strawson would take such 'statements' as "don't cares" , and quandaries concerning their truth-values to not even arise, van Fraassen looked for a distinct take on the problem. One of his main aspirations, for instance, was not to render extraneous such 'statements', when fitting them into the patterns of classical *Predicate Logic*. He puts the matter in the following terms:

> "For the sake of perspicuity, let us consider an argument with English sentences:
>
> 4. a. Mortimer is a man.
>   b. If Mortimer is a man, then Mortimer is mortal.
>   ―――――――――――――――――――――――
>   c. Mortimer is mortal.
>
> Should our present view, that 4a is neither true nor false if 'Mortimer' does not refer, cause us to qualify our precritical reaction that 4 is a valid argument? I think not. This naive reaction is not based simply on the conviction inculcated by elementary logic courses that questions of validity can be decided on the basis of syntactic form. It can also be based quite soundly on the semantic characterization of validity found in many logic texts:
>
> > 5. An argument is valid if and only if, were its premises true, its conclusion would be true also.
>
> The fact that Mortimer has to exist for the premises of 4 to be true is just as irrelevant to the validity of that argument as any other factual precondition for the truth of those premises. Were Mortimer a man and were it the case that if he is a man then he is mortal, then it would be the case that he is mortal-this is exactly why 4 is valid." (van Fraassen, 1966, pp.487-483).

So what happens when a *first-order language* contains, besides a set of predicates, of variables, quantifiers and boolean connectives, also a set of singular terms, including non-referring ones? How to render an interpretation of such a language?

To simplify matters, let us put aside quantifiers and variables, and only work with a language L_s containing a set of one-place predicates, the 'singular terms' a and b, and the *boolean connectives* $\neg$ and $\vee$. Grammar will be straightforward: if α is a 'singular term', and Φ is a predicate, then $\ulcorner \Phi\alpha \urcorner$ is an *atomic formula*; If ϕ, ψ are *formulas*, then $\ulcorner \neg\phi \urcorner$ and $\ulcorner \phi \vee \psi \urcorner$ are formulas.

We can start by defining "an interpretation of L_s" as a pair $\langle D, f \rangle$, where D is a non-empty set of objects (the *domain of discourse*), and f is an *interpretation function.* Traditionally, f is intended as a function mapping every predicate of L_s to a subset of D, and every 'singular term' to an element of D. Thus normally, given an interpretation $\langle D, f \rangle$ of L_s, we could define *the truth-value* $[\![\phi]\!]_{\langle D,f \rangle}$ *of a formula* ϕ, *according to* $\langle D, f \rangle$, as:

For an atomic formula:

$$[\![\Phi\alpha]\!]_{\langle D,f \rangle} = \begin{cases} \text{True}, & \text{if } f(\alpha) \in f(\Phi) \\ \text{False}, & \text{if } f(\alpha) \notin f(\Phi) \end{cases} \tag{1.27}$$

Boolean connectives:

$$[\![\neg\phi]\!]_{\langle D,f \rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]_{\langle D,f \rangle} = \text{False} \\ \text{False}, & \text{otherwise} \end{cases} \tag{1.28}$$

$$[\![\phi \vee \psi]\!]_{\langle D,f \rangle} = \begin{cases} \text{False}, & \text{if } [\![\phi]\!]_{\langle D,f \rangle} = [\![\psi]\!]_{\langle D,f \rangle} = \text{False} \\ \text{True}, & \text{otherwise} \end{cases} \tag{1.29}$$

But what happens when we have non-referring 'singular terms'? van Fraassen's strategy will then come to the following: given an interpretation $\langle D, f \rangle$ of L_s, we define *the* v_1 *truth-value* $[\![\phi]\!]^{v_1}_{\langle D,f \rangle}$, and *the* v_2 *truth-value* $[\![\phi]\!]^{v_2}_{\langle D,f \rangle}$, *of a formula* ϕ, *according to* $\langle D, f \rangle$, as:

For an atomic formula $\Phi\alpha$*, where* α *is a referring 'singular term'*:

$$[\![\Phi\alpha]\!]^{v_1}_{\langle D,f\rangle} = \begin{cases} \text{True}, & \text{if } f(\alpha) \in f(\Phi) \\ \text{False}, & \text{if } f(\alpha) \notin f(\Phi) \end{cases} \tag{1.30}$$

$$[\![\Phi\alpha]\!]^{v_2}_{\langle D,f\rangle} = \begin{cases} \text{True}, & \text{if } f(\alpha) \in f(\Phi) \\ \text{False}, & \text{if } f(\alpha) \notin f(\Phi) \end{cases} \tag{1.31}$$

For an atomic formula $\Phi\alpha$*, where* α *is a* _nonreferring_ *'singular term'*:

$$[\![\Phi\alpha]\!]^{v_1}_{\langle D,f\rangle} = \Big\{ \text{True} \tag{1.32}$$

$$[\![\Phi\alpha]\!]^{v_2}_{\langle D,f\rangle} = \Big\{ \text{False} \tag{1.33}$$

Boolean connectives:

$$[\![\neg\phi]\!]^{v_1}_{\langle D,f\rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^{v_1}_{\langle D,f\rangle} = \text{False} \\ \text{False}, & \text{otherwise} \end{cases} \tag{1.34}$$

$$[\![\neg\phi]\!]^{v_2}_{\langle D,f\rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^{v_2}_{\langle D,f\rangle} = \text{False} \\ \text{False}, & \text{otherwise} \end{cases} \tag{1.35}$$

$$[\![\phi \vee \psi]\!]^{v_1}_{\langle D,f\rangle} = \begin{cases} \text{False}, & \text{if } [\![\phi]\!]^{v_1}_{\langle D,f\rangle} = [\![\psi]\!]^{v_1}_{\langle D,f\rangle} = \text{False} \\ \text{True}, & \text{otherwise} \end{cases} \tag{1.36}$$

$$[\![\phi \vee \psi]\!]^{v_2}_{\langle D,f\rangle} = \begin{cases} \text{False}, & \text{if } [\![\phi]\!]^{v_2}_{\langle D,f\rangle} = [\![\psi]\!]^{v_2}_{\langle D,f\rangle} = \text{False} \\ \text{True}, & \text{otherwise} \end{cases} \tag{1.37}$$

Let us take some stock. Both the v_1 and the v_2 *truth-values*, the way we defined above, behave just as the simple *truth-value* $[\![\phi]\!]_{\langle D,f\rangle}$ (what traditionally would be done, as we mentioned) would behave, whenever ϕ is an arbitrary formula containing no occurrences of nonreferring 'singular terms' (be it a complex formula or not). The difference comes when an atomic formula has an occurrence of a nonreferring 'singular term'. In that case, the v_1-*truth-value* will simply output *True*, while the v_2-*truth-value* will output *False*. Now you may ask where this is going, and we will promptly show now.

We will now introduce the notion of *a supervaluation over an interpretation*. This is how we do it: given an interpretation $\langle D, f\rangle$ of L_s, and given the aforementioned definitions of the v_1 and v_2 *truth-values*, we define *a supervaluation s over $\langle D, f\rangle$*, as a function s mapping every formula of the language in the following way: *the s truth-value of ϕ, according to $\langle D, f\rangle$*, for every formula of the language L_s – denoted by $[\![\phi]\!]^s_{\langle D,f\rangle}$ – is defined as follows:

$$[\![\phi]\!]^s_{\langle D,f\rangle} = \begin{cases} \text{True,} & \text{if } [\![\phi]\!]^{v_1}_{\langle D,f\rangle} = [\![\phi]\!]^{v_2}_{\langle D,f\rangle} = \text{True} \\ \text{False,} & \text{if } [\![\phi]\!]^{v_1}_{\langle D,f\rangle} = [\![\phi]\!]^{v_2}_{\langle D,f\rangle} = \text{False} \\ \text{undefined,} & \text{otherwise} \end{cases} \tag{1.38}$$

Thus basically, whenever we have an interpretation $\langle D, f\rangle$ for a language containing nonreferring 'singular terms', the *supervaluation s* over $\langle D, f\rangle$, will yield *truth-value gaps* for every atomic formula containing these nonreferring 'singular terms' (since the v_1 and v_2 *truth-values* will arbitrarily never accord in their outputs). As a result, these nonreferring 'statements', if we view them as such, are neither *true* nor *false* according to a *supervaluation*.

So let us suppose that our language contains exactly, the referring 'singular term' a, the nonreferring 'singular term' b, and the one-place predicate F. Let us further assume that an interpretation $\langle D, f\rangle$ is such that $f(a) \in f(F)$. The following diagram will show us exactly how a *supervaluation s* would compare in outputs with those yielded by v_1 and v_2.

	v_1	v_2	s
Fa	T	T	T
$\neg Fa$	F	F	F
Fb	T	F	–
$\neg Fb$	F	T	–
$Fb \vee \neg Fb$	T	T	T

Figure 1.11: *Supervaluation* outputs for 'referring' Fa and 'nonreferring' Fb

So this is what happens. Since Fb contains a nonreferring 'term', its v_1*truth value* is arbitrarily outputted *True*, and its v_2-*truth value* is *False*. Thus, the *supervaluation function s* will neither yield *True* nor *False* (what we represent through the dashes). If we follow the recursive definition of truth for a negated formula, then despite the fact that the v_1-*truth value* of Fb is arbitrarily *True*, it will nevertheless recursively yield *False* for $\neg Fb$, since it is not the case that $[\![Fb]\!]^{v_1}_{\langle D,f \rangle} = False$. And the converse happens concerning v_2; it arbitrarily yields *False* for Fb, and then it renders *True* for $\neg Fb$. If we now follow the definition for disjunctive formulas, then it will happen that both the v_1 and the v_2-*truth values* will be *True* for the formula $Fb \vee \neg Fb$; and thus, s will also yield *True* for this formula. And things start getting really interesting when we come across two notions of validity:

> **(C-validity).** A formula ϕ of L_s is *C-valid*, if, for every *interpretation* $\langle D, f \rangle$, both the v_1 and the v_2-*truth values* of ϕ, based upon $\langle D, f \rangle$, is *True*.
>
> **(S-validity).** A formula ϕ of L_s is *S-valid*, if, for every *interpretation* $\langle D, f \rangle$, the s-*truth value* of ϕ based upon $\langle D, f \rangle$ is *True*.

As we have already noticed, *classical valuations* and *supervaluations* bear a major contrast: different from the former, which never yield *truth-values gaps*, *supervaluations* will admit such circumstances. Nonetheless, a surprising outcome will directly fall out from the resulting definitions of validity: the set of *C-valid* formulas will be exactly the same as the set of *S-valid* formulas!

So we have run through quite a lengthy exposition of van Fraassen's technique, without even touching yet anything that looks as resembling *Temporal Logic* or the semantics for the future-tense. But the most important thing was to observe that his technique would basically permit that atomic formulas and their negations could be both neither true nor false, while 'excluded middle' would turn out valid, and even instances embedding these formulas, 'neuter' in truth-values, would be outputted as *True* by *supervaluations*.

And it was exactly that feature of van Frassen's method that immediately caught attention from logicians who were working with *Tense Logic* and 'branching-time structures'. In the very paper, van Fraassen hinted at a major clue. This is what he said:

> "[W]e may distinguish between the logical law of the excluded middle and the semantic law of bivalence. The first says that any proposition of the form $P \vee {\sim} P$ is logically true. The second says that every proposition is either true or false, or, equivalently, that one of P and ${\sim} P$ is true, the other false.
>
> Clearly the law of bivalence fails for supervaluations and, indeed, for all the interpretations other than that based on classical valuations. But all our interpretations agree that $P \vee {\sim} P$ is logically true. This shows that (contrary to usage) the two laws must be strictly distinguished." (p.493).

A little later, he adds:

> "One interpretation of Aristotle's remarks on future contingencies is that he wished to deny the law of bivalence while retaining the law of the excluded middle." (p.493).

Four years later, Richmond Thomason came up with a complete assessment on how to put together van Frassen's method of *supervaluations* into the context of 'branching-time structures'. It is to this theory that we now turn our attention.[20]

1.9.2 Thomason's take on the problem

Let us refresh our memory by recapitulating some of the main concepts involved in the *Ockhamist approach*: especially that of a 'history' and a set of 'histories' through a moment. First, we start with a 'branching structure'. At this point, we then provide an *assignment* mapping every atomic formula (of the language) to a subset containing moments of time. Finally, the compositional semantics for all expressions of the language takes the form of a recursive definition of truth relative to a 'structure', an *assignment* over it, a moment of time, and a history passing through that moment.

As we saw, this interpretation was able to make sense of the distinction between the *factual* and the *modal* senses of talk about the future. But this would only work at the expense of too high a price to pay (at least too high for an indeterminist). The problem there, and what became subjected to the determinist's decisive sting (as we discussed), was the idea of provisionally positing a 'history', relative to which a simple future-tensed formula would be accounted *true* or *false* at a moment. "The choice of a 'history' ", we said there, would have to be entirely *prima facie*. It would come in as a trojan horse, and implode the good sense of indeterminism that 'branching structures' sought to represent. So how could we remedy this situation?

When Thomason published his *Indeterminist Time and Truth-value gaps* (1970), he alluded to a different kind of approach concerning the problem. But it wasn't just a

[20] Before moving on, a brief note. We have here largely deviated in both the notation and the original definitions provided by van Fraassen. In his paper, he defines *a classical valuation v over a model*. By 'model' he is referring to what we have called an interpretation; a pair with a domain D and an *interpretation function f*. A *classical valuation v* over such a pair, would then be a function defined over formulas (of the language) which would accord in what counts as *truth in a model*, whenever there is no occurrence of nonreferring 'terms' (in compliance with what was done through the definitions of the v_1 and v_2 *truth-values*, according to an interpretation $\langle D, f\rangle$ - or a model as he calls it). And further, he adds that "if there is any name e that has no referent in the domain of a given model and if $A(e)$ is an atomic statement in which e occurs, then there are at least two classical valuations over this model: one which assigns T to $A(e)$ and one which assigns F to $A(e)$" (van Fraassen, 1966, p.486). But in essence, we thought that it would be much more straightforward, and perhaps easier to grasp the concept, if we just defined directly two 'classical valuations' through the definitions of the v_1 and v_2 *truth-values*, and directly making them arbitrarily output the truth-values when nonreferring 'singular terms' were at stake.

"quick fix" of 'branching-time theories'; the technical implementations were now providing a fresh new outlook on the very problem of future contingents.

He starts the paper with a brief introduction about 'branching-like structures', and introduces the two main theories coined by Prior: the *Peircean* and the *Ockhamist*. After presenting the *Ockhamist* theory, he writes:

> "Since we may often be in situations in which we have made no suppositions concerning which of a variety of possible futures will come about, it should also often be the case that certain statements in the future tense are neither true nor false. But the present theory provides us with no way to accomplish this" (Thomason, 1970, p.271).
>
> Briefly after, he complements:
>
> "I now want to prepare a response to this last objection. None of the materials used in my proposal are new, though as far as I know the combination is one that has not been suggested [...] Recently a very general way of providing for such truth-value gaps has been developed by B. van Fraassen" (pp.271-272).

Let us then show how the *supervaluationist* approach to the problem of future contingents goes (we will introduce it with already used notation). It will prove useful to start with some diagrams:

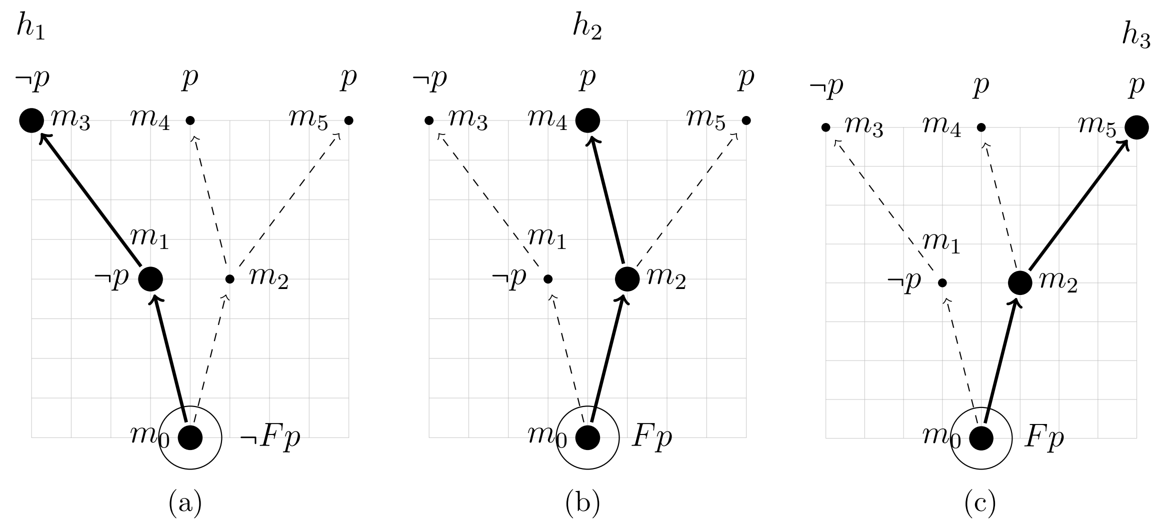


Figure 1.12: the *true* formulas relative to m_0 and *each* 'history' passing through it

Here all three diagrams are depicting the same 'branching structure'. We are also representing an *assignment* mapping the atomic formula p only to the subset containing the moments m_4 and m_5. The first figure in the left is highlighting one of the histories passing through m_0: we are calling it h_1.

Thus, given such structure, and according to the represented *assignment* (let us call it V), the V*-truth value* of $\neg Fp$ is true relative to the pair comprising m_0 and the 'history' h_1, since first of all, (1) the V*-truth value* of Fp is false at the same pair $(\langle m_0, h_1\rangle)$; and second, (2) Fp is false at that pair because there is no later moment belonging to h_1 such that p is true. The second picture is highlighting the fact that Fp is true at $\langle m_0, h_2\rangle$ (since m_4 belongs to h_2, and p is true at $\langle m_0, h_2\rangle$), and the third picture is representing the fact that Fp is also true at $\langle m_0, h_3\rangle$.

According to the *Ockhamist* semantic clause concerning future-tensed formulas and formulas preceded by $\Box$, we saw back then that the V*-truth value* of $F\phi$ and $\Box\phi$ would then behave in the following way:

$$[\![F\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \text{True,} & \text{if } [\![\phi]\!]^V_{\langle m',h\rangle} = \text{True, for some } m' \in h, \text{ such that } m < m' \\ \text{False,} & \text{otherwise} \end{cases} \tag{1.39}$$

$$[\![\Box\phi]\!]^V_{\langle m,h\rangle} = \begin{cases} \text{True,} & \text{if } [\![\phi]\!]^V_{\langle m,h'\rangle} = \text{True, for every } h' \in H_m \\ \text{False,} & \text{otherwise} \end{cases} \tag{1.40}$$

We now have the needed material to introduce the following definition concerning a *supervaluation* over an interpreted 'branching-time structure':

Definition 10 (**Supervaluationist** s**-truth value**). *Given a 'branching structure'* $\langle M, <\rangle$*, and an* assignment function V *over it, and the recursive definition of the* V-truth value $[\![\phi]\!]^V_{\langle m,h\rangle}$ *of* ϕ *for every formula* ϕ *of the language; the* s-truth value *of* ϕ *at a moment* m *of* M *(denoted by* $[\![\phi]\!]^s_m$*), for every formula* ϕ *of the language, is defined as follows:*

$$[\![\phi]\!]^s_m = \begin{cases} \textit{True,} & \textit{if } [\![\phi]\!]^V_{\langle m,h\rangle} = \textit{True, for every } h \in H_m \\ \textit{False,} & \textit{if } [\![\phi]\!]^V_{\langle m,h\rangle} = \textit{False, for every } h \in H_m \\ \textit{undefined,} & \textit{otherwise} \end{cases} \tag{1.41}$$

This brief definition will deliver a batch of important consequences, so let us keep track of each of them:

(**Bivalence and Excluded Middle**). Let us word the *Principle of Bivalence* as the semantic law stating that for every formula ϕ of the language, either ϕ is *true*, or ϕ is *false*; or equivalently, stating that for every ϕ, one of either ϕ or $\neg\phi$ is *true*, and the other is *false* (but only "equivalently" here, because of the truth-conditions for $\neg$). The *Principle of Excluded Middle*, in its turn, let us state it as the logical law that every instance of the formula $\phi \vee \neg\phi$ is *valid* (following terminology we have been using).

If we look back at the last diagrams, we have it now that the *supervaluationist* truth-value of Fp at the moment m_0, is neither *True*, nor *False*. This works because despite of the fact that the V*-truth value* of Fp is *True* at both pairs $\langle m_0, h_2\rangle$ and $\langle m_0, h_3\rangle$, the V*-truth value* of the same formula is *False* at $\langle m_0, h_1\rangle$. But additionally, the negated formula $\neg Fp$ will also be neither true nor false, since it is false in both $\langle m_0, h_2\rangle$ and $\langle m_0, h_3\rangle$. In effect, with *supervaluations*, *Bivalence* as we have stated won't hold up here.

This kind of result is also in line with something we said way back when we started our thesis. There we quoted MacFarlane: "[i]f there are objective possible futures which would make the prediction true and others which would make it false, symmetry considerations seem to forbid counting it either true or false" (MacFarlane, 2003a, p.321). He called this kind of intuition as *the indeterminacy intuition*. So we are now finally able to fit the semantics of a *temporal language* into this very intuition. None of the competing theories had managed to do that, so this definitely counts in favor of the *Supervaluationist* solution. But there is even more.

Observe that the formula $Fp \vee \neg Fp$, an instance of excluded middle, will be given the value *True* by *supervaluations*, since the V*-truth value* of this formula is always *true* relative to any pair where m_0 is its moment, and h is a 'history' passing through it; at some, the first disjunct is *true*, at others, the second is *true*. In effect, if we rerun a usual notion of ***validity*** here, provided some minor adjustments, the formula $\phi \vee \neg\phi$ will be *S-valid*!

> **(*Supervaluationist* validity).** A formula ϕ is *supervaluationally valid* if, relative to all *branching structures* $\langle M, <\rangle$, and relative to all *assignments* V (with respect to $\langle M, <\rangle$): the *s-truth value* $[\![\phi]\!]^s_m$ of ϕ at m, is *True* for every $m \in M$.

And again, this feature also looks in line with a widely endorsed interpretation concerning Aristotle's *Chapter 9* of *De Interpretatione*: that the solution he was there providing, was that of rejecting *Bivalence* while retaining *Excluded Middle*[21]. The evi-

[21]'Widely endorsed', yet certainly not undisputed; for instance, Martha Kneale rejects such a reading: "In chapter 9 of *De Interpretatione* Aristotle questions the assumption that every declarative sentence is true or false. It might seem that he is clearly committed to this thesis already, but this is not so; for when he says that to be true or false belongs to declarative sentences alone, this may be taken to mean that only these are capable of being true or false not that they necessarily are. . . . Given the definitions of truth which we have quoted, the principles [of Bivalence and of Excluded Middle] are, however, obviously equivalent; for if 'It is true that P' is equivalent to 'P', 'P or not-P' is plainly equivalent to 'It is true that P or it is false that P' " (Kneale, 1962, pp.46-47 *apud* van Fraassen, 1966, p.493).

dences in favor of this interpretation are sprinkled all over Aristotle's text. Particularly telling, a famous passage we come across in *18b26*:

> "These and others like them are the absurdities that follow if it is necessary, for every affirmation and negation either about universals spoken of universally or about particulars, that one of the opposites be true and the other false, and that nothing of what happens is as chance has it, but everything is and happens of necessity" (*De Int.*18b26 [Ackrill's translation, 2002, p.51], emphasis added).

And perhaps the most suggestive passages, lending support for the interpretation that a distinction (between both principles) is upholding in Aristotle's argument, we find in both *19a23* and *19a39*:

> "[...] everything necessarily is or is not, and will be or will not be; but one cannot divide and say that one or the other is necessary.
>
> [...] So, since statements are true according to how the actual things are, it is clear that wherever these are such as to allow of contraries as chance has it, the same necessarily holds for the contradictories also. This happens with things that are not always so or are not always not so. With these it is necessary *for one or the other* of the contradictories to be true or false – not, however, this one or that one, but as chance has it; or for one to be true rather than the other, yet not already true or false" (*19a23*, emphasis added).

> "Clearly, then, it is not necessary that of every affirmation and opposite negation one should be true and the other false. For what holds for things that are does not hold for things that are not but may possibly be or not be; with these it is as we have said" (*19a39*, emphasis added).

(Conserving *Linear* and *Ockhamist* validities). If we remember the notion of *Ockhamist* validity, it stated that a formula ϕ would be valid if relative to any 'branching structure', and to any *assignment*, and to any moment $m \in M$ and history h passing through it, its V*-truth value* would remain being *True*.

Thus, it falls out directly from both notions of validity, that everything of which the *Ockhamist* system is a conservative extension, a *Tense Logic* built on supervaluations would also be a conservative extension of it – given the fact that the set of *Okhamist* valid formulas will be exactly the set of *Supervaluationally valid* formulas. They simply collapse. This heredity property will thus make every validity of linear tense logic, and every validity of *Ockhamist* logic, to be *supervaluationally* valid. A representative example is the formula $\phi \supset PF\phi$, which is both *linear* and *Ockhamist* valid, and thus, also valid here. But there is something different going on here, that we ought to remark.

While the scheme $\phi \supset PF\phi$ is *supervaluationally valid* (the whole material conditional), this doesn't imply that if ϕ is *true* at a moment, then $F\phi$ must have been *true* at some m', such that $m' < m$. Take for instance the satisfaction of formulas at the point m_2 of the preceding diagrams:

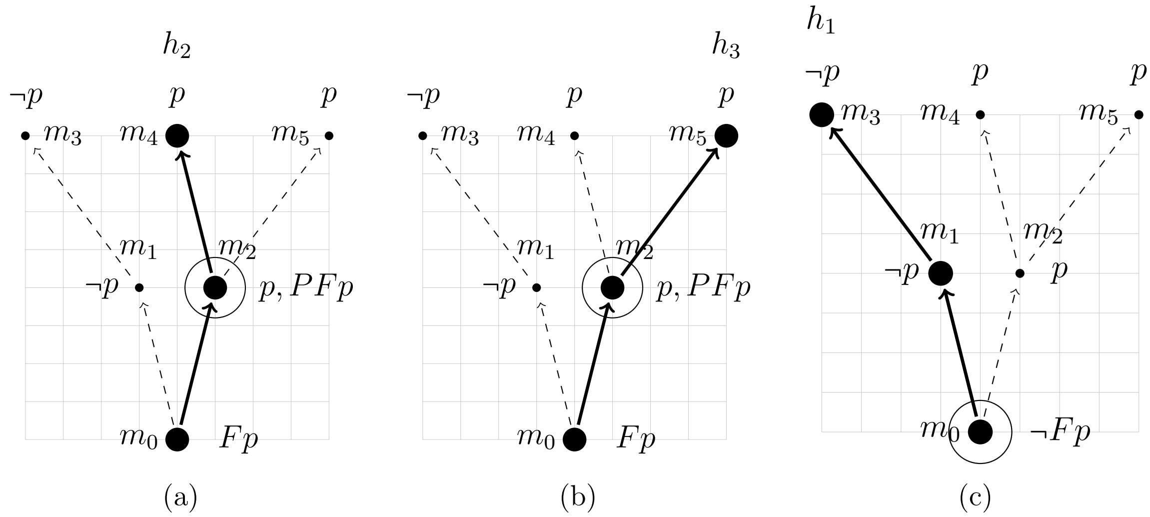


Figure 1.13

The *s-truth value* of p at m_2 is *True*, because the V-truth value of p is *True* at both $\langle m_2, h_2\rangle$ and $\langle m_2, h_3\rangle$ (thus true with respect to all $h \in H_{m_2}$). Exactly the same happens with PFp being *true* at m_2. As a result, $p \supset PFp$ is true at m_2 (in effect it is valid, and in particular true here under a circumstance where both antecedent and consequent are true).

Nevertheless, the fact that PFp is *supervaluationally true* at m_2, implies only that for every 'history' h passing through m_2, there *must have been* one earlier moment belonging to h, such that the *V-truth value* of Fp is *True* at that moment and h. Yet this alone doesn't imply that there is some $m' < m_2$ where the *s- truth value* of Fp is *True*. For instance, there is only m_0 standing earlier than m_2, and Fp is *supervaluationally* neither true nor false there (because of h_1).

(Dodging the sting). It was one of the main features of *Ockhamisim* that it could discriminate between the *factual* and the *modal* senses of a future-tense. Thus, it was easy to provide a structure and a certain *assignment*, where the *V-truth value* of a formula Fp would be *True* at a pair $\langle m, h\rangle$, while that of $\Box Fp$ would be *False* (or equivalently, where $\Diamond\neg Fp$ would be *True*). But it rendered such move at the expense of a high cost, as we said: it had to provisionally posit a 'history'. So what is happening here with this respect?

In the *supervaluationist* approach, we have no such provision of a 'history', thus it seems that it circumvents this kind of objection. Yet if we lend some more thought on it, it looks that something will be missed in the long run. For instance, this kind of workaround has made room for a situation where *'There will be a sea battle tomorrow'* may be neither true nor false; precisely, in circumstances where the future is open with respect to a sea fight taking place tomorrow. But then it would also suggest that truth simply collapses with necessity; that is, whenever the *s-truth value* of a formula $F\phi$ is defined (so that it gets either the value *True* or *False*), then either it is necessary that ϕ will happen (or equivalently, that $\Box F\phi$), or else impossible that ϕ will happen (equivalently, $\neg \Diamond F\phi$). So how come now things happen *as chance has it*?

But this kind of outcome is representative of a quite unorthodox feature we find here. The way then that *supervaluationism* thwarts this kind of argument will have to rest elsewhere. We have already provided a notion of *supervaluationist validity*, so we must now also couple to it a notion of *logical entailment*; this will throw some light on the matter:

> **(*S*-entailment).** A formula ϕ is *s-entailed by* a set Γ (which we denote by $\Gamma \vDash_s \phi$) if, relative to all *branching structures* $\langle M, < \rangle$, and relative to all *assignments* V (with respect to $\langle M, < \rangle$): for every $m \in M$, **If** the *s-truth value* $[\![\gamma]\!]^s_m$ is *True* (for every $\gamma \in \Gamma$), **Then** the *s-truth value* $[\![\phi]\!]^s_m$ is also *True*.

Following this notion of *S-entailment*, we thus have that $F\phi$ *s-entails* $\Box F\phi$ (or in notation, that: $F\phi \vDash_s \Box F\phi$). Nevertheless (and this is the 'elsewhere' where the approach ought to sever the determinist's objection), the formula $F\phi \supset \Box F\phi$ is not here *S-valid*. In particular, this means that in situations where, say, a sea battle taking place tomorrow will only succeed *as chance has it*, then the whole material conditional $F\phi \supset \Box F\phi$ won't be *supervaluationally true*, since either in one 'history' the antecedent will be true ($F\phi$) yet the consequent will be false ($\Box F\phi$) - thus the conditional is false -; or the conditional will be true because the antecedent is false. Thus, when we *supervaluate* over all histories, the *s-truth value* of the whole conditional will be undefined, and therefore not true.

Therefore, the *supervaluationist* solution may even be said to show why the determinist's argument looks so irresistible: it brings sense to the determinist through the notion of $F\phi$ *s-entailing* $\Box F\phi$ (and $\neg F\phi$ *s-entailing* $\neg \Diamond F\phi$). Yet it thwarts its argument by bringing sense of indeterminism through the notion that the whole conditional $F\phi \supset \Box F\phi$ is not *s-valid*.

But is all this convincing? If you are not so sure, then you are in good company. We will present in the next section some of the main charges that the *supervaluationist* approach will face, and finally culminate in the kind of charge that MacFarlane sought to leverage against it (though perhaps with not so much success). We should move on.

1.9.3 Charges against it

In this subsection, we will just briefly address two of the main charges that has run in literature against the *Supervaluationist* approach, since these charges will be revisited later, when we will have at our disposal more resources to assess them. Particularly, because we have not yet put *context-sensitive* languages into the picture, nor any notion of *truth at a context*.

But let us just start with a first charge directly linked to what we have just said in our previous section. When we showed that the approach made $F\phi$ *s-entail* $\Box F\phi$, while it would also make $F\phi \supset \Box F\phi$ not *s-valid*, you might have wrinkled your nose. But not for a bad reason, in effect. This kind of behavior shows that its *logic* will run counterexamples to a classical rule: *Conditional Proof*. That means that the following inference will not be guaranteed here:

Conditional Proof **If** $\phi \vdash \psi$**, then** $\vdash \phi \supset \psi$

Actually, the impact goes well beyond just violating *Conditional Proof*, since it is for similar reasons that the *Supervaluationist* approach will also run counterexamples to *Reductio ad Absurdum*, *Case Argument* and *Contraposition*. Nevertheless, what we have to say concerning *Conditional Proof*, will also extend equally to these other case.

MacFarlane has claimed, in (2008: p.88), that the extent of this charge may be manageable, since *Conditional Proof* is primarily not a semantic claim, but a proof claim. And he further states that since it mainly makes reference to a *subproof*, in the sense that it avows for a conditional to be deduced from a *subproof* that has started with its antecedent and ended with its consequent, then nothing would bar us from just amending the proof system and render unlicensed any move from ϕ to $\Box\phi$ occurring in *subproofs*. Moreover, as MacFarlane puts the point, this kind of charge against the approach would at most yield qualms about its proof theory, but not threat it *qua* a semantic theory. But we are not sure how forceful this argument is, since it doesn't look that taking an approach *qua* a semantic theory will insulate it from any disturbances coming from its proof system. In any case, the charge looks as a tradeoff that the *Supervaluationist* will have to face: accept the approach, but lose Conditional Proof.

Now concerning a second charge against *Supervaluationism*, is that it will treat disjunction non-truth-functionally. This criticism was first leveraged by T. Williamson (1994, p.152), and it trades on the fact that the approach will render a disjunction *true*, while neither of its disjuncts are *true*; we saw before one such example: instances of *Excluded Middle*, $\phi \vee \neg\phi$. Same will happen if we take formulas $F\phi$ and $F\psi$, say, concerning future-tensed statement which, while not contradicting each other, they may be mapped in a way by the *assignment* such that they are never jointly true. Thus it will also happen a case where the disjunction may be *true*, while neither disjunct is *supervaluationally true*. But there we saw, that such feature was exactly what made the *Supervaluationist solution* outperform any of its rival theories. But in any case, as we already alerted, we will later revisit all these charges, when the supervaluationist approach will be taken from a different perspective.

1.10 Concluding remarks (so far)

We now have expounded at length the main theories which we find in literature concerning the problem of future contingents. Having argued for both what we took as merits of each theory, and also their main adversities, we hope to have prepared the reader for identifying the main problems which are at stake. But we shall now move on, and finally take care of our own view of the problem. We will also start giving an account for languages now containing *temporally indexical* operators, and we will provide a general framework able to accommodate all the different views we have expounded so far.

Chapter 2

Leading to $T \times W$ Theories

This chapter is devoted to showing what $T \times W$ structures amount to, and it will build the foundations upon which some of the main preceding interpretations may be critically evaluated in comparison to each other. Once we provide the definition of such structures, and display some of its properties, we will show how to enrich them with a set of contexts, and thus deliver a way to start speaking about *assertions* being true or false.

2.1 Why $T \times W$ structures?

We have seen so far some of the main attempts at providing the semantics involved in our ‘talk about the future’. We have also seen that all of those who have cast their theories through indeterminist representations of time - the *Peircean*, the *Ockhamist* and the *Supervaluationist* approaches - have been relying on ‘branching-time’ kinds of structures.

We also haven’t yet considered in full any *temporal-modal languages* containing context-sensitive expressions, and how to work out the ‘implementation details’ of a definition of truth for sentences containing them. We have only mentioned them briefly and offered some hint at how the semantics could have been taken care of. But we will now take this task in full.

In the following sections, we will finally arrive at the kinds of structures which will basically ground our own assessments of the problem of future contingents. We will basically rely on a theory built over structures first devised by Richmond Thomason in his *Combinations of Tense and Modality* (1984): $T \times W$ structures. But why is this our target notion, and why should we undertake such a task or even bother? Why not simply continue on ‘branching’ kinds of structure? There are many things to be said on this respect.

First of all, the basis for $T \times W$ theories is that they assume all worlds share a same temporal ordering, which makes it easier to account for the modal-status of sentences embedding *temporally indexical* operators – for instance, to evaluate ‘It is *possible* that it *will* rain, *tomorrow*’. As a second point, $T \times W$ structures will provide us with a smooth link between distinct definitions of *truth at a context*, and the recursive definitions within the compositional semantics, which handles truth-conditions for every expression of the *temporal-modal language*. Every distinct solution to the problem of future contingents will provide a different answer as to what counts as being *true at a context*, but they all work with the same compositional semantics that we will define.

Additionally, $T \times W$ Theory has many of the results that branching theories will lack in full generality. For instance, it is a much simpler theory for which complete and consistent axiomatizations have already been accomplished.

On the side of *Ockhamist* theories, however, the hardships on handling them have been noticed at a very early stage of its development. As Thomason recalls some of the troubles involved, he remarks that Burgess had once sketched an early proof for *Ockhamist* validity being recursively axiomatizable (in 1979), but that Kripke had "challenged the proof, and Burgess has been unable to substantiate all the details" (1984, p.152). He briefly later remarks that, as far as a proof of decidability is concerned, "Gabbay's completeness techniques do not seem (at first glance, anyway) to extend to the treelike case" - 'treelike' here referring to 'branching' structures (p.152). Thomason is here referring to Gabbay's *Irreflexivity Lemma*, which provided a straightforward method for constructing irreflexive models.

In fact, it was always a problem within branching-time theories to induce irreflexivity of the ordering relation by means of an axiom. There is simply no way. But Gabbay's technique would trade on a different strategy; it resorts on adding an inference rule, rather than providing an axiom. Even so, it was only in 1997 that a full proof of $T \times W$ completeness was published, by von Kutschera (1997). As expected, the proof was given through the medium of Gabbay's technique.

2.2 Defining $T \times W$ structures

By now, it might have got clear that when we define temporal structures, bearing certain properties, we are entrenching some of our pre-theoretic notions of time and objective possibility, by defining what constraints ought to be imposed in the relations concerning instants of times and possible worlds. Once we provide this kind of stage, we are then able to determine how an instant relates to another as being earlier or later, and we can also determine in what way possible worlds access each other, relative to a specific time. Further, when we define an *assignment* over a $T \times W$ structure, we will be able to give a recursive truth definition for all sentences containing the temporal and modal operators of our language; that is, a *compositional semantics* for the language. Let us then start by defining $T \times W$ structures, and show some of its features.

Definition 11 (Thomason's $T \times W$ **Structures).** *A* $T \times W$ Structure *is a quadruple* $\langle W, T, <, \simeq \rangle$*, where* W *and* T *are non-empty sets, of respectively, possible worlds and times;* $<$ *is a binary,* irreflexive*,* transitive*, and* linear *relation on* T[1]*; and* $\simeq$ *is a ternary relation on* $T \times W \times W$*, such that: (1)* $\forall t \in T$*,* $\simeq_t$ *is an equivalence relation on* W*, and (2)* $\forall w_1, w_2 \in W$*, and* $\forall t, t' \in T$*, if* $w_1 \simeq_t w_2$ *and* $t' < t$*, then* $w_1 \simeq_{t'} w_2$.[2]

Intuitively, given any such structure we might picture it visually as a kind of a grid that at each temporal level (at each $t \in T$), "chains" or "links" the possible worlds which are accessible at that level (the *worlds* that are *t-accessible*). As we move upwards, say moving from t_0 to t_1 (where $t_0 < t_1$) the chains tying possible worlds at these later levels get narrower, so that from a perspective of a world w and a world w', such that w' is *accessible from* w*, at* t_0, it might well happen that when moving to t_1, w' is not anymore *accessible from* w*, at* t_1. As usual, it will prove useful to introduce some figures depicting just such a situation:

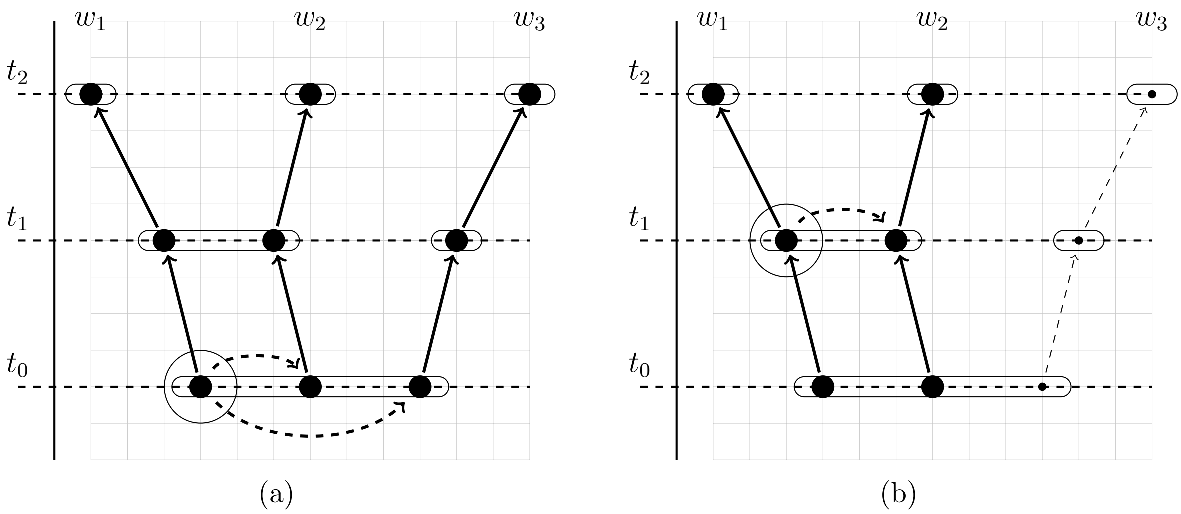


Figure 2.1: A $T \times W$ structure

Both figures are representing exactly the same $T \times W$ structure; it is a structure composed by the *possible worlds* $w_1, w_2, w_3 \in W$, and times $t_0, t_1, t_2 \in T$ which are being *linearly ordered* by the relation $<$, such that for every instant of time, all the times above it are *later*, and all below it are *earlier*.

Yet, the 'dots' that we see here represented, they are not anymore standing for distinct moments, as we depicted before when expounding 'branching-like' structures. Instead, what each of these 'dots' do represent here through this grid-like representation, is a pair composed by a *time* and a *world* (like coordinates in a plane).

[1]Meaning: $\forall t \in T : t \nless t$ (irreflexive); $\forall t, t', t'' \in T$, if $t < t'$ and $t' < t''$, then $t < t''$ (transitive); $\forall t, t' \in T$, either $t = t'$, or $t < t'$, or $t' < t$ (linear)

[2]Cf. Thomason, 1984, p.146 - Definition (6).

So in the leftmost figure ('a'), the circled 'dot' is representing the pair $\langle t_0, w_1 \rangle$, while in the rightmost figure ('b'), the circled 'dot' is representing the pair $\langle t_1, w_1 \rangle$. Thus, albeit both diagrams are representing the same structure, we are highlighting the fact that from the perspective of the circled pair in Figure 'a' (which is $\langle t_0, w_1 \rangle$), all three worlds are accessible from its time (t_0). While when we look from the perspective of the circled pair in Figure 'b' ($\langle t_1, w_1 \rangle$), only the worlds w_1 and w_2 remain accessible from that "time level" (which is t_1).

Thinking about each dot as representing a pair, when looking at such structures, will prove useful for evaluating the truth-conditions of formulas with modals ('necessity' and 'possibility'), in the *compositional semantics* – which will be defined in our next section. It proves useful because the definition of truth for formulas, at this stage of the semantics, is defined *relative to* exactly a pair composed by a *time* and a *world*.

In effect, these "chains" that are tying the dots are showing you how at each time, a possible world is accessing another one in the structure. Thus for instance, at t_0 all three worlds are accessible to each other. This means for example, that w_2 *is accessible from* w_1*, at* t_0 (or using the notation provided, we denote this through $w_1 \simeq_{t_0} w_2$); and also that $w_1 \simeq_{t_0} w_3$ – besides also, the fact that $w_2 \simeq_{t_0} w_1$ and $w_3 \simeq_{t_0} w_1$ (because of $\simeq$ being *symmetric*), and the fact that all w_1, w_2, w_3 access themselves at t_0, and in effect at every t (because of $\simeq$ being *reflexive*). Now when we move upward to t_1, then w_3 is not anymore accessible from neither w_1 nor w_2, though these two remain accessing each other at t_1. Therefore, now w_3 only accesses itself at t_1. In effect, when we move to t_2, all worlds are only accessing themselves.

2.3 A *context-sensitive* language, and the *compositional semantics*

So having already defined $T \times W$ structures, we could finally provide an *interpretation* for a *temporal-modal language*, through the medium of an *assignment function* V, mapping every and each atomic formula of the *temporal-modal language* to a subset of $T \times W$ (given a structure). But since we want to handle the context-sensitive operator *Tom*, we will have to make a provision on Thomason's original definition, by enriching our structures with a set comprising *contexts*. So before we move on, let us speak a little about *contexts*.

A standard framework we find in both Lewis's (1980) and Kaplan's (1989) style of semantics, works out by relativizing truth of a sentence to a triple: a *context*, a *circumstance of evaluation* (as in Kaplan) or an *index* (in Lewis), and an *assignment*. Each one these will play some role in handling expressions in the object language, be it in fixing semantic values for indexical terms (as in Kaplan's *Logic of Demonstratives*), or in giving truth conditions for quantifiers and sentential operators.

This *assignment* here is not to be confused with the terminology we will use with respect to an *interpretation function* for a language. In this case of Kaplan's and Lewis's systems, the *assignment* is a function which assigns variables to objects in a *domain of discourse*, and they are needed to handle quantifiers within a *first-order language*. But since we won't work with such a language, we can just put aside this kind of parameter.

Now, a 'context' here may be viewed as a sequence of parameters representing specific features of a possible occasion of use of a sentence: for instance, a possible occasion of use of the sentence '*Jake will visit Paris tomorrow*'. Depending on what expressions a language contains, distinct features of a context may become semantically relevant, and should take place as some value in the sequence filling a context.

Finally, an '*index*' (or Kaplan's '*circumstance of evaluation*') is a sequence or collection of coordinates which are either shiftable by sentential operators of the language, or somehow play a direct role in their truth-conditions. The whole package, comprising a *context*, and an *index*, will be here called a *point of evaluation*, following the very terminology used by MacFarlane (2003b; 2014). At this stage then, of defining our *compositional semantics*, sentences will have their truth evaluated with respect to such *points of evaluation*.

Our natural first task then, is to introduce a *temporal-modal language* and a grammar governing the construction of arbitrarily complex formulas (given this language). Let us then introduce a simple language $\mathscr{L}$ as one consisting of: (1) a set of atomic formulas $p, q, \ldots$ (each one standing for some present-tensed sentence - as we have seen in our previous section); (2) the *boolean connectives* $\neg, \wedge, \vee, \supset$; (3) the one-place *temporal operators* F ("It will be the case that"), P ("It was the case that"), and *Tom* ("It will be the case tomorrow, that"); and finally, (4) the one-place *historical modal operators* $\Diamond$ ("It is *possible* that"), and $\Box$ ("It is *necessary/settled* that").

We also use the greek letters $\phi, \psi, \ldots$ to stand for arbitrary formulas (either atomic or molecular). The grammar will be straightforward: if ϕ is a formula, then $\ulcorner \neg\phi \urcorner$, $\ulcorner F\phi \urcorner$, $\ulcorner P\phi \urcorner$, $\ulcorner Tom\, \phi \urcorner$, $\ulcorner \Diamond\phi \urcorner$, $\ulcorner \Box\phi \urcorner$ are formulas (we will sometimes mention formulas like $F\phi$ and $P\phi$ as 'future-tensed formulas' and 'past-tensed formulas', respectively); also if ϕ, ψ are formulas, then $\ulcorner \phi \wedge \psi \urcorner$, $\ulcorner \phi \vee \psi \urcorner$, and $\ulcorner \phi \supset \psi \urcorner$ are formulas. So let us now flesh out in detail everything needed to run the *compositional semantics*.

Let us then take a $\langle C, T, W, <, \simeq \rangle$ structure to be a $T \times W$ structure now coupled with a non-empty set C of 'contexts'. The only constraint we should impose at this stage, is to make some way to handle a *temporally indexical* operator such as *Tomorrow*. Thus, we further fix that for every member c (a 'context') of the set C, it has at least one parameter informing us what time (of T!) counts as being *the time of the context*. In order to achieve this, we can simply do as in Kaplan (1978; 1989), when he adds the following condition: "If $c \in \mathscr{C}$, then (ii) $c_T \in \mathscr{T}$" (Kaplan, 1978, p.88), where $\mathscr{C}$ is the set of contexts, c_T is *the time of context* c, and $\mathscr{T}$ is the set of times.

Now we have all the material to provide the definitions of an *assignment*, and thereof provide a recursive definition of truth for every formula, relative to a *point of evaluation.* Here is how we can phrase the definitions:

Definition 12 (*TW* **assignment).** *Given a* $T \times W$ *structure* $\langle T, W, <, \simeq \rangle$*, a* $T \times W$ assignment *(based upon the structure) is a function* V*, such that: (1)* V *maps each atomic formula of the* temporal-modal language *to a subset of* $T \times W$*; and, (2)* ***whenever*** $w \simeq_t w'$ *and* $t_1 \leqslant t$*,* ***then*** *for every atomic formula* ϕ*:* $\langle t_1, w \rangle \in V(\phi)$ *iff* $\langle t_1, w' \rangle \in V(\phi)$

Condition (2) will here secure that whenever the *assignment* maps an atomic formula p to a subset containing the pair $\langle t, w \rangle$, it should also map p to every other pair $\langle t, w' \rangle$, such that w' is *accessible from* w*, at* t. This is to reflect the idea that every world accessible at a time must match in all present and past truths.

Now the following definition will give us a recursive definition of truth for all formulas of the language, relative to a 'structure', a 'context', and an 'index' comprising a pair with a time and a world.

Definition 13 (V**-truth value**). *For all sentences* ϕ *of* $\mathscr{L}$*, the* V*-truth value* $[\![\phi]\!]^c_{\langle t,w \rangle}$ *of* ϕ *at the* point of evaluation $c, \langle t, w \rangle$ *is defined as follows:*

For an atomic formula p*:*

$$[\![p]\!]^c_{\langle t,w \rangle} = \begin{cases} \textit{True}, & \textit{if } \langle t, w \rangle \in V(p) \\ \textit{False}, & \textit{if } \langle t, w \rangle \notin V(p) \end{cases} \tag{2.1}$$

Boolean connectives:

$$[\![\neg\phi]\!]^c_{\langle t,w \rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t,w \rangle} = \textit{False} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.2}$$

$$[\![\phi \wedge \psi]\!]^c_{\langle t,w \rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t,w \rangle} = [\![\psi]\!]^c_{\langle t,w \rangle} = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.3}$$

$$[\![\phi \vee \psi]\!]^c_{\langle t,w \rangle} = \begin{cases} \textit{False}, & \textit{if } [\![\phi]\!]^c_{\langle t,w \rangle} = [\![\psi]\!]^c_{\langle t,w \rangle} = \textit{False} \\ \textit{True}, & \textit{otherwise} \end{cases} \tag{2.4}$$

$$[\![\phi \supset \psi]\!]^c_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t,w\rangle} = \textit{False, or,} [\![\psi]\!]^c_{\langle t,w\rangle} = \textit{True} \\ \textit{True}, & \textit{otherwise} \end{cases} \tag{2.5}$$

Temporal operators F and P:

$$[\![F\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',w\rangle} = \textit{True, for some t' such that } t < t' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.6}$$

$$[\![P\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',w\rangle} = \textit{True, for some t' such that } t' < t \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.7}$$

Temporal operator Tom *("Tomorrow"):*

$$[\![\,\textit{Tom}\ \phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t_c+1,w\rangle} = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.8}$$

Possibility and Necessity:

$$[\![\Diamond\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t,w'\rangle} = \textit{True, for some } w' \textit{ such that } w \simeq_t w' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.9}$$

$$[\![\Box\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t,w'\rangle} = \textit{True, for every } w' \textit{ such that } w \simeq_t w' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{2.10}$$

Let us take a look at a diagram concerning the truth of a sentence compounding $\Diamond$, $\neg$ and *Tom*. So suppose we want to evaluate the truth of $\ulcorner\Diamond\neg\textit{Tom}\ p\urcorner$ ("It is possible, that it won't be the case *tomorrow*, that p"). We can then represent a $T \times W$ structure coupled with an *assignment*:

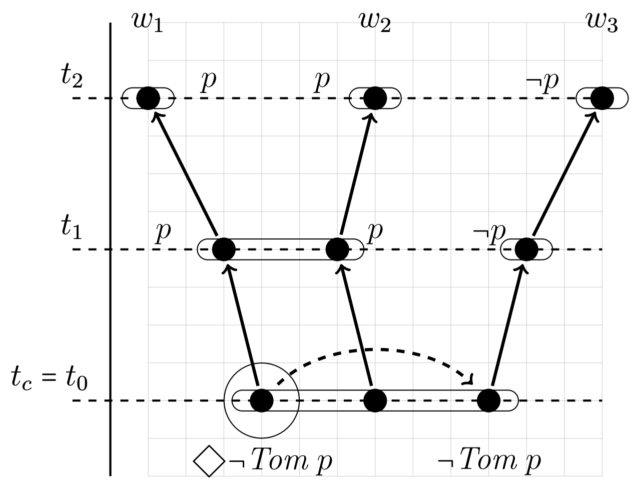


Figure 2.2: Evaluating truth of '$\Diamond\neg\, Tom\, p$' at the *point* '$c, \langle t_0, w_1\rangle$'

We are here representing a structure where t_c (the *time of the context*) is the time t_0 in T. The assignment then is running in the following way: p is being mapped to a subset containing (at least) the pairs $\langle t_1, w_1\rangle$, $\langle t_1, w_2\rangle$, $\langle t_2, w_1\rangle$, and $\langle t_2, w_2\rangle$. And according to V, p is false at both $\langle t_1, w_3\rangle$ and $\langle t_2, w_3\rangle$. Let us further suppose the distance between each time here represented to be "one day ahead". If we then follow the diagram, we have it that $\Diamond\neg\, Tom\, p$ is true at the circled pair $\langle t_0, w_1\rangle$, because there is a world - namely w_3 - such that $w_1 \simeq_{t_0} w_3$ and $\neg\, Tom\, p$ is true at the pair $\langle t_0, w_3\rangle$.

We will see in the sections that follow, how the main theories will provide distinct solutions to the problem of future contingents, by resorting to distinct definitions of *truth at a context* and how it interacts with the kind of compositional semantics we have just defined. But before, we will make a brief inquire into the birth of the very notion. This exposition will provide us with crucial notions to assess the differences between each solution to the problem of future contingents, and what is it that comes novel in MacFarlane's solution.

2.4 *Truth at a context*: the beginnings

In what different ways are contexts *semantically relevant*? So far, we have only considered one manner that contexts can be said semantically relevant: they have been playing a role when handling the compositional semantics of expressions containing either the *Now* or the *Tomorrow* operators. As we saw for instance, a formula $Tom\, \phi$ is true at a *point* $c, \langle t, w\rangle$, *just in case* ϕ is true at the *point* $c, \langle t_c + 1, w\rangle$. At this stage then, we only restricted attention to the effects of a context within a notion of *truth relative to a point of evaluation* (as we called – following MacFarlane – a tuple comprising a *context* and an *index*). As an interesting feature in terms of their truth-conditions, we could also notice that whenever $N\phi$ or $Tom\, \phi$ were true at a *point* (given a structure and an *assignment* V), they would remain true at every other *point* differing at most in its temporal coordinate, since both N and *Tom* always retrieve a same value given by the temporal parameter of a context, in order to evaluate truth of the scoped sentence. Nothing, no expression, connective or operator in the language will shift the initial parameters given by a context. This is one prime aspect of contexts, yet only one feature among many they might have.

But as we will see later, the key notion that contexts play concerning the problem of future contingents, is the one involved in different definitions of *truth at a context*. Thus so far, we have only been speaking about a sentence's *truth-at-a-point* profile, the kind of notion that is primarily involved in a *compositional semantics*, which takes the form of a recursive definition of truth for every sentence of the language. This stage of a semantic theory is where we describe the 'meanings' of expressions of a language, by showing how they contribute to truth of sentences containing them. But it is idle in providing us with the pragmatically relevant notion of a sentence being true *as uttered in a context*, which gives us a notion of a sentence's *truth-at-a-context* profile.

For instance, we happen to not primarily work with a *first order language* – one containing quantifiers, variables, singular terms and predicates, among perhaps other things (as temporal and modal operators). Yet in Kaplan's *theory of demonstratives* (1978; 1989), who works with such a language, contexts are also semantically relevant in fixing or supplying the semantic values (or extensions) of certain indexical expressions, such as the nouns 'I' and 'here', which may occur in 'term'-positions of a predicative sentence in a language containing demonstratives.

Contrary to what one might expect, the way to fix such extensions will not greatly depart from the traditional treatments we already find in *Predicate Logic*. Yet the path leading to such a refinement, even eventually revealing itself to not greatly depart from traditional ways, has faced initially many obstacles and failed attempts. These kinds of missteps were actually crucial to ultimately motivate a notion of *truth at a context* which would come apart from the familiar notion of truth at a *point of evaluation*, but which nonetheless, would still relate closely to the latter notion of *truth at a point*, in a very particular way.

These developments revealed a second and different way that contexts can be said to be *semantically relevant*: when we want to know whether a sentence is *true at a context* c – full stop –, then contexts will have a role of "initializing" the appropriate *point of evaluation* relative to which the sentence should have its truth evaluated. In effect, this ancillary notion of *truth at a context*, split from that of *truth at a point*, would yield many distinctive outcomes when coupled with a notion of validity associated with *truth at (every) context*. But why was that kind of role eventually needed? It helps to get a grip of the whole concept if we spend some time on the very story behind such developments.

So suppose we take sentences in a predicative form, but which contain the demonstratives 'I' (the first-person pronoun) and 'here' occurring in 'term'-positions. Let us take two such sentences in separate:

(1) 'I am hungry now'; and,

(2) 'Pete is here now'

We ask ourselves: how do we evaluate the conditions under which they bear a specific truth-value? Or maybe even better: how do we make them fit into the patterns of ordinary *first-order logic*? It seems that in order to square them with standard procedures, we only need a way to work around how the extensions of 'I' and 'here' would be determined. And we may already hint at what kind of notion would convey extensions for those kinds of terms: a *context* representing a possible occasion of use of such sentences.

But let us just take a step back at this point, and momentarily suspend everything we know so far about *contexts*, as a technical notion used by a semantic theory. So how would we think at first – in perhaps the most straightforward way – about working out and supplying the extensions of such demonstratives? Or how could this be done in light of how things are already worked out when we have a *first order* language containing modal and temporal operators, but containing no demonstratives?

As we know, providing a semantic theory for a *first-order* language (equipped with tense and modal operators) will require encoding truth for all sentences of the language, including those in a predicative form. Since the rules governing well-formed expressions (of a language containing an infinite set of predicates) will usually render an infinite number of atomic formulas, we usually make the truth-value of an atomic formula depend on a general rule, concerning a relation holding between its constitutive elements, according to an *interpretation for the language* – usually a pair comprising a *domain of discourse*, and a *function* which will determine the extensions of elements that take part as constituents of an atomic formula. These elements occurring in an atomic formula are 'terms' (like 'variables' and 'singular terms') and an n-place predicate (relating the 'terms'). Thus in order to evaluate truth of sentences in a predicative form, the 'general rule' will ultimately make the output of a *truth-value* depend on the extensions of 'terms' either belonging, or not, to the extension of the 'predicate'. And as we said, it is an *interpretation for a language* which gives us the *domain of discourse* (a set of 'objects', or 'individuals') and determines the *extension* of every n-place predicate (which will be a set of n-tuples of 'objects' of the domain), and also the *extension* of every 'singular term' of the language (which will be an 'object' of the *domain*).

When we further have a language containing variables and quantifiers, we have an additional trouble to be taken care of. Essentially, we will have to move along two successive steps. The reason is simple: suppose we have a sentence (a closed formula, with no free variables), such as '$\forall x Qx$'. If we want to systematically account for truth of every sentence of the language, then we ought to resort to a recursive definition of truth for complex formulas which might contain quantifiers (like the one above), and we are required to compute its truth-value in terms of the truth-value of its components. But this component, when we strip off quantification, will eventually come to an atomic formula now containing an occurrence of a free variable: in the aforementioned case, 'Qx'. So how do we proceed from this?

The first step will then amount to the following. In order to make things work, we have to treat a formula (containing a *free* variable) in a similar way that we would treat formulas containing 'singular terms', for which an *interpretation* would provide an appropriate extension (an 'object' of the domain). To achieve this, we then have to make truth of an atomic formula be relativized to three things: (i) an *interpretation for the language*, fixing the extensions of 'predicates' and 'singular terms'; (ii) a *point of evaluation* (let us say, just containing a time t and a world w – enough to handle a language containing tense and modal operators); and finally, (iii) also a function f (with respect to a given structure and an *interpretation*) mapping each 'variable' (of the set of variables of the language) to an object of the *domain of discourse*. Kaplan calls this function an *assignment* (1978; 1989)[3].

At this stage then, having provided a notion of *assignment-relative* truth, we are then allowed to move forward to a second step where we can now systematically define truth of *sentences* containing the universal quantifier $\forall$, for instance. The truth-conditions for a sentence containing a universal quantifier, will then amount to the following clause:

> Where x is a variable, ϕ a formula, and w, t, f are respectively a *world*, a *time* and an *assignment* mapping all variables.
>
> $\ulcorner \forall x \phi \urcorner$ is true at $\langle w, t, f \rangle$ (in a given structure), *iff*, ϕ is true at every *index* $\langle w, t, f' \rangle$ (in that structure), where f' differs from the *assignment* f, at most (and only) in the value it assigns to the variable x.

A very important thing to notice here, is that since we have resorted to an *assignment-relative* notion of truth in order to handle quantifiers, we would have to make every other recursive definition of truth (for the other connectives and operators) to also be relativized to an *assignment.* This happens because the general rule of truth for formulas containing a quantifier has taken the form of a *recursive* definition of truth, and we would have eventually to compute the truth value of a complex formula in terms of sub-formulas which might contain operators and connectives. So even if we are initially considering *closed* formulas where all variables are bound by quantifiers, we can only output a truth value

[3]Not to be confused with the notion of *assignment* that we have been using so far. Kaplan's *assignment* refers to a mapping of 'variables' to objects of the *domain of discourse*; yet in our terminology, it has been referring to mappings of propositions and sentences to elements of a 'structure', like subsets of the set of moments, or subsets of the set of pairs comprising an element of set T (of times) and an element of the set W (of possible worlds).

for the whole formula, if we have the means to output the truth value of its sub-formulas. And in some cases, when we are required to evaluate the truth value of a sub-formula, we might reach an occasion where an operator or connective is directly operating on an *open* formula.

For example, suppose we have *closed* formulas like '$\forall x \neg Qx$', '$\forall x \Box Qx$', and '$\forall x \exists y(Qx \wedge Sy)$', where all variables are bound by quantifiers. In order to output a truth value for these formulas, we need to know what truth values their sub-formulas – '$\neg Qx$', '$\Box Qx$', and '$\exists y(Qx \wedge Sy)$' – would have, at every *index* containing an *assignment* which differs at most in the value it assigns to the variable x. But notice then, that these sub-formulas all have operators and connectives ($\neg$, $\Box$ and $\wedge$) that are now immediately operating on a formula with a *free* variable: Qx.

That is why we need to repeat relativization to an *assignment*, for every other connective or operator present in a *language* which contains variables and quantifiers. This is a point well reminded and stressed by MacFarlane (2003b, p.3). We also find a remark in Kaplan (1989), touching on a similar factor, concerning necessity operators. He says:

> "For example, consider
>
> (0) $\exists x(Qx \wedge \sim \Box Qx)$
>
> This sentence would not be taken by anyone to express a singular proposition. But in order to evaluate the truth-value of the component
>
> $\Box Qx$
>
> (under some assignment of an individual to the variable 'x'), we must first determine whether the *proposition* expressed by its component
>
> Qx
>
> (under an assignment of an individual to the variable 'x') is a necessary proposition. So in the course of analyzing (0), we are required to determine the proposition associated with a formula containing a *free* variable. [...] In determining a semantical value for a formula containing a free variable we may be given a *value* for the variable" (Kaplan, 1989, pp.483-484 – with adapted notation: using predicate letter Q, instead of F).

Thus in some sense, even when it comes to a formula having a *free* variable, we end up driven by the need to associate a proposition with such a formula – and we make it by relativizing truth to a *time*, a *world* and an *assignment* to the variable, given a 'structure' and an *interpretation*. But we might ask here: what does it mean to evaluate an *open* formula as being true [or false]? Is it meaningful? What is really happening with *open* formulas, formulas containing variables unbound by any quantifier? In fact, it is just in order to make a systematic (and *recursive*) account of truth-conditions that we provide some way to evaluate truth of a formula containing a *free* variable; and this can be done by making the recursive clauses resort to a notion of truth relative to an *assignment* f. This was Tarski's primary "insight", as MacFarlane puts the matter: "[o]ne of Tarski's great conceptual insights was the realization that in order to give a systematic account of the way in which the truth values of quantified sentences depend on their construction and the semantic values of their parts, we need to use a *relativized* notion of truth: truth on an assignment of values to the variables, or, as Tarski called it,"satisfaction." It is this relativized notion, and not truth *simpliciter*, that is defined in the recursive clauses of a Tarskian truth theory." (MacFarlane, 2003b, p.3).

Hence, we ultimately make reference to *assignment-relative* truths, just and solely as an expedient for systematically evaluating truth of every sentence (including *quantified* expressions). But doing that doesn't mean that we would also 'make sense' of utterances of 'sentences expressing *open* formulas', just because we have now the means to give it a *semantic content* that would output – *qua* a function – a truth-value relative to an *assignment* (and a *time* and a *world*).

In fact, we may ultimately provide a notion of *truth* full stop (truth *simpliciter*), stripped of any relativizations to *assignments* of variables, if we assume our relevant target notion to be that of *truth of a sentence*, that is, if we think that it relevantly makes sense to amount for truth [or falsehood] only in the case of *closed* formulas. If we make such an assumption, we are then free to move to a notion of *truth* full stop by opting for one of either three definitions:

Definition of *truth* (full stop)

If ϕ is a sentence (a *closed* formula),

then ϕ is *true* (full stop), *iff* (a) ϕ is true on *every* assignment; or,
(b) ϕ is true on *some* assignment; or,
(c) ϕ is true on *the* assignment f_i (arbitrarily fixed)

Why it doesn't matter to pick either of the three options? Primarily because whenever we have a closed formula like $\forall x_i \forall x_j ... \phi$, where all 'variables' $x_i, x_j, ...$ occurring in it are bound by a quantifier, then **if** this sentence is true [false] on a fixed assignment f_i (or *on at least one assignment*), **then** the very same sentence would be true [false] *on every assignment.* So we can freely shift between either of the three options.

But we may from now on put aside quantifiers and *assignment-relative* definitions of truth, since as we suggested, our prime interest here in this section relies on how to account for truth of predicative sentences containing demonstratives in 'term'-positions.

Indeed, the argument we presently want to show would still work within a very simple language just possessing 'predicates', 'singular terms', and indexical terms such as 'I' and 'here'. As we said, our aim here is to show how semanticists initially sought a way for providing truth-conditions for sentences such as (1) 'I am hungry now', and (2) 'Pete is here now'.

So if we once more confine attention to ordinary semantics for *Quantified Modal Logics*, we already have a good sense on how to work things out when only 'singular terms', like 'proper names', are occurring in 'term'-position, so we better first take a look at these.

In brief, we say that where $Q\alpha$ is an atomic formula (and where Q is a one-place predicate, and α a 'singular term' of the language), $\ulcorner Q\alpha \urcorner$ is *true* at a time t and a world w, according to an *interpretation*, just in case, the *interpretation* is mapping the referent of the term α (at t and w), as *belonging* to the extension of the predicate Q (at t and w). Here, the crucial notions are that an *interpretation* provides extensions, and that we determine these extensions relative to both a time and a world.

It is this kind of relativization which will allow for example for a same sentence (already without any occurrences of demonstratives) to vary in truth along distinct *indices* containing distinct times and worlds. Even if we restrict ourselves to evaluations of truth only with respect to *indices* containing a same and fixed possible world, we could take a demonstrative-free sentence, such as 'Angela Merkel is sleeping', to already vary in truth along distinct times. So essentially, such a sentence may happen to be, at a single possible world, true relative to some times, and false relative to others.

The initial thought then, was that when we look at sentences (1) and (2), we could also take them to vary in truth when expressed by different agents (or speakers), even at a same possible world. Intuitively, (1) would be true when expressed by an agent that happens to be hungry at the time when he uses the sentence, while it would be false if expressed by another agent who is not hungry when uttering the sentence, or even if uttered by the same agent at a distinct time when he happens to not be hungry. A similar thing happens with (2): it is true if the referent of the proper name 'Pete' happens to be at the same place where the utterer is located, and it is false otherwise. Thus at a first glance, as many have initially thought, perhaps things wouldn't be that different from what already occurred with demonstrative-free sentences when they varied in truth along distinct times. And it was just such an insight which first sprang to mind among those working with the semantics of a language containing demonstratives – Kaplan included.

Speaking with more precision, the strategy would then amount to the following: when we look at sentences containing demonstratives, such as (1) and (2), and we ask how to account for their truth-values, why don't we just enrich our *indices* with additional coordinates, and start determining extensions relative to coordinates beyond just times and worlds? At first sight, this kind of solution looked just what was needed, and it surely seemed to incorporate a much less steep route up to a *Logic of Demonstratives*, when departing from ordinary *Quantified Modal Logics*. As Kaplan puts the point (*section VII* of (1989)), if we view the 'meanings' of nonindexical expressions as "a function from circumstances to extensions"; and since the 'meanings' of indexical expressions just looked to be functions from *contextual factors* to extensions, then "[f]rom this point of view" – writes Kaplan – "it may appear that the addition of indexicals requires no new *logic*, no sharp distinction between contexts and circumstances, just the addition of some special new *features* ('contextual' features) to the circumstances of evaluation. (For example, an *agent* to provide an interpretation for 'I'.)" (Kaplan, 1989, p.508). And here is a more thoroughgoing passage where Kaplan details some of the earlier motivations:

> "When it was noticed that contextual factors were required to determine the extension of sentences containing indexicals, a still more general notion was developed and called an "index." The extension of an expression was to be determined with respect to an index. The intension of an expression was that function which assigned to every index, the extension at that index.
>
>> The above example supplies us with a statement whose truth-value is not constant but varies as a function of $i \in I$. This situation is easily appreciated in the context of time-dependent statements; that is, in the case where I represents the instant of time. Obviously the same statement can be true at one moment and false at another. For more general situations one must not think of the $i \in I$ as anything as simple as instants of time or even possible worlds. In general we will have
>>
>> $$i = (w, t, p, a, ...)$$
>>
>> where the index i has many *coordinates*: for example, w is a *world*, t is a *time*, $p = (x, y, z)$ is a (3-dimensional) *position* in the world, a is an *agent*, etc. All these coordinates can be varied, possibly independently, and thus affect the truth-values of statements which have indirect references to these coordinates. [From the *Advice* of a prominent logician.]" (Kaplan, 1989, p.508).

So for instance, as much as a sentence such as 'Angela Merkel is sleeping' will vary in truth along distinct times, and thus be true relative to some choices of an *index* (given an *interpretation*) – and false relative to others –, we would also say that sentences such as (1) 'I am hungry now', and (2) 'Pete is here now', could be true relative to some choices of *indices* $\langle w, t, p, a \rangle$ (where p is a *location*, and a is an *agent*), and false relative to other *indices*, depending on what values are figuring for *agent* and *location* coordinates (even when *time* and *world* coordinates are standing fixed).

Thus instead of supplementing the theory with the technical notion of a *context*, and eventually screening it off from the notion of an *index*, we could just slip directly into the very notion of an *index* some additional coordinates representing semantically relevant features of an occasion of use of a sentence. This seemed the kind of direction to go for, when we for instance said that a sentence like ‘I am hungry now’ “would be true when expressed by an agent that happens to be hungry at the time when he uses the sentence”. But as we see at this very early stage of the framework, no significant cost seemed to be imposed at first sight when considering that sentences would be true relative to an *index* $\langle w, t, p, a\rangle$ (according to a provided structure and an *interpretation*), *if, and only if*, in the world w, the agent a would belong to the extension associated with the property “being hungry”, at time t and world w. And as we said, the ‘implementation details’ did also not seem to greatly depart from what was traditionally done, since we have just supplemented the theory with a more general and expanded notion of an *index*, containing coordinates beyond just times and worlds[4]. But it wouldn’t take long for this kind of solution to eventually find itself dragged into a tight spot.

To start, let us first consider sentence (1), ‘I am hungry now’, as being true relative to a choice of an *index* where, let us say, the *agent* (or, the *speaker* of the sentence) is ‘Horace’ and the time is ‘April 4^{th} 1982’. It then looks that the *content* of ‘I am hungry now’ - *what would be expressed* by an utterance of that sentence – would fare on a par with the *content* of the sentence:

(3) ‘Horace is hungry on April $4^{th}, 1982$’

Sentence (3) looks contingent, and so would have to be considered sentence (1) relative to that choice of an *index*. Similarly, if we take sentence (2), ‘Pete is here now’, to be true at an *index* where, let us say, the *agent* is (again) ‘Horace’, the time is (again) ‘April 4^{th} 1982’, and the *location* is ‘New York’, then we would say its *content* would fare on a par with the *content* of

(4) ‘Pete is in New York on April 4^{th} 1982’

Again, (4) will be taken as contingent, and so we would expect to treat sentence (2). But now consider the following sentence:

[4] Of course, we are not saying there was no expectation for objections being leveraged against the idea of admitting *indices* containing highly unorthodox coordinates, as ones taking values for *agents* or *locations*. Even those who took times (beyond possible worlds) as coordinates already faced serious troubles, and that is why we frequently run into arguments for the admissibility of coordinates (Kaplan’s ‘operator argument’ – in favor of admitting tense operators and *time* coordinates – is one such argument). Perhaps in the case of admitting times as coordinates, it doesn’t look that large a step to argue for the feasibility of *time-neutral* contents; contents which would vary in truth along distinct times. Yet we could wonder about the immediate rebuttals raging against those taking a sentence such as ‘I am hungry’ to express agent-neutral contents. That looks a different beast to be taken care of. But for a present-day (and highly unorthodox) defense moving along these lines, see especially Isidora Stojanovic’s “Semantic Content” (2009).

(5) I am here now

As Kaplan puts the matter, "it is obvious that for many choices of index – i.e. for many quadruples $\langle w, x, p, t\rangle$, where w is a possible world, x is a person, p is a place, and t is a time - [(5)] will be false. In fact, [(5)] is true only with respect to those indices $\langle w, x, p, t\rangle$, which are such that in the world w, x is located at p at the time t." (Kaplan, 1978, p.82). So for example, for choices where, say, in world w and time t Pete is not in New York, then the index containing w and t, and also coordinates a and p corresponding to Pete as the agent, and New York as the place, would make sentence (5) false. But this verdict concerning that special sentence, looks odd.

As Kaplan notices, if we allow for that kind of sentence to express something false, then "we have missed something essential to our understanding of demonstratives. Intuitively, [(5)] is deeply, and in some sense universally, true. One need only understand the meaning of [(5)] to know that it cannot be uttered falsely." (1978, p.82).

He then asks if we could impose some constraint (perhaps in the definition of a structure), that would include only *proper* indices; that is, if we imposed that there are only indices where "$\langle w, x, p, t\rangle$ is such that in the world w, x is located at p at the time t."

But he readily acknowledges that this maneuver, when coupled with the behavior of a modal $\Box$ (quantifying universally over worlds), would make the sentence '$\Box$ I am here now' be also accounted 'logically true', or 'universally true'. Yet this looks wrong. For instance consider '$\Box$ I am here now' as *uttered* by Pete in New York, on April 4^{th} 1982. It seems then, that the *content* expressed by that sentence, would fare on a par with a sentence stating that 'it is necessary that Pete is in New York on April 4^{th} 1982'. And because of the constraint, we would have to consider that sentence as being true. And that looks wrong. So how could we get out of that?

It is here, where the notion of a sentence *being true at a context* (or equivalently, of 'a sentence-taken-in-a-context' is true) becomes crucial. But in order to understand this, we will have to introduce some new notions.

So, when putting the matter in terms of distinct *sentences* whose *contents* would both "fare on a par with each other", we have run into notions whose concepts may be made less informal by introducing some new terminology. According to Kaplan, "[t]he Content of a sentence in a context is, roughly, the proposition the sentence would express if uttered in that context." (Kaplan, 1978, p.91).

So first, Kaplan will take a *structure* for his *LD* (his *Logic of Demonstratives*) to be a *tuple* $\langle C, W, U, P, T, I\rangle$, where C is a non-empty set comprising *contexts*, embedding features of an 'occasion of use of a sentence' which are semantically relevant, depending on expressions you have in your language. So for instance, it can have a parameter for *the time of the context*; in Kaplan, it additionally has a parameter informing *the world* *of the context*, *the agent/speaker of the context*, *the place...*, and whatever other features you might want.

The gist of Kaplan's proposal, is then to impose the envisaged constraints in terms of the members c of the set of contexts C; that is, we make in terms of the *parameters* of a context, constraints such as: (i) *the agent/speaker of* c[5] must be at *the place of* c, at *the time of* c, in *the world of* c, etc.

A key concept here is that in order to make the right predictions concerning a sentence such as 'I am here now', we should resort to a definition of a *sentence* being *true at a context c*. The way to render this is quite straightforward. We first entrench the notion of "the *content* expressed by a sentence ϕ at a context c", denoting it by '$\{\phi\}_c$' – you can think of it as denoting "the proposition that would be expressed, had the sentence been uttered at c".

Additionally (and this is crucial), whenever we have a *content* '$\{\phi\}_c$', we can associate with it *the intension* of this *content* (or, the *intension* of the proposition expressed). And this intension will be that function f, such that for every *time/world* pairs, $f(t,w) = [\![\phi]\!]^c_{\langle t,w\rangle}$, where $[\![\phi]\!]^c_{\langle t,w\rangle}$ is defined in the *compositional semantics*. So for example, the semantics takes the form of a recursive definition of truth for sentences ϕ (sentence-truth, as some like to call). But the *intension* of the proposition expressed is that *function* which when applied to a *circumstance of evaluation* (a pair with a time and a world), will give you the same value that is given in terms of sentence-truth: the value outputted by $[\![\phi]\!]^c_{\langle t,w\rangle}$.

Then, we can have now a definition of *truth at a context c*. Basically, it comes to the following:

(*truth at a context*).

A sentence ϕ is *true at a context c*, just in case:

$$\{\phi\}_c(t_c, w_c) = \textit{True}, \quad \text{where:} \quad \begin{array}{l} t_c \text{ is } \textit{the time of the context } c \\ w_c \text{ is } \textit{the world of the context } c \end{array}$$ [6]

Many things to notice here. The idea is that a sentence is *true at a context*, if the *intension* of the proposition expressed (by that sentence), yields the truth-value *True*,

[5] It is perhaps more natural to think of a *speaker* of a sentence, but talk of a 'potential user' involves a weaker and more adequate requirement. If the target notion was that of a speaker performing utterances, there would be no room for evaluating truth of multiple (potential) occurrences of a sentence at a same context (with a same agent, time, world, etc.). As Kaplan puts the matter, "it is important to distinguish an *utterance* from a *sentence-in-a-context*. The former notion is from the theory of speech acts, the latter from semantics. Utterances take time, and utterances of distinct sentences cannot be simultaneous (i.e., in the same context). But in order to develop a logic of demonstratives we must be able to evaluate several premises and a conclusion all in the same context. We do not want arguments involving indexicals to become valid simply because there is no possible context in which all the premises are uttered, and thus no possible context in which all are uttered truthfully" (Kaplan, 1989, p.522).

[6] The notation '$\{\phi\}_c(t_c, w_c)$' expresses *the function* representing the *intension* associated with the *content* '$\{\phi\}_c$'. Where you see '$\{\phi\}_c(t, w)$', you see: that function f, such that,.. and so on.

when the function is applied at the pair containing the *time* and the *world* of the context. In practice, this would come equivalent to a definition in the following lines:

(truth at a context).

A sentence ϕ is *true at a context* c[7], just in case:

$[\![\phi]\!]^c_{\langle t_c, w_c \rangle}$ = *True*, where: t_c is *the time of the context c*
w_c is *the world of the context c*

The most important thing here, is to observe this "initializing" effect of the definition of *truth at a context*, in the sense that it "initializes" (with values given by the context) the relevant *point of evaluation* relative to which the truth of the sentence should be assessed, according to the definition given in *the compositional semantics*. Notice that it is the *compositional semantics* which gives you the conditions of truth for expressions; that is, which defines what makes " $[\![\phi]\!]^c_{\langle t_c, w_c \rangle}$ = *True*", or not. But the notion of *truth at a context* interacts with the *compositional semantics*, in terms of " initializing" the *point* which is relevant for truth of the sentence.

Now, because of the constraints imposed in a context, this kind of definition will render the desired outcomes. For instance, it will make a sentence such as 'I am here now' to be *true at every context* (*LD-valid*), but it won't make '$\Box$ I am here now' *LD-valid* (true at every context).

Our findings then, up to now, should be summarized in the following. First, we now have two ways that contexts can be said to be *semantically relevant*: we can say it is *locally relevant* (that is MacFarlane's terminology), when any of its parameters plays a role in the truth-conditions within the *compositional semantics*. For example, context are *locally relevant* in our framework, because of the operator *Tom*, which retrieves values from the context.

But there is then a novel way that we can say that a context is *semantically relevant*: we can say it is *globally relevant*, by meaning that it has a role of "initializing" the relevant *point of evaluation* relative to which we evaluate truth of a sentence.

In effect, we are now prepared to explain (finally) what MacFarlane's solution amounts to. Let's move on.

[7] We put aside relativization to a 'structure' and *assignments to the variables*

Chapter 3

MacFarlane's solution

In this chapter we finally arrive at MacFarlane's original solution to the problem of future contingents, in its first appearance (2003), and also his later option to argue for a different improvement over *supervaluations*: providing better verdicts for retrospective assessments of assertions claiming what would *actually* happen in the future, which he understood as claims containing the operator '*Actually*', an *assessment-sensitive*, *necessity-like* operator. As we will see, this is the first appearance of a modality which is sensitive to *contexts of assessment.*

3.1 Giving two intuitions their due

So the whole gist of distinct solutions to the problem of future contingents is how they provide an answer as to what counts for an 'assertion' to be expressing something *true*, *false*, or even *neither true nor false.* Essentially, they will make this decision through distinct definitions of *truth at a context.*

But before moving on to each solution, let us first bring a standard picture about the notions of 'assertions' and *utterance-truth.* First of all, if 'assertions' may be qualified as being true or false, they are so said in virtue of disclosing true or false sentences, or as having expressed a *content* (through a sentence) that is true or false. Sometimes, we say that it is *contents* that are the primary bearers of truth, by understanding them as "what has traditionally been called a proposition" – as Kaplan remarks (1989, p.500). In parallel, we say that 'assertions' taking place at a *context* will disclose sentences whose *contents* are true or false. Thus a commonplace notion of an 'assertion' being true or false can be made through the notion of *utterance-truth*: truth or falsehood of a sentence as occurring or taken in a *context of utterance.*

If we take a semantic theory such as Kaplan's (1978; 1989), for example, rendered to handle a language containing certain *indexical expressions* – such as the first-person pronoun 'I' – a same sentence may well express or yield distinct *contents*, and be assigned distinct truth values on different *contexts* (occasions of use of a sentence); yet the theory is designed in such a way, that a *sentence as taken in a context* will provide a definite answer as to whether it is true or false, according to the *content* it expresses and the truth-value that it bears.

Now suppose someone makes an assertion by uttering the sentence 'Tomorrow, a sea battle will rage in the Persian Gulf'[1]. Can this assertion be true or false? Should it be assigned a definite truth-value? According to MacFarlane (2003), the problem looks pressing because we face two conflicting answers. In one sense, when we stem from the assumption that our future is objectively open, it looks as these assertions ought not to be understood as being either true or false, or equivalently, that whatever *contents* are expressed through such assertions, they equally ought not to bear any definite truth-values before the future "unfolds".

So this suggests a first answer to the question: No! if we are under the assumption that the future is objectively open, then it ought to be fully treated as such; the histories must be treated as equally possible alternatives, and nothing should "break [that] symmetry" (as MacFarlane speaks). Therefore, these 'assertions' ought to be *neither* assigned as being true, *nor* false. MacFarlane designates this kind of rationale as 'the indeterminacy intuition' (2003, p.321).

The kind of theory which manages to make sense of this intuition is *Supervaluationism.* The essence of the solution is to take an 'assertion episode' of a future contingent to be taking place at some definite time (the *time of the context*) but also as taking place – all at once – at distinct worlds overlapping at the *context of utterance.* In order to meet this provision, we will then further let a *context* determine a *set of overlapping worlds*, and introduce the following terminology (due to MacFarlane), which tries to reflect the assumption that in an occasion of 'assertion', the future is objectively open with respect to future contingent statements. Thus we have:

> **W(c).** Where c is a context, let $W(c)$ denote *the set of worlds overlapping at* c (Cf. MacFarlane, 2008: p.91; 2014, p.208); and let it respect the following constraint:
>
> **Mutual accessibility.** *For every* $c \in C$ *and* $w, w' \in W$*:* $w, w' \in W(c)$ iff $w \simeq_{t_c} w'$ ("w' *is accessible from* w *at* t_c.")[2].

[1] Quick (and related) historical fact: on November 28^{th}, 1980, during Iran-Iraq war, Iranian Navy almost depleted entire Iraqi Navy, in a sea battle fought in the Persian Gulf (around 80% of loss of Iranian Navy frigates). So just imagine the admiral addressing this sentence, on November 27^{th}, to his subordinates versed in the problem of future contingents.

[2] This constraint is different from the one we find in MacFarlane (2014, p.208) under the same label. There he phrases: "*For all* $w_1, w_2 \in W(c)$: $w_1 \simeq_{t_c} w_2$ ("w_1 *is accessible from* w_2 *at* t_c.") (2014, p.208). But this will just guarantee that if two worlds are members of the *set* $W(c)$, then they are accessible according to the relation $\simeq$ of the structure; so you might still have worlds accessible at t_c which are not members of $W(c)$. The idea of the constraint in $W(c)$ must be that it selects some "clump" of worlds in accordance with $\simeq$.

And finally, Supervaluationist's definition of *truth at a context* will amount to the following:

(Supervaluationism). A sentence ϕ is *true* [*false*] *at* c, just in case, for every *point of evaluation* $c, \langle t_c, w\rangle$:

$[\![\phi]\!]^c_{\langle t_c,w\rangle}$ = *True* [*False*], where: t_c is *the time of the context*
w is a world in the set $W(c)$

This kind of solution will make sense of the 'indeterminacy intuition', in the following way: suppose at a context c, the future is assumed to be objectively open with respect to a sentence ϕ being true *tomorrow*. Then an occasion of 'assertion' at c, concerning ϕ happening tomorrow, will be viewed as taking place – all at once – at all the worlds overlapping at that context c (where in some of them, ϕ will be the case tomorrow, while in other worlds it won't). Therefore, according to the *supervaluationist* definition, this episode of utterance is viewed as expressing neither truth nor falsity – since neither '*Tom* ϕ' is *true at* c, nor is it *false at* c. It might prove useful to assess the prediction through a diagram. So consider a context c, such that w_1, w_2, w_3 are worlds in $W(c)$, as represented in the following diagram:

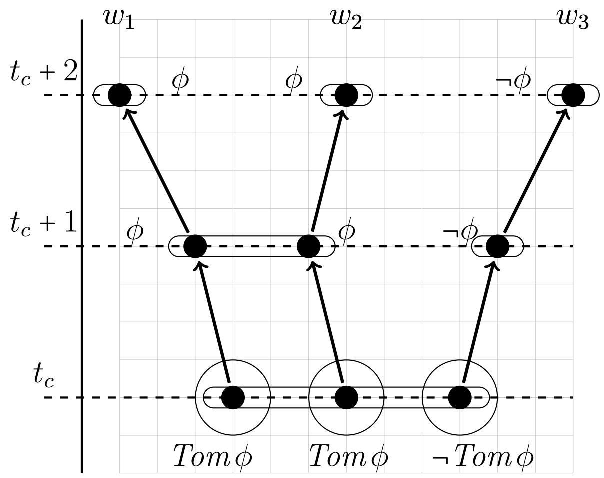


Figure 3.1: The *Supervaluationist* solution: evaluating truth at every pair, containing a world overlapping at c

Here we see a 'structure' coupled with an *assignment*, where ϕ is being mapped to a subset containing both $\langle t_c + 1, w_1\rangle$ and $\langle t_c + 1, w_2\rangle$, while according to the assignment, ϕ is false at the pair $\langle t_c + 1, w_3\rangle$. Consequently, $Tom\phi$ is false at $\langle t_c, w_3\rangle$, while it is true at $\langle t_c, w_1\rangle$ (and also at $\langle t_c, w_2\rangle$). Thus in that case, we count $Tom\phi$ as being neither *true* at c, nor *false* at c.

Yet here comes the problem for that kind of solution: no matter what distinct perspectives we take in assessing such an *utterance* (thus, *qua* a 'sentence taken in a determinate context'), this episode will always have to be taken as having expressed something which is *neither* true, *nor* false. It can never cease to be viewed as such, precisely because we are taking it *qua* an *utterance* or 'assertion episode'. So for example, let us remember an example we brought in the very outset of this paper:

> "Suppose Thomas is on his way to meet Jake, carrying an 'agreement contract' concerning the sale of his business. Just before knocking on Jake's door, Thomas eavesdrop on a conversation between Jake and his partners. Amidst some altercation, he overhears Jake saying: 'I will meet Thomas today'. [...] suppose now that Thomas has finally made up his mind. He knocks the door, enters the room and finally shakes Jake's hand. What are our thoughts now concerning Jake's assertion?"

As we said, sometimes we face occasions where time has "unfolded" one way or the other, and we may start retrospectively assessing past assertions concerning the future. And from these privileged perspectives, assessing concrete assertions which have taken place in the past, it starts looking unbearable not to assign these assertions a definite truth-value. Take a look again at the portrayed aftermaths of the episode, as a good case in point; how come we not take Jake as having spoken truthfully? It just looks unbearable not to consider his 'assertion' as having expressed truth, or having expressed a true *content.*

So then, let us now take off from a distinct perspective. By having introduced contexts into the picture, we now have the resources to evaluate one additional approach concerning future contingents: the *Thin Red Line* solution[3]. And as an alternative for providing a correct judgment respecting the 'determinacy intuition', we might resort to the definition provided by the *Thin Red Line* solution:

(Thin Red Line). A sentence ϕ is *true at c*, just in case:

$[\![\phi]\!]^{c}_{\langle t_c, w_c\rangle}$ = *True*, where: t_c is *the time of the context*
w_c is that *unique* world (or 'history') determined by c (the 'thin red line')

This alternative now makes sense of the 'determinacy intuition'. Different from the *supervaluationist* approach, this solution won't take a context to determine a set of *overlapping worlds.* Instead, it understands a context to determine a unique world (*the world of the context*), the same way for instance that it determines a *time of the context.*

[3]The theory is originally proposed in Belnap, Perloff and Xu's *Facing the Future. Agents and Choices in Our Indeterminist World* (2001). See references.

So when we assess Thomas's utterance (or his 'episode of assertion') from a future perspective, we asses it again *qua* a 'sentence taken in a determinate context'. But now the theory will correctly predict that the sentence is true at that context, since it always gives a definite answer as to what counts as being *the world of the context*.

But the problem now bends to the other pole of the spectrum. When we assess the *utterance* just before Thomas makes up his mind (and is still deciding whether to meet Jake or not), the *Thin Red Line* solution will equally have to assign the utterance a definite truth-value: in this case, *True*. So how come the future be open? We have now lost sight of 'indeterminacy'.

So how could any of the theories presented so far give due to both the determinacy and indeterminacy intuitions? In order to do this, they must provide some way to account for present 'assertions' of a future contingent as lacking a truth-value (because the future is objectively open), and they also ought to provide some way to account past 'assertions' as bearing determinate truth-values.

Could any of the previous theories do this? According to MacFarlane, the data so far suggests a simple answer: they just can't. At most, each theory is able to make sense of only one of the intuitions. For example, both Łukasiewicz's and Supervaluationist's solutions are able to save the 'indeterminacy intuition', but then they immediately lose track of 'determinacy'. Peircean and Thin Red Line solutions, in their turn, will succeed in saving 'the determinacy intuition', but will be unable to make sense of 'indeterminacy'. But why does that happen? Put simply, because of their limitations when accounting for *utterance-truth*. These theories don't have enough space to handle a definition of *truth at a context* that is able to respect both intuitions, precisely because they have nothing beyond *contexts of utterance* to undertake further relativizations of truth for 'assertion' episodes.

Thus for example, in presence of considerations such as Aristotle's, or of any advocate of objective indeterminism, a theory such as *Supervaluationism* is designed in a way that makes good sense for accounting 'assertions' of future contingents as expressing neither truth nor falsehood, and it will do this through the technique of quantifying over every possible world (or history) overlapping at the *context of utterance*. But this is exactly the spot where it gets stuck: having made such an option, 'assertions' of future contingents will then remain from any distinct perspective as being *a sentence taken in a context*, and it will be always evaluated as being *neither true nor false*, from whatever distinct perspectives you may want to assess the assertion episode.

To see this with clarity, suppose we have an 'assertion' where $Tom\phi$ is a future contingent, and c is the *context of utterance* of $Tom\phi$. From the technical perspective adopted by *Supervaluationism* for instance, it doesn't matter if this 'assertion'[4] is seen

[4]Notice that when we talk about *this* (or *that*) 'assertion', it doesn't matter if a concrete speech episode has in fact taken place; the relevant notion is that of a potential speech act (expressing the same

prospectively at the same time of utterance, or retrospectively from any future perspective, because this 'assertion' is always simply *viewed* as '*the sentence 'Tom ϕ' as taken in context* c', and the theory has already been designed in a way to yield a definite answer: it is *neither true nor false*.

Now take for instance the *Thin Red Line* approach. In presence of considerations such as those present in 'the determinacy intuition', this theory will in its turn provide a good way to make '*the sentence 'Tomϕ' as taken in context* c' as bearing a definite truth-value. As we have seen, the way it does this is by making a context provide a parameter for *the world of the context*.

So in this case, when an utterance taken in a context c is *assessed* retrospectively, the theory will provide a definite answer as to which of the worlds overlapping at that context is the one relevant for evaluating truth of the sentence; so the future contingent will now bear a definite truth value. But that again, is exactly what will backlash and wash off any good sense for the 'indeterminacy intuition': when presently *assessing* an utterance taking place *right here, right now*, the 'assertion' will also have to be seen as bearing a definite truth-value. So no sense for objective indeterminism will be anymore cogent.

Summing up, there is no space to square with both intuitions. Every theory, irrespective of which intuition they seek to reconcile with, will be led into the exact same tight spot; they will inevitably have to make a choice as to what makes an assertion true, and as soon as the theory makes that choice to make sense of one intuition, it immediately blocks all the roads leading to coherent views for the other intuition. So what then would be the solution?

According to MacFarlane, it is the tension between both intuitions which is firmly inlaid with the problem of future contingents: the problem of assigning (or not) definite truth-values to future contingents. From a retrospective perspective, past assertions concerning the future look as having a definite truth-value. But prospectively, looking from perspectives concomitant to present assertions about the future, how could they bear definite truth values without breaking the symmetry implied by assuming that the future is objectively open and indeterminate?

In his (2003), MacFarlane suggested a curious prescription for solving the problem: the same tension which makes the problem spring into life – this relativity of distinct perspectives from which we assess an 'assertion act' – could be itself the very key to its solution. Thus in order to *break the spell*, we just have to provide a way to "give both intuitions their due" (p.322).

And according to MacFarlane, in order to do this once and for all, we need to break the assumption of 'absoluteness of utterance-truth'; that is, we need to take an additional step that overcomes the assumption that *truth* of future contingent statements doesn't

content), which *could have* taken place at a past time.

get relativized to anything beyond just *contexts of utterance*. Hence, the solution will require relativizing *utterance-truth* in some way. But how should we do that?

In essence, for MacFarlane, we just ought to make room for a new kind of *context sensitivity*: a sentence's truth-value must be determined not only by a *context of use*, but also be determined by a *context of assessment*. Once we provide such a notion, the truth-value of 'a sentence *uttered at a context of use*' will then be determined in terms of (and made relative to) a *context of assessment*; so it may well vary in its truth-status. That is the sense MacFarlane speaks of, when he talks about breaking the assumption of 'absoluteness of utterance truth'. A *sentence-taken-in-a-context* will not anymore be seen as bearing an absolute and determinate truth-value (as all the competing theories predict), but rather it will be seen as only bearing truth-values when also relativized to a *context of assessment*. Hence, the truth-status of an *utterance* may well vary along distinct *contexts of assessment* from which a *sentence-taken-in-a-context* is being assessed – thus a same *utterance* (a same occasion of 'assertion'), may have distinct truth-values relative to distinct *contexts of assessment*.

For MacFarlane then, the required improvement linked to the notion of truth of a sentence must be that of countenancing relativizations of truth not only to a *context of use* of the sentence, but also to a *context of assessment* of the use of the sentence. However, the implementation of the solution is in effect quite straightforward from a technical perspective. The first thing we need to do, is to bring a notion of worlds *overlapping* at both a *context of use* and a *context of assessment*. Thus we first have:

$W'(c_1|c_2)$. Where c_1 is a *context of use* and c_2 is a *context of assessment*, let $W'(c_1|c_2)$ work out in the following way:

$$W'(c_1|c_2) = \begin{cases} W(c_1), & \text{if}\, W(c_1) \cap W(c_2) = \varnothing \\ W(c_1) \cap W(c_2), & \text{otherwise} \end{cases} \tag{3.1}$$

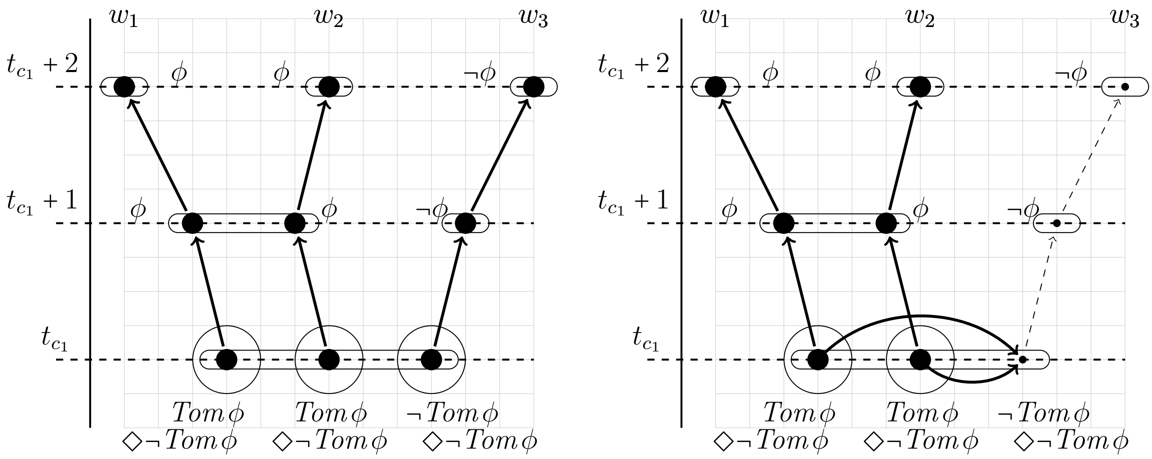
where $W(c)$ is nothing else than we have already defined, and thus respects the constraints we have imposed (**mutual accessibility**).

We may then bring about the following relativist definition of *truth*:

(Relativism). A sentence ϕ is *true* [*false*] as *used at* c_1 and *assessed from* c_2, just in case, for every *point of evaluation* $c_1, \langle t_c, w\rangle$:

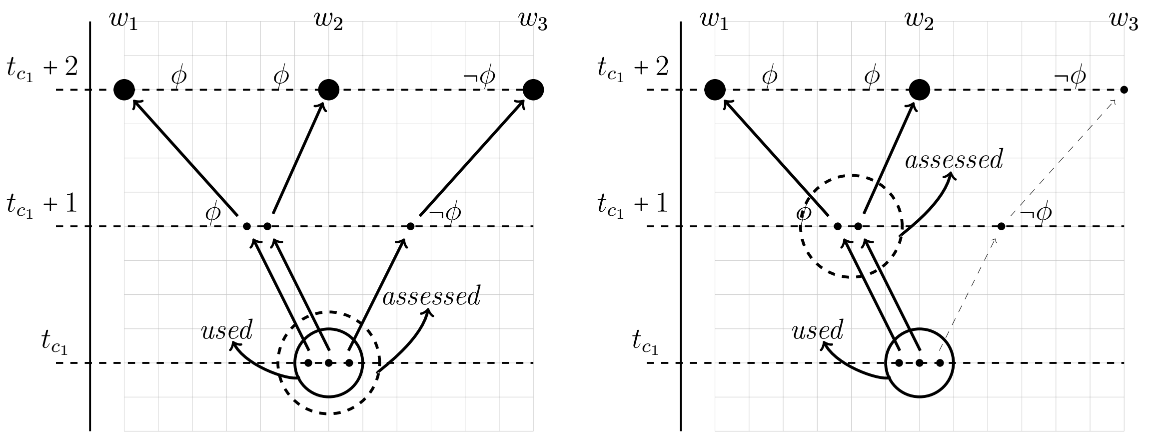
$[\![\phi]\!]^{c_1}_{\langle t_{c_1}, w\rangle}$ = *True* [*False*], where: t_{c_1} is *the time of the context of use* c_1
w is a world in the set $W'(c_1|c_2)$

Again, it helps bringing up an example. Let us then represent the two kinds of situation which show how MacFarlane's solution would be able to give both intuitions their due: the 'determinacy' and 'indeterminacy' intuitions. Let first c_1 be a context in which all the three worlds w_1, w_2, w_3 are overlapping at that context (thus, such that $w_1, w_2, w_3 \in W(c_1)$). Now suppose '*Tom*ϕ' is *uttered* at that context c_1. We then have two representative pairs of diagrams we ought to show:



(a) *used at* c_1, *assessed from* c_1 (b) *used at* c_1, *assessed from* c_2

Figure 3.2: MacFarlane's solution: truth of formulas



(a) *used at* c_1, *assessed from* c_1 (b) *used at* c_1, *assessed from* c_2

Figure 3.3: MacFarlane's solution: the two perspectives *assessing* a same *context of use*

Suppose we first want to evaluate '*Tom*ϕ' *as used at* c_1, and *assessed from* (the same) *context* c_1. According to the definition we have provided, '*Tom*ϕ' would be true just in case it is true in every *point* $c_1, \langle t_{c_1}, w \rangle$, where w is a member of $W'(c_1|c_1)$. The first diagram 'a' in Figure 3.2, is representing all the worlds which get collected by $W'(c_1|c_1)$:

all of w_1, w_2, w_3, showing up through the worlds with highlighted arrows. The circled points then represent all the pairs we ought to evaluate '$Tom\phi$'.

Immediately below it, there is a corresponding diagram ('a', of Figure 3.3), showing how all three worlds end up being selected: the inner circle represents the *context of use*, and it is revolving around all the worlds that are overlapping from the perspective of the *utterer* (so to sepak). On the other hand, the larger dashed circle represents the *context of assessment*, and it revolves around the worlds overlapping from the perspective of the *assessor*. But then notice that in this first case, when we depict a situation where both the contexts of *use* and *assessment* are c_1, all worlds are selected.

In fact, it is easy to see that the relativist definition will be able to output the exact same result as *supervaluationism* already provided us with: all the three worlds end up being collected, and so '$Tom\phi$' is here *not true*, since it is false when we evaluate it at the *point* containing w_3 – as one can observe in Figure 3.2. But neither is it false, since it is true at points containing either w_1 or w_2. The depicted situation shows us then how the definition is able to comply with the 'indeterminacy intuition', and in effect, how it is in agreement with the same verdict already predicted by the *supervaluationist* solution.

Yet now suppose a later context c_2 is such that only the worlds w_1 and w_2 are overlapping at that context, and we are now *retrospectively* assessing the utterance that has taken place at c_1, from this privileged perspective of c_2. In this case, only the worlds w_1 and w_2 are intersecting at both c_1 and c_2, and thus $W'(c_1|c_2)$ will only include those worlds.

This is exactly the situation represented by both diagrams 'b' of Figures 3.2 and 3.3. The worlds selected are again just those with highlighted arrows. So now, when we evaluate whether '$Tom\phi$' is true *as used at* c_1*, and assessed from* c_2, we only restrict attention to *points* containing w_1 and w_2. And in that case, '$Tom\phi$' will now come out simply true, full stop.

Hence, by using the same definition provided in **(Relativism)**, we are now also able to give the 'determinacy' intuition its due; an intuition which up to now, was only provided by the *Thin Red Line* solution, at the undesired cost of not making sense of 'indeterminacy'. A corresponding cost, as we have seen, would apparently also affect *supervaluationism*: though it could render future contingents as being *neither true nor false* at a context, it wouldn't be able to retrospectively assess an utterance as bearing a definite truth-value. Yet the *relativist* definition, in its turn, would be able to additionally preserve this verdict, without committing itself to that additional cost.

Before moving on to a conclusion, we want to show an additional fact these diagrams are calling for. Notice the curved arrows in diagram 'b' of Figure 3.2., that are reaching w_3 from both worlds w_1 and w_2. This is to show that regardless of the relativist definition of truth, w_3 is still *accessible at* t_c (in the sense of $\simeq$) from both w_1 and w_2. As we have defined the *compositional semantics*, truth of a formula $\Diamond\phi$ at a *point of evaluation* with a time t and world w (a formula containing the possibility modal), depends on whether the operated formula ϕ is true at some point with a world w', such that $w \simeq_t w'$. In the case we see here, the formula $\Diamond\neg Tom\ \phi$ is true at both points containing w_1 and w_2, because they both have w_3 as an accessible world at t_{c_1}. Thus, the formula even gets outputted as *true* according to the *relativist* definition, because it is true at all the point relevant to the evaluation. In fact, any formula having a modal with primary scope has a determinate truth-value in both the definitions provided by *supervaluationism* and *relativism*; that is, they can never be rendered *neither true nor false*.

Now just to sum up some concluding remarks, we have seen that MacFarlane's treatment would be able to give both the 'determinacy' and 'indeterminacy' intuitions their due. When $Tom\phi$ is evaluated prospectively, *as used at* c_1*, and (also) assessed from* c_1, it will be accounted as *supervaluationism* already predicted: as being *neither true, nor false*. So the 'assertion' episode can still be viewed as expressing neither truth nor falsehood from this prospective viewpoint. Yet this approach will also be able to meet the 'determinacy intuition', when the assertion is still evaluated *as used at* c_1, but is now retrospectively *assessed from* the distinct and later context c_2. In this case, the past 'assertion' episode is evaluated as being simply *true*, full stop.

This was essentially what MacFarlane's solution to the problem amounted to, in his original (2003). Yet here comes now a problem. In 2008, two distinct articles were published, arguing that *Supervaluationism* could already make sense of retrospective judgments of truth for past assertions of future contingents – just like MacFarlane intended. One of these articles was Berit Brogaard's *Sea Battle Semantics* (2008); the other article, was authored by MacFarlane himself – *Truth in the Garden of Forking Paths* (2008).

This kind of acknowledgement made MacFarlane, also in his (2008), propose a distinct improvement over *Supervaluationism*, and that was the fact that it wouldn't be able to handle judgments of truth for past assertion concerning what would *actually* hold in the future. This kind of approach resorted to the treatment of a modal-like *Actually* operator. But in his most recent exposition of the problem (2014), he opted for a different kind of defense for an improvement, apparently abandoning the argument from the *Actually* operator.

In this latest monograph of his, MacFarlane basically claims that while *supervaluationism* could make sense of retrospective judgments of *truth* for the propositions expressed by assertions of future contingents (and we will see how that works), it would never be able to retrospectively judge such past assertions as having been *accurate*, even if the prediction contained in the assertion comes about true. However, as he argues, his relativist theory would be able to make such judgments of accuracy. And the reason why such a thing would be a problem for *supervaluationism* would be when pragmatic norms governing *retractions* of assertions were at stake. The idea of *retraction* is that of "taking back" an assertion one has previously made, and according to MacFarlane, one is always compelled to *retract* his own past assertions whenever it can be judged as having been *inaccurate*. So while *supervaluationism* predicts that one could make sense that the content expressed by an assertion was true, it also would predict that one is compelled to retract the assertion; "take it back".

So we think now would be the right time to move on to our final and concluding batch of sections, where we will finally start talking about assessment-sensitive modal operators. We will do it in the following way: first, (1) we will take of care of some remaining aspects related to MacFarlane's approach, by revisiting a charge leveraged against *supervaluationism*.

After this, (2) we show in what sense *supervaluationism* was initially said to already respond well to claims about retrospective judgments of truth; in this case, we will bring the topic of *direct speech reports*, as discussed by Berit Brogaard, and also the device suggested by MacFarlane himself (2008), *monadic truth ascriptions*. Then, (3) we will show what different route MacFarlane took in (2008) to claim for a different improvement over *supervaluationism*: handling sentences embedding the *Actually* operator. This is where MacFarlane started to talk about an 'assessment-sensitive' modal-like operator. Finally, (4) we will present our own contribution to the topic: we motivate and develop a framework able to handle *assessment-sensitive* historical modals, and support its features by connecting it to a problem involving *indicative* uses of future-oriented statements, embedded under possibility modals.

3.1.1 Revisiting a charge against *Supervaluationism*

As we recall, Williamson leveraged a kind of charge against the *Supervaluationist* approach, trading on the fact that it yielded some disjunctions true, while neither of its disjuncts were true. Particularly telling, as we remember, was the fact that it allowed occasions where instances of *Excluded Middle* were true, while neither disjuncts were true. This happened exactly in occasions where both disjuncts were contingently true.

But now we have come to two kinds of truth-profiles of sentences (so to speak): the *truth-at-a-context* and the *truth-at-a-point* profiles. So the question now is: with respect to which notion is it sensible to ask whether a theory violates treating disjunction truth-functionally?

When we first assessed the Supervaluationist's approach, perhaps the absence of contexts and a notion of truth at a context has foreshadowed any attempts at devising two kinds of truth-profiles, and in consequence, it would forestall us from even making sensible such a kind of reply to Williamson's argument: – concerning your charge, where exactly is it sensible to ask whether a theory violates treating disjunction truth-functionally?

So we now have at our disposal an additional contrast: it is one thing to ask for the *truth-at-a-context* profile of a sentence, and it is another thing to ask for its *truth-at-a-*

point (or *index*, or *circumstance*) profile. Putting the question another way (and perhaps with more precision): when we ask whether a connective (such as disjunction) is not being treated *truth*-functionally, to what notion of truth is this charge referring to; is it referring to the notion of *truth at a context*, or is it referring to that of *truth at a point*?

So here is a plausible reply to the charge: in terms of *truth at point*, supervaluationism entirely respects *truth*-functionality of disjunction. First of all, there is no occasion where a disjunction is true at a *point* $c, \langle t, w \rangle$, while neither disjuncts are true at that very point. Additionally to settle the matter, truth or falsity of a disjunction (at a *point*) yields entirely truth-functionally from the truth status of its constituting disjuncts (at that very *point*) – it is a function of truth of the sub-formulas. Whenever two formulas are false at a point, the disjunction built from these formulas will be also false at that point; if otherwise, both are true or at least one of them is true, the whole disjunction will be true.

Perhaps it would help noticing this fact if we phrase a general principle concerning what it means for a connective to be *truth*-functional. MacFarlane actually offers such a principle to back his argument that Williamson's charge can be resisted. This is how he phrases such a principle and yields the "sense in which disjunction can be said to be truth-functional":

> **(Truth-functionality of a connective).** "A binary connective ☆ is *truth-functional* iff for every point of evaluation π and all formulas Φ, Ψ, the truth value of $\ulcorner \Phi \star \Psi \urcorner$ at π is a function of the truth values of Φ and of Ψ at π" (MacFarlane, 2008, p.87).

In the sense of such a condition, we see that *Supervaluationism* would preserve the orthodox meaning of disjunction in terms of truth-functionality, as far as *truth at a point* is concerned. MacFarlane additionally argues that the sense of *truth functionality* as concerns Williamson's charge, ought to be only sensible in terms of *truth at a point*, but not in terms of *truth at a context*. His argument comes (more or less) to the following.

He first calls attention to the fact that in ordinary theories for *first order* languages (with *context-sensitivity* – such as Kaplan's *LD*), in which a formula is true or false relative to an *assignment to the variables*; disjunctions containing *open* atomic formulas (with a *free* variable, like the formula $Fx \vee \neg Fx$), are as truth-functional as disjunctions containing *closed* formulas (with no *free* variables). So we pick any 'structure' and *assignment to the variables*, any context, any sequence of shiftable parameters, and if we evaluate any disjunctive formula (including those containing *open* formulas as disjuncts), the truth of the disjunctive formula *will be* a function of truth of the disjuncts, even if these disjuncts are *open* formulas – since the *assignment* provides us with a way to evaluate truth of an open formula.

But that doesn't mean it makes sense to charge such a theory of not treating disjunctions truth-functionally, when it would come to the truth-at-a-context profile of *open* sentences, simply because there ain't such a profile for open formulas. The theory is not designed to make an account of assertions, utterances or other speech act involving *open* sentences! And we have already seen that the only reason why we provide a way to account for truth of an open formula (or to associate a proposition to it) is to handle *recursively* the definitions of truth for quantified expressions.

So we don't ask whether an open formula would be true *at a context*, because there is no such thing as "the assignment of the context". *Contexts* don't "initialize" the *assignment* parameter. So if it is sensible to ask whether *LD* treats disjunctions truth-functionally, we only test the theory at its *truth at a point* profile, so to speak. That is why MacFarlane argues that the right notion of truth concerned in Williamson's charge ought to be *truth at a point*, since if it concerned the notion of *truth at a context*, we could equally leverage the charge against it[5].

There is also an additional way to support the reply: so again, when we say (or charge) that a theory doesn't make sense of truth-functionality of disjunction, what is required? Is it required that 'not making *any* sense of truth-functionality' is yielded when a theory has a notion of truth that violates it - even when it has a notion of truth where no such violations occur? Perhaps we would be reasonably bent at saying that *not making sense of* would come to not making sense *at all*. And if we frame the question this way, then the charge could (perhaps) be put to rest.

So it is true that concerning the notion of *truth at a context* (or the truth-at-a-context profile), a disjunction may be true while neither disjuncts are true [nor false either] (in effect, this is actually not even seen as a defect, but on contrary, it is actually the main feature that Supervaluationists will argue for their view, when it comes to the problem of future contingents). But moreover, unless the argument comes additionally with an independent argument for why the charge extends to the notion of *truth at a context*, even if the approach is already making sense of the truth-functionality when it comes to *truth at a point*, the charge will be pending a substantive reason for being raised. Of course in this case, the burden of proving that Supervaluationists distort the conventional

[5]Of course, Kaplan's notion is of a formula being true at a context (under a 'structure' and an *assignment*), since 'structures' and *assignments* are required when formalizing *LD*, in order to yield logical *validities* and *consequence relations* concerning sentences of a *first order language* containing demonstratives. But as much as we can ask whether an utterance of a sentence such as 'I am Here now' would be true at a context with *such and such features*, or whether such a sentence is *logically valid*, up to *the logic of demonstratives* (which it is), we could equally charge *LD* of not treating disjunction truth-functionally, because it makes the sentence 'x is a dog *or* it is not the case that x is a dog' *true at a context with such and such features*, while it makes neither 'x is a dog', nor its negation true at that context. Of course, this perverts the original meaning of providing assignments just in order to yield *recursive* truth-conditions for quantified expressions, but perhaps this is precisely the point for arguing that the charge is restricted at most to the notion of *truth at a point*, and in that case, the charge is equally nonsense if it were leveraged against Kaplan's analysis.

meaning of disjunction, would rest entirely on who's levering the charge; and in this case, the burden would rest on Williamson. It is not a task for the Supervaluationists to argue why it should make sense of truth-functionality of disjunction with respect to the notion of *truth at a context*, if it already makes sense of it in terms of truth-at-a-point."I already make sense of truth-functionality of disjunction", so might argue the Supervaluationist.

3.2 A problem for MacFarlane: *direct speech reports* and *monadic truth ascriptions*

We have seen that a great deal of MacFarlane's defense for his own solution rested on charging *supervaluationism* with not being able to square with the 'determinacy intuition'; that is, in its inability to evaluate past assertions as having expressed something true or false (bearing a determinate truth-value).

In brief, the argument went something like this: the whole problem for traditional accounts to the problem of future contingents is when they make a decision as to what makes an *utterance* express truth or falsehood. But once they have made that decision, and provided the way that a context determines the relevant *points of evaluation* (relative to which truth of the used sentence is evaluated), then the truth-status of the assertion episode will be simply unaffected by any future speech situation assessing it. And the argument continues: yet suppose someone asserted yesterday the sentence "There will be a sea battle tomorrow", and it turned out that such a battle is raging today at the sea; then it looks plain that the earlier assertion was true at the time of utterance, and not *neither true nor false*, as *supervaluationism* would expect us to evaluate.

But are there really no ways for ascribing a determinate truth-value to assertions that were made when the future was open? In fact, it turns out that very simple devices could be used to make sense of ascribing truth to past assertions, even if we just have at our disposal the notion of a *context of utterance*. One such way is through *direct speech reports*, as was shown by Brogaard (2008); the other is through *monadic truth ascriptions*, as shown by MacFarlane himself (2008; 2014). Let us then speak about the former, and in what sense it could be used by the *supervaluationist* approach to meet the challenge of giving due to the 'determinacy intuition'.

So suppose Jake makes an assertion at time t_1, by uttering the sentence "There will be a sea battle tomorrow". Now suppose we are at a later time t_2, where a sea battle is in fact raging at the sea, and we start wondering what kind of claims about Jake's assertion we could be making.

First thing to recall here is that we only have *contexts of utterance* around, so whatever we wonder about what shall make sense of ascribing truth to Jake's past *utterance*, it should be made through the *use* of some sentence at that future speech situation t_2.

But in fact, a very simple kind of device would accomplish in making judgments of truth for past assertions. For instance, consider that we utter at t_2 the following *speech report*:

(1) 'The sentence "There will be a sea battle tomorrow", as uttered by Jake yesterday, was true.'

This sentence simply amounts to a *direct speech report*; it reports the sentence asserted by Jake, by *directly* quoting the very sentence used by him. Now the key question is whether this report, which is taking place at t_2, would be saying something correct when it qualifies as *having been true* Jake's assertion of the quoted sentence when he uttered it at t_1 – the time of Jake's *context of use*. At first glance, if we take *supervaluationism* as our chosen framework, we perhaps would be moved to say that the report is incorrect, since the sentence would have been rendered as being *neither true nor false* at the context of Jake's speech act, and the report is qualifying it as *having been true*. But here is the thing: the professed accuracy in the verdict "betrays a mistaken view of direct speech reports", as Brogaard argues (2008, p.329).

To ease exposition, let us first try to make some clarifications. There are two distinct *contexts of utterance* playing roles here: first, there is the *context of the report* which is taking place at the time t_2 (remember that we are considering the report as being used through an *utterance*); the other distinct context is the *context of assertion* which is being reported – that is, Jake's assertion which took place at the time t_1.

What Brogaard has shown, is that by exploiting an interpretation of the semantics of *direct speech reports*, first suggested by Recanati (2004), we could make sense of retrospective judgments of truth using nothing else than the standard *supervaluationist* semantics. In order to understand the argument, let us speak about the two key features of these reports, that make it useful for *supervaluationist*. The first of them, present at Recanati's analysis, is that the sentence reported through quotations in a *speech report* is not just *mentioned*, but is in fact *used* in the *context of report*. So albeit the fact that we are quoting, some of the features of the *context of report* must play some role in determining truth of what is being reported. This is the first key point.

But what about the quoted material itself and the expressions contained in it? For example, it contains the indexical 'tomorrow'. What is the right semantic value to assign for it? Here is where we have the second key feature of Recanati's analysis: according to him (as Brogaard argues), these linguistic devices "create a shifted context that determines the semantic values of the indexicals in the report" (2008, p.330). So for example, suppose Paula makes the following direct report of an assertion made by Jake, using quotations:

(2) 'While we were dancing at the party, Jake said "I love Jane" '

We never see the indexical 'I' inside the quoted material as referring to Paula, who is uttering the report; that is, we don't see Jake as *having said* that Paula loves Jane. What we see is simply Jake having said that he himself loves Jane. So this is a clear symptom that the expressions in the quoted sentence must be determined by the *context of Jake's assertion.* Thus, we have expressions in the report which are determined differently, by features of distinct contexts. So for example, in the matrix of the report (outside the quotation) the expressions are sensitive to the *context of report*; for instance, whether both were dancing, whether in fact Jake said what she reports him to have said, whether it was asserted while they were dancing, etc. But the *content* expressed by what is inside the quotation is determined by a distinct context shifted by the report – in this case, Jake's assertion.

Now comes two additional points relevant for understanding how such a speech report present in (1) could be said to make justice to the 'determinacy intuition', and be making a retrospective judgment of truth for Jake's *utterance.* The first thing is that it is directly making a claim about Jake's 'assertion', through the device of a speech report – so it can respectfully be said to be assessing Jake's *utterance.*

The second thing is that in order for the report to be making a correct judgment of truth for Jake's *utterance*, it must be evaluated as having said something true and correct at *the context of the report.* That is, the primary and relevant *point of evaluation* relative to which we evaluate truth of sentence (1) must be "initialized" by the *the context of the report*, and all the indexicals outside the quotation are determined by that context. So the "yesterday" and "was true" embedded in the matrix of the report determines the relevant *point of evaluation* to contain a time coordinate which corresponds to "one day before" the *context of report.* What about the world coordinate? Since the framework we want to test is that of *supervaluationism*, then truth of the report ends up depending on truth at *every point of evaluation* containing the time corresponding to yesterday, and a world *overlapping at the context of report.*

Now for what concerns the reported sentence, the shifting effect makes the indexical 'tomorrow', contained in the quoted material, be interpreted as retrieving a semantic value not from the *context of the report*, but rather from a context that was shifted by the report – which is Jake's *context of assertion.* And relative to Jake's context, the semantic value assigned to Jake's use of the indexical 'tomorrow' turns out to be t_2, which is exactly *the time of the report.* So in practice, the truth of the report ends up depending on whether a sea battle is raging at the time of report and all worlds overlapping at its context. And since we assume it is, then the report makes a correct prediction about Jake's assertion yesterday. In a sense, the idea is that "what Jake said" – the *content* he expressed, or equivalently, the proposition that he expressed – is in fact simply and outright *true* from the perspective of the report.

We have provided an intuitive grasp of the idea. But there is also another way to view the same example, if we are not to picky about speaking from a metalanguage perspective. So first, we can view the report as attributing truth for the *content*, or proposition, expressed by the quoted material – which is Jake's *utterance* at t_1. So according to what we have already showed, the shifting effect of speech reports makes the quoted material be appraised at the context of its utterance – and not of the report – in order to grasp the proposition that was expressed. Thus, the proposition we must associate with the quoted sentence should be *that content* which is expressed by the sentence used by Jake at the *context of assertion*. But how could we make a more precise sense out of all that?

Perhaps one good way to accomplish this, and provide a coherent link between the correctness of a speech report and the proposition expressed by the quoted sentence (at the relevant shifted context), is to bring here once more Kaplan's notions of (i) the '*content* of a sentence taken in a context', and also, (ii) the *intension* associated with that *content*.

We may then recall that where ϕ is a sentence, and c is a context, we denote by '$\{\phi\}_c$', "the *content* of ϕ in c". The *intension* associated with the *content* that was expressed (the *intension* of the proposition), is that function from *time/world* pairs to truth-values, such that $\{\phi\}_c(t, w) = [\![\phi]\!]^c_{\langle t,w\rangle}$, for every time t and world w.

With this notion, we have a way to evaluate from a metalanguage perspective, the claims that *direct speech reports* make about the propositions expressed in past assertions. So then, one straightforward way to understand correctness of a speech report, within the framework of *supervaluationism*, can be phrased through the following:

> A *direct speech report* taking place at c', qualifying truth of a quoted sentence ϕ as used at c, is correct, just in case, for every world w overlapping at the context c' of the report, and where $t_{c'}$ is the time of the *context of report* c', it happens that $\{\phi\}_c(t_{c'}, w) =$ *True*.

The idea here (a very loose one, but instructive enough) is that we evaluate a report as being correct if the proposition expressed by a past assertion is true at all the points that matter at the *context of the report*; that is, if the *intension* of the proposition yields truth when applied to all *time/world* pairs which are relevant at the *context of the report*. If *supervaluationism* can make it through this, then it can be properly said to be making sense of judging a past assertion as simply true, full stop.

Now, let the quoted sentence be represented by '*Tom* ϕ' and let Jake's *context of assertion* be c_1 (where t_1 is as expected, the time of context c_1). In essence, we are taking Jake's *utterance* to be a sentence containing the indexical operator *Tom* operating on the sentence ϕ, which we could take as simply being the sentence "a sea battle is taking place". Further, let the report be taking place at the context c_2, a context in which a sea battle is raging, and such that t_2 is the time of the *context of report*.

So first, we have it that in the report, its shifting effect makes us take $\{\mathit{Tom}\ \phi\}_{c_1}$ as the relevant *content* we are predicating truth of; that is, $\{\mathit{Tom}\ \phi\}_{c_1}$ is here denoting "the proposition expressed by the use of sentence '$\mathit{Tom}\ \phi$' at c_1". The *intension* then of that proposition, is that function from *time/world* pairs to truth values, such that $\{\mathit{Tom}\ \phi\}_{c_1}(t,w) = [\![\mathit{Tom}\ \phi]\!]^{c_1}_{\langle t,w\rangle}$, for every time t and world w.

When we apply then the above rule in order to evaluate whether the speech report is correct, the *intension* of the proposition is evaluated only with respect to pairs containing the time of the report, and worlds overlapping at the report; that is, we are interested in knowing whether the function always outputs the *true* truth-value, when we input times and worlds relevant at the *context of report*. Since the relevant time at the report is t_2, then we always start evaluating the function when it is applied to a pair containing t_2. Now, having fixed that time, we know that for whatever world w we feed in the function, the value given by $\{\mathit{Tom}\ \phi\}_{c_1}(t_2,w)$ will be the same value as given by $[\![\mathit{Tom}\ \phi]\!]^{c_1}_{\langle t_2,w\rangle}$.

Now remember that because of the indexical behavior of *Tom*, the truth-value of the whole sentence will depend on truth of ϕ relative to a time which is "one day ahead" from the *context* of the *point of evaluation*, which is c_1 – so it trumps the initial value of t_2 that was given, and ends up depending on truth relative to "one day ahead" the time of the *context of assertion*. So now, the value given by $\{\mathit{Tom}\ \phi\}_{c_1}(t_2,w)$ will be the same as the value given by $[\![\phi]\!]^{c_1}_{\langle t_1+1,w\rangle}$. But then, since $t_1 + 1$ ('tomorrow' from the perspective of Jake) is simply t_2 (the time of the report), this simply amounts to the truth-value of ϕ at a *point* containing t_2 as its time coordinate. Therefore, it comes equivalent to the values given by $[\![\phi]\!]^{c_1}_{\langle t_2,w\rangle}$. But since a sea battle is in effect taking place at the *context of report*, and ϕ is representing "a sea battle is taking place", then it happens that at all worlds overlapping at the *context of report*, ϕ will be true at the selected pairs. Therefore, the speech report will be correct in evaluating Jake's *utterance* as having been true, in the precise sense of taking it as having expressed a proposition which is true from the perspective of the *context of the report*.

In that sense, we show a first device which is already available for *supervaluationism* to make sense of retrospective judgments of truth for past assertions; indeed, one which is already connected to a coherent and quite standard view, which can relate *utterance-truth*, to the truth of the proposition expressed by the *utterance*. Essentially, we make a way to 'talk of *sentence-truth at a context*' (that is, *utterance-truth*) through the medium of '*proposition-truth*'. And since we still got *contexts of utterance* lying around, we use that context to make claims about past assertions, by claiming truth of *what he asserted* – the proposition one expressed, etc.

As we have said, in MacFarlane's (2008), he also acknowledged that *supervaluationism* could respect the same judgments through another device: *monadic truth ascriptions*. These amount to ascriptions of the form "What he said is/was true", where "what he said" would denote a proposition (with an *intension*), and we are attributing the property of *being true* to that proposition. The idea is simple and in fact very similar.

We would first have to provide the object language with a single predicate "*True*", which behaves as a one-place predicate (thus *monadic*) whose extension relative to a world is the set of propositions *true at that world*. So the first step, since we are concerned with such a predicate in the object language, is to provide its truth conditions in the *compositional semantics*; which as usual, would amount to a recursive definition of truth (for sentences containing the predicate) relative to a *point of evaluation* (a context, a time and a world). We could just provide an intuitive grasp of that concept, through the notion of the extension of that predicate:

(Predicate "true"). $[\![\text{"}true\text{"}]\!]^{c}_{\langle t,w\rangle} = \{x \mid x \textit{ is a proposition and } x \textit{ is true at } \langle t,w\rangle\}$

So now take P to denote the *content* expressed by Jake yesterday at c_1, when he used '*Tomϕ*'. Recall that we take a *content* to be the proposition encoded by the use of a sentence at a context. And for every such, we have a corresponding *intension* of the proposition, as we have frequently been using here. Since we are working with *supervaluationism*, we just want to know what truth value an assertion of "What Jake said yesterday is true" would have as uttered at c_2.

According to the definition, '*true P*' is *true at* c_2, just in case:

$[\![true\ P]\!]^{c2}_{\langle t_{c_2},w\rangle} = True$, for every world w in $W(c_2)$

By the semantics of "*true*", this comes equivalent to:

P *is true at* $\langle t_{c_2}, w\rangle$, for every world w in $W(c_2)$

Now remember that P amounts to *that* proposition expressed by ' '*Tomϕ*' *as-taken-in-c_1*'. So the *intension* of this *proposition* is that function f, such that $f(t,w) = [\![Tom\phi]\!]^{c_1}_{\langle t,w\rangle}$, for every t, and w. But because of the behavior of *Tom*, the values given by that function f will be such that: $f(t,w) = [\![\phi]\!]^{c_1}_{\langle t_{c_1}+1,w\rangle} = [\![\phi]\!]^{c_1}_{\langle t_{c_2},w\rangle}$, for every t and w (because notice that "one day ahead" from Jake's context, is simply the time t_{c_2} of the *context of ascription*).

Now recall that the predicate "true" is running in the *compositional semantics*, so in essence, predicating "true" of P, *is true* at a *point* containing $\langle t_{c_2}, w\rangle$, <u>if in practice</u>, $[\![\phi]\!]^{c_1}_{\langle t_{c_2},w\rangle} = True$, also according to the *compositional semantics*. And so the whole *ascription* will be *true at context* c_2, depending on whether:

$$[\![\phi]\!]^{c_1}_{\langle t_{c_2}, w\rangle} = True, \text{ for every } w \in W(c_2)$$

Which again, is the case. So the *truth ascription* to "what Jake said", carried in an assertion happening in c_2, can respectfully be treated as having been truthful. And thus, *supervaluationism* again may be qualified as making justice to retrospective judgments of truth, for past assertions.

Enough for reports and truth ascriptions. We will see in the next section how this kind of acknowledgment made MacFarlane opt for an entirely distinct approach, now claiming for an improvement when handling sentences containing the modal-like (and assessment-sensitive) operator *Actually*. This will provide our first source of the defense we want to make: that the most essential, and radically intrinsic feature of truth-relativist frameworks – the feature which largely splits it from any other kind of approach – is giving due to assessment-sensitive modals.

3.3 The case of *Actually*: an *assessment-sensitive*, *necessity-like* modal

So we have seen in the previous section how *supervaluationism* could be said to make sense of retrospective judgments of truth, through *direct speech reports* and *monadic truth ascriptions*. In (2008), MacFarlane also acknowledged that *supervaluationism* could respect the same judgments also through another device: *monadic truth ascriptions*. These amount to ascriptions of the form "What he said is/was true", where "what he said" would denote a proposition (with an *intension*), and we are attributing the property of *being true* to that proposition.

As we have already mentioned, it was also in (2008) where MacFarlane opted for a new defense of an improvement, and that was the fact that even if *supervaluationism* provided some way to make judgments of truth for past assertions of future contingents (such as "There will be a sea battle tomorrow"), it wouldn't be able to handle the same correct judgments about sentences stating what *actually* would be the case. He understood these kind of sentences to contain the modal-like operator *Actually*, which would quantify over worlds. It is here where he first mentions a modal which would be sensitive to *contexts of assessment*, the same way future contingents were qualified.

But before anything, let us tell the story which led MacFarlane to start talking about a modal-like operator (the operator *Actually*) to be 'assessment-sensitive', and thus making sentences embedding such modals to vary in truth-value along distinct *contexts of assessment*, while they are taken *as used* in a same *context of utterance*.

So first of all, MacFarlane's main solution to the problem of future contingents rested on allowing these sentences to be 'assessment-sensitive' (this is his terminology).

This doesn't mean that every sentence of a language is assessment-sensitive; just that some of them are – for instance, future contingents. So first we need to understand what exactly makes an expression be 'assessment-sensitive', in the sense of MacFarlane's theory.

Basically as one could expect, being assessment-sensitive simply amounts to an expression, as already taken *as used in a context*, to further vary in truth-value along distinct *contexts of assessment*. So for example, in MacFarlane's theory, future-tensed sentences are assessment-sensitive because when these kinds of sentences are *taken as used* in a context, they can vary in truth-value relative to distinct *contexts of assessment*. We have just seen how this worked: when '*Tom* ϕ' is taken *as used at* c_1, it is 'neither true nor false' *as assessed from* c_1'; yet it is simply 'true' when *assessed from* c_2'. So we have a 'sentence-as-used-in-a-context' varying in its truth-status along distinct *contexts of assessment*, and that is why it is accounted as being 'assessment-sensitive'. In his (2003), this was the said feature of his approach which would be able to give due to both intuitions, and thus make sense of retrospective judgments of truth for past 'assertions' concerning the future. Yet in the standard *supervaluationist* framework, there are no such things as 'assessment-sensitive' expressions: a 'sentence taken in a context' has an absolute truth-value, and it won't vary in truth along anything beyond a *context of utterance*. But this, as we saw, was no obstacle for the standard and traditional theory to make room for judgments of truth for past assertions; we just had to shift from "talk about sentences and utterances" to "talk about proposition-truth".

So MacFarlane then called for a distinct kind of obstacle. While in his (2003) there was no account for a monadic truth predicate, and nothing about an *actuality* operator, his new approach now claimed "that the real problem with standard supervaluationism is its inability to make good sense of "actually," not its treatment of retrospective assessments of predictions" (2008, p.82).

His argument went something like this: though supervaluationism is able to make sense of judgments of truth for past assertions expressing what *would* happen, it wouldn't be able to make the same judgment of truth for a past assertion expressing what would *actually* happen, since there is no kind of information in the *point of evaluation* (no coordinate and no contextual parameter), to make an operator only quantify over the worlds overlapping at the episode of the speech report. Because remember, the report will take the *expressed* proposition to be appraised at the *context of assertion*, and whose *intension* would be *that function* according in all values given by a *point of evaluation* containing the *context of assertion* as its contextual parameter. In fact, that is the kind of spot where MacFarlane intends to thwart strategies such as those present at an account of *direct speech reports*. Let us then show how both theories would finally come apart when handling such an operator, and how they would disagree in judgments for past assertions containing such a modal operator.

First of all, we need to ask what kinds of very general (and non-negotiable) requirements a respectful *Actually* operator ought to meet. Perhaps then, a good starting point would be to inspect some of the semantic properties already displayed by such an operator in traditional frameworks. One example is for instance Kaplan's sentential operator A ("it is actually the case that") in both his (1978) and (1989). Its behavior works in the following way: first of all, in Kaplan's kind of framework a context will always determine *the world of the context*, in the same way it determines *the time of the context*. So we then first render the truth conditions running in the *compositional semantics* (of sentences containing A):

$$[\![A\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^c_{\langle t,w_c\rangle} = \text{True, (where } w_c \text{ is } \textit{the world of context } c) \\ \text{False}, & \text{otherwise} \end{cases} \tag{3.2}$$

According to MacFarlane, one kind of constraint which seems plausible for an *actuality* operator to comply with, is a condition he calls *Initial Redundancy* (2008, p.98). It amounts to the following:

> "An operator ★ is initial-redundant just in case for all sentences ϕ, ★ϕ is true at exactly the same contexts of use (and assessment) as ϕ (equivalently: each is a logical consequence of the other)" (2008, p.98 – adapting some notation).

Kaplan's A operator is an example of an operator respecting such a constraint. As we have seen, Kaplan's definition of *truth at a context* for sentences, is that where c is a context, ϕ is *true at* c, if and only if, $[\![\phi]\!]_{c,f,t_c,w_c} = \textit{True}$. Thus, by the fact that the definition "initializes" the *point of evaluation* with c and *the world of* c, and since the behavior of A in the *compositional semantics* is to retrieve the world of context and evaluate truth of the operand relative to that world, then whenever ϕ is *true at* c (for an arbitrary c), $A\phi$ will be also *true at* c (and vice versa).

From the perspective of a semantic theory, it is easy to see that the definition will comply with the constraint; but technical features apart, the intuitive idea of the constraint is that if one could *utter* a sentence ϕ that is true at his context of speech, then a corresponding sentence *claiming* that ϕ is true in the actual world, ought also to be true at that same speech situation. In this respect, MacFarlane writes:

> "If "*Actually :*" were not initial-redundant, it might sometimes happen that you could truly utter a sentence S, but not $\ulcorner \textit{Actually} : S \urcorner$ (or perhaps vice versa). But that does not seem to be possible. When you can truly say,

> "It will be sunny tomorrow," you can truly say, "It will actually be sunny tomorrow," and when you can truly say, "It will actually be sunny tomorrow," you can truly say, "It will be sunny tomorrow." This is not because "actually" has no effect on truth conditions, but because of a delicate relation between the semantics for "actually" and the definition of sentence truth at a context." (p.98)

So what are then the options for *supervaluationism* to account for an "Actually" operator, which simultaneously complies with *Initial Redundancy*? Here things start to get interesting. First of all, the theory works on the assumption that a context determines a *set of overlapping worlds*, as we have seen. So a straightforward way to work out the semantics for '*Actually*' (henceforth, using symbol A) would come to this: first we should augment the *compositional semantics* with truth-conditions for operator A. It could be something in the following lines:

$$[\![A\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^c_{\langle t,w'\rangle} = \text{True, } \underline{\text{for every}}\ w' \text{ in } W(c) \\ \text{False}, & \text{otherwise} \end{cases} \tag{3.3}$$

This much will be enough to respect the constraint. Recall that by the *supervaluationist* definition of *truth at a context*, a sentence ϕ is *true at* c, just in case $[\![\phi]\!]^c_{\langle t_c,w\rangle} = \mathit{True}$, for every world w *overlapping at* c (every member of $W(c)$). Hence, for every context, ϕ is true at that context, if and only if, $A\phi$ is also true at that context (in effect, both will be *logical consequences* of each other).

Up to now, everything looks good. But let us now suppose that instead of asserting "There will be a sea battle tomorrow", Jake makes a distinct assertion at his context, by uttering the sentence "It is *actually* the case, that *tomorrow* there will be a sea battle". We could take its logical form to be that of '$ATom\phi$'. The first distinctive outcome is that now *supervaluationism* will account Jake as asserting something *false at* c – so not *neither true nor false*, but outright simply *false*. But since there is nothing to break the symmetry implied by the assumption that the future is open at Jake's *context of assertion*, this in effect looks fine.

But what about assessing Jake's *utterance* from a privileged future speech situation, where a sea battle is raging? What is out there for *supervaluationism*? The first thought would be to bring devices such as *direct speech reports*; and to all appearances, it just looks enough, since it would have the advantage to take the *content* of Jake's assertion (the proposition he expressed) from a privileged *context of report*, in which ϕ is *actually true* – notice: not only is it *true* at the context of report, it is additionally *actually* true at this context, in the sense of the operator A. Yet, this kind of workaround will not do. In fact, it is precisely by exploiting the very feature peculiar to *speech reports* – to directly evaluate the *expressed* proposition – that this kind of device ends up helpless in judging Jake's past assertion as *having been true*.

To see that, let again the *context of Jake's assertion* be c_1, and the *context of report* be c_2. Further, let the quoted sentence be now represented by '$ATom\phi$'. The shifting effect of the report will then take $\{ATom\ \phi\}_{c_1}$ as the proposition we are assessing through the report. So the *intension* of that proposition, is that function from *time/world* pairs to truth values, such that $\{ATom\ \phi\}_{c_1}(t, w) = [\![ATom\ \phi]\!]^{c_1}_{\langle t,w\rangle}$, for every time t and world w.

By applying now the rule governing correctness of speech reports, we end up having to evaluate whether $\{ATom\ \phi\}_{c_1}(t_2, w) =$ *True*, for every world w member of $W(c_2)$. First thing we know, is that the *intension* of that proposition always accords with the values that are given by $[\![ATom\ \phi]\!]^{c_1}_{\langle t_2,w\rangle}$. But then observe the following: once we make Jake's *context of assertion* c_1 be present at the *point of evaluation*, the behavior of A will simply trump any initial world coordinate, and make truth of the whole sentence depend on the truth of '*Tom* ϕ' at *all* the worlds overlapping at c_1. And since that context was open with respect to sea battles, '$ATom\ \phi$' is now just false at all the relevant *points of evaluation*.

So how would we move away from that tight spot? How could we enable retrospective judgments of truth for sentences containing *actually* operators? MacFarlane's proposal would then come straightaway: make A sensitive to *contexts of assessments*; make it quantify over the set of overlapping worlds at $W'(c_1|c_2)$. But in order to make this work, we will need some new tweaks.

The sensible one (in effect, the only substantial one) will be to now include *contexts of assessment* as an independent parameter in *points of evaluation*, and thus change the overall definition of the V*-truth-value*, which should now be relativized to a point containing a context c_1 (for a *context of utterance*), a context c_2 (for a *context of assessment*), and again a *time* and a *world*. So here is an adapted definition that will work:

Definition 14 (V**-truth value**). *For all sentences ϕ of $\mathscr{L}$, the V-truth value $[\![\phi]\!]^{c_1,c_2}_{\langle t,w\rangle}$ of ϕ at the* point of evaluation $c_1, c_2, \langle t, w\rangle$ *is defined as follows:*

For an atomic formula p:

$$[\![p]\!]^{c_1,c_2}_{\langle t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } \langle t, w\rangle \in V(p) \\ \textit{False}, & \textit{if } \langle t, w\rangle \notin V(p) \end{cases} \tag{3.4}$$

Temporal operator Tom *("Tomorrow"):*

$$[\![\, Tom\ \phi]\!]^{c_1,c_2}_{\langle t,w\rangle} = \begin{cases} True, & if\ [\![\phi]\!]^{c_1,c_2}_{\langle t_{c_1}+1,w\rangle} = True \\ False, & otherwise \end{cases} \tag{3.5}$$

'Actually' operator A ("It is actually the case that"):

$$[\![A\phi]\!]^{c_1,c_2}_{\langle t,w\rangle} = \begin{cases} True, & if\ [\![\phi]\!]^{c_1,c_2}_{\langle t,w'\rangle} = True,\ \underline{for\ every}\ w'\ in\ W(c_1|c_2) \\ False, & otherwise \end{cases} \tag{3.6}$$

Additionally for a final step, we also have to make the definition of *truth at a context* now "initialize" *points of evaluation* containing the new and independent contextual parameter. Hence:

(Relativism 2). A sentence ϕ is *true* [*false*] as *used at* c_1 and *assessed from* c_2, just in case, for every *point of evaluation* $c_1, c_2, \langle t_{c_1}, w\rangle$:

$[\![\phi]\!]^{c_1,c_2}_{\langle t_{c_1},w\rangle} = True\ [False]$, where: t_{c_1} is *the time of the context of use* c_1
w is a world in the set $W'(c_1|c_2)$

There are many things to say here. Let us then start by taking a look at the adapted *compositional semantics*. Concerning atomic formulas and sentences containing *Tom*, nothing essential has changed. An atomic p still has its *truth-value* conditioned to the V *assignment* function, and '*Tom*' also behaves essentially in the same way: it retrieves a value from the *context of use* c_1, and makes truth of the whole formula depend on truth relative to "one day ahead" of that time. The main difference here is when we evaluate truth of sentences containing A: in this case, *contexts of assessment* c_2 now start playing a semantic role already in the *compositional semantics*. Besides, for what may concern *Initial Redundancy*, it is easy to see that this definition will respect the constraint.

But the most distinctive feature here, is that we finally provide a way to make Jake's *utterance* having been true, as assessed from a privileged *context of assessment* c_2. In fact, it would preserve two distinct judgments concerning *utterance-truth* of Jake's assertion: when we take Jake's *utterance* at c_1, as *assessed from* the same context c_1, the definition will judge Jake as having said something *false*, since we end up selecting all worlds overlapping at c_1; yet when we assess the same utterance from the future perspective of c_2, it will judge Jake as having said something true.

Moreover, this approach could now countenance an additional feature: it could also preserve the same two judgments in terms of *speech reports* or *truth ascriptions*. Essentially, this happens because the *relativist* framework can now fully accredit *contexts of assessment* as being semantically relevant from the "internal" perspective of the *compositional semantics* – the stage that gives meanings for expressions by showing how they contribute to the truth of sentences containing them. So it could take Jake's *content* to be associated with an *intension* that outcomes as *that function* f (from *time/world* pairs to truth-values), such that $f(t,w) = [\![A\,Tom\,\phi]\!]^{c_1,c_2}_{\langle t,w\rangle}$. And in effect, that is the only way it can make the judgments work. The A operator will still trump any initial world coordinate, but at least, it now quantifies over worlds overlapping at both contexts.

But notice that this kind of feature, first appearing in (2008), amounts to a major theoretical shift: it is a substantial departure even from MacFarlane's own past frameworks. For instance, the differences between MacFarlane's original solution and *supervaluationism* wouldn't show up in terms of *the compositional semantics* and *intensions* of propositions. There was no split at that level, and both theories would agree on the truth-values of sentences at every *point of evaluation*. The only place where they would finally come apart was in their distinct judgments concerning *truth at contexts*. But here, we are taking an additional and entirely distinct step: we are taking *contents* themselves (the very propositions) to be sensitive to *contexts of assessment*.

Chapter 4

The fading light of contingencies

This is our final chapter, which includes our own contribution to the topic. Having critically evaluated the main approaches concerning the problem of future contingents, including MacFarlane's distinctive approach to the truth-status problem, we now claim for a sense of a 'possibility' modal which behaves as being sensitive to *contexts of assessment*. The suggestion, we believe, becomes sensibly compelling in light of a puzzle involving some assertions containing 'possibility' modals, and which behave differently when phrased in the *indicative*, or the *subjunctive* mood.

4.1 A puzzle

Let us be frank, what do we really mean when we assert that something *can* or *may* happen in the future? What hangs on this? Halfway this thesis, we have put aside any substantive talk about historical modals, and started to be concerned only with the truth-status problem of future contingents. In fact, the whole gist of MacFarlane's solution amounted to an account for the truth-status of *utterances* of future contingents, and how his proposed picture would outperform competing theories in giving due to two distinct intuitions. Yet eventually, modals started to become salient again when MacFarlane opted for a different kind of improvement, resting on the behavior of the *Actually* operator. Different from the historical modals present at traditional theories, this modal would be *assessment-sensitive* – that is, a same *use* of a sentence containing the modal could be assessed false from one context, but retrospectively assessed true from a future perspective.

I believe there is much more to modals bearing assessment-sensitivity than we have been told about. In fact, I think it is illuminating to an extent that has not yet been fully explored. Up to now, we have only come across a necessity-like modal that bears assessment-sensitive – the case of *Actually*. But here in this section, we would like to put much more stress on its dual: a possibility-like modal bearing *assessment-sensitivity*. Our whole defense and discussion will be driven by a puzzle. Consider the following two sentences, as *uttered* by someone:

(1) It *can be true now*, that *tomorrow* there *will be* a sea battle.

(2) It *could be true now*, that *tomorrow* there *would be* a sea battle.

First of all, these sentences are compounding a possibility modal with a future-oriented statement. But observe the different moods in phrasing. In (1), the scoped sentence is in an indicative mood (*will be*), while in (2) it is in the subjunctive (*would be*). Additionally, the differences in mood of the sentences seem to be flexed in order to match the moods in the very phrasings of the modals – *can be* looks indicative, while *could be* looks in a subjunctive mood. First question: do (1) and (2) differ in meaning? Do they bear distinct conditions of truth? Second question: what is the proper sense that each bear? Do they bear a *metaphysicial* sense – a claim concerning objective possibility –, or do they bear a distinct sense? Let us explain why these questions are so crucial. Consider now the following sentence being *uttered* by someone:

(3) There won't be a sea battle tomorrow, although *it can be true now*, that *tomorrow* there will be a sea battle.

In a brief inspection, the conjunction in sentence (3) looks odd-sounding and defective. In what sense could someone say that something won't happen but that it can be true now that it will happen? If it *really* can happen, then it is because it is not already the case that it won't happen. Agreed. But let us start slowly: when we glimpse at the oddness of (3), it doesn't look that the *contents* of each conjunct seem to be readily contradicting each other – say, by their very meanings. For example, you see someone just asserting the first conjunct: "There won't be a sea battle tomorrow", then silence. Is this odd? Certainly not! Now, suppose instead the same speaker would only assert the second conjunct: "*It can be true now*, that *tomorrow* there will be a sea battle." Is it odd? Again, certainly not. So rather, it looks that when the conjunction is carried out all at once through an *assertion*, they look incompatible. It comes close to instances of Moore-like paradoxes, with the epistemic *might* operator. Consider:

(4) It is not raining, but it *might* be raining.

(4) looks defective, but it is not good a move to take the oddness of these instances to result from an incompatibility or contradictoriness in the semantic sense; that is, by taking each conjunct to be truth-conditionally incompatible with the other. This would

come at too high a price, because it would make truth of 'It *might* be raining' *entail* truth of 'It is raining'. However, how come "being compatible with one's knowledge" imply truth? That is surely not the right bullet to bite – even if we are happy at biting some bullet or other. In fact, a good explanation is available straightaway. Concerning Moore-like sentences such as (4), Yalcin puts the matter in a precise and simple way:

> "[...] we have a grip on *why* Moore-paradoxical sentences are defective: they involve the speaker in some kind of pragmatic conflict. For instance, if it is conventionally understood that, in making an assertion in a normal discourse context, one usually represents oneself as knowing what one says, then in uttering [sentence (4)], one will end up representing oneself as both knowing something and also as knowing that one does not know it. It is not coherent to intend to represent oneself in this way, and so one therefore expects [sentence (4)] to strike us as defective." (Yalcin, 2007, p.984).

The defectiveness can in effect be exploited through many distinct ways. Just to give an example. Suppose we take the epistemic possibility operator $\Diamond_e$ as a sentential operator, scoping over sentences whose contents can be entertained by agents. The semantics might be simple enough: a formula $\Diamond_e \phi$ is true, just in case, the sentence ϕ expresses something compatible to one's *stock of knowledge*. One way to do this more readily is to make the *compositional semantics* take the form of a recursive definition of truth, relative to a *point* containing a context c, a world w, and a non-empty set s which contains the possible worlds representing *live epistemic possibilities* for an agent – or equivalently, worlds which are not *ruled out* by one's knowledge. We could impose some additional constraints – for example, whether or not the actual world of one's context must be included in the set s, etc. But we can readily see how a theory could provide a verdict of defectiveness for sentences like (4). For example, the semantics could convey semantic rules such as:

For an atomic formula p:

$$[\![p]\!]^c_{\langle w,s\rangle} = \begin{cases} \text{True}, & \text{if } w \in V(p) \\ \text{False}, & \text{if } w \notin V(p) \end{cases} \tag{4.1}$$

For epistemic operator $\Diamond_e$ ("It might be that"):

$$[\![\Diamond_e \phi]\!]^c_{\langle w,s\rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^c_{\langle w',s\rangle} = \textit{True, for some world } w' \textit{ in } s \\ \text{False}, & \text{otherwise} \end{cases} \tag{4.2}$$

Thus an atomic formula is true at a *point of evaluation*, if the world in the coordinate is included in the set mapped by V when applied to p. Additionally, the epistemic possibility operator is true, if the scoped formula is true relative to some *point* having a world of s in its coordinate. So we could then devise a certain *rule* governing appropriateness of assertions, trying to reflect the kind of remark made by Yalcin. It could be as simple as:

(Rule of assertion). An agent at c is permitted to assert ϕ at c, just in case, for every *point of evaluation* $c, \langle w, s \rangle$:

$[\![\phi]\!]^c_{\langle w,s_c \rangle} = True$, where: s_c is *the stock ok knowledge of* c*'s agent/speaker*
w is a world in the set s_c

This kind of rule will deliver an explanation for the defectiveness when *uttering* a sentence such as (4). If we take its logical form to be that of '$\neg\phi \wedge \Diamond\phi$', then it is assertable at a context c, just in case, the whole conjunction is true at each and every *point* containing s_c and a world $w \in s_c$. But then, at every choice of $\langle w, s_c \rangle$, it must happen that $[\![\neg\phi]\!]^c_{\langle w,s \rangle} = True$, and also simultaneously that $[\![\Diamond_e\phi]\!]^c_{\langle w,s \rangle} = True$. Concerning the negated formula, this will amount to ϕ being *false* at every world. But since every such world is a member of s_c, then $\Diamond\phi$ can never be satisfied. Thus the scheme is never considered *assertable*.

But how does that relate to the problem we are concerned with, in light of a sentence such as (3), that makes a claim about the future? Here is the problem: Keith DeRose (in both (1998); (1999)) has taken examples such as (3) to be evidence that sentences claiming possibility of future-oriented statements *in indicative form* cannot bear any meaning other than the *epistemic* sense – the same sense we express through uses of expressions such as in (4), like in: "Jane *might be* at the party right now". As we have seen, the modal is qualified as *epistemic* in the sense of appealing to a meaning in the lines of: "For all I know (the evidence I have), nothing rules out Jane being at the party right now".

Thus DeRose takes modal sentences involving the future, to be of an *invariably epistemic character* (1998, p.67). For him then, as soon as we compound a possibility modal $\Diamond$ with a future statement *in indicative form* (such as the way sentence in (1) was phrased – using *will be*), the modal $\Diamond$ cannot bear any other meaning than the *epistemic sense* – *metaphysical* sense included. The main reason why DeRose embraces this route is because it gives a ready and natural explanation for why we see a self-defeating character in conjunctions such as in (3) – through notions such as that conveyed by the **Rule of Assertion**. So he takes the sense of modal in (1) to be invariantly that of the *epistemic sense*.

But this looks wrong! Wait, let us take some stock: granted, it looks clear that nothing prevents one from using a sentence such as (1) in an *epistemic sense*. Of course, one can use the sentence as conveying a similar meaning as if he had used something in the lines of "For all I know, nothing rules out a sea battle being fought tomorrow". But it is a substantial claim to say that the modal must bear this sense in general (or in most cases). On his side though, DeRose has a strong defense for the claim: if the sense is the *epistemic*, we have a ready explanation for defectiveness as in (3); if you want it to be able to bear a *metaphysical* sense – a claim about objective possibility of an open future – then it is on your side the burden of explaining oddness in (3). Good luck.

But let us now look again at the pair of sentences, by focusing on (2). Does it look as conveying the same as in (1)? Does the differences in mood suggest any vestige of difference in meaning? It looks that something tells for a difference, but it is hard to precise what it is. Fortunately for us, Stalnaker has provided a very interesting clue. He hints at a kind of flexibility that examples such as in (2) lend us, for taking them as expressible – and not necessarily odd-sounding. This is Stalnaker's passage in his *Inquiry* (1984), relevant for our puzzle:

> "*Might*, of course, expresses possibility, *John might come to the party* and *John might have come to the party* each say that it is possible, in some sense, that John come, or have come, to the party. I think the most common kind of possibility which this word is used to express is epistemic possibility. Normally, a speaker using one of the above sentences will be saying that John's coming, or having come, to the party is compatible with the speaker's knowledge. But <u>might sometimes expresses some kind of nonepistemic possibility</u>, *John might have come to the party* could be used to say that it was within John's power to come, or that it was not inevitable that he not come. The fact that the sentence *John might come to the party, although he won't* is somewhat strange indicates that the epistemic is the dominant one for this example. <u>There is less strangeness</u> <u>in</u> <u>*John could come to the party, although he won't*</u>. The epistemic interpretation seems less dominant in the past tense example: *John might have come to the party, although he didn't is not so strange*." (Stalnaker, 1984, p.143 – emphasis added).

Precisely! There is *less strangeness* when *could* is involved, and it surely need not bear an epistemic flavor. In fact, all the evidence points in favor of it being nonepistemic. For instance, consider someone asserting a variant of the conjunctive sentence (4):

> (5) There *won't* be a sea battle tomorrow , although *it could be* that there *would be* a sea battle tomorrow.

Sentence (5) doesn't look odd-sounding for someone to assert; the very least, it is certainly not as odd-sounding as (3), by any standards. Granted. But still, (3) looks very defective. Let us then abbreviate the sentence in (3) by 'not-will-ϕ, yet *Can be that* will-ϕ'; and (5) by 'not-will-ϕ, yet *Could be that* will-ϕ'. So the puzzle first comes to meeting two desiderata:

(D1) 'not-will-ϕ, yet *Can be that* will-ϕ' sounds defective.

(D2) 'not-will-ϕ, yet *Could be that* will-ϕ' doesn't look defective.

We should start with (D1). How could we explain that? How can we explain the defectiveness? Should we just stick to the explanation provided by **Rule of Assertion**, and give up speaking of *can* as bearing any *metaphysical* sense? In fact, there is already a quite simple explanation as to why (3) sounds odd, while still fully granting that it expresses the historical possibility modal – *supervaluationism* is enough to provide us with a good verdict.

We should first recall the truth conditions for historical possibility:

Possibility:

$$[\![\Diamond\phi]\!]^c_{\langle t,w\rangle} = \begin{cases} \text{True,} & \text{if } [\![\phi]\!]^c_{\langle t,w'\rangle} = \text{True, for some } w' \text{ such that } w \simeq_t w' \\ \text{False,} & \text{otherwise} \end{cases} \tag{4.3}$$

Also, we should recall *supervaluationist's* definition of *truth at a context*:

(Supervaluationism). A sentence ϕ is *true* [*false*] *at* c, just in case, for every *point of evaluation* $c, \langle t_c, w\rangle$:

$[\![\phi]\!]^c_{\langle t_c,w\rangle}$ = *True* [*False*], where: t_c is *the time of the context*
w is a world in the set $W(c)$

Suppose then we propose that speakers abide to a norm governing assertions amounting to the following rule:

(Truth Rule). An agent at c is permitted to assert ϕ at c, just in case, ϕ is *true at* c.

Now we have again a good explanation for why asserters would take someone *uttering* (3), with the indicative *can*, as having imparted something defective, while still sticking to the fact that the vehicled modal stands for a *metaphysical* sense of objective possibility. Thus suppose someone asserts (3) at a context c which is contingent with respect to ϕ obtaining tomorrow. Further, we take its logical form to be that of '$\neg$ *Tom* $\phi \land \Diamond$ *Tom* ϕ'. Thus, in order for the assertion to count as being permitted to be uttered at c, we require *truth at* c for the whole conjunction. So, for each and every choice of a *point* containing a

w overlapping at c, it must happen that both $[\![Tom\ \phi]\!]^c_{\langle t_c,w\rangle} = False$ (because of negation), and $[\![\Diamond Tom\ \phi]\!]^c_{\langle t_c,w\rangle} = True$. But then there will be no accessible world from w at t_c such that '$Tom\phi$' is true. Hence, '$\Diamond Tom\ \phi$' can't be satisfied.

So far so good. But what about complying with (D2)? In this case, *supervaluationism* is of no help. If we only have the above mentioned **Truth Rule**, and if we only have one type of possibility modal, there is no room for understanding why speakers would take assertions of (5) to be non-defective – or at least, to be certainly not as odd-sounding as (3).

It is here where MacFarlane's approach may provide us with a better verdict. But the explanation will have to run through a much broader theory involving the significance of assertions, and what speakers commit themselves to when they make assertions concerning the future.

Let us explain: in his (2003), a great deal of MacFarlane's defense for taking future contingents as bearing assessment-sensitivity, and why speakers would assert such assessment-sensitive contents, rested on connecting it to an alternative picture of assertions – one in which speakers impart to others that they commit themselves to respond to challenges and defend their claims, when they assert future contingents. But before reaching that, we need to take some steps back.

So here is what we want to defend (and if there are any novel contributions here, perhaps this is the one): first of all, (i) we claim that both the modals in (1) and (2) are plain historical modals, bearing a *metaphysical* sense – that is, the claims therein are speaking of *objective* possibilities. Second, (ii) we claim that albeit both the modals convey this *sense*, the data so far suggest they must differ in meaning; otherwise, we wouldn't take the conjunction in (5) to be "less strange" or not odd-sounding as (3) strikes us. But we have some grip on what is going on here: the indicative mood when phrasing (1) suggests it is "directing to facts" – it must concern in some way the worlds of the speaker, whatever way the future might "unfold" relative to him. This gives us a good hint. So finally, (iii) we will defend that the possibility modal embedded in (1) (*can be*) is *assessment-sensitive*, whereas the one we find in (2) (*could be*) is only *use-sensitive*; that is, it only involves worlds overlapping at the *time of the context of utterance*. So let's move on and see how we could meet all these points.

4.2 The semantics of a 'possibility' that "fades away"

Basically, there are two ways to provide *assessment-sensitivity* for a possibility modal. One way is to treat it *indexically*, in the sense that it retrieves values from *contexts of assessment*. This is the way we have seen that *Actually* works; only difference is that a possibility modal would be the dual of *Actually*, quantifying existentially over the worlds overlapping at a *context of assessment*– rather than universally. As we have seen, this option will have to add these contexts to *points of evaluation*, and rework the *compositional semantics* as to include truth conditions for the operator.

The other way is to retreat from including *contexts of assessment* in points of evaluation, but opt instead to add a second temporal coordinate in *points of evaluation* – which gets initialized by *contexts of assessment* in the relativist definition of truth–, and then make the truth conditions of the operator depend on that coordinate. So let us provide the two ways to do this.

First we have an *indexicalist* interpretation of possibility, dual of necessity:

Operator $\Diamond_a$ ("It *can be* the case that"):

$$[\![\Diamond_a \phi]\!]^{c_1,c_2}_{\langle t,w\rangle} = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^{c_1,c_2}_{\langle t,w'\rangle} = \text{True}, \underline{\text{for some}}\ w' \text{ in } W'(c_1|c_2) \\ \text{False}, & \text{otherwise} \end{cases} \tag{4.4}$$

Then we have the second option, which will require reworking both the *compositional semantics*, and also the definition of *truth at a context*. Let us start with our first task – taking care of the definition of truth relative to a *point of evaluation*:

Definition 15 (V**-truth value**). *For all sentences ϕ of $\mathscr{L}$, the V-truth value $[\![\phi]\!]^c_{\langle t',t,w\rangle}$ of ϕ at the* point of evaluation $c, \langle t', t, w\rangle$ *is defined as follows:*

For an atomic formula p:

$$[\![p]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ \langle t,w\rangle \in V(p) \\ \mathit{False}, & \mathit{if}\ \langle t,w\rangle \notin V(p) \end{cases} \tag{4.5}$$

Boolean connectives:

$$[\![\neg\phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]^c_{\langle t',t,w\rangle} = \mathit{False} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{4.6}$$

$$[\![\phi \wedge \psi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \mathit{True}, & \mathit{if}\ [\![\phi]\!]^c_{\langle t',t,w\rangle} = [\![\psi]\!]^c_{\langle t',t,w\rangle} = \mathit{True} \\ \mathit{False}, & \mathit{otherwise} \end{cases} \tag{4.7}$$

$$[\![\phi \vee \psi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{False}, & \textit{if } [\![\phi]\!]^c_{\langle t,w\rangle} = [\![\psi]\!]^c_{\langle t',t,w\rangle} = \textit{False} \\ \textit{True}, & \textit{otherwise} \end{cases} \tag{4.8}$$

$$[\![\phi \supset \psi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t,w\rangle} = \textit{False, or,} [\![\psi]\!]^c_{\langle t',t,w\rangle} = \textit{True} \\ \textit{True}, & \textit{otherwise} \end{cases} \tag{4.9}$$

Temporal operators F and P:

$$[\![F\phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t'',w\rangle} = \textit{True, for some t" such that } t < t'' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.10}$$

$$[\![P\phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t'',w\rangle} = \textit{True, for some t" such that } t'' < t \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.11}$$

Temporal operator Tom *("Tomorrow"):*

$$[\![\,\textit{Tom}\ \phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t_c+1,w\rangle} = \textit{True} \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.12}$$

Subjunctive possibility ("It <u>could</u> be the case that"), and its dual:

$$[\![\Diamond_s\phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t,w'\rangle} = \textit{True, for some } w' \textit{ such that } w \simeq_t w' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.13}$$

$$[\![\Box_s\phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t,w'\rangle} = \textit{True, for every } w' \textit{ such that } w \simeq_t w' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.14}$$

Indicative possibility ("It <u>can</u> be the case that"), and its dual:

$$[\![\Diamond_i \phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t,w'\rangle} = \textit{True, for some } w' \textit{ such that both: } w \simeq_t w' \textit{ and } w \simeq_{t'} w' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.15}$$

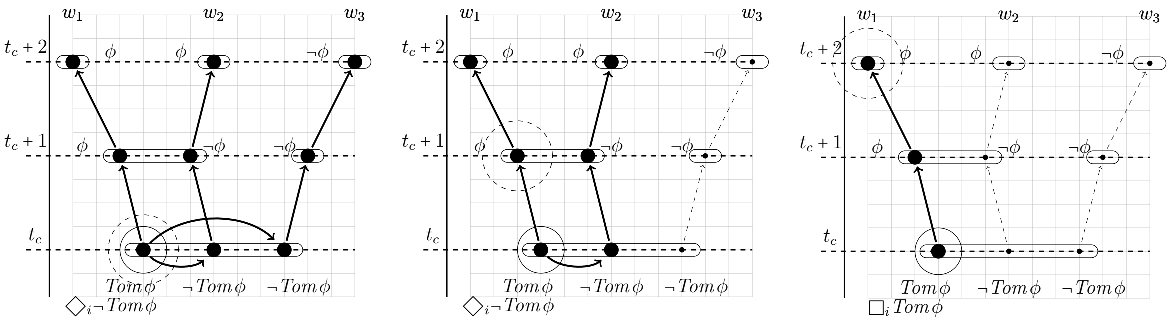
$$[\![\Box_i \phi]\!]^c_{\langle t',t,w\rangle} = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^c_{\langle t',t,w'\rangle} = \textit{True, for every } w' \textit{ such that both: } w \simeq_t w' \textit{ and } w \simeq_{t'} w' \\ \textit{False}, & \textit{otherwise} \end{cases} \tag{4.16}$$

There are many things to detail here. First notice that for atomic formulas, we still output truth conditioned to the same V assignment, just as is defined in the standard recursive definitions in $T \times W$ models. In effect, we have it that whenever an atomic p is true relative to a point $\langle t', t, w\rangle$, it will be true relative to all other triples $\langle t'', t, w\rangle$ which differ from the first triple at most in the value of the first temporal coordinate. In essence, the definition tries to make the second temporal coordinate t and the world coordinate w, behave as it did with $T \times W$ structures. And in most cases of operators, the first temporal coordinate t' in the triple remains idle in terms of truth-value of formulas.

Second, observe that the *indices* i and s serve to indicate whether the modal is the *indicative* one, or the *subjunctive* one, and note how the *subjunctive* modals behave in a similar vein compared to historical modals as have been defined in $T \times W$ structures. These operators only consult accessible worlds relative to one time coordinate in the triple. Third, notice that here only *Tom* behaves indexically, in the sense of retrieving a value from the context, and replacing (only) the second time coordinate by this value.

Now, things get mildly different in terms of what we called *Indicative operators*. These operators will be sensitive to both temporal coordinates; that is, the modals will consult the worlds which are both accessible at t and also at t'. Another noticeable feature is the fact that no operators will properly shift the first time coordinate; what happens is that they play a role when the *indicative* modals consult the worlds that are accessible.

That being said, let us provide (once again) a full grasp of the concept, by appealing to a visual representation of how this new 'possibility' modal, that we defined, behaves in terms of its truth-conditions, and how we ought to evaluate truth relative to distinct *points of evaluation*.



(a) '$\Diamond_i \neg Tom\phi$' is *true* (b) '$\Diamond_i \neg Tom\phi$' is *true* (c) '$\Diamond_i \neg Tom\phi$' is *false*

Figure 4.1: The fading light of $\Diamond_i$

Now, you might hint at what is going on here. Let us first start fresh, by remembering that an *index of evaluation* consists of a triple $\langle t', t, w\rangle$. Now, let us take the smaller circle to be indicating two of the coordinates of the triple that concerns us: it shows us its 'possible world' coordinate, and also its second 'time' coordinate (reading from left to right). By the same token, take the larger dashed circle to be indicating the first 'time' coordinate of the triple.

Thus for instance, in the first diagram above, 'a', we are evaluating truth of formulas, relative to the triple $\langle t_c, t_c, w_1\rangle$. The curved arrows, (and equally the highlighted "forward looking" arrows), are showing us which possible worlds the modal $\Diamond_i$ would quantify over, given the triple that is being informed by the circles. So for example, the formula '$\Diamond_i \neg Tom\phi$' is true relative to the triple $\langle t_c, t_c, w_1\rangle$, since there is *at least* one accessible possible world, such that the scoped formula is true at the triple containing that world, and the same time coordinates. In fact, $\neg Tom\phi$ is true at both $\langle t_c, t_c, w_2\rangle$ and $\langle t_c, t_c, w_3\rangle$.

However, even the slightest change in the first 'time' coordinate of a triple – for example, when moving to a triple now containing $t_c + 1$ as the first temporal coordinate – may affect the range of accessibility for the modal to quantify over.

This is mainly what is going on in both the second and third diagrams. When having a coordinate allocated at t_c+1 – as in diagram 'b' – the truth of the formula '$\Diamond_i \neg Tom\phi$', at $\langle t_c + 1, t_c, w_1\rangle$ will have to depend on whether the scoped formula ψ is true *at least in one* of either $\langle t_c + 1, t_c, w_1\rangle$ or $\langle t_c + 1, t_c, w_2\rangle$ [1]. In this case, there is still a way to satisfy the condition, since we can access w_2. Yet come now the third and last diagram, when we finally move to the temporal perspective of t_c+2, and the same formula is now evaluated as *false*, since it is not anymore possible that "tomorrow ϕ *won't* happen" is *true*, from the perspective of the only coordinate that this operator is able to shift – *viz.* the second coordinate t_c.

One could think – especially because of the behavior of *Tom* – that the second temporal coordinate is the one which relates to the *time of use* of a sentence, while

[1]Recall that $\simeq$ is – among other things – an *equivalence* relation, which means that for any time t, any world accesses itself at that time t.

the first, relates to something else – *the time of contexts of assessment*? Wait, not so fast. Eventually, one could perhaps think in these terms, but it is misleading, since there is nothing going on here, in the *compositional semantics*, which tells anything more substantial than just providing an extensionally correct truth definition.

Yet in some sense, it seems to already invoke some notion of a possibility "fading away" from a certain temporal perspective, as we start moving the other coordinate of the triple. In fact, we are quite close now to accomplish – in full – such a notion; we just need to add to our theory a definition of *truth at contexts* for sentences, determining the precise way that it "initializes" the relevant *point of evaluation* (the one relative to which truth of the sentence is evaluated).

So let us take stock. We have already seen how the definition worked in the case of *Actually*. It went like this:

(Relativism 2). A sentence ϕ is *true* [*false*] as *used at* c_1 and *assessed from* c_2, just in case, for every *point of evaluation* $c_1, c_2, \langle t_{c_1}, w\rangle$:

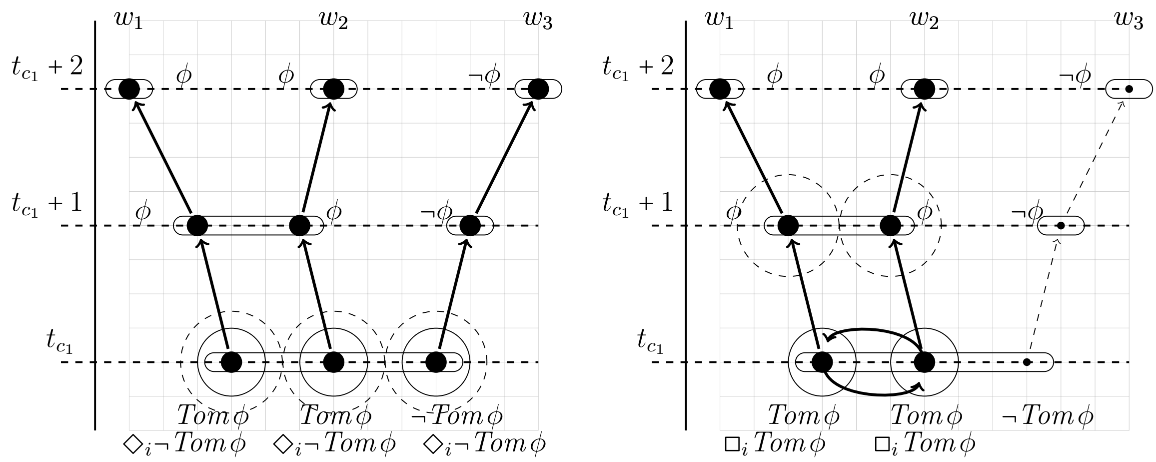
$[\![\phi]\!]^{c_1,c_2}_{\langle t_{c_1},w\rangle}$ = *True* [*False*], where: t_{c_1} is *the time of the context of use* c_1
w is a world in the set $W'(c_1|c_2)$

But since we want to work with the variant *compositional semantics* we have defined (with triples), we will have to spell out a third and independent definition of *truth at contexts*, which relates to that distinct framework. And the way it "initializes" the *point of evaluation* will be (we hope) illuminating. So here it goes:

(Relativism 3). A sentence ϕ is *true* [*false*] as *used at* c_1 and *assessed from* c_2, just in case, for every *point of evaluation* $c_1, \langle t_{c_2}, t_{c_1}, w\rangle$:

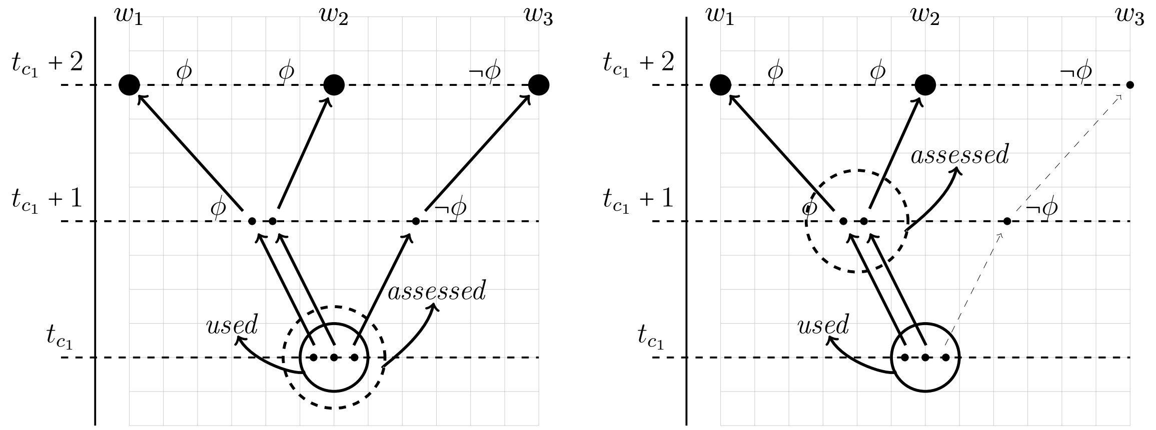
$[\![\phi]\!]^{c_1}_{\langle t_{c_2},t_{c_1},w\rangle}$ = *True* [*False*], where: t_{c_1} is *the time of the context of use* c_1
t_{c_2} is *the time of the context of assessment* c_2
w is a world in the set $W'(c_1|c_2)$

So first, observe that the relevant worlds initialized are still those overlapping at both the *context of use* and the *context of assessment*, just as it happened in MacFarlane. So for example, it still gives the verdict of retrospectively assessing past assertions of future contingents as having been true. Basically because here, the worlds selected are also the ones overlapping at both a *context of use* and a *context of assessment*. Yet this definition will provide us with some additional features when modals come into the picture. To see this, let us first consider some diagrams:



(a) *used at* c_1, *assessed from* c_1 (b) *used at* c_1, *assessed from* c_2

Figure 4.2: $T \times T \times W$ interpretation, with *assessment-sensitivity*



(a) *used at* c_1, *assessed from* c_1 (b) *used at* c_1, *assessed from* c_2

Figure 4.3: Two distinct perspectives when *assessing* a same *context of use*

The main difference here is that now a *context of assessment* will not only play a role in selecting the worlds overlapping at two distinct contexts, but it will also play a role in initializing a time coordinate in the very *index*. That is why the curved arrows representing accessibility (as in diagram 4.2.b), will now only relate to worlds which are also accessed by the time of the *context of assessment*.

Notice how in the case of the *indicative* modal of ‘possibility’ – which we claim to be *assessment-sensitive* –, as soon as a *future contingent* is secured to hold some truth-value, the *indicative* ‘possibility’ modal “fades away”, as if it ceased to ever become actualized.

On the other hand, if we restrict attention only to the fragment of the language without the *indicative* modal (that is, only with *subjunctive* possibilities), our definition will still preserve all outcomes of MacFarlane’s framework, and how it behaves in terms of *logical validites* and *logical consequences*, compared to *supervaluationism*.

Let us explain. In terms of *supervaluationism*, we saw a way to define *logical validity* and *logical consequence* in the following lines:

(s-Logical consequence). Γ *s-implies* ϕ (denoted by $\Gamma \vDash_s \phi$), just in case, for every context c, if every member in Γ is *true at* c, then ϕ is *true at* c.

(s-Validity). ϕ is *s-valid* (denoted by $\vDash_s \phi$), just in case, for every context c, ϕ is *true at* c.

Yet within MacFarlane's framework, we may have two distinct pairs of notions concerning *logical consequences* and *validities.* The first pair of notions, which he qualifies as being *diagonal* (2014), will amount to the following:

(*diagonal* logical consequence). Γ *d-implies* ϕ[2] (denoted by $\Gamma \vDash_d \phi$), just in case, for every context c, if every member in Γ is *true as used at* c*, and assessed from* c, then ϕ is *true as used at* c*, and assessed from* c.

(*diagonal* validity). ϕ is *d-valid* (denoted by $\vDash_d \phi$), just in case, for every context c, ϕ is *true as used at* c*, and assessed from* c.

First crucial thing to observe: because of the way that MacFarlane's framework is designed, *being true at* c (in the sense of *supervaluationism*), equates with *being true as used at* c*, and assessed from* c (in the sense of MacFarlane). Thus, every *supervaluationist* **s-validity** is here a **d-validity**. Additionally, every **s-Logical consequence** will be here a **diagonal logical consequence**. This, remember, in terms of the fragment containing only *subjunctive necessity*[3].

Thus for example, in *supervaluationism*, it happens that ϕ *s-implies* $\Box\phi$, and $\phi\vee\neg\phi$ is an *s-validity.* So here, ϕ also *d-implies* $\Box_s\phi$, and $\phi\vee\neg\phi$ is a *d-validity.* The difference comes when we are concerned with the remaining pair with respect to MacFarlane's framework. They are:

(*absolute* logical consequence). Γ *ab-implies* ϕ (denoted by $\Gamma \vDash_{ab} \phi$), just in case, for every *context of utterance* c_1 and *context of assessment* c_2, if every member in Γ is *true as used at* c_1*, and assessed from* c_2, then ϕ is *true as used at* c_1*, and assessed from* c_2.

[2] Or equivalently, ϕ is a *d-logical consequence* of Γ.

[3] Always remember that in MacFarlane, there are only modalities of the *subjunctive* kind. There are no *indicative* ones, the way we termed and defined.

(***absolute*** **validity).** ϕ is *ab-valid* (denoted by $\vDash_{ab} \phi$), just in case, for every *context of utterance* c_1 and *context of assessment* c_2, ϕ is *true as used at* c_1, *and assessed from* c_2.

So here, it doesn't happen anymore that $\Box_s\phi$ is an *absolute logical consequence* of ϕ. In fact, we have already seen how we can construct counter-models where '$Tom\phi$' would be (relativist) true, while it would be possible that *Tom* wouldn't hold – that is, that '$\Diamond\neg\, Tom\,\phi$' would also be (relativist) true (thus, '$\Box\, Tom\phi$' would be false).

Hence, we have a peculiar outcome concerning the *absolute* sense of logical validity, which calls for a major upshot related to our puzzle. For instance, let us briefly forget the definitions we have provided, and work with the original definitions of truth-relativism, coupled with the standard $T \times W$ *compositional semantics*. So consider a diagram representing the truth-status of both formulas '$\neg\, Tom\phi$' and '$\Diamond_s\, Tom\phi$', as used at c_1 and assessed from a later (privileged) c_2.

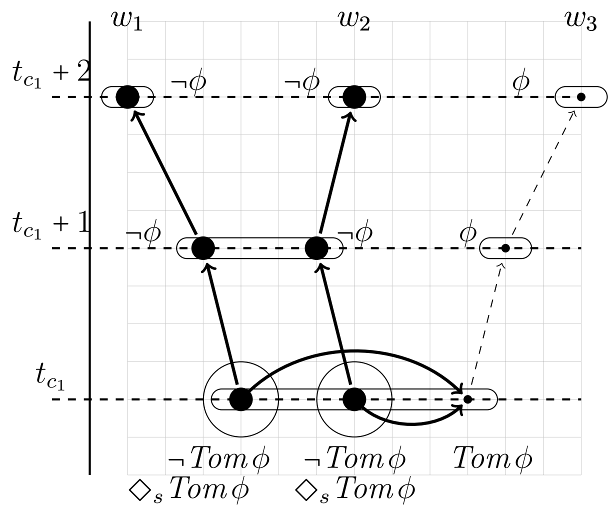


Figure 4.4: '$\neg\, Tom\phi \wedge \Diamond_s\, Tom\phi$' is true, *as used* at c_1, *and assessed* from c_2

When we "initialize" the relevant points to evaluate a sentence's truth-status, we only select worlds overlapping at both c_1 and c_2 – that is, w_1 and w_2. And in all such worlds '$\neg\, Tom\phi$' is true, so a past assertion of the future contingent would be evaluated as having been true. Yet because the modal (the only one around) always consults accessible worlds in terms of $\simeq_t$, it could still reach a world where '$Tom\phi$' is false.

In fact, this gives us some hint at why we would take assertions of the conjunction (5) not to be as odd-sounding as the one in (3). Remember, an assertion of (5) would state: "There *won't* be a sea battle tomorrow, although *it could be* that there *would be* a sea battle tomorrow." This means that from a future perspective, this assertion can be – and often is – assessed as having been true. This is a verdict which truth-relativism is able to give us, though *supervaluationism* is certainly not able to provide (and notice, because of the modal, it doesn't even get to resort to truth ascriptions or speech reports – it simply won't work)). So at least, there is some way for an assertion of (5) be assessed as true.

Granted, but what about the oddness of (3), when using *can be*? Here things start to get interesting again. For example, consider: how can MacFarlane make the same judgment *supervaluationism* is able to give, through the norm of **Truth Rule** governing assertions? It seems that the way to make this idea precise is through MacFarlane's distinct proposal of a norm governing assertions. It amounts to the following:

> **(Reflexive Truth Rule).** An agent at c is permitted to assert ϕ at c, just in case, ϕ is *true as used at c, and assessed from c.*

This kind of norm gives us an explanation for why asserters would take someone *uttering* (3) as having imparted something defective. Yet this kind of norm will make the same prediction concerning assertions of (5). So both (3) and (5) would have to be taken as being odd-sounding for one to assert. But this is not what we want. How do we get out of this?

Perhaps the best way, is to take a look at what MacFarlane has spoken about a correlate problem. The fact that even simple assertions of future contingents would violate the *Reflexive Truth* norm when the future is contingent with respect to some outcome.

First of all, we need to understand the 'distinct picture of assertions' we have mentioned, that MacFarlane claims to give support to his relativist theory, and why speakers would assert assessment-sensitive expressions such as future contingents.

But before, we should appreciate what standard responses there are for explaining what assertions entail; that is, what is itself imparted or implicated when speakers assert. A good starting point could be Lewis's view in (1980). There we find an oft-cited and illuminating passage:

> "The foremost thing we do with words is to impart information, and this is how we do it. Suppose (1) that you do not know whether A or B or. ..; and (2) that I do know; and (3) that I want you to know; and (4) that no extraneous reasons much constrain my choice of words; and (5) that we both know that the conditions (1)-(5) obtain. Then I will be truthful and you will be trusting and thereby you will come to share my knowledge. I will find something to say that depends for its truth on whether A or B or ... and that I take to be true. I will say it and you will hear it. You, trusting me to be willing and able to tell the truth, will then be in a position to infer whether A or B or...." (Lewis, 1980, p.80).

Lewis's central claim amounts to the relevance of speakers knowing how a context contributes for truth of a sentence uttered at that context. It is that kind of knowledge, plus an expectancy of speakers being truthful, that makes us impart information, and take ourselves to understand what someone has spoken. We find a similar point in John Searle (also oft-cited):

> "The point or purpose of the members of the assertive class is to commit the speaker (in varying degrees) to something's being the case, to the truth of the expressed proposition" (Searle, 1979, p.12 *apud* MacFarlane, 2003, p.334, n.15).

Thus one option for understanding what one implies when he asserts, may amount to the following:

(Implicature of Assertions).

When speakers assert ϕ at c they imply: - that ϕ is *true at c*

Thus for example, we would take someone asserting "There *will* be a sea battle tomorrow" to *imply* that it is *true* at his context, and so we would necessarily take him as having said that the future is settled with respect to the content he imparted – either if *supervaluationism* or *truth-relativism* were the correct theory. But then speakers would never be truthful when asserting future contingents, precisely because the future is contingent with respect to the asserted sentence. And it wouldn't help either taking the **(Truth Rule)** or the **(Reflexive Truth Rule)**.

Already in his (2003), MacFarlane proposed an entirely different view of assertions, and what speakers would imply when they asserted future contingents. Thus he writes:

> "What is it, then, to make an assertion? What is one doing when one asserts a sentence? One must have certain intentions and produce certain noises, but there is no assertion unless one thereby brings about a certain kind of change in normative status. One commits oneself to the truth of the sentence asserted (at its context of utterance). But what kind of a commitment is this? When one commits oneself to the truth of a sentence, what exactly is one committed to doing?"

And then he finally tells us what he understands that speakers commit themselves to, which connects firmly to his proposal of relativizing truth to *contexts of assessment*:

> "I suggest that one is committed to producing a justification, that is, giving adequate reasons for thinking that the sentence is true (relative to its context of utterance and the asserter's current context of assessment), whenever the assertion is challenged." (2003, p.334).

The main idea, as he tells us, is inspired by Robert Brandom's view of assertions, contained in *Making it Explicit* (1994). But the difference here is that MacFarlane deployed a semantically relevant significance to the very act of "defending one's claim", by making truth of an utterance depend on the *context of assessment* – since one "defends a claim" that was carried through an *utterance*, by assessing it at a *context of assessment.* That is the novelty in MacFarlane.

In fact, the *locus* he claims truth-relativism to substantially come apart from any other *contextualist* theory (*supervaluationism* included), is manifested not in norms "for the makings of assertions" (such as the Reflexive Truth Rule), but rather in norms that target speech acts. One such norm is what he calls the norm of *Retraction.* By retraction, he means "the speech act one performs in saying "I take that back" or "I retract that." The target of a retraction is another speech act, which may be an assertion" (2014, p.108). And the main implication of retracting an assertion, is that "one disavows the assertoric commitment undertaken in the original assertion" (p.108). The norm governing retraction can be given through rules such as:

> **(Retraction Rule).** An agent in context c_2 is required to retract an (unretracted) assertion of p made at c_1 if p is not true as used at c_1 and assessed from c_2.

But there is still more that is needed, in order to understand how people could defend their claims (and not be obliged to retract) when assertions of future contingents would be targeted and challenged. That is basically where MacFarlane opts to finally make his case for truth-relativism. His idea, is that one has always room to defend the claim, if the future contingent can *in principle* be assessed true – even if it is not yet definitely true, from one's *context of assessment.* When putting forward the matter, he first writes: "the relativist semantics implies that one should never assert a future contingent, and that one should retract an assertion when its content is shown to be still unsettled". He then continues: "This may seem unreasonably stringent. We assert future contingents all the time" (2014, p.230).

His argument to explain the case of asserting future contingents, goes in the following way (Cf. 2014, pp.231-232): suppose that a speaker asserts "I'll arrive on the 9:30 train," and is immediately challenged by someone who replies: "[e]ven if there is a strike or accident on the rails?". At this point, according to MacFarlane, the speaker must respond to this challenge by adopting one of either three actions: either he should (1) *retract* his assertion; or he should (2) *back up* his assertion by giving reasons why an accident or a strike couldn't happen; or just (3) *clarify* that what he meant through his assertion, was something much weaker than what its semantic content suggests. That what he meant, then, was that he would *very likely* arrive on the 9:30 train, or that he would arrive on the 9:30 train, "barring strikes, accidents, or other rare and unpredictable mishaps" (2014, p.231).

Agreed. But of course, one could only defend a claim, if it in principle could be defended. Thus, the strategy largely depends on asserters being able to always back their assertions. This line of explanation will provide a good answer as to why we take someone asserting (5) as not imparting something odd. It is because the speaker can in principle defend the claim, the same way he can defend the future contingent in the first conjunct. So for example, the same strategies he has for defending an assertion of "There won't be a sea battle tomorrow", are strategies for defending the whole conjunction (5) "There *won't* be a sea battle tomorrow , although *it could be* that there *would be* a sea battle tomorrow." – since the modal is the *subjunctive* possibility.

This is OK. It in effect amounts to a good answer. What is not OK is when we are concerned with the oddity of (3). First of all, we have claimed that the *indicative* modal in both sentences (1), and the problematic conjunction (3), do not bear the *epistemic sense*; as we said, we don't mean to rule out that someone could use the sentence in such a way. But what we claim is that speakers often express it in the *metaphysicial* sense. So they do express something concerning the possible and *objective* outcomes that the future might unfold.

If I say now, "It *can be* that Barcelona will win the match", I am making a claim about *objective* possibility. I am not just speaking that "according to my evidence and knowledge, nothing rules out Barcelona winning the match". No. It just means that it can be true, here in our world, that Barcelona will win the match.

Notice further, that if I solely claim "It *can be* that Barcelona will win the match" I can always defend my claim with ease. In fact, with much more ease than if I had asserted "Barcelona *will* win the match". Yet, if we only had the meaning provided by the *subjunctive* $\Diamond_s$, this alone wouldn't explain why we would defend an assertion of (5) (in the same lines we would defend the simple future contingent), while we take the conjunction in (3), bearing the *can*, to be outright odd.

In his (2014), there is a passage which I believe to settle the question about oddness of (3). It comes through two principles concerning judgments of rationality for assertions. We read there:

> **Reflection-Assertion I.** *One cannot rationally assert that p now if one expects that one will later acquire good grounds for retracting this assertion.*
>
> **Reflection-Assertion II.** *One cannot rationally assert that p now if it is generally expected that one will later acquire good grounds for retracting this assertion.* (Cf. 2014: pp.306-307). [4]

[4] Some additional clarification is needed here. In its original context, where MacFarlane refers to these

Thus, either by taking principle I or II, we have an explanation of why an assertion of (3) sounds odd (unlike (5)) because we immediately take it to be indefensible; we cannot expect to ever have grounds to defend it, if we happen to be challenged. That is why we don't claim things as (3), and yet we are happy to claim, and stand for, an assertion of (5). Equally, we are happy at asserting (1), with the *indicative* sense. And we will always have the means to stand for the claim, if possibilities are still out there. In other words, if *the light of contingency has not already faded*.

Notice however, that this explanation depends on countenancing the *assessment-sensitive* modal '*can*'. If we only have the *subjunctive* sense of the possibility modal, we cannot explain through principles I and II why people would feel odd about hearing or asserting the conjunction in (3), while they wouldn't feel the same about (5). Thus, it is indispensable that some sense of a possibility modal is linked to the norm governing **Retractions** in a way that (5) is not. A conjunction such as in (5) doesn't violate principles I or II (though it does violate *Reflexive Truth Rule*). Whereas taking the conjunction in (3) to involve the *indicative 'can'*, shows us how principles I and II are violated.

Thus in conclusion to answer our puzzle, we have a good explanation, and one supported by the same data provided by MacFarlane. We take it, and we stand to our claim, that '*Can*', and what is imparted through (1), has a *metaphysical* sense, but it is *assessment-sensitive*; whereas the possibility in (2) amounts to the standard notion of historical possibility.

One more thing. One might worry that this answer views *assessment-sensitivity* itself as bearing a *metaphysical sense*, as a *property* of worlds – whatever that means. But this is a confusion, in whatever meaning one wishes to convey. What is *metaphysicially* assumed is that worlds are *objective* possibilities (and that indeed, is a metaphysical stance – that's OK). To put this into perspective: if one claims that *can* behaves in an *assessment-sensitive* way (and we do claim that), this only tells for the relations obtaining between speakers when they assert, or target *utterances* of third-parties. But *objective* possibility is an assumption of a quality holding between *worlds*; *assessment-sensitivity* in its turn, is a theory about how we ought to judge someone as having asserted something

two principles, he is inquiring about – and very likely anticipating – an objection that could be leveraged against him, concerning assertions that his theory would predict oneself often to feel entitled to make – in light of norms such as the *Reflexive Truth Rule* – but which would be generally expected (or perhaps inevitably expected) to be later assessed as *false*, by the very asserter. What he has in mind involves assertions containing epistemic modals – which he also classes as being *assessment-sensitive* – like for example "It might be a boy and it might be a girl.", as said by someone who *doesn't know* whether the soon-to-be-born child is a boy or a girl. The case here is different, because both conjunctions – either when embedding *can* or *could* – are already *false* when *used* and *assessed* from a same context. In fact, the puzzle here is a cousin of his own worry, but lies instead in the opposing extreme; that is, MacFarlane is wishing to explain why one would assert (or believe) something presently *true* – as *assessed from* his context – with the acknowledged expectancy that he will later *assess* it as false; whereas in our case, we want to explain and understand why someone would feel entitled to impart a presently false claim – when *could* is involved –, but not ever feel entitled to make similar claims with '*can*'.

true at his context. The confusion, I believe, rests on the fact that a semantic theory makes a decision about what it takes for an asserter to have imparted something true, and it connects the explanation to truth conditions relative to possible worlds, or set of overlapping worlds (the case here). But this alone doesn't imply anything about possible worlds *qua* "worlds".

Still, it is perfectly reasonable to ask: what makes it *assessment-sensitive*? For a guess: because of the phrasing, because of the *indicative mood*, and the sense this mood normally implies. Especially, the sense of "directing to facts" implied by the indicative mood, is what makes its truth depend on assessors of the assertion. Simply because: whatever stances you want to take about the status of possible worlds (e.g, if you are a *modal realist* as Lewis, or not), there is at least one thing that is certainly safe and reasonable to assume: that *utterers* and *assessors* certainly inhabit the same place.

4.3 Some additional logical outcomes

Having made the central claim of our thesis, we will now just speak about some subtleties related to the *assessment-sensitive indicative* necessity $\Box_i$, and the *subjunctive* $\Box_s$. First of all, notice that $\Box_i$ would give us similar outcomes that the *Actually* operator renders. Even in the non-indexical interpretation. But not necessarily if we compound it with subjunctive possibility. We explain.

When MacFarlane devises the *Actually* operator, he has in mind Lewis's *indexical* argument of *actuality*. The idea is that it behaves as a *pure* indexical, and it always "leaps to the front" even in embedded contexts. This means that you can scope any operators over it, and still its truth-value won't vary because it always retrieves an unchangeable value from the context parameter. In this respect, for example, MacFarlane speaks: "No matter how deeply embedded we are, no matter how far the world of evaluation has been shifted, the actuality operator returns it to the world of the context of use." (2008, p.98). This, amounts to the *indexical* sense of "actually".

But the indicative operator $\Box_i$ can provide us with a *shifty* sense of "Actually", which Lewis himself talks about in his *Anselm and Actuality* (1970). Thus for Lewis, for example, "we can distinguish primary and secondary senses of "actual" by asking what world "actual" refers to at a world w in a context in which some other world v is under consideration. In the primary sense, it still refers to w, as in "If Max ate less, he would be thinner than he actually is". In the secondary sense it shifts its reference to the world v under consideration, as in "If Max ate less, he would actually enjoy himself more" " (Lewis, 1970, p.185).

Of course, Lewis takes the *indexical* reading (the primary sense) to be the accurate one, though he acknowledges this *shifty* use of "Actually", which changes reference to another world as if it had been the *actual* world. Our framework is able to give both readings, when we make use of *subjunctive* possibility and *indicative* necessity. The kind of sentences we bring to make our point are the following. Suppose it is July 8, 2014, just after we have seen Germany beat Brazil by scoring seven goals, against one. We have two distinct sentences:

(1) It *can have been* true, that Brazil would actually win the match.

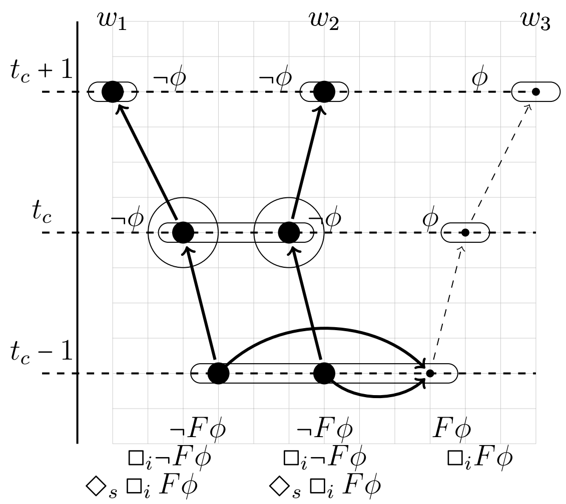
(2) It *could have been* true, that Brazil would actually win the match.

For this to work, we first have to suppose that it was *metaphysically* possible for Brazil to win the match (even if this is hard to suppose, do it for philosophy). Let us take *Actually* to be the *indicative* necessity $\Box_i$, *can* being the indicative possibility, and *could* being the *subjunctive* possibility. So we can take their logical forms as being respectively:

(1') $P \Diamond_i \Box_i F\phi$. (where P is the past operator, F the future).

(2') $P \Diamond_s \Box_i F\phi$.

We can represent this through a diagram.



(a)

Let us first see why (1) is false. Suppose then we "initialize" the relevant *points* as being $\langle t_c, t_c, w_1\rangle$ and $\langle t_c, t_c, w_2\rangle$, worlds where Brazil actually lost the match by the time t_c of the context. Now, notice that at the points $\langle t_c, t_c - 1, w_1\rangle$ and $\langle t_c, t_c - 1, w_2\rangle$ it is "actually" true that Brazil would not win the match, since $\Box_i$ quantifies over accessible worlds *accessible* at both t_c and $t_c - 1$. This works because the *indicative necessity* makes w_1 access only itself or w_2 (similarly, the same applies to w_2). But observe then, that $\Diamond_i$

won't reach w_3. So (1) is not true. But the *subjunctive* possibility $\Diamond_s$ is able to access every world accessible at $t_c - 1$, so it reaches $\langle t_c, t_c - 1, w_3\rangle$ from both w_1 and w_2. Now notice: at $\langle t_c, t_c{-}1, w_3\rangle$, it is "actually" true (in the shifty sense provided by *indicative necessity*) that Brazil would win the match, since w_3 only accesses itself with these temporal coordinates, when $\Box_i$ is at stake. Therefore, (2) is true. We then have: "It *could have been* that Brazil would *actually* win", though "it *can't have been* that Brazil would *actually* win".

Concluding remarks (and prospects for future work)

First and foremost, we hope to have contributed to as much a meaningful experience we could provide. The beauty of the problem of future contingents has always been, since its birth, that kind of spark (inherent to the problem) which seems to taunt our beliefs of an objectively open future. In one sense, the problem seems to live in that foggy land where *Logic*, and our *sciences of meanings*, make contact with "our surroundings".

As we said at the outset of this dissertation, part of the goal within this project was to endow the reader with good foundations, in order to critically evaluate every approach to the problem. This was mainly done in our *Chapter 1* ('Theories of time'), and we have spared no effort in making the formal definitions palatable to as much broad an audience we could – especially when it comes to visual representations.

The second objective of this dissertation was to argue for an interpretation of a 'possibility' modal which is sensitive to *contexts of assessment*, in the sense of MacFarlane's theory. This suggestion, we claimed, becomes sensibly compelling in light of a puzzle involving defectiveness of assertions containing that modal, and it provides a second option for interpretation, besides the one taking the modals to invariably bear an epistemic sense – when mood of the phrasing is in the *indicative*.

As far as prospects for future work are concerned, we may refer to two promising lines of investigation, on which we have been working: the first one, concerns the so-called 'Gibbardian standoffs', which is a problem involving disagreements over assertions of pairwise incompatible indicative conditionals (Gibbard, 1981). One way to deal with the problem is to propose that *indicative conditionals* are *assessment sensitive* expressions, along similar lines in which we have been working out here, in this dissertation.

The other (quite related) line of investigation stands in an overlap between *conditionals* and *time*, and involves again whether *assessment sensitivity* is of any help. We may provide a glimpse of the problem. For instance, in a well-known footnote in his *Time in Counterfactuals* (1978), M. Slote writes:

> "Imagine a completely undetermined random coin. Your friend offers you good odds that it will not come up heads; you decline the bet, he flips, and the coin comes up heads. He then says: "you see; if you had bet (heads), you would

> have won." I know of no theory of counterfactuals that can adequately explain why such a statement seems natural and correct" (Slote, 1978, p.27).

But what if, in fact, we had asserted an *indicative conditional* before the coin's landing; what if we had asserted "If you bet heads, you will win"? Should we assess the conditional as true? Should we assess a previous assertion as having been accurate? In one way, there seems to be something right about that story – perhaps for instance, by saying that one was right *in claiming* what the conditional expresses. But in another sense, there also seems to be something quite strange or wrong about that story – for instance, that there is something wrong in saying that one was right *to claim* what the conditional expresses. Question: how can we elaborate such distinctions?

Additionally, we have another kind of interesting related problem. In her (1997), Dorothy Edgington tells us that what a counterfactual of the form " "If p had been the case... then q would have been the case" expresses at a later time, is what "If p is the case... then q will be the case" expressed at an earlier time." (Edgington, 1997, p.108).

Edgington mainly wishes to establish the above connection because she is an advocate of an *expressivist* account of conditionals; that is, a non-truth-conditional view. And since "Gibbard kind" of standoffs are often used in support of an *expressivist* view of conditionals, she wants to render the same impact of Gibbard cases (for indicatives) to counterfactual conditionals. It then becomes a question whether an *assessment-sensitive* theory may provide an alternative view. This is a line of investigation that may be promising, and we plan to undertake it in the future (an open and contingent one, we hope).

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Appendix

Some updated discussions concerning MacFarlane's approach

In the midst of writing this dissertation, some material appeared in literature, which involved (in one way or another) MacFarlane's approach. We thus have especially prepared this Appendix, which contains some evaluations of these fresh material.

The ***Subvaluationist*** alternative

This theory has been propounded very recently (Ciuni & Proietti, 2013), and to our knowledge it hasn't yet gained the attention it deserves. In the original paper, the approach is not linked to any definition of *truth at a context*, so we have provided here a standard formulation showing how it would comply to such a notion.

The main and sensible difference between the *Thin Red Line*, the *Supervaluationist's*, and the *Subvaluationist's* approaches concerns distinct characterizations of the *truth-at-a-context* profile. In the first two, this profile differed as to whether the approach accounts it in terms of a sentence being true [or false] at *the point of evaluation* containing *the* 'moment of the context' and *the* 'history of the context' (which is definite and given by context) – in the case of *Thin Red Line* –, or whether this profile concerns a sentence being true [or false] at *every point* containing *the* moment and *each* history passing through that moment.

The *subvaluationist* approach comes to the remaining case where the truth-at-a-context profile of a sentence is determined by it being true [or false] *at least* in one point containing *the* moment of the context, and *some* of the histories passing through that moment. Again, we can make this precise:

So, this approach will face the same challenge when it comes to giving both intuitions their due. It classes with *Thin Red Line* in giving the determinacy intuition its due, but it equally stand as not making sense of the indeterminacy intuition, since in case of contingency, it will not be the case that a future contingent is *neither true nor false* as is predicted within Supervaluationism. Actually, not only won't it yield neither truth-value for future contingents, but it will also yield both affirmation and negation true. The definition is now the following:

(Subvaluationism). A sentence ϕ is *true* [*false*] *at* c, just in case, for 'at least one' *point of evaluation* $c, \langle t_c, w\rangle$:

$[\![\phi]\!]^{c}_{\langle t_c,w\rangle}$ = *True* [*False*], where: t_c is *the time of the context*
w is a world in the set $W(c)$

Different from *supervaluationism*, which quantifies universally among *overlapping worlds* in $W(c)$, this option quantifies existentially: a sentence is *true at a context* c, just in case *at least one* of the relevant points of evaluation (containing an overlapping world) assigns the value *True* to that sentence; and it is *false at a context* c, just in case one of them assigns *False*. Thus in this case we might ask again, does *subvaluationism* meet the challenge of giving both intuitions their due? Let us see.

So let us first ask whether it complies with 'the determinacy intuition'? At first sight we could be moved into claiming that it can, since it could after all retrospectively assign to the past claim the value *True*. Granted, it does. But it does so at the expense of also assigning the value *False*. So if we follow this approach, then it would be also correct to asses Jake's past utterance as expressing something false. How come? On what kind of sense would rest yielding Jake's past prediction as having been false, while we see both of them shaking hand? By saying that it is also true? It just doesn't look good.

But let us now move to the interesting case. We may ask: does it make sense of 'the indeterminacy intuition'? We think the problem looks pressing. At first sight one could argue that it doesn't, since it won't assign the utterance as being *neither true nor false*. Yet it does assign the episode as expressing both truth and falsity. "And what more of a full-blooded objectivist view of indeterminism could one wish for, than assigning the episode as bearing both true and false values?". Some could argue in these lines perhaps, and it is not ruled out as a defense to make. We have to remember that all these inquiries are seeking for what *semantic theory* best reflects our pre-theoretic intuitions concerning time. That is why we always add the condition "under the assumption that the future is objectively open", and the like. Thus under such an assumption, doesn't *subvaluationism* qualify as a better sense of time objectively branching?

We think not, for a simple reason: MacFarlane's theory already makes room for the sense of an *utterance* being accounted *true* and *false*, exactly in virtue of his relativistic framework.

Conditional Proof and the problem of *retrogradation of truth*

One aspect claimed by proponents of *subvaluationism* as outperforming its *supervaluationist* cousin, is by showing that they are able to handle the *problem of retrogradation of* *truth* without losing some classical inferential schemata (which are lost by *supervaluationism*). In brief, the problem concerns the impacts when providing enough expressive power to express *in the object language* that a proposition "would come true", while "it wasn't determinate" that the proposition would come true – this is made possible through the expansion of the language as to include the necessity operator $\Box$. Yet once this is done, the increase of expressive power will backlash at the approach, by providing counter-examples to the rules of *Conditional Proof, Argument by Cases*, and *Reductio ad Absurdum.*

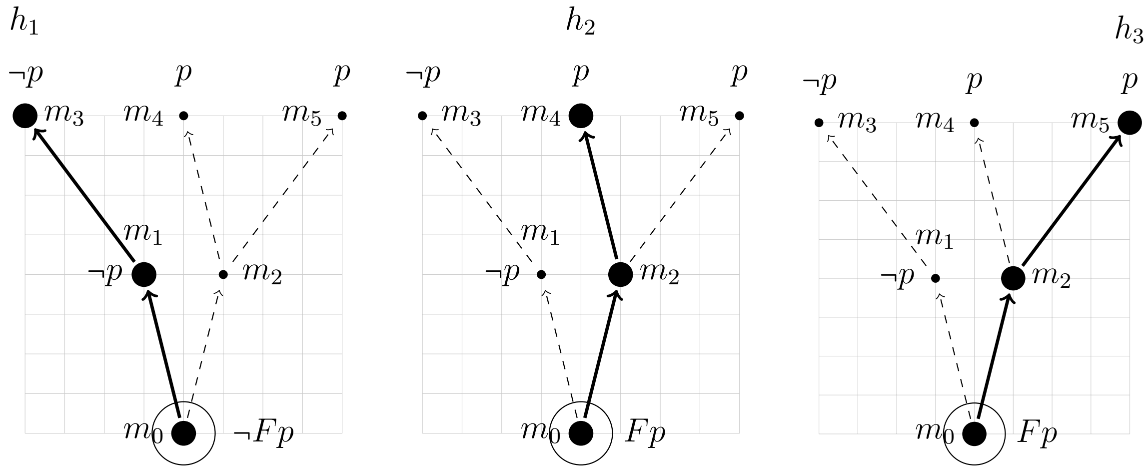
Yet one aspect which has come to our attention is the fact that MacFarlane's doubly relativized notion of *utterance-truth* won't provide the same instance of counter-example to *Conditional Proof*, when augmenting its expressive power in the same way. The simple reason is that through MacFarlane's notion, the representative *logical consequence* which serves as a counter-example to the rule of *Conditional Proof* in *supervaluationism* (the consequence $F\phi \vDash \Box F\phi$), is not itself anymore a *logical consequence* in MacFarlane's system, if we tailor it with respect to the notion of *truth* relative to a *context of use* and a *context of assessment.* Thus with respect to making sense of *retrogradation of truth*, MacFarlane's analysis would stand equally well as *subvaluationism*, yet with the additional benefit of giving due to both the 'indeterminacy' and 'determinacy' intuitions. Let us then start by showing what the problem comes to.

First of all, we may remember that it is one feature of *supervaluationism*, as originally presented by Thomason in his (1970), that a formula Fp might be *neither true nor false* at a moment m, while at some moment m' later than m, the formula PFp may turn out *supervaluationally* true at that moment m'. According to Ciuni & Proietti, this feature would reflect the good sense, available to *supervaluationism*, to claim for *retrogradations of truth.* The sense seems to be that of being able to express *in the object language* (from an *internal* perspective), that a *gappy* formula Fp at some moment m, has "turned out to have been true" at some later moment m'; that is, that PFp is *true* at m'. To illustrate this fact, let us bring back again the definition of the *s-truth value* of a formula (being true at a moment m), and also some pictures. So first we have:

Definition 16 (Supervaluationist s-truth value). *Given a 'branching structure' $\langle M, < \rangle$, and an* assignment function V *over it, and the recursive definition of the* V-truth value $[\![\phi]\!]^V_{\langle m,h\rangle}$ *of ϕ for every formula ϕ of the language; the* s-truth value *of ϕ at a moment m of M (denoted by $[\![\phi]\!]^s_m$), for every formula ϕ of the language, is defined as follows:*

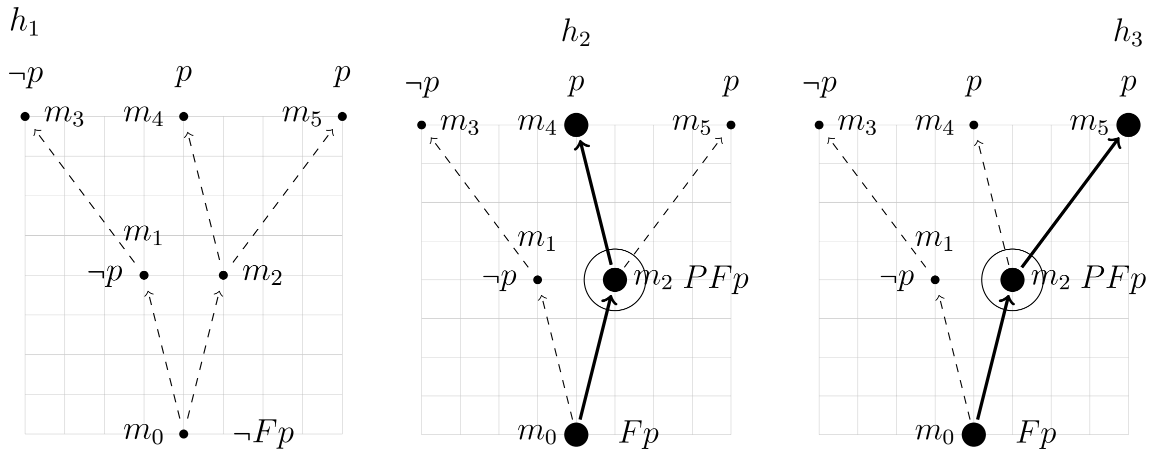
$$[\![\phi]\!]^s_m = \begin{cases} \textit{True}, & \textit{if } [\![\phi]\!]^V_{\langle m,h\rangle} = \textit{True}, \ \textit{for every } h \in H_m \\ \textit{False}, & \textit{if } [\![\phi]\!]^V_{\langle m,h\rangle} = \textit{False}, \ \textit{for every } h \in H_m \\ \textit{undefined}, & \textit{otherwise} \end{cases} \tag{4.17}$$

Now consider the following diagrams:



(a) (b) (c)

These diagrams show a model where the formula Fp is *gappy* at m_0 (or equivalently: $[\![Fp]\!]^s_{m_0}$ = *undefined*), since the V*-truth value* of the formula is true at both $\langle m_0, h_2\rangle$ and $\langle m_0, h_3\rangle$, yet it is false at $\langle m_0, h_1\rangle$. Yet if we look now from the perspective of m_2, *retrogradation of truth* takes place:



(a) (b) (c)

Here, the V*-truth value* of PFp at every pair consisting of moment m_2, and a history passing through m_2 (only h_2 and h_3), is *True*; thus in consequence, PFp is now *s-true* at m_2. But according to Ciuni & Proietti, we now face a problem. When allowing for *gaps* at a moment m and later *retrogradations of truth* at a later m', the framework must provide some way for expressing from an "internal perspective" (in the object language), that although it is true at m_2 that 'it would have been true that p' (PFp), it wasn't determinately true at the earlier moment m_0, that it would have been true that p (or speaking in another way, that 'it was *gappy* at m_0, that Fp'). And the only way for *supervaluationism* to achieve this, is to expand the language as to include the historical modal $\Box$. If we do that, and determine the recursive definition of the V*-truth value* of $\Box\phi$ the way we have ($[\![\Box\phi]\!]^V_{\langle m,h\rangle}$ = True, *iff*, $[\![\phi]\!]^V_{\langle m,h'\rangle}$ = True, for every $h' \in H_m$), then we have a way to differentiate between 'it *was going to be* true' (PFp), and 'it was *determinate* that it would be true' - ($P\Box Fp$). As in the case shown above, though PFp is true at m_2, $P\Box Fp$ is false at m_2 (thus providing a way to express *in the object language* that it was *gappy* – not *determinately* true). And just to remeber why this happens, it is because at m_0, there is the 'history' $h_1 \in H_{m_0}$ where Fp is false at $\langle m_0, h_1\rangle$; thus at any pair containing a history passing through m_0, it is false that $\Box Fp$. According to Ciuni & Proietti, the problem of expanding the language in such a way to handle the situation (differentiating between two meanings), is that the inclusion of $\Box$ will allow for counter-examples to some classical inferential schemata. In effect, we have already discussed such a problem. Because of the necessity modal, supervaluationism will run counter-examples to some classical rules, like: *Conditional Proof, Argument by Cases*, and *Reductio ad Absurdum*:

$$\Gamma, \phi \vDash \psi \textit{ implies } \Gamma \vDash \phi \supset \psi \qquad \textbf{(\textit{Conditional Proof})}$$
$$\Gamma, \phi \vDash \psi \text{ and } \Gamma, \zeta \vDash \psi \textit{ implies } \Gamma, \phi \vee \zeta \vDash \psi \qquad \textbf{(\textit{Argument by cases})}$$
$$\Gamma, \phi \vDash \psi \wedge \neg\psi \textit{ implies } \Gamma \vDash \neg\phi \qquad \textbf{(\textit{Reduction ad Absurdum})}$$

Let us here concentrate in the case of *Conditional Proof.* We have already seen why supervaluationism violates the rule: it is because $F\phi$ here *logically implies* $\Box Fp$ (according to the notion of *s-validity*), while it is not the case that the conditional $Fp \supset \Box Fp$ is *s-valid.* To see a counter-example to its *s-validity*, we may just consult the pictures above. Pick just the pair $\langle m_0, h_2\rangle$; the conditional $Fp \supset \Box Fp$ is false at this point, so the conditional is also not *s-true* at m_0.

But as Ciuni and Proietti shows us in the case of the *Abundance* interpretation (how they call the *subvaluationist* approach), there is a way to provide for an expansion of the language, to handle *retrogradation* without violating these rules. We will show how this works in the case of *Conditional Proof.* First consider the way we adapt to our notation their *subvaluationist* definition of *truth at a moment* m:

$$[\![\phi]\!]^a_m = \begin{cases} \text{True}, & \text{if } [\![\phi]\!]^V_{\langle m,h\rangle} = \text{True}, \text{ at some } h \in H_m \\ \text{False}, & \text{if } [\![\phi]\!]^V_{\langle m,h\rangle} = \text{False}, \text{ at some } h \in H_m \end{cases} \tag{4.18}$$

First, last us fix some additional terminology, by saying that ϕ is *subtrue* at m, when $[\![\phi]\!]^a_m$ = *True*; and by saying that ϕ is *subfalse* at m, when $[\![\phi]\!]^a_m$ = *False.* Thus for example, it might happen at an m that a formula is both *subtrue* and *subfalse.* But we can also say that at an m, a formula is **not *subtrue***, meaning that it is only *subfalse* (only false); and we can also say it is **not *subfalse***, meaning it is only *subtrue.* If we compare to *supervaluationism*, we could run a dual case: we would say ϕ is *supertrue* at m, when $[\![\phi]\!]^s_m = True$, and *superfalse* when $[\![\phi]\!]^s_m = False$; and that in some cases, it is *neither supertrue nor superfalse.*

The occasion Ciuni & Proietti now want to make sense of, comes to the following dual case of *supervaluationism*: even if we can express in the object language, that at m_2 "it wasn't true that p *was not going to* happen" – since it did happen (through $P\neg Fp$ being *not subtrue* at m_2), "how can we express, from the point of view of $[m_2]$, the fact that 'there will be a sea battle' was *glutty* before (for instance, at $[m_0]$)?" (Ciuni & Proietti, 2013, p.16). That is, how can we make sense at m_2, that it "wasn't true that p wouldn't happen", and still say that "it was before subtrue that p wouldn't happen"? Another way to express this: *supervaluationism* wants to be able to speak of *retrogradations of supertruth* that were not supertruths before (nor were them superfalsehoods). While *Abundance* wants to be able to speak of *retrogradations of* ***not subtruths*** that were subtruths before.

The only way as they notice, in order to make sense of this perspective of *retrogradation*, is by expanding the language with the possibility operator $\Diamond$. With this, they are able to differentiate *from an internal perspective*, the fact that at m_2 it was *false that* p would happen ($P\neg Fp$), and also that it was "*glutty* true" – or *sub-true* – that p would not happen ($P \Diamond \neg Fp$).

But the interesting fact they further provide, is that even with this move, the expansion of the language as to include $\Diamond$ won't violate *Conditional Proof* – as happened in the case of *Supervaluationism*, when adding $\Box$. The proof is simple. First of all, notice that here, we can run a notion of *logical consequence* and *validity* as we provided for *supervaluationism* (let us call them here *a-implying* and *a-validity*):

> **(*a*-implying).** A set Γ of formulas *a-imply* ϕ (which we denote by $\Gamma \vDash_a \phi$) if, relative to all *branching structures* $\langle M, < \rangle$, and relative to all *assignments* V (with respect to $\langle M, < \rangle$): for every $m \in M$, **If** $[\![\gamma]\!]^a_m = True$ (for every $\gamma \in \Gamma$), **Then** $[\![\phi]\!]^a_m$ is also *True.*

> **(*a*-validity).** A formula ϕ is *a-valid* (which we denote by $\vDash_a \phi$) if, relative to all *branching structures* $\langle M, < \rangle$, and relative to all *assignments* V (with respect to $\langle M, < \rangle$): for every $m \in M$, $[\![\gamma]\!]^a_m = True.$

First thing to notice here, is that unlike in *supervaluationism* – where Fp *s-implies* $\Box Fp$ –, it won't be anymore the case here that Fp *a-implies* $\neg \Diamond \neg Fp$. That is, it won't happen that whenever Fp is *a-true* at a moment m, then $\neg \Diamond \neg Fp$ is also true at that moment.

To see this, pick m_0 as in the last diagram. Fp is evaluated as *a-true* at m_0, because there is *at least one* 'history' passing through m_0 (for example h_2), where $[\![Fp]\!]^V_{\langle m_0,h_2\rangle} =$ *True*. As a result, $[\![Fp]\!]^a_{m_0} =$ *True*. Yet notice that the formula $\neg \Diamond \neg Fp$ is *a-false* at m_0 (in fact, it is both *a-false* and 'not *a-true*'), since there is the 'history' h_1, where $[\![\neg Fp]\!]^V_{\langle m_0,h_2\rangle} =$ *True*, and therefore $\Diamond\neg Fp$ is V*-true* at every pair with m_0 and a history in H_{m_0}, and thus $\neg \Diamond \neg Fp$ is V*-false* at every such pair. Therefore, $[\![\neg \Diamond \neg Fp]\!]^a_{m_0} =$ *False*. We have proved that $Fp \nvDash_a \neg \Diamond \neg Fp$.

Now the proof of *Conditional Proof* holding here runs as follows: suppose that $\Gamma, \phi \vDash_a \psi$ holds. There are only two cases concerning the hypothesis of such a *consequence* holding, and the status of *a-truth at m* for the formulas in the premises. So either, (1) for some $h \in H_m$, $[\![\phi]\!]^V_{\langle m,h\rangle} =$ *False*. Or, (2) for every $h \in H_m$, $[\![\phi]\!]^V_{\langle m,h\rangle} =$ *True*. Consider case (1): if $[\![\phi]\!]^V_{\langle m,h\rangle} =$ *False* for some $h \in H_m$, then fix h^* as that 'history' passing through m such that $[\![\phi]\!]^V_{\langle m,h^*\rangle} =$ *False*. By the clauses of the V*-truth value* for the conditional $\supset$, we then have that $[\![\phi \supset \psi]\!]^V_{\langle m,h^*\rangle} =$ *True* (because the antecedent is false). Following then the definition of *a-truth*, we then have that $[\![\phi \supset \psi]\!]^a_m =$ *True*.

In the second case, (2) we have that for every $h \in H_m$, $[\![\phi]\!]^V_{\langle m,h\rangle} =$ *True*. From this it first falls out directly that $[\![\phi]\!]^a_m =$ *True*. Coupled with our hypothesis (that $\Gamma, \phi \vDash_a \psi$ holds), we then have also that $[\![\psi]\!]^a_m =$ *True*. But now it follows that for some $h \in H_m$, $[\![\psi]\!]^V_{\langle m,h\rangle} =$ *True*. Since in case (2), we are assuming that for every $h \in H_m$, $[\![\phi]\!]^V_{\langle m,h\rangle} =$ *True*, then there is at least one 'history' passing through m such that both ϕ and ψ are V*-true*. Thus it follows that $[\![\phi \supset \psi]\!]^a_m =$ *True*.

So with these findings, Ciuni & Proietti argue that compared to *supervaluation*, their move to express *retrogradation of truth* in the object language will run less impacts on classical inferential schemata[5]. But how would this kind of move compare to MacFarlane's approach, when handling the *problem of retrogradation of truth*? Or, in what ways MacFarlane's could be said to make sense of *retrogradation* and what impacts would result from it?

Before answering how MacFarlane's solution handles *retrogradation*, let us observe a neat fact about it. Different from *supervaluationism*, the consequence relations holding between ϕ and $\Box\phi$ won't run as a counter-example to *Conditional Proof*, for a simple fact: If we tailor the notion of validity in MacFarlane, with respect to *truth at contexts*, ϕ won't anymore *logically imply* $\Box\phi$. Let us see this by first remembering how the *supervaluationist* notion of *truth at a context* yields such a consequence relation. We first provide the notions of *consequence* and *validity*:

[5]One classical rule which still stands violated in both *Supervaluationism* and *Abundance* is *Modus Tollens*: $\Gamma, \phi \vDash \psi$ *implies* $\Gamma, \neg\psi \vDash \neg\phi$. The counter-example rests on the fact that $\Diamond Fp \vDash_a Fp$, yet $\neg Fp \nvDash_a \neg \Diamond Fp$.

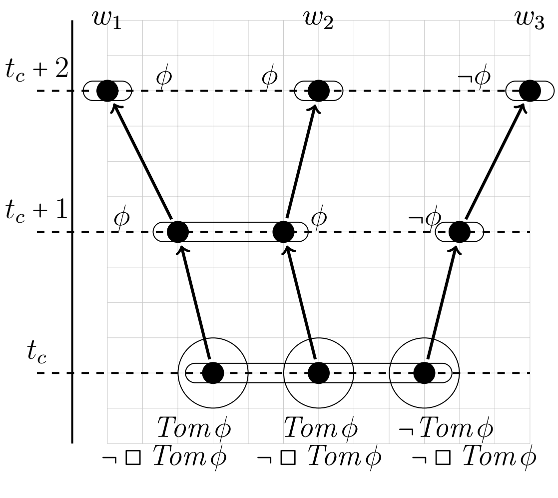
(Logical consequence). Γ *s-implies* ϕ (denoted by $\Gamma \vDash_s \phi$), just in case, for every context c, if every member in Γ is *true at* c, then ϕ is *true at* c

(s-Validity). ϕ is *s-valid* (denoted by $\vDash_s \phi$), just in case, for every context c, ϕ is *true at* c

In effect, if we check the consequence relations holding between the formulas $Tom\phi$ and $\Box Tom\phi$, using these notions, we can show that it will deliver a counter-example to *Conditional Proof*. Here is the proof we give:

First we show that $Tom\phi \vDash_s \Box Tom\phi$. Take a context c such that $Tom\phi$ is *s-true at* c. This means, according to the definition we have provided, that $[\![Tom\phi]\!]^c_{\langle t_c,w\rangle} = True$, for every $w \in W(c)$. But then, by the fact that '$\forall w, w' \in W(c)$: $w \simeq_{t_c} w'$', and by the definition of truth for $\Box$ ($[\![\Box\phi]\!]^c_{\langle t,w\rangle} = True$, *iff*, $[\![\phi]\!]^c_{\langle t,w'\rangle} = True$, for every w' such that $w \simeq_t w'$), we then have it that $[\![\Box Tom\phi]\!]^c_{\langle t_c,w\rangle} = True$, for every $w \in W(c)$ – since as we should remember, t_c is a member of T in a 'structure'. It then follows that $\Box Tom\phi$ must also be *s-true at* c.[6].

We can now run the counter-example to *Conditional Proof*, by showing that $Tom\phi \supset \Box Tom\phi$ is not *s-valid* – or, $\nvDash_s Tom\phi \supset \Box Tom\phi$. Consider a context c, such that $w_1, w_2, w_3 \in W(c)$ as represented in the following diagram:



(a)

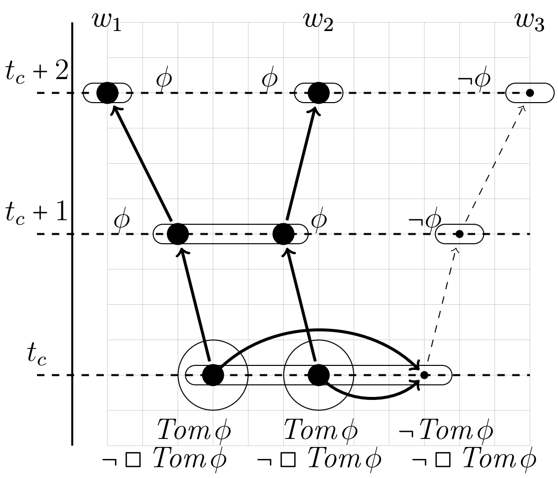
Here we see a 'structure' coupled with an *assignment*, where ϕ is being mapped to a subset containing both $\langle t_c+1, w_1\rangle$ and $\langle t_c+1, w_2\rangle$, while according to the assignment, ϕ

[6]Notice that the truth clause for $\Box$ only refers to worlds accessible at the time t standing as a coordinate (from a world w, it looks at every world w' such that $w \simeq_t w'$). But since the notion of *truth at a context* "initializes" this coordinate by t_c, plus the fact that $w, w' \in W(c)$, *iff*, $w \simeq_{t_c} w'$, it then follows that $[\![\Box Tom\phi]\!]^c_{\langle t_c,w\rangle} = True$, for every $w \in W(c)$.

is false at the pair $\langle t_c + 1, w_3\rangle$. Consequently, *Tom*$\phi$ is false at $\langle t_c, w_3\rangle$, while it is true at $\langle t_c, w_1\rangle$ (and also at $\langle t_c, w_2\rangle$). Notice additionally that $\Box$*Tom*ϕ is false at $\langle t_c, w_1\rangle$, because it is not the case that for every w', such that $w_1 \simeq_{t_c} w'$, *Tom*ϕ is true – since $w_1 \simeq_{t_c} w_3$, and $[\![Tom\phi]\!]^V_{\langle t_c, w_3\rangle} = False$. So we have a *time* and *world* pair where the conditional *Tom*$\phi \supset \Box$*Tom*ϕ is false at $\langle t_c, w_1\rangle$. Therefore, *Tom*$\phi \supset \Box$*Tom*ϕ ***is not true*** *at* c[7].

Yet what about MacFarlane's notion of *truth at a context*? In fact, using the same model we can easily show that in his case, it doesn't happen that *Tom*ϕ *logically implies* $\Box$*Tom*ϕ. So we cannot run the same counter-example to *Conditional Proof* by resorting to this instance.

First of all, a formula ϕ is said *true [false] as used at* c_1*, and assessed from* c_2, just in case, $[\![\phi]\!]^c_{\langle t_c, w\rangle}$ = *True* [*False*], for every $w \in W(c_1|c_2)$. Consider for instance, using the same structure and *assignment* represented in the diagram above, that now $w_1, w_2, w_3 \in W(c_1)$, yet only $w_1, w_2 \in W(c_2)$; consequently, $W(c_1|c_2)$ will include only w_1 and w_2. To ease evaluation, consider such a representation:



(a)

What happens here is that now *Tom*ϕ is considered *true as used at* c_1*, and assessed from* c_2, because in every world which is a member of $W(c_1|c_2)$ (both w_1 and w_2), *Tom*ϕ is true at the relevant pairs. But because $\Box$ will always consult the time coordinate to retrieve the accessible worlds (through the relation $\simeq$), it will still be able to reach w_3 from both w_1 and w_2 – which here are represented by the curved arrows. It then follows that $\Box$*Tom*ϕ is evaluated as false at both $\langle t_c, w_1\rangle$ and $\langle t_c, w_2\rangle$ (or equivalently, $[\![\Box Tom\phi]\!]^c_{\langle t_c, w_1\rangle}$ = *False*, and, $[\![\Box Tom\phi]\!]^c_{\langle t_c, w_2\rangle}$ = *False*). And since we only look at w_1 and w_2 here, it follows in addition that $\Box$*Tom*ϕ is *false as used at* c_1*, and assessed from* c_2. Consequently, *Tom*ϕ doesn't *logically imply* $\Box$*Tom*ϕ.

[7]Observe that the conditional is ***not false*** at c either, since the antecedent *Tom*ϕ is false at the pair $\langle t_c, w_3\rangle$, and thus the whole conditional is V*-true* at that pair. Consequently, we have the conditional being V*-true* at some relevant pair, V*-false* at another relevant pair, and thus being ***not false*** at c either. Still, it runs as a counterexample to the *s-validity* of *Tom*$\phi \supset \Box$*Tom*ϕ, because we have a c where the conditional is ***not true*** *at* c.

A short historical note on alleged precursors of MacFarlane's solution

It has very recently been suggested by Gila Sher (2015) that some key aspects of MacFarlane's solution (especially his notion of *contexts of assessments*) were already present in an essay written around 1985. Gila Sher is referring to David Foster Wallace's 'honors thesis' *Richard Taylor's "Fatalism" and the Semantics of Physical Modality* ([1985] 2010), a brilliant and perspicuous piece of philosophical writing, remarkably written by a young philosophy undergraduate, and soon to become some years later, one of the greatest novelists in recent american literature. Thus writes Sher relating Wallace's insights to MacFarlane's approach, offered almost twenty years laters:

> "MacFarlane, thus, can be viewed as offering a vindication of Wallace's claim that the truth of some statements requires relativity to multiple contexts, specifically, in the case of free will and fatalism, relativity to time of occurrence and to time of evaluation.
>
> Bringing Wallace to the contemporary scene, we might say that, in a sense, his essay anticipated MacFarlane. Wallace's "context of evaluation" is a forerunner of MacFarlane's "context of assessment," and Wallace's "time of evaluation" is one of the possible parameters in MacFarlane's context of assessment. In this sense, then, Wallace was ahead of his time." (Sher, 2010, pp.84-85).

Wallace was indeed ahead of his time, but his contribution sides much alike with distinct solutions than MacFarlane's, and we here in this brief note want to argue why this is the case. The main and straight simple reason is that Wallace's approach concerns temporally indexed modalities, whereas MacFarlane's approach concerns relativity of utterance-truth (or sentence-truth) of future contingent statements. Another feature which really makes Wallace's work come apart from MacFarlane's solution, is that not only it doesn't concern any senses for *utterance-truth* (but we have to remind, this is someone writing in 1985!), but it also won't concern any notion that takes part in a theory involving speech acts and the *significance* of uses and evaluations of assertions of future contingents; for instance, things like the 'rules' governing retractions of past assertions, or the 'truth' norms governing assertions, all of which are entirely due to MacFarlane's development of his theory in all these years.

Yet something other came along the road during our researches. In the course of writing this thesis, and running through some literature, we have come across a much closer resemblance between MacFarlane's solution and anything written before, that we have found so far. So albeit not properly in Wallace's work, nor in any literature already cited, it is actually in a twelve-page paper published in (1980), by Michael J. White – a scholarly trained philosopher mostly dealing with Aristotle's works by the time –, that the suggestions of correlation to MacFarlane's solution start to get interesting. In his paper, White's main concern was actually not about *truth relativity* of a sentence as taken in a context (though he does talk about it!), but rather about modalities whose truth conditions were doubly relativized to two distinct moments of time. But just to give a glimpse at what kind of resemblance we are talking about, here is what White writes in the first pages of his paper:

> "This paper begins with the development of a technique for dealing with temporal indexicals within a modal propositional logic in which the alethic modalities are temporally interpreted. More specifically, it develops semantic accounts of the 'temporally rigidly referring' senses of several temporal operators by means of the mechanism of 'double indexing'." (White, 1980, p.287)

So as we said, double-indexing alone would hardly count for enough correlation, more than for example Wallace's or so many other's approaches could already be accounted as resembling MacFarlane's work. Yet the startling passage comes a few pages later. First, White writes:

> "A *bivalent evaluation* $V_M^h(\phi, \langle w_i, w_j\rangle)$ is a quaternary function from an Aristotelian model M, a world history h of that model (i.e., $h \subset W$), a [modal propositional calculus] wff ϕ, and an ordered pair of points [of time] $\langle w_i, w_j\rangle$ such that $w_i, w_j \in h$ [,] into [the] set $\{v, f\}$, the elements of which represent truth and falsity, respectively."[8] (White, 1980, p.289)

And immediately after, he complements:

> "The first member in the pair of points or 'possible times' represents a 'context of use' for the modal PC wff ϕ being evaluated, the second a 'context or possible time of evaluation' for the modal PC wff-in-context of use." (White, 1980, p.289)

Two points are noteworthy here: first, is that unlike previous doubly-indexed theories relating to both Tense Logic and the problem of future contingents, where we find nothing like an explicit move to a pragmatically salient notion of truth (at least in the Kaplanian way of defining truth at a context), here contexts are explicitly acknowledged, and given a role within the semantics. White's use of the expression 'context of use' is no accident; he does explicitly acknowledge that it is being used in the familiar Kaplanian sense.

And we should remark, this is a paper being published in 1980, at a time when Kaplan's works concerning indexicals would mainly circulate among quite specific audiences, but didn't have yet the spreading attention it would gain a few years later. And we shouldn't also forget, it is a paper being published almost twenty-three years before MacFarlane's (2003) paper. But we should add that his mention to contexts (though it comes explicitly) is still very incipient, undeveloped, and without any further relevant

[8] content between brackets added by us

qualifications. Qualifications that would certainly be needed to take the correlations to be of some substance.

The second point is that he also doesn't make further qualifications of how truth *at an Aristotelian Model* (as he terms it) relates to the pragmatically relevant notion of truth at a context (or relative to both a context of use, and a context of evaluation, in case of relativism). Perhaps had White persevered in the very notion of what he coins 'a context of possible time of evaluation', he would very likely be on his way to meet something very close to MacFarlane's truth relativism. But the fact is that he didn't.