UNIVERSIDADE ESTADUAL DE CAMPINAS
Faculdade de Engenharia Elétrica e de Computação

Reginaldo Nunes de Souza

Estatísticas de Segunda Ordem para Retransmissões Transparentes

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Tese apresentada à Faculdade de Engenharia Elétrica e de Computação da Universidade Estadual de Campinas como parte dos requisitos exigidos para a obtenção do título de Doutor em Engenharia Elétrica, na Área de Telecomunicações e Telemática.

Orientador: Prof. Dr. José Cândido Silveira Santos Filho

Este exemplar corresponde à versão final da tese defendida pelo aluno Reginaldo Nunes de Souza, e orientada pelo Prof. Dr. José Cândido Silveira Santos Filho.

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A ata de defesa, com as respectivas assinaturas dos membros da Comissão Julgadora, encontra-se no SIGA (Sistema de Fluxo de Dissertação/Tese) e na Secretaria de Pós-Graduação da Faculdade de Engenharia Elétrica e de Computação.

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Resumo

Comunicações baseadas em retransmissão (*relaying*) emergiram como uma técnica promissora de diversidade espacial (referida como diversidade cooperativa) para combater o desvanecimento do canal e assim melhorar o desempenho das transmissões em sistemas de comunicação sem fio. Esse novo paradigma de transmissão tem se tornado um componente fundamental no processo de padronização de redes sem fio modernas como 3GPP LTE-Advanced e IEEE 802.16j e, muito provavelmente, a próxima geração de redes sem fio se beneficiará do uso de técnicas de *relaying*. A maioria dos trabalhos sobre sistemas *relaying* encontrados na literatura se limitam a analisar o desempenho desses sistemas em termos de estatísticas de primeira ordem, tais como probabilidade de *outage* e taxa de erro de bit. De fato, muito pouco se sabe sobre as estatísticas de ordem superior do canal *relay*, apesar da relevância prática de tais estatísticas. A taxa de cruzamento de nível e a duração média de desvanecimento, por exemplo, são estatísticas de segunda ordem essenciais para caracterizar o comportamento dinâmico do processo de desvanecimento associado ao canal de comunicação, complementando assim a caracterização estática obtida a partir das estatísticas de primeira ordem, além de fornecerem uma avaliação mais detalhada do desempenho das técnicas de *relaying*. Em particular, existem alguns critérios no projeto de redes cooperativas (alocação de potência e de largura de banda em cada salto, requisitos de latência máxima, dentre outros) para os quais as estatísticas de primeira ordem se mostram insuficientes. Este trabalho visa preencher essa lacuna ao analisar as estatísticas de segunda ordem da relação sinal-ruído fim-a-fim de redes cooperativas com *relays* do tipo amplifica-e-encaminha com ganho variável. O protocolo amplifica-e-encaminha mostra-se atraente por sua simplicidade e relativa facilidade de implementação, e a versão com ganho variável constitui o padrão ouro desse protocolo. Mais especificamente, dois cenários são abordados: redes *half-duplex* com múltiplos saltos com *relays* e redes com dois saltos com *relay full-duplex*. Em ambos os casos são obtidas e analisadas expressões exatas e assintóticas para a taxa de cruzamento de nível e para o tempo médio de desvanecimento.

Palavras-chaves: amplifica-e-encaminha; comunicações cooperativas; *full-duplex*; ganho variável; *half-duplex*; taxa de cruzamento de nível; tempo médio de desvanecimento.

Abstract

Relay-based communications have emerged as a promising spatial diversity technique (referred to as cooperative diversity) for combating channel fading and thus improving the transmission performance in wireless communication systems. This new transmission paradigm has become an essential component in the standardization process of modern wireless networks such as 3GPP LTE-Advanced and IEEE 802.16j and, most likely, the next generation will benefit from the deployment of relaying techniques. Most of the literature dealing with relaying networks is limited to the performance analysis in terms of first-order statistics, such as outage probability and bit error rate. In fact, very little is known regarding the higher-order statistics of the relay channel, despite their practical relevance. The level crossing rate and the average fade duration, for example, are fundamental second-order statistics that characterize the dynamic behavior of the fading process associated with the communication channel, thus complementing the characterization provided by first-order statistics, and rendering a more detailed performance assessment on relaying techniques. In particular, some design criteria exist (power and bandwidth allocation over multiple hops, maximum latency requirements, among others) for which the first-order statistics are insufficient. This work aims to fill this gap by analyzing the second-order statistics of the end-to-end signal-to-noise ratio of cooperative networks with variable-gain amplify-and-forward relays. The amplify-and-forward protocol proves attractive for its simplicity and relative ease of implementation, and the variable-gain version is its gold standard. More specifically, two scenarios are addressed: multi-hop half-duplex networks and dual-hop full-duplex networks. In both cases, exact and asymptotic expressions are derived and analysed for the level crossing rate and the average fade duration.

Keywords: amplify-and-forward; average fade duration; cooperative communications; full-duplex; half-duplex; level crossing rate; variable gain.

Lista de Figuras

Lista de Acrônimos

3GPP	*Third Generation Partnership Project*
AF	*amplify-and-forward*
AFD	*average fade duration*
a.k.a.	*also known as*
CDF	*cumulative distribution function*
CSI	*channel state information*
DF	*decode-and-forward*
EGBMGF	*extended generalized bivariate Meijer-G function*
FD	*full-duplex*
FG	*fixed-gain*
HD	*half-duplex*
LCR	*level crossing rate*
LI	*loop interference*
LTE	*Long Term Evolution*
MIMO	*multiple-input multiple-output*
PDF	*probability density function*
RV	*random variable*
SINR	*signal-to-interference-plus-noise ratio*
SNR	*signal-to-noise ratio*
VG	*variable-gain*

Lista de Símbolos

$\Gamma(\cdot)$	função gamma
$\gamma(\cdot)$	função gamma incompleta inferior
$\mathrm{E}[\cdot]$	esperança estatística
$\max\{\cdot\}$	máximo de uma função ou de uma lista de valores
$\min\{\cdot\}$	mínimo de uma função ou de uma lista de valores
$f_{A,B}(\cdot,\cdot)$	PDF conjunta das RVs A e B
$f_{A\|B}(\cdot\|\cdot)$	PDF condicional de A dado B
$f_A(\cdot)$	PDF de uma RV A
$\dot{A}$	derivada temporal de uma RV A
T	tempo médio de desvanecimento
N	taxa de cruzamento de nível
P	probabilidade de *outage*
P_{T}	potência média de transmissão
N_0	potência média do ruído
γ_{th}	limiar de SNR
$f_{m,n}$	deslocamento de frequência Doppler máximo no n-ésimo salto
m_n	índice de desvanecimento Nakgami-m no n-ésimo salto
Ω_n	ganho de potência médio do canal no n-ésimo salto
α_n	amplitude do canal no n-ésimo salto
G_n	fator de amplificação do n-ésimo relay
R_n	n-ésimo relay
S	nó fonte
D	nó de destino
h_n	coeficiente complexo do canal no n-ésimo salto
Γ_n	SNR média recebida no n-ésimo salto
Γ_0	SNR transmitida
Γ_e	SNR fim-a-fim exata
Γ_a	SNR fim-a-fim aproximada
c	ganho de codificação
d	ganho de diversidade

Lista de Publicações

Artigos em revista

- R. N. de Souza, E. E. B. Olivo, C. R. N. da Silva, L. C. F. Ferreira e J. C. S. S. Filho, “On Second-Order Statistics of Full-Duplex Relaying”, *IEEE Commun. Lett.*, pp. 1–1, Dec. 2020.
- R. N. de Souza, E. E. B. Olivo, L. C. F. Ferreira, C. R. N. da Silva e J. C. S. S. Filho, “Second-Order Statistics of Amplify-and-Forward Relaying: Variable Gain and Multiple Hops”, *IEEE Trans. Veh. Technol*, em processo de revisão.
- R. N. de Souza, E. E. B. Olivo, L. C. F. Ferreira, C. R. N. da Silva e J. C. S. S. Filho, “Impact of Channel Diversity on the Second-Order Statistics of Multi-Hop Transparent Relaying”, *IEEE Wireless Commun. Lett.*, em processo de submissão.

Artigo em congresso

- R. N. de Souza, E. E. B. Olivo, L. C. F. Ferreira, C. R. N. da Silva e J. C. S. S. Filho, “High-SNR Second-Order Statistics of Amplify-and-Forward Relaying with Variable Gains and Multiple Hops”, em *XXXVIII Simpósio Brasileiro de Telecomunicações*, Florianópolis, SC, Brazil, Nov. 2020.

Sumário

1 Introdução

Vários são os desafios a serem superados pela próxima geração de redes sem fio. Dentre eles, destacam-se o suporte a um crescimento massivo do número de dispositivos conectados, bem como a um conjunto vasto e diversificado de serviços e aplicações que demandam taxas de dados cada vez mais elevadas — a exigência de cobertura ubíqua e confiável, capaz de prover ao usuário o acesso à informação em qualquer lugar e a qualquer momento, além de um consumo eficiente de energia [1]. Portanto, as próximas gerações de redes sem fio devem ser capazes de contornar os diferentes fatores de degradação de desempenho impostos pelo canal de comunicação, quais sejam a perda de percurso, o sombreamento e o desvanecimento [2].

Em sistemas de comunicação sem fio modernos, em que predomina o ambiente urbano com terminais móveis, o desvanecimento de pequena escala destaca-se como um dos principais fatores de degradação de desempenho. O desvanecimento de pequena escala é causado pela propagação multipercurso, em que o sinal transmitido chega no receptor via múltiplos caminhos com diferentes atrasos devido à reflexão, difração e espalhamento oriundos dos obstáculos localizados no trajeto percorrido pelo sinal. Essas componentes de multipercurso podem se somar construtivamente ou destrutivamente, dependendo do atraso de cada componente. Uma técnica efetiva de combate aos efeitos detrimentais do desvanecimento é o esquema MIMO (*multiple-input multiple-output*), caracterizado pelo uso de múltiplas antenas no transmissor e/ou receptor. No entanto, em muitas aplicações com dispositivos móveis (como, por exemplo, sistemas celulares), limitações de tamanho e de custo são obstáculos à implantação de múltiplas antenas nos dispositivos. Como alternativa, as redes cooperativas foram propostas como um método efetivo para obter ganhos de diversidade sem utilizar terminais com múltiplas antenas.

A ideia básica das comunicações cooperativas é permitir aos diferentes nós em uma rede sem fio colaborar e compartilhar seus recursos através da transmissão e processamento distribuídos da informação. Mais especificamente, os nós chamados de *relays*, que tipicamente possuem uma única antena, podem transmitir, além da sua própria informação, a informação proveniente de um outro nó da rede. Desse modo, um arranjo virtual de antenas é emulado, obtendo-se múltiplas réplicas do sinal de interesse, provenientes dos múltiplos *relays* espacialmente distribuídos. Gera-se assim um novo tipo de diversidade espacial, chamada de diversidade cooperativa. Mesmo aqueles nós providos de uma única antena podem usufruir dos benefícios oferecidos por sistemas MIMO, como graus de liberdade espaciais, eficiência espectral melhorada e diversidade [3].

As técnicas de comunicações cooperativas evoluíram consideravelmente desde os primeiros resultados de Teoria de Informação sobre o canal *relay* clássico com três terminais (uma fonte, um *relay* e um destino), apresentados por Van der Meulen em [4], e os teoremas de capacidade apresentados por Cover e El Gammal em [5], seguidos dos trabalhos pioneiros sobre diversidade por cooperação e protocolos de *relaying*, apresentados por Sendonaris et al. [6] e Laneman et al. [7], até se chegar a uma fase em que aplicações práticas surgem em sistemas celulares modernos [8]. Em particular, a comunicação com múltiplos saltos (*multi-hop*) por meio de *relays*, envolvendo a implantação de estações rádio-base fixas de baixa potência para auxiliar na comunicação entre fonte e destino, já foi considerada nas especificações de padrões de redes de quarta geração (4G), tais como 3GPP LTE-Advanced (*Releases* 10 a 12) e WiMAX (IEEE 802.16j e IEEE 802.16m) [9], [10]. Tanto o enlace direto quanto o enlace reverso devem se beneficiar da implantação de *relays*.

Em geral, os protocolos de *relaying* são classificados em dois esquemas principais: *amplify-and-forward* (AF) e *decode-and-forward* (DF) [6]. No primeiro esquema, também conhecido como *relaying* transparente, o *relay* envia para o destino uma versão amplificada do sinal recebido proveniente da fonte. No segundo, também conhecido como *relaying* regenerativo, o *relay* recebe e decodifica a informação proveniente da fonte e a codifica novamente antes de retransmiti-la ao destino. Comparando-se esses dois protocolos, uma complexidade de implementação reduzida faz com que o protocolo AF seja amplamente estudado em esquemas variados de diversidade por cooperação. De acordo com o ganho de amplificação usado pelo *relay*, o protocolo AF é classificado ainda em duas categorias: (i) *relaying* com ganho variável, que usa a estimativa instantânea do estado do canal (CSI – *channel state information*) entre fonte e *relay*, a fim de compensar as flutuações do canal devido ao desvanecimento e garantir um valor constante de potência transmitida instantânea na saída do *relay*; e (ii) *relaying* com ganho fixo, que aplica um ganho constante ao sinal recebido proveniente da fonte, baseado nas estatísticas de longo prazo do canal entre fonte e *relay*, de modo a garantir um determinado valor de potência transmitida média na saída do *relay*. Existe um compromisso entre esses dois protocolos: enquanto o protocolo AF de ganho variável oferece um melhor desempenho, o de ganho fixo oferece uma maior simplicidade e facilidade de implementação. Por outro lado, o protocolo AF de ganho fixo pode apresentar problemas de saturação na potência de saída do *relay*, dependendo da condição do estado do canal [11].

Outro aspecto importante das redes cooperativas é a capacidade do *relay* em transmitir e receber informação ao mesmo tempo. No modo de operação denominado *half-duplex* (HD), o relay transmite e recebe informação por meio de canais ortogonais, ou seja, a transmissão e recepção não ocorrem de forma simultânea no mesmo canal físico.

Por outro lado, no modo *full-duplex* (FD), a transmissão e recepção ocorrem ao mesmo tempo e na mesma banda de frequência [12]. O modo FD não só aumenta a eficiência espectral, mas também reduz o atraso fim-a-fim do sistema quando comparado com o modo HD [13]. Entretanto, ao operar no modo FD, o *relay* sofre auto-interferência (LI – *loop-interference*), ou seja, o sinal transmitido pelo *relay* é recebido por sua própria antena receptora, interferindo assim no sinal de interesse. Portanto, para que o modo FD seja realizável, é necessário que o sinal LI seja suprimido a níveis aceitáveis, ao mesmo tempo em que o sinal desejado seja mantido o mais forte possível [13], [14]. Felizmente, os avanços na tecnologia de antenas e nas técnicas de processamento de sinal resultaram em níveis aceitáveis de supressão do sinal LI [13], [15].

Inúmeros trabalhos têm investigado o desempenho de redes cooperativas baseadas em *relaying* (veja, por exemplo, alguns trabalhos em sistemas HD [7], [16]–[23] e em sistemas FD [12], [15], [24], [25]). Apesar desse vasto número de trabalhos na área, a ampla maioria desses estudos limitam sua análise à avaliação do desempenho do sistema em termos da probabilidade de *outage* ou da taxa de erro de bit. Embora essas métricas sejam importantes para avaliar a confiabilidade de redes cooperativas, elas refletem apenas as estatísticas de primeira ordem do processo aleatório associado ao desvanecimento, ou seja, apenas o comportamento estático do canal para um dado instante fixo de tempo. Tais métricas não proveem informação alguma sobre o comportamento dinâmico do canal ao longo de diferentes instantes de tempo, causado pelo movimento relativo entre fonte, *relay* e destino, assim como pelo movimento de objetos circundantes.

Nesse sentido, as estatísticas de segunda ordem são fundamentais para caracterizar a natureza dinâmica, variante no tempo, do processo aleatório associado ao desvanecimento, obtendo-se assim, uma avaliação completa do seu efeito sobre o desempenho de sistemas de comunicação sem fio. Em especial, existem vários critérios no projeto de redes sem fio e, em particular, de redes cooperativas — por exemplo, alocação de potência e de largura de banda em cada salto, seleção do comprimento do bloco de símbolos, requisitos de latência máxima, e profundidade de entrelaçamento em códigos corretores de erro — para os quais a probabilidade de *outage* e a taxa de erro de bit são insuficientes [26]. Essas questões podem ser abordadas através de métricas de desempenho como a taxa de cruzamento de nível (LCR – *level crossing rate*) e a duração média de desvanecimento (AFD – *average fade duration*). A LCR e a AFD são estatísticas de segunda ordem que permitem quantificar o efeito dinâmico do canal de desvanecimento sobre o desempenho do sistema e, portanto, são métricas essenciais quando considerados esquemas de alocação de recursos ou de transmissão adaptativa. A LCR estabelece o número médio de cruzamentos por segundo, para cima ou para baixo, de um processo aleatório por um dado limiar, enquanto que a AFD estabelece o intervalo de tempo médio que o processo aleatório permanece abaixo desse limiar após cruzá-lo.

1.1 Motivação e trabalhos relacionados

Embora as estatísticas de segunda ordem tenham sido estudadas em profundidade para o canal de comunicação ponto-a-ponto — isto é, considerando apenas o enlace direto, sem uso do enlace auxiliar via *relays* — entre fonte e destino [27]–[30], estudos para o caso do canal *relay* são escassos. Em particular, para sistemas *relaying* FD, a escassez é extrema, pois não há trabalhos reportados na literatura que analisam as estatísticas de segunda ordem desses sistemas. Já para sistemas *relaying* HD, a maioria dos trabalhos encontrados na literatura focam em estudos das estatísticas de segunda ordem apenas para redes cooperativas *dual-hop* (isto é, com dois saltos) [31]–[39]. Mesmo para esse cenário com apenas um *relay*, a análise das estatísticas de segunda ordem se mostra complexa. Poucos trabalhos analisaram essas estatísticas para o caso geral com múltiplos saltos (*multi-hop*) [40]–[44]. Mais especificamente, em [40], considerando os cenários em que o sistema é limitado por ruído ou limitado por interferência, soluções exatas em forma fechada foram determinadas para a LCR e AFD de redes DF em ambientes generalizados de desvanecimento. Em [41], soluções em forma integral, cuja dimensionalidade é igual ao número de saltos da rede, foram calculadas para a LCR e AFD de redes AF de ganho fixo sob desvanecimento Rayleigh. Como alternativa às formulações integrais com elevado custo computacional, formulações aproximadas foram também propostas, com base no teorema de aproximação de Laplace multivariado. O trabalho em [42] explora o mesmo método para obter as estatísticas de segunda ordem da capacidade do canal de uma rede AF de ganho fixo com múltiplos saltos sob desvanecimento Nakagami-m. Em [43], soluções em forma fechada foram obtidas para redes AF de ganho fixo sob desvanecimento log-normal. Em [44], são obtidas expressões em forma fechada para a LCR e AFD de uma rede AF de ganho fixo com múltiplos saltos e desvanecimento Nakagami-m por meio de uma aproximação log-normal para o produto de processos aleatórios Nakagami-m independentes. Embora os resultados em [41], [43] e [44] sejam contribuições relevantes para as estatísticas de segunda ordem de redes AF de ganho fixo com múltiplos saltos, esses trabalhos analisam a LCR e AFD para a amplitude equivalente (fim-a-fim) do canal, ou seja, não contabilizam o acúmulo do ruído ao longo dos múltiplos saltos. De fato, é a relação sinal-ruído fim-a-fim (SNR – *signal-to-noise ratio*) que, em última instância, rege o desempenho do sistema. Mesmo em [42], em que a LCR e AFD são calculadas para a SNR fim-a-fim, o ruído acumulado no destino é suposto Gaussiano, o que constituiu uma aproximação grosseira. Na verdade, a análise exata dessas estatísticas de segunda ordem em termos da SNR para redes AF com múltiplos saltos mostra-se bastante complexa, mesmo para o caso mais simples com ganho fixo, considerado em [42]. Portanto, a análise dessas métricas para a SNR fim-a-fim do canal com múltiplos saltos e *relays* AF com ganho variável permanece um problema em aberto. Cabe ainda destacar que, dentre todos os trabalhos mencionados [31]–[44], o único que analisa a LCR e AFD para o caso AF com ganho variável foi apresentado em [36]. Mas, como informado anteriormente, considerou apenas o cenário dual-hop.

1.2 Objetivos

Tendo em vista o contexto descrito na seção anterior, este trabalho visa preencher a referida lacuna na literatura, tendo como objetivo principal analisar as estatísticas de segunda ordem para a SNR fim-a-fim de redes cooperativas com *relays* AF de ganho variável. Mais especificamente, será feita a análise da LCR e da AFD para redes HD com *relays* AF de ganho variável com múltiplos saltos sob desvanecimento Rayleigh e Nakagami-m. Além disso, também será feita a análise de tais estatísticas para redes FD com *relays* AF de ganho variável com dois saltos sob desvanecimento Nakagami-m. É importante ressaltar que a abordagem neste trabalho engloba tanto uma análise exata, com expressões em formato integral, quanto uma análise aproximada para alta SNR, com soluções em formato fechado. A análise em alta SNR é especialmente atrativa, pois revela com simplicidade e clareza como cada parâmetro do sistema afeta o desempenho sob condições adequadas de operação.

1.3 Organização da Tese

Os capítulos a seguir são réplicas de artigos publicados ou submetidos em revistas e congressos ao longo das atividades de pesquisa. As permissões para reproduzir os artigos publicados estão no Apêndice A. A seguir, apresenta-se um resumo de cada capítulo.

O Capítulo 2 reproduz o artigo intitulado "*High-SNR Second-Order Statistics of Amplify-and-Forward Relaying with Variable Gains and Multiple Hops*", que foi publicado no XXXVIII Simpósio Brasileiro de Telecomunicações (SBrT 2020) [45]. Esse trabalho investiga o desempenho em alta SNR da LCR e AFD de uma rede cooperativa *half-duplex* com *relays* AF de ganho variável e com múltiplos saltos. Considera-se que todos os canais estão sob desvanecimento Rayleigh plano. Como mencionado anteriormente, a maioria dos trabalhos que tratam das estatísticas de segunda ordem de redes *multi-hop* consideram *relays* AF com ganho fixo e obtêm essas estatísticas apenas para a amplitude fim-a-fim do canal. Essa análise, embora matematicamente mais simples, ignora o acúmulo de ruído ao longo dos saltos. Por sua vez, o nosso trabalho leva em consideração tal acúmulo de ruído ao fazer a análise da LCR e AFD para a SNR fim-a-fim, que determina em última instância o desempenho do sistema. Além disso, consideramos *relays* AF com ganho variável. Obtivemos expressões assintóticas simples e em forma fechada para as estatísticas de segunda ordem investigadas e, como nobre subproduto, para a probabilidade de *outage*. Nossos resultados mostram que os ganhos de diversidade da LCR e AFD valem ambos 1/2, independentemente do número de saltos. Também é mostrado que, quando todos os saltos são identicamente distribuídos, o ganho de codificação da LCR é inversamente proporcional ao número de saltos ao quadrado, enquanto que o ganho de codificação da AFD é independente do número de saltos.

O Capítulo 3 reproduz o artigo intitulado "*Second-Order Statistics of Amplify-and-Forward Relaying: Variable Gain and Multiple Hops*", atualmente sob revisão na revista IEEE *Transactions on Vehicular Technology* [46]. Esse trabalho amplia os resultados do Capítulo 2 ao fornecer expressões exatas para a LCR e AFD (em formato integral) para uma rede HD *multi-hop* com *relays* AF de ganho variável. Assim como no Capítulo 2, a análise das estatísticas de segunda ordem é feita considerando a SNR fim-a-fim do sistema. As expressões exatas e assintóticas para a LCR e AFD são validadas via simulação de Monte Carlo.

O Capítulo 4 reproduz o artigo intitulado "*Impact of Channel Diversity on the Second-Order Statistics of Multi-Hop Transparent Relaying*", atualmente sob revisão na revista IEEE *Wireless Communications Letters* [47]. Este trabalho amplia os resultados do Capítulo 2 ao considerar que cada canal está sujeito a um desvanecimento do tipo Nakagami-m. Portanto, esse trabalho investiga o desempenho em alta SNR de uma rede HD *multi-hop* com *relays* AF de ganho variável e canais sob desvanecimento Nakagami-m. Em particular, são obtidas expressões assintóticas simples e em forma fechada para a LCR e AFD que revelam claramente como o desempenho do sistema em termos de estatísticas de segunda ordem é afetado pela ordem de diversidade do canal. As expressões obtidas mostram que o desempenho em alta SNR é governado pelo canal com o desvanecimento mais severo, correspondente ao menor parâmetro de desvanecimento, digamos, m^*. Mais especificamente, demonstra-se que o ganho de diversidade da LCR vale $m^* - 1/2$, enquanto que o ganho de diversidade da AFD vale 1/2, não sendo pois afetado pela diversidade do canal.

O Capítulo 5 reproduz o artigo intitulado "*On Second-Order Statistics of Full-Duplex Relaying*", publicado na IEEE *Communications Letters* [48]. Esse trabalho investiga as estatísticas de segunda ordem de uma rede *dual-hop* com *relay full-duplex* do tipo AF com ganho variável e canais sob desvanecimento Nakagami-m. São obtidas expressões exatas em forma integral e expressões assintóticas em forma fechada para a LCR e AFD. Como subproduto, é obtida uma solução geral assintótica para a probabilidade de *outage*, até então disponível apenas para valores inteiros do parâmetro m de desvanecimento [24], [25]. Todos os resultados analíticos foram validados por simulações de Monte Carlo. As expressões obtidas mostram que, na região de alta SNR, nem a LCR nem a AFD são afetadas pelas condições do canal entre o *relay* e o destino. Por outro lado, essas métricas são governadas pela potência média e mobilidade do canal entre fonte e *relay*, pela auto-interferência residual, e, no caso da LCR, também pela severidade do desvanecimento no canal entre fonte e *relay*. Além disso, como já conhecido para a probabilidade de outage em sistemas *relaying* FD [24], [25], a ordem de diversidade do sistema é zero.

O Capítulo 6 apresenta considerações finais e perspectivas de desdobramento para este trabalho.

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2 High-SNR Second-Order Statistics of Amplify-and-Forward Relaying with Variable Gains and Multiple Hops

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Abstract

We analyze the high-SNR behavior of the level crossing rate and average fade duration for a variable-gain amplify-and-forward relaying system composed of an arbitrary number of hops under Rayleigh fading. Most studies on the second-order statistics of multi-hop amplify-and-forward relaying assume fixed gains at the relays and analyze the end-to-end channel gain (from source to destination). Such analysis leads to the so-called cascaded product channel, being mathematically attractive, yet ignoring the accumulation of noise along the hops. Here, in turn, we take the noise into account by analyzing the end-to-end SNR, which ultimately governs the system performance. Also, aiming at the benchmark for transparent relaying, we assume variable gains at the relays. We obtain simple closed-form asymptotic expressions for the investigated second-order statistics and, as a byproduct, for the associated outage probability.

2.1 Introduction

Second-order statistics are used to characterize the dynamic nature of the wireless channel in mobile communications systems, serving essential design issues such as packet length, symbol rate, and transmission time interval [1]. Those statistics complement the static characterization given by their first-order counterparts, e.g., outage probability and bit-error rate. Key second-order statistics are the level crossing rate (LCR) and the average fade duration (AFD). The former provides the temporal rate at which the fading channel crosses a given threshold, either upwards or downwards; the latter, the average amount of time the channel remains below that threshold.

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Despite their practical importance, very little is known about second-order statistics in relaying networks. Due to tractability reasons, most related studies are limited to the dual-hop scenario [2]–[5]. Very few works have addressed the general scenario with multiple hops [6]–[10]. Although the results in [7], [9], [10] are noteworthy contributions on the second-order statistics for multi-hop fixed-gain amplify-and-forward (AF) relaying, those works analyzed the LCR and AFD for the end-to-end channel gain (a.k.a. the cascaded product channel), which is mathematically appealing while neglecting the accumulation of noise along the multiple hops. Even in [8], where the LCR and AFD are given for the end-to-end signal-to-noise ratio (SNR), the resulting noise at the destination was assumed to be Gaussian — an approximation for rendering the problem more tractable. Anyway, the second-order statistics for variable-gain (VG) AF relaying systems with multiple hops remain open for investigation. Our primary aim here is to help fill this gap.

Even for the simplest scenario with two hops, an exact analysis of the second-order statistics for AF relaying systems proves intricate. Existing solutions appear usually in multi-fold integral form, bringing little or no insight into the system performance. In this work, we aim to shed light on the second-order statistics of multi-hop VG-AF relaying by deriving simple closed-form asymptotic solutions at high SNR.

2.2 System Model

Consider a multi-hop AF relaying system composed of N hops, as depicted in Fig. 2.1, in which the communication process between the source S and the destination D occurs through $N-1$ VG-AF relays $\{R_n\}_{n=1}^{N-1}$. There is no direct link between S and D. Each relay R_n receives the information signal coming from the preceding relay R_{n-1} (or from S, in the case of R_1), amplifies the signal by a factor of G_n, and then forwards it to the next node. This process is carried on hop by hop, up to the destination D. All terminals, including the source, are assumed to transmit with the same average power P_{T}. The amplification factor at the nth relay is given by $G_n^2 = \left(\alpha_n^2 + \Gamma_0^{-1}\right)^{-1}$ [11, eq. (9)], where α_n is the channel amplitude of the nth hop, and $\Gamma_0 \triangleq P_{\mathrm{T}}/N_0$ is the average transmit SNR at the source and relays, with N_0 being the mean power of the additive white Gaussian noise at the relays and destination. In such a case, the end-to-end SNR is obtained as $\Gamma_e = [\prod_{n=1}^{N}(1+\Gamma_n^{-1})-1]^{-1}$, where $\Gamma_n = \Gamma_0\alpha_n^2$ is the received SNR at the nth hop [12]. A widely used upper bound on Γ_e is [12]

$$\Gamma \triangleq \left[\sum_{n=1}^{N}\frac{1}{\Gamma_n}\right]^{-1} \simeq \Gamma_e, \quad \text{for} \quad \Gamma_0 \to \infty, \tag{2.1}$$

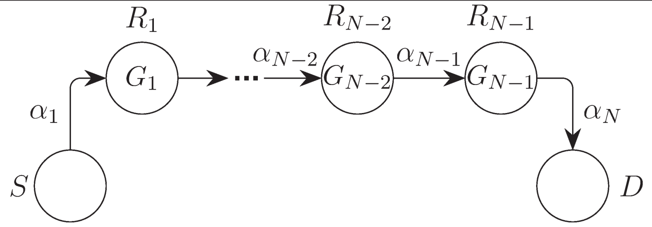

Figure 2.1 – Variable-gain amplify-and-forward relaying system with multiple hops.

where "$\simeq$" denotes asymptotic equivalence, indicating that Γ increasingly approaches Γ_e at high SNR. The upper bound in (2.1) serves as a benchmark for practical transparent relaying systems and will provide a basis for the high-SNR analysis attained in the next section.

We assume that the channel amplitudes $\{\alpha_n\}_{n=1}^N$ of the hops are independent, non-necessarily identically distributed Rayleigh random variables (RV). Hence, each received SNR Γ_n follows an exponential distribution with mean value $\bar{\Gamma}_n = \Gamma_0 \Omega_n$, where $\Omega_n = \mathrm{E}\left[\alpha_n^2\right]$ is the corresponding average channel power gain, with $\mathrm{E}\left[\cdot\right]$ denoting statistical expectation. Finally, considering an isotropic propagation environment, it is known that the time derivative $\dot{\alpha}_n$ of α_n is statistically independent of it and follows a zero-mean Gaussian distribution with variance $\sigma_{\dot{\alpha}_n}^2 = \pi^2 f_{m,n}^2 \Omega_n$, where $f_{m,n}$ is the maximum Doppler shift at the nth hop [1], [7].

2.3 High-SNR Second-Order Statistics

2.3.1 Preliminaries

A system outage occurs whenever the end-to-end SNR Γ_e drops below a certain threshold γ_{th}. The LCR $N_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)$ gives the average temporal rate at which outage events take place, which can be obtained from Rice's formula [1]

$$N_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right) = \int_0^\infty \dot{\gamma}\, f_{\Gamma_e,\dot{\Gamma}_e}\left(\gamma_{\mathrm{th}}, \dot{\gamma}\right)\, d\dot{\gamma} \tag{2.2}$$

in terms of the joint probability density function (PDF) $f_{\Gamma_e,\dot{\Gamma}_e}\left(\cdot,\cdot\right)$ of Γ_e and its time derivative $\dot{\Gamma}_e$. The AFD $T_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)$, in turn, gives the average duration of an outage event, being obtained as [1]

$$T_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right) = \frac{P_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)}{N_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)}, \tag{2.3}$$

where

$$P_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right) = \int_0^{\gamma_{\mathrm{th}}} f_{\Gamma_e}\left(\gamma\right) d\gamma \tag{2.4}$$

is the outage probability, with $f_{\Gamma_e}(\cdot)$ denoting the PDF of Γ_e.

2.3.2 Main Contributions

It is worth noting that there is no closed-form exact solution for either the first- or second-order statistics of VG-AF relaying systems. Regarding the outage probability, a two-fold integral-form solution was provided in [13] for an arbitrary number of hops, when the channels undergo Rayleigh, Rice, or Nakagami-m fading. Regarding the LCR and AFD, a two-fold integral-form solution was provided in [4] considering the existence of a direct link between source and destination, but for two hops only. In light of the mathematical complexity associated with VG-AF relaying, and aiming at better insights into the impact of each channel parameter on the system performance, we provide here a high-SNR analysis that yields simple closed-form expressions for the LCR, AFD, and, as a byproduct, outage probability. Our solutions apply to an arbitrary number of hops.

In the high-SNR regime ($\Gamma_0 \to \infty$), each investigated metric can be expressed asymptotically as $k_{\Gamma_e}(\gamma_{\text{th}}) \simeq (c_k\Gamma_0)^{-d_k}$, $k \in \{P, N, T\}$, as required, where d_k is called the diversity gain, and c_k is called the coding gain [14]. After a lengthy derivation process outlined in Section 2.3.4, we obtain remarkably simple high-SNR expressions for the outage probability, LCR, and AFD of multi-hop VG-AF relaying systems operating over Rayleigh fading:

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left(\frac{1}{\gamma_{\text{th}} \sum_{n=1}^{N} \frac{1}{\Omega_n}} \Gamma_0\right)^{-1} \tag{2.5}$$

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{1}{2\pi\gamma_{\text{th}} \left(\sum_{n=1}^{N} \frac{f_{m,n}}{\sqrt{\Omega_n}}\right)^2} \Gamma_0\right]^{-1/2} \tag{2.6}$$

$$T_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{2\pi}{\gamma_{\text{th}} \left(\frac{\sum_{n=1}^{N} \frac{1}{\Omega_n}}{\sum_{n=1}^{N} \frac{f_{m,n}}{\sqrt{\Omega_n}}}\right)^2} \Gamma_0\right]^{-1/2}. \tag{2.7}$$

2.3.3 Remarks

The asymptotic expressions in (2.5)–(2.7) reveal how the diversity and coding gains of each investigated metric are influenced by the average channel powers $\{\Omega_n\}_{n=1}^{N}$ and maximum Doppler shifts $\{f_{m,n}\}_{n=1}^{N}$ at the various hops. To our best knowledge, these expressions are new.

The parameters $\{\Omega_n\}_{n=1}^{N}$ and $\{f_{m,n}\}_{n=1}^{N}$ represent the channels' strength and the nodes' mobility, respectively. Note that these parameters play no role in the diversity gains of the outage probability, LCR, and AFD, namely, $d_P = 1$, $d_N = 1/2$, and $d_T = 1/2$. In particular, as known, the diversity gain of the outage probability mirrors the number of independent copies of information that achieve the destination [14], [15]. On the other hand, the impact on the coding gains vary for each metric: the higher the values of $\{\Omega_n\}_{n=1}^N$ (the stronger the channels), the higher the values of c_P (less likely fades), c_N (less frequent fades), and c_T (shorter fades); the higher the values of $\{f_{m,n}\}_{n=1}^N$ (the higher the mobility of the nodes), the smaller the values of c_N (more frequent fades) and the higher the values of c_T (shorter fades). As expected, the Doppler shifts (a dynamic feature) have no impact on the outage probability (a static metric).

2.3.4 Derivation Outline

In this section, we outline the derivation process of (2.5)–(2.7). First, we derive integral-form expressions for the LCR and AFD of Γ, the upper bound on Γ_e defined in (2.1). Then we simplify those expressions under the assumption of a high-SNR regime. For the lack of space, some details are omitted.

2.3.4.1 Approximate Analysis

To calculate the LCR of Γ, we start by rewriting (2.2) as

$$N_\Gamma(\gamma_{\text{th}}) = \int_0^\infty \dot{\gamma}\, f_{\dot{\Gamma}|\Gamma}(\dot{\gamma}|\gamma_{\text{th}})\, f_\Gamma(\gamma_{\text{th}})\; d\dot{\gamma}, \tag{2.8}$$

where $f_{\dot{\Gamma}|\Gamma}(\cdot|\cdot)$ is the PDF of $\dot{\Gamma}$ conditioned on Γ, and $f_\Gamma(\cdot)$ is the PDF of Γ. The latter can be obtained as follows. In [16], Brennan proposed a method to calculate the PDF of a generic sum of N non-negative, arbitrarily correlated, arbitrarily distributed RVs. In our case, let us consider the sum

$$Z = \sum_{n=1}^{N} Z_n, \tag{2.9}$$

where $\{Z_n\}_{n=1}^N$ are non-negative, independent RVs. From Brennan's results, the PDF of Z is given by [16]

$$f_Z(z) = \int_0^z \int_0^{z-z_N} \cdots \int_0^{z-\sum_{n=3}^N z_n} f_{Z_1}\left(z - \sum_{n=2}^{N} z_n\right) \prod_{n=2}^{N} f_{Z_n}(z_n)\, dz_2 \ldots dz_{N-1} dz_N. \tag{2.10}$$

To express (2.1) as in (2.9), we introduce the relationships

$$Z \triangleq \Gamma^{-1} \tag{2.11}$$

$$Z_n \triangleq \Gamma_n^{-1}, \tag{2.12}$$

with the respective PDFs being related as $f_\Gamma(\gamma) = \gamma^{-2} f_Z(\gamma^{-1})$ and $f_{Z_n}(z_n) = z_n^{-2} f_{\Gamma_n}(z_n^{-1})$. Using this into (2.10), the PDF of Γ is then obtained as

$$f_\Gamma(\gamma) = \frac{1}{\gamma^2} \int_0^{\frac{1}{\gamma}} \int_0^{\frac{1}{\gamma}-z_N} \cdots \int_0^{\frac{1}{\gamma}-\sum_{n=3}^N z_n} \frac{1}{\left(\frac{1}{\gamma} - \sum_{n=2}^N z_n\right)^2} f_{\Gamma_1}\left(\frac{1}{\frac{1}{\gamma} - \sum_{n=2}^N z_n}\right) \times \prod_{n=2}^N \frac{1}{z_n^2} f_{\Gamma_n}\left(\frac{1}{z_n}\right) dz_2 \ldots dz_{N-1} dz_N. \tag{2.13}$$

Now, to obtain $f_{\dot{\Gamma}|\Gamma}(\cdot|\cdot)$, we first determine from (2.1) the time derivative of Γ:

$$\dot{\Gamma} = \frac{\partial \Gamma}{\partial t} = \frac{\sum_{n=1}^N \frac{1}{\Gamma_n^2} \dot{\Gamma}_n}{\left(\sum_{n=1}^N \frac{1}{\Gamma_n}\right)^2} = \frac{2\sqrt{\Gamma_0} \sum_{n=1}^N Z_n^{3/2} \dot{\alpha}_n}{\left(\sum_{n=1}^N Z_n\right)^2}, \tag{2.14}$$

since $\dot{\Gamma}_n = \partial \Gamma_n / \partial t = 2\sqrt{\Gamma_0}\sqrt{\Gamma_n}\dot{\alpha}_n$ and $\Gamma_n = Z_n^{-1}$. Hence, it becomes apparent that $\dot{\Gamma}$ is a weighted sum of $\{\dot{\alpha}_n\}_{n=1}^N$, zero-mean Gaussian RVs with variances given by $\sigma_{\dot{\alpha}_n}^2 = \pi^2 f_{m,n}^2 \Omega_n$. The weights depend on $\{Z_n\}_{n=1}^N$. As a result, conditioned on Γ and $Z_2, \ldots, Z_N$, $\dot{\Gamma}$ is also a Gaussian RV, with zero mean and variance computed from (2.14) as

$$\sigma^2_{\dot{\Gamma}|\Gamma, Z_2, \ldots, Z_N} = 4\Gamma_0 \gamma_{\text{th}} \left[\left(1 - \gamma_{\text{th}} \sum_{n=2}^N z_n\right)^3 \sigma_{\dot{\alpha}_1} + \gamma_{\text{th}}^3 \sum_{n=2}^N z_n^3 \sigma_{\dot{\alpha}_n}\right]. \tag{2.15}$$

By integrating (2.8) with respect to $\dot{\gamma}$ while taking into account the Gaussianity of $f_{\dot{\Gamma}|\Gamma, Z_2, \ldots, Z_N}(\cdot|\cdot, \ldots, \cdot)$, we have [17]

$$\int_0^\infty \dot{\gamma}\, f_{\dot{\Gamma}|\Gamma, Z_2, \ldots, Z_N}(\cdot|\cdot, \ldots, \cdot)\, d\dot{\gamma} = \sqrt{\frac{\sigma^2_{\dot{\Gamma}|\Gamma, Z_2, \ldots, Z_N}}{2\pi}}. \tag{2.16}$$

Then, by applying (2.13) and (2.16) into (2.8), we obtain an approximate analytical expression for the LCR of a multi-hop VG-AF relaying system over Rayleigh fading as

$$N_\Gamma(\gamma_{\text{th}}) = \frac{\sqrt{2\pi\gamma_{\text{th}}}}{\Gamma_0^{N-1/2}} \int_0^{\frac{1}{\gamma_{\text{th}}}} \int_0^{\frac{1}{\gamma_{\text{th}}}-z_N} \cdots \int_0^{\frac{1}{\gamma_{\text{th}}}-\sum_{n=3}^N z_n} \times \sqrt{f_{m,1}^2 \Omega_1 \left(1 - \gamma_{\text{th}} \sum_{n=2}^N z_n\right)^3 + \gamma_{\text{th}}^3 \sum_{n=2}^N f_{m,n}^2 \Omega_n z_n^3} \times \frac{\exp\left[-\frac{\gamma_{\text{th}}}{\Omega_1 \Gamma_0 \left(1-\gamma_{\text{th}} \sum_{n=2}^N z_n\right)} - \sum_{n=2}^N \frac{1}{\Gamma_0 \Omega_n z_n}\right]}{\left(1 - \gamma_{\text{th}} \sum_{n=2}^N z_n\right)^2 \Omega_1 \prod_{n=2}^N z_n^2 \Omega_n} dz_2 \ldots dz_{N-1} dz_N. \tag{2.17}$$

To calculate the AFD of Γ as in (2.3), we need to find the associated outage probability. By substituting (2.13) into (2.4), we obtain an approximate analytical expression for the outage probability of a multi-hop VG-AF relaying system over Rayleigh fading as

$$P_\Gamma(\gamma_{\text{th}}) = \frac{1}{\Gamma_0^N} \int_0^{\frac{1}{\gamma_{\text{th}}}} \int_0^{\frac{1}{\gamma}} \int_0^{\frac{1}{\gamma}-z_N} \cdots \int_0^{\frac{1}{\gamma}-\sum_{n=3}^N z_n} \times \frac{\exp\left[-\frac{\gamma}{\Omega_1\Gamma_0\left(1-\gamma\sum_{n=2}^N z_n\right)} - \sum_{n=2}^N \frac{1}{\Gamma_0\Omega_n z_n}\right]}{\left(1-\gamma\sum_{n=2}^N z_n\right)^2 \Omega_1 \prod_{n=2}^N z_n^2\Omega_n} dz_2 \ldots dz_{N-1} dz_N d\gamma. \tag{2.18}$$

Then, by replacing (2.17) and (2.18) into (2.3), we obtain a corresponding expression for the AFD.

2.3.4.2 Asymptotic Analysis

The final step is to derive asymptotic representations for (2.17), (2.18), and the corresponding AFD as $\Gamma_0 \to \infty$. In such cases, a popular approach is to replace the exponential function in the integrands by its Maclaurin series expansion, to drop the terms beyond the second one (i.e., $\exp(-x_i) \simeq 1 - x_i$), and to solve and simplify the integral. But this approach alone does not work in our case; the resulting integrals turn out to diverge, since their integrands contain poles on each integration limit. To overcome this, we split the integration interval into two parts containing a single pole each, and then we perform certain mathematical manipulations to ensure the integral will converge when the exponential function is approximated.

But applying the above method to (2.17) and (2.18) proves quite involved if done directly for an arbitrary number of hops N. Rather, we apply the method for two and three hops first, and then, building on these results, we generalize them by induction for any number of hops. Next, due to the lack of space, we present the derivation steps only for the LCR under two hops. The same rationale can be also applied for the outage probability, as well as for a larger number of hops. Finally, using (2.3), those results can be combined into corresponding AFD expressions.

The LCR for the dual-hop case is obtained by setting $N = 2$ in (2.17), yielding

$$N_\Gamma(\gamma_{\text{th}}) = \frac{\sqrt{2\pi}\sqrt{\gamma_{\text{th}}}}{\Omega_1\Omega_2\Gamma_0^{3/2}} \int_0^{\frac{1}{\gamma_{\text{th}}}} \frac{\sqrt{(1-z_2\gamma_{\text{th}})^3\Omega_1 f_{m,1}^2 + (z_2\gamma_{\text{th}})^3\Omega_2 f_{m,2}^2}}{(1-z_2\gamma_{\text{th}})^2 z_2^2} \times \exp\left[-\frac{\gamma_{\text{th}}}{(1-z_2\gamma_{\text{th}})\Gamma_0\Omega_1} - \frac{1}{z_2\Gamma_0\Omega_2}\right] dz_2. \tag{2.19}$$

A high-SNR asymptotic expression for (2.19) cannot be obtained by directly using the Maclaurin representation of the exponential function and dropping the terms beyond the first, since the resulting integral will diverge due to the poles on $z_2 = 0$ and $z_2 = 1/\gamma_{\text{th}}$. To overcome this, as mentioned before, we split the integration interval into two parts with only one pole each, and then a change of variable is made to ensure the integral will converge when the exponential function is approximated. Specifically, after the change of variables $u \triangleq z_2\gamma_{\text{th}}$, (2.19) can be split as

$$N_\Gamma\left(\gamma_{\text{th}}\right) = \frac{\sqrt{2\pi}\gamma_{\text{th}}^{3/2}}{\Omega_1\Omega_2\Gamma_0^{3/2}}\left(I_1 + I_2\right), \tag{2.20}$$

where

$$I_1 \triangleq \int_0^{1/2} \frac{\sqrt{\left(1-u\right)^3\Omega_1 f_{m,1}^2 + u^3\Omega_2 f_{m,2}^2}}{\left(1-u\right)^2 u^2}\exp\left[-\frac{\gamma_{\text{th}}}{\Gamma_0\Omega_1\left(1-u\right)} - \frac{\gamma_{\text{th}}}{\Gamma_0\Omega_2 u}\right]du \tag{2.21}$$

$$I_2 \triangleq \int_{1/2}^{1} \frac{\sqrt{\left(1-u\right)^3\Omega_1 f_{m,1}^2 + u^3\Omega_2 f_{m,2}^2}}{\left(1-u\right)^2 u^2}\exp\left[-\frac{\gamma_{\text{th}}}{\Gamma_0\Omega_1\left(1-u\right)} - \frac{\gamma_{\text{th}}}{\Gamma_0\Omega_2 u}\right]du. \tag{2.22}$$

By changing also $v \triangleq \frac{(\gamma_{\text{th}}-2u\gamma_{\text{th}})}{(\Gamma_0\Omega_2 u)}$ in (2.21), and $w \triangleq \frac{(-\gamma_{\text{th}}+2\gamma_{\text{th}}u)}{(\Gamma_0\Omega_1-\Gamma_0\Omega_1 u)}$ in (2.22), we have

$$\begin{aligned} I_1 =& \frac{\Gamma_0\Omega_2}{\gamma_{\text{th}}}\int_0^\infty \frac{\left(2\gamma_{\text{th}}+v\Gamma_0\Omega_2\right)^2}{\left(\gamma_{\text{th}}+v\Gamma_0\Omega_2\right)^2}\sqrt{\frac{\left(\gamma_{\text{th}}+v\Gamma_0\Omega_2\right)^3\Omega_1 f_{m,1}^2}{\left(2\gamma_{\text{th}}+v\Gamma_0\Omega_2\right)^3} + \frac{\gamma_{\text{th}}^3\Omega_2 f_{m,2}^2}{\left(2\gamma_{\text{th}}+v\Gamma_0\Omega_2\right)^3}} \\ &\times \exp\left[-v - \frac{2\gamma_{\text{th}}}{\Gamma_0\Omega_2} - \frac{2\gamma_{\text{th}}^2 + v\gamma_{\text{th}}\Gamma_0\Omega_2}{\Gamma_0\Omega_1\left(\gamma_{\text{th}}+v\Gamma_0\Omega_2\right)}\right]dv \end{aligned} \tag{2.23}$$

$$\begin{aligned} I_2 =& \frac{\Gamma_0\Omega_1}{\gamma_{\text{th}}}\int_0^\infty \frac{\left(2\gamma_{\text{th}}+w\Gamma_0\Omega_1\right)^2}{\left(\gamma_{\text{th}}+w\Gamma_0\Omega_1\right)^2}\sqrt{\frac{\gamma_{\text{th}}^3\Omega_1 f_{m,1}^2}{\left(2\gamma_{\text{th}}+w\Gamma_0\Omega_1\right)^3} + \frac{\left(\gamma_{\text{th}}+w\Gamma_0\Omega_1\right)^3\Omega_2 f_{m,2}^2}{\left(2\gamma_{\text{th}}+w\Gamma_0\Omega_1\right)^3}} \\ &\times\exp\left[-w - \frac{2\gamma_{\text{th}}}{\Gamma_0\Omega_1} - \frac{2\gamma_{\text{th}}^2 + w\gamma_{\text{th}}\Gamma_0\Omega_1}{\Gamma_0\Omega_2\left(\gamma_{\text{th}}+w\Gamma_0\Omega_1\right)}\right]dw. \end{aligned} \tag{2.24}$$

Since $1/\Gamma_0 \to 0$ as $\Gamma_0 \to \infty$, we can apply the Maclaurin series to the whole integrand in (2.23) and (2.24), and take only the first term to track the asymptotic behavior:

$$I_1 \simeq \frac{\Omega_2\sqrt{\Omega_1 f_{m,1}^2}\Gamma_0}{\gamma_{\text{th}}}\int_0^\infty \exp\left[-v\right]dv = \frac{\Omega_2\sqrt{\Omega_1 f_{m,1}^2}\Gamma_0}{\gamma_{\text{th}}} \tag{2.25}$$

$$I_2 \simeq \frac{\Omega_1\sqrt{\Omega_2 f_{m,2}^2}\Gamma_0}{\gamma_{\text{th}}}\int_0^\infty \exp\left[-w\right]dw = \frac{\Omega_1\sqrt{\Omega_2 f_{m,2}^2}\Gamma_0}{\gamma_{\text{th}}}. \tag{2.26}$$

Then, by substituting (2.25) and (2.26) into (2.20), we obtain a high-SNR asymptotic expression for the LCR of a dual-hop VG-AF relaying system:

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{1}{2\pi\gamma_{\text{th}}\left(\frac{f_{m,1}}{\sqrt{\Omega_1}} + \frac{f_{m,2}}{\sqrt{\Omega_2}}\right)^2}\Gamma_0\right]^{-1/2}. \tag{2.27}$$

As already mentioned, the procedure described in (2.19)–(2.27) can be also applied for the outage probability and for any number of hops. For these cases, due to space constraints, we only reproduce the final expressions. The high-SNR asymptotic expression for the outage probability with $N = 2$ is obtained as

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{1}{\gamma_{\text{th}}\left(\frac{1}{\Omega_1} + \frac{1}{\Omega_2}\right)}\Gamma_0\right]^{-1}, \tag{2.28}$$

while the high-SNR asymptotic expressions for the LCR and outage probability with $N = 3$ are given respectively as

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{1}{2\pi\gamma_{\text{th}} \left(\frac{f_{m,1}}{\sqrt{\Omega_1}} + \frac{f_{m,2}}{\sqrt{\Omega_2}} + \frac{f_{m,3}}{\sqrt{\Omega_3}} \right)^2} \Gamma_0 \right]^{-1/2} \tag{2.29}$$

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{1}{\gamma_{\text{th}} \left(\frac{1}{\Omega_1} + \frac{1}{\Omega_2} + \frac{1}{\Omega_3} \right)} \Gamma_0 \right]^{-1}. \tag{2.30}$$

Expressions similar to (2.27)–(2.30) can be found for $N > 3$. All in all, building on these results, and using (2.3), we arrive by induction at the general solution given in (2.5)–(2.7).

2.4 Numerical Results

The analytical expressions derived in the previous section are now evaluated for sample scenarios. To validate our analysis, simulations results are also provided for the exact and approximate end-to-end SNRs. For illustration purposes and without loss of generality, the SNR threshold and the average channel power at each hop are set to unity, i.e., $\gamma_{\text{th}} = 0$ dB and $\Omega_1 = \cdots = \Omega_N = 1$. Also, we consider that all nodes have the same mobility, so that $f_{m,1} = \cdots = f_{m,N} = f_m$.

Figs. 2.2 and 2.3 show the normalized LCR (N_{Γ_e}/f_m) and normalized AFD ($T_{\Gamma_e} \times f_m$) versus the average transmit SNR (Γ_0), respectively, for a number of hops ranging from two to five. The approximate curves, labeled as "Approximate SNR (Analysis)", were obtained from (2.17) and (2.18), and the asymptotic curves, from (2.6) and (2.7). Note the tightness of the proposed approximations at medium to high SNR, regardless of the number of hops. As expected, the proposed approximations become less accurate at low SNR, and this mismatch increases with the number of hops. It can be also observed how the number of hops affects the performance of VG-AF relaying in itself. Although a broader coverage area is normally attained as the number of hops is increased, the outage events become more frequent (see Fig. 2.2). Yet, the penalty due to an extra hop diminishes with the number of hops. On the other hand, as shown in Fig. 2.3, the number of hops barely affects the duration of the outage events at medium to high SNR.

Note that the above observations can be straightforwardly assessed via our main analytical results in (2.5)–(2.7). In particular, for the homogeneous scenario addressed in the numerical examples, the coding gains therein specialize to $c_P = \Omega/(N\gamma_{\text{th}})$, $c_N = \Omega/(2\pi f_m^2 N^2 \gamma_{\text{th}})$, and $c_T = 2\pi f_m^2 \Omega/\gamma_{\text{th}}$.

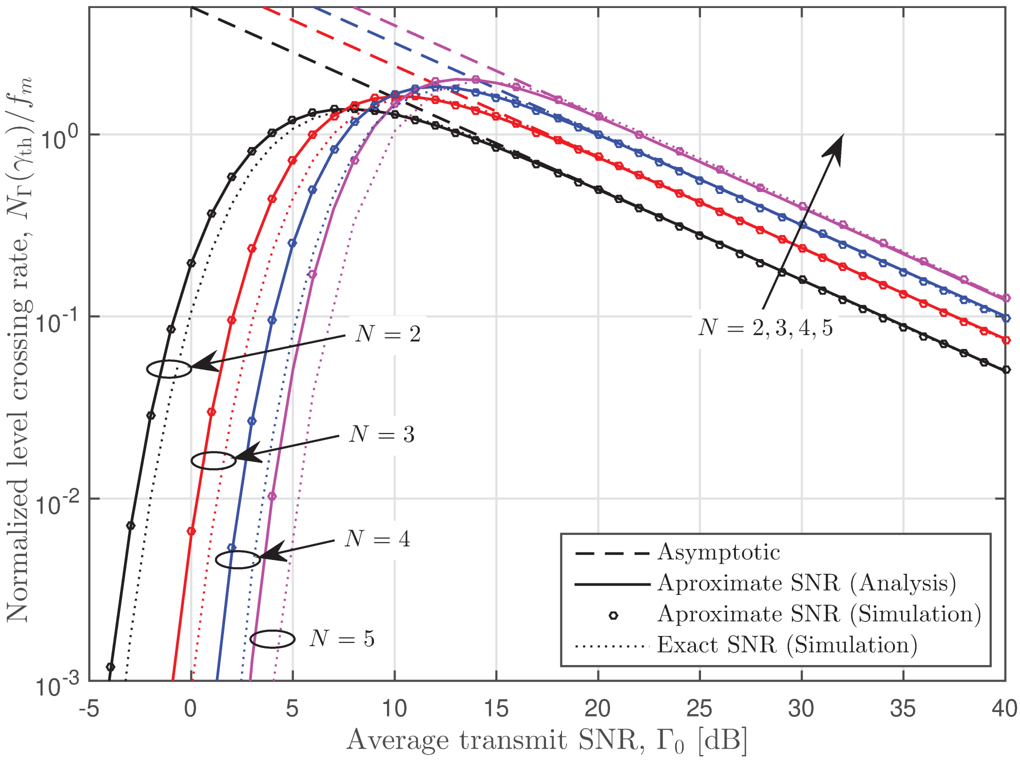

Figure 2.2 – Normalized level crossing rate for a multi-hop VG-AF relaying system: $\Omega_1 = \cdots = \Omega_5 = 1$ and $\gamma_{\text{th}} = 0$ dB.

2.5 Conclusions

We investigated the second-order statistics of a variable-gain amplify-and-forward relaying system containing an arbitrary number of hops subject to Rayleigh fading. Our main results are novel high-SNR expressions for the LCR, AFD, and, in passing, outage probability, of the end-to-end SNR. These asymptotic results circumvent the integral-form formulations that typically emerge in related exact analyses, while shedding light on the subject by means of simple closed-form expressions that unveil how each system parameter roughly affects the performance. In particular, our results show that the diversity gains of the LCR and AFD are both 1/2, regardless of the number of hops. Also, when all the hops are identically distributed, the coding gain of the LCR is inversely proportional to the squared number of hops, whereas the coding gain of the AFD is not affected whatsoever.

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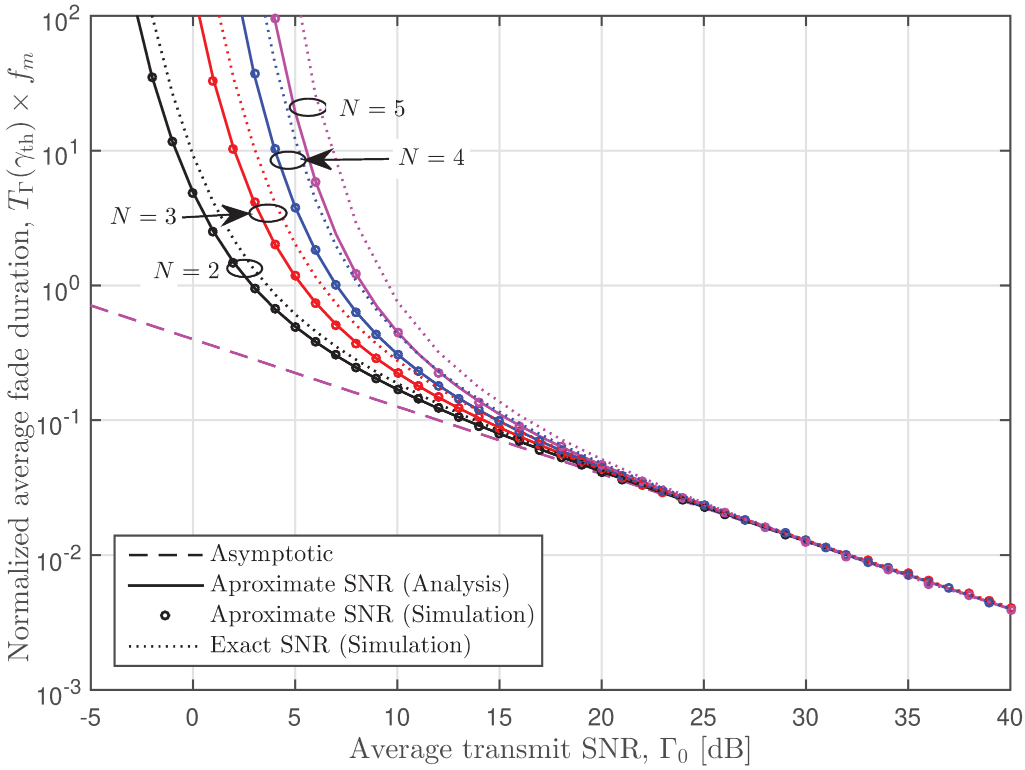

Figure 2.3 – Normalized average fade duration for a multi-hop VG-AF relaying system: $\Omega_1 = \cdots = \Omega_5 = 1$ and $\gamma_{\text{th}} = 0$ dB.

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3 Second-Order Statistics of Amplify-and-Forward Relaying: Variable Gain and Multiple Hops

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Abstract

We analyze the level crossing rate and the average fade duration of a variable-gain amplify-and-forward relaying system composed of an arbitrary number of hops undergoing non-necessarily identically distributed Rayleigh fading. Unlike previous related studies, which analyze second-order statistics for the end-to-end envelope (the so-called cascaded fading channel), herein we analyze those statistics for the channel metric that ultimately governs the system performance: the end-to-end signal-to-noise ratio. Moreover, while earlier works focus on the fixed-gain variant of the amplify-and-forward protocol, we address the variable-gain one—a benchmark for transparent relaying. Our analysis provides exact and asymptotic solutions. The diversity gain is found to be 1/2 for both level crossing rate and average fading duration, regardless of the number of hops. The coding gain, in turn, proves quite different in each case. In particular, for identically distributed hops, the coding gain for the level crossing rate is inversely proportional to the squared number of hops, while for the average fade duration it is not affected whatsoever. As a byproduct, we derive an asymptotic expression for the system's outage probability.

3.1 Introduction

Over the last two decades, relay-based communications have been established as a promising technology to counteract shadowing and multipath fading, thereby improving coverage and capacity in wireless networks. Traditionally, relaying schemes employ two basic protocols: decode-and-forward (DF) and amplify-and-forward (AF) [1]. The AF relaying protocol is particularly appealing because of its low implementation complexity.

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It has been one of the most investigated relaying protocols in a plethora of scenarios (see [2], [3], and references therein). In the AF protocol, the relay forwards to the destination an amplified version of the signal received from the source, without performing any decoding operation. This protocol can be further classified into two categories, according to the amplification factor used at the relay: (i) fixed-gain (FG) relaying, whereby a constant gain is applied to the signal received from the source, based on the long-term channel statistics between the source and relay, in order to guarantee a certain average transmit power at the relay output; and (ii) variable-gain (VG) relaying, whereby an estimate of the instantaneous channel state between the source and relay is used to compensate the channel fluctuations due to fading, thus ensuring a certain instantaneous transmit power at the relay output. There is a trade-off between these two variant protocols: VG-AF relaying offers a better performance, whereas FG-AF relaying offers more simplicity and ease of implementation. The FG-AF relaying protocol may present saturation issues at the relay's amplifier, depending on the channel conditions [4].

Many studies have investigated the performance of cooperative networks based on relaying [1], [5]–[11]. Most of these studies have focused on the analysis of first-order statistics, e.g., outage probability and bit-error rate. Although such performance metrics are essential, they provide only system snapshots, failing to capture the temporal variation of the wireless channel. In mobile communications systems, the channel gains change over time due to the relative motion among the nodes, as well as due to the motion of surrounding objects.

The time-varying nature of fading can be described by second-order statistics. The most popular ones are level crossing rate (LCR) and average fade duration (AFD). The former provides the rate at which the fading channel crosses a given threshold, either upwards or downwards; the latter, the average amount of time the channel remains below that threshold. These statistics contain key information for the design of wireless communications systems, e.g., in the proper dimensioning of the packet length, symbol rate, and transmission time interval [12].

Despite their practical importance, very little is known about second- or higher-order statistics for relaying networks. Most related studies are limited to the dual-hop scenario, containing a single relay [8], [13]–[20]. And even for this scenario, an analysis of the second-order statistics proves rather intricate. Very few works have addressed the general scenario with multiple hops [21]–[25]. More specifically, in [21], considering noise- and interference-limited scenarios, exact closed-form solutions were determined for the LCR and AFD of a multi-hop DF system in generalized fading environments. In [22], exact solutions for the LCR and AFD of a multi-hop FG-AF system under Rayleigh fading were derived in integral form, the dimension of which equals the number of relays. As an alternative to those integral-form solutions, which incur a high computational complexity, approximate closed-form expressions were also presented, using the multivariate Laplace approximation theorem. The work in [23] also used the latter approach to obtain the second-order statistics for the channel capacity of a multi-hop FG-AF system subject to Nakagami-m fading. In [24], closed-form solutions for the LCR and AFD were obtained for an FG-AF system under log-normal fading. In [25], closed-form expressions for the LCR and AFD of a multi-hop Nakagami-m fading channel were provided, by formulating a log-normal approximation to the product of a given number of independent Nakagami-m random processes. Although the results in [22], [24], and [25] are noteworthy contributions on the second-order statistics for multi-hop FG-AF relaying, those works analyzed the LCR and AFD for the end-to-end fading envelope, i.e. without addressing the accumulation of noise over multiple hops. Of course, the end-to-end signal-to-noise ratio (SNR) is the quantity that ultimately governs the system performance. Even in [23], where the LCR and AFD are given for the end-to-end SNR, the accumulated noise at the destination was assumed to be Gaussian with an arbitrary mean power, thus disregarding the cascaded fading effect along the multi-hop relay channel. Indeed, the exact analysis of these second-order statistics in terms of SNR for a multi-hop AF relay system proves to be rather involved, even for the simplest case with fixed gain, considered in [23]. Therefore, the second-order statistics for VG-AF relaying systems with multiple hops remain open for investigation. Our primary aim is to fill this gap. It is worthwhile to mention that, among all these works [8], [13]–[25], the only study that analyzes the LCR and AFD for the VG-AF case was presented in [8]. However, as mentioned earlier, it considered only the dual-hop scenario.

In this work, we provide an analytical framework for evaluating the second-order statistics of a VG-AF relaying system composed of an arbitrary number of hops. More specifically, the following are our main contributions:

- we obtain exact integral-form expressions for the LCR and AFD of a multi-hop VG-AF relaying system;
- we obtain simple high-SNR asymptotic expressions for the referred metrics;
- we show that, at high SNR and for an homogeneous scenario, the LCR varies proportionally to the number of hops, while the AFD is unaffected; and
- as a byproduct, we derive an asymptotic expression for the outage probability of a multi-hop VG-AF relaying system.

The remainder of this paper is organized as follows. In Section 3.2, the system model of multi-hop VG-AF relaying is presented. In Section 3.3, we derive exact and asymptotic expressions for the second-order statistics of the system under study. Numerical results and discussions are presented in Section 3.4. Finally, overall conclusions are drawn in Section 3.5.

3.2 System Model

Consider an AF relaying system consisting of N hops, as depicted in Fig. 3.1, in which the communication process between the source S and the destination D occurs through $N-1$ VG-AF relays $\{R_n\}_{n=1}^{N-1}$. We assume there is no direct link between S and D. Each relay R_n receives the information signal coming from the preceding relay R_{n-1} (or from S, in the case of R_1), amplifies the signal by a factor of $\{G_n\}_{n=1}^{N-1}$, and then forwards it to the next node. This process is carried on hop by hop, up to the destination D. All terminals, including the source, are assumed to transmit with the same average power P_{T}. To comply with this assumption, we set the amplification factor at the nth relay to [1, eq. (9)]

$$G_n^2 = \frac{\frac{P_{\mathrm{T}}}{N_0}}{\frac{P_{\mathrm{T}}}{N_0}\alpha_n^2 + 1}, \tag{3.1}$$

where $\{\alpha_n\}_{n=1}^{N}$ are the fading amplitudes of the hops and N_0 is the mean power of the additive white Gaussian noise at each node. All in all, the end-to-end SNR Γ_e is obtained as [5]

$$\Gamma_e = \left[\prod_{n=1}^{N}\left(1+\Gamma_n^{-1}\right) - 1\right]^{-1}, \tag{3.2}$$

where the received SNRs $\{\Gamma_n\}_{n=1}^{N}$ at the hops are given by

$$\Gamma_n = \Gamma_0\alpha_n^2, \tag{3.3}$$

with $\Gamma_0 \triangleq P_{\mathrm{T}}/N_0$ being the average transmit SNR at the source and relays.

We assume that the channel amplitudes $\{\alpha_n\}_{n=1}^{N}$ are independent, non-necessarily identically distributed Rayleigh random variables (RV). Hence, each received SNR Γ_n follows an exponential distribution, given by

$$f_{\Gamma_n}(\gamma_n) = \frac{1}{\bar{\Gamma}_n}\exp\left[-\frac{\gamma_n}{\bar{\Gamma}_n}\right], \tag{3.4}$$

where $\bar{\Gamma}_n = \mathrm{E}[\Gamma_n] = \Gamma_0\Omega_n$ is the average received SNR at the nth hop, and $\Omega_n = \mathrm{E}[\alpha_n^2]$ is the corresponding average channel power gain, with $\mathrm{E}[\cdot]$ denoting statistical expectation. Finally, considering an isotropic propagation environment, it is known that the time derivative of the channel amplitude $\dot{\alpha}_n$ is independent of α_n itself, and follows a zero-mean Gaussian distribution with variance [12]

$$\sigma_{\dot{\alpha}_n}^2 = \pi^2 f_{m,n}^2\Omega_n \tag{3.5}$$

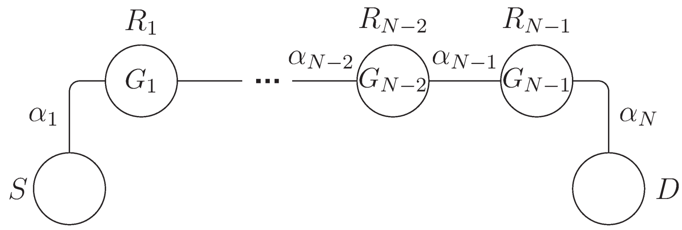

Figure 3.1 – Variable-gain amplify-and-forward relaying with multiple hops.

depending on the average channel power gain Ω_n and on the maximum Doppler frequency shift $f_{m,n}$ at the nth hop [22].

3.3 Second-Order Statistics

In this section, using both exact and asymptotic approaches, we obtain analytical expressions for the second-order statistics of the VG-AF relaying system under study. We begin by revisiting some basic formulas for second-order statistics. Afterwards, we calculate the LCR and AFD of the exact end-to-end SNR given in (3.2). Finally, we provide corresponding asymptotic expressions at high SNR.

3.3.1 Preliminaries

A system outage occurs whenever the end-to-end SNR Γ_e drops below a certain threshold level, say, γ_{th}. The LCR $N_{\Gamma_e}(\gamma_{\text{th}})$ gives the average temporal rate at which outage events take place, which can be obtained from Rice's formula [12]

$$N_{\Gamma_e}(\gamma_{\text{th}}) = \int_0^\infty \dot{\gamma}\, f_{\Gamma_e,\dot{\Gamma}_e}(\gamma_{\text{th}}, \dot{\gamma})\; d\dot{\gamma} \tag{3.6}$$

in terms of the joint probability density function (PDF) $f_{\Gamma_e,\dot{\Gamma}_e}(\cdot,\cdot)$ of Γ_e and its time derivative $\dot{\Gamma}_e$. The AFD $T_{\Gamma_e}(\gamma_{\text{th}})$, in turn, gives the average duration of an outage event, being obtained as [12]

$$T_{\Gamma_e}(\gamma_{\text{th}}) = \frac{P_{\Gamma_e}(\gamma_{\text{th}})}{N_{\Gamma_e}(\gamma_{\text{th}})}, \tag{3.7}$$

where

$$P_{\Gamma_e}(\gamma_{\text{th}}) = \int_0^{\gamma_{\text{th}}} f_{\Gamma_e}(\gamma)\, d\gamma \tag{3.8}$$

is the outage probability, with $f_{\Gamma_e}(\cdot)$ denoting the PDF of Γ_e.

3.3.2 Exact Analysis

To evaluate the exact second-order statistics of the investigated system, we resort to Brennan's approach in [26]. This approach relies on geometric arguments to formulate the PDF and CDF (cumulative distribution function) of a generic sum of positive

RVs in terms of the summands' joint PDF. As detailed in Appendix 3.A, we first need to represent (3.2) as an equivalent sum of RVs. We eventually arrive at an exact expression for the LCR of multi-hop VG-AF relaying systems, as shown in the theorem next.

Theorem 3.1 *An exact analytical expression for the LCR of a multi-hop VG-AF relaying system is written as*

$$\begin{aligned}
N_{\Gamma_e}(\gamma_{\rm th}) &= \frac{\sqrt{2\pi\gamma_{\rm th}}}{\Gamma_0^{N-1/2}\prod_{n=1}^{N}\Omega_n}\int_0^{\ln\left(1+\frac{1}{\gamma_{\rm th}}\right)}\int_0^{\ln\left(1+\frac{1}{\gamma_{\rm th}}\right)-z_N}\cdots\int_0^{\ln\left(1+\frac{1}{\gamma_{\rm th}}\right)-\sum_{n=3}^{N}z_n}\\
&\times\left\{e^{-\sum_{n=2}^{N}z_n}\left[1+\gamma_e\left(1-e^{\sum_{n=2}^{N}z_n}\right)\right]^3\Omega_1 f_{m,1}+\gamma_e(1+\gamma_e)^2\sum_{n=2}^{N}e^{-2z_n}(e^{z_n}-1)^3\Omega_n f_{m,n}\right\}^{\frac{1}{2}}\\
&\times\frac{1}{\left(1+\gamma_{\rm th}-\gamma_{\rm th}e^{\sum_{n=2}^{N}z_n}\right)^2\prod_{n=2}^{N}(e^{z_n}-1)^2}\\
&\times\exp\left[-\frac{\gamma_{\rm th}}{\Gamma_0\Omega_1\left(-\gamma_{\rm th}+(1+\gamma_{\rm th})\,e^{-\sum_{n=2}^{N}z_n}\right)}-\sum_{n=2}^{N}\left(\frac{1}{\Gamma_0\Omega_n(e^{z_n}-1)}-2z_n\right)\right]\\
&\times dz_2\ldots dz_{N-1}dz_N. \qquad (3.9)
\end{aligned}$$

Proof See Appendix 3.A.

As can be seen in (3.7), the outage probability is required to compute the AFD. Exact integral-form solutions for the outage probability of multihop VG-AF relaying are available in terms of moment-generating functions [5] or characteristic functions [27]. In the theorem that follows, as a byproduct of our second-order statistical analysis, we provide an alternative, PDF-domain solution for the outage probability.

Theorem 3.2 *An exact analytical expression for the outage probability of a multi-hop VG-AF relaying system is written as*

$$\begin{aligned}
P_{\Gamma_e}(\gamma_{\rm th}) &= \int_0^{\gamma_{\rm th}}\int_0^{\ln\left(1+\frac{1}{\gamma_e}\right)}\int_0^{\ln\left(1+\frac{1}{\gamma_e}\right)-z_N}\cdots\int_0^{\ln\left(1+\frac{1}{\gamma_e}\right)-\sum_{n=3}^{N}z_n}\\
&\times\frac{e^{-\frac{\gamma_e}{\Gamma_0\Omega_1\left[-\gamma_e+(1+\gamma_e)e^{-\sum_{n=2}^{N}z_n}\right]}-\sum_{n=2}^{N}\frac{1}{\Gamma_0\Omega_n(e^{z_n}-1)}-2z_n}}{\Gamma_0^N\left(1+\gamma_e-\gamma_e e^{\sum_{n=2}^{N}z_n}\right)^2\Omega_1\prod_{n=2}^{N}\Omega_n(e^{z_n}-1)^2}\,dz_2\ldots dz_{N-1}dz_N d\gamma_e. \qquad (3.10)
\end{aligned}$$

Proof The outage probability in (3.10) is obtained by substituting (3.26) (from Appendix 3.A) into (3.8).

A corresponding expression for the AFD is readily obtained by replacing (3.10) and (3.9) into (3.7).

3.3.3 Asymptotic Analysis

In the previous section, we provided exact analytical expressions for the LCR and AFD. Despite their accuracy, these expressions were given in integral form and thus do not provide much insight about the influence of each system parameter on the transmission performance. It is worth noting that there exists no closed-form exact solution for the first- and second-order statistics of VG-AF relaying systems. Regarding the outage probability, a two-fold integral-form solution was provided in [27] for an arbitrary number of hops, when the channels undergo Rayleigh, Rice, or Nakagami-m fading. Regarding the LCR and AFD, a two-fold integral-form solution was provided in [8] considering the existence of a direct link between source and destination, but for two hops only.

In light of the mathematical complexity associated with VG-AF relaying, and aiming at better insights into the impact of each system parameter on performance, we provide next a high-SNR analysis that yields simple closed-form expressions for the LCR, AFD, and, as a byproduct, outage probability. Our solutions apply to an arbitrary number of hops.

In the high-SNR regime ($\Gamma_0 \to \infty$), each investigated metric can be expressed asymptotically as

$$k_{\Gamma_e}(\gamma_{\text{th}}) \simeq (c_k \Gamma_0)^{-d_k}, \tag{3.11}$$

$k \in \{N, T, P\}$, where d_k is known as diversity gain (or diversity order), and c_k, as coding gain [28]. Here, $f(\cdot) \simeq g(\cdot)$ denotes asymptotic equivalence, i.e., $f(\cdot) \simeq g(\cdot) \iff \lim_{\Gamma_0 \to \infty} f(\cdot)/g(\cdot) = 1$. In the following theorem, we provide simple closed-form high-SNR expressions for the LCR, AFD, and outage probability of a VG-AF relaying system with multiples hops.

Theorem 3.3 *At high SNR, the asymptotic LCR, AFD, and outage probability of a multi-hop VG-AF relaying system are respectively written as*

$$N_{\Gamma_e}(\gamma_{th}) \simeq \left[\frac{1}{2\pi\gamma_{\text{th}}\left(\sum_{n=1}^{N}\frac{f_{m,n}}{\sqrt{\Omega_n}}\right)^2}\Gamma_0\right]^{-1/2}, \tag{3.12}$$

$$T_{\Gamma_e}(\gamma_{th}) \simeq \left[\frac{2\pi}{\gamma_{\text{th}}\left(\frac{\sum_{n=1}^{N}\frac{1}{\Omega_n}}{\sum_{n=1}^{N}\frac{f_{m,n}}{\sqrt{\Omega_n}}}\right)^2}\Gamma_0\right]^{-1/2}, \tag{3.13}$$

$$P_{\Gamma_e}(\gamma_{th}) \simeq \left(\frac{1}{\gamma_{\text{th}}\sum_{n=1}^{N}\frac{1}{\Omega_n}}\Gamma_0\right)^{-1}. \tag{3.14}$$

Proof See Appendix 3.B.

These simple asymptotic expressions reveal how each investigated metric is chiefly governed by the average channel power gains $\{\Omega_n\}_{n=1}^N$ and the maximum Doppler frequency shifts $\{f_{m,n}\}_{n=1}^N$ of the hops. To the best of our knowledge, these results are new.

Remark 3.1 *The parameters $\{\Omega_n\}_{n=1}^N$ and $\{f_{m,n}\}_{n=1}^N$ represent the hops' strength and mobility, respectively. Note that these parameters play no role in the diversity gains of the outage probability, LCR, and AFD, namely, $d_P = 1$, $d_N = 1/2$, and $d_T = 1/2$. As known to be the case in general, the diversity gain of the outage probability mirrors the number of independent copies of information that achieve the destination [28], [29]. On the other hand, the coding gains vary in each case: the higher the values of $\{\Omega_n\}_{n=1}^N$ (the stronger the channels), the higher the values of c_P (less likely fades), c_N (less frequent fades), and c_T (shorter fades); the higher the values of $\{f_{m,n}\}_{n=1}^N$ (the higher the mobility of the nodes), the smaller the value of c_N (more frequent fades) and the higher the value of c_T (shorter fades). As expected, the Doppler shifts (a dynamic feature) have no impact on the outage probability (a static metric).*

Next, we specialize (3.12)–(3.14) to a homogeneous scenario in which all hops are identically distributed.

3.3.3.1 Identically Distributed Hops

When the hops have the same strength ($\Omega_1 = \cdots = \Omega_N \triangleq \Omega$) and the same mobility ($f_{m,1} = \cdots = f_{m,N} \triangleq f_m$), we can rewrite (3.12)–(3.14) as

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left(\frac{\Omega}{2\pi f_m^2 N^2 \gamma_{\text{th}}}\Gamma_0\right)^{-1/2}, \tag{3.15}$$

$$T_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left(\frac{2\pi f_m^2 \Omega}{\gamma_{\text{th}}}\Gamma_0\right)^{-1/2}, \tag{3.16}$$

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left(\frac{\Omega}{N\gamma_{\text{th}}}\Gamma_0\right)^{-1}. \tag{3.17}$$

Remark 3.2 *This is the main takeaway: at high SNR, as the number of hops is increased, the level crossing rate increases proportionally, whereas the average fading duration keeps unchanged. Whenever the number of hops is doubled, an additional 6 dB of per-hop average transmit power is required to maintain the same average outage rate.*

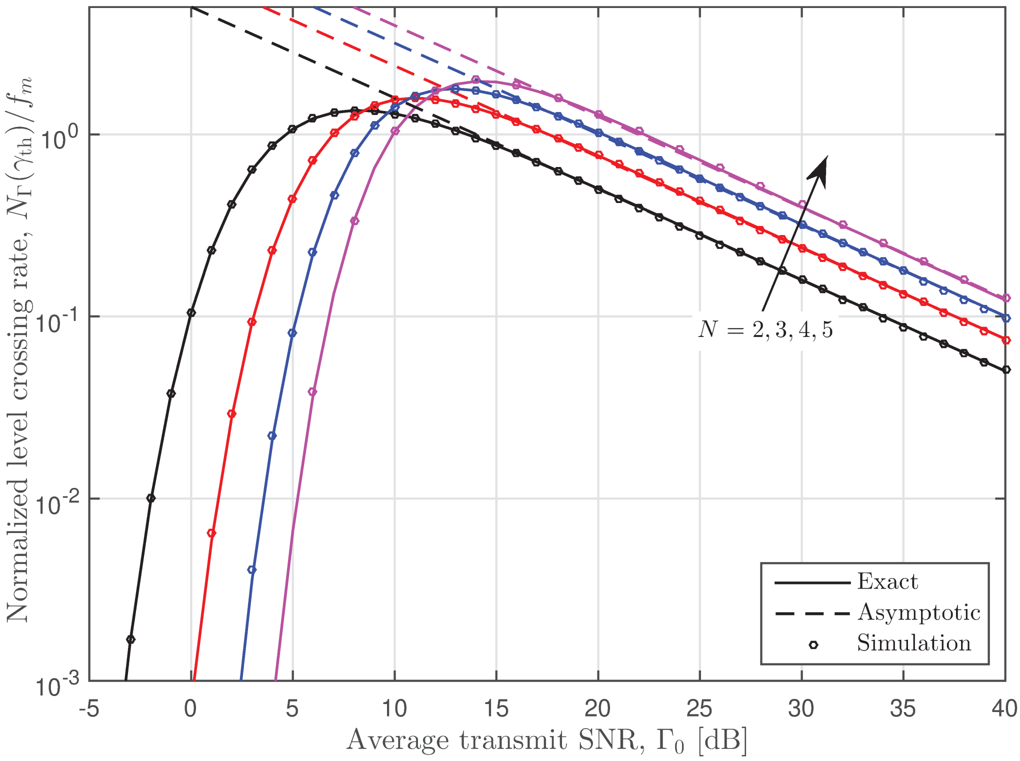

Figure 3.2 – Level crossing rate of variable-gain amplify-and-forward relaying with multiple hops: $\Omega_1 = \cdots = \Omega_N = 1$ and $\gamma_{\text{th}} = 0$ dB.

3.4 Numerical Results

The analytical expressions derived in the previous sections are now evaluated for sample scenarios. In all cases, simulations were performed that fully validate our analysis. For illustration purposes, we set $\gamma_{\text{th}} = 0$ dB, $\Omega_1 = \cdots = \Omega_N = 1$, and $f_{m,1} = \cdots = f_{m,N} = f_m$.

Figs. 3.2 and 3.3 show the normalized LCR (N_{Γ_e}/f_m) and the normalized AFD ($T_{\Gamma_e} \times f_m$) versus the transmit SNR, respectively, for a number of hops ranging from two to five. The exact curves were obtained from (3.9) and (3.10), and the asymptotic ones, from (3.12) and (3.13). Note how our simple asymptotic expressions provide an excellent fit to the exact performance at medium to high SNR. Also, note how the performance in this range is affected by the number of hops N. As N increases, the outage rate (LCR) increases as well, although the penalty due to an extra hop diminishes. On the other hand, the outage duration (AFD) is barely affected, as its asymptote does not depend on N whatsoever. In short, as the number of hops increases, outage events become more likely (for any fixed time) and more frequent (over time) in the same proportion, causing the average outage duration to remain constant.

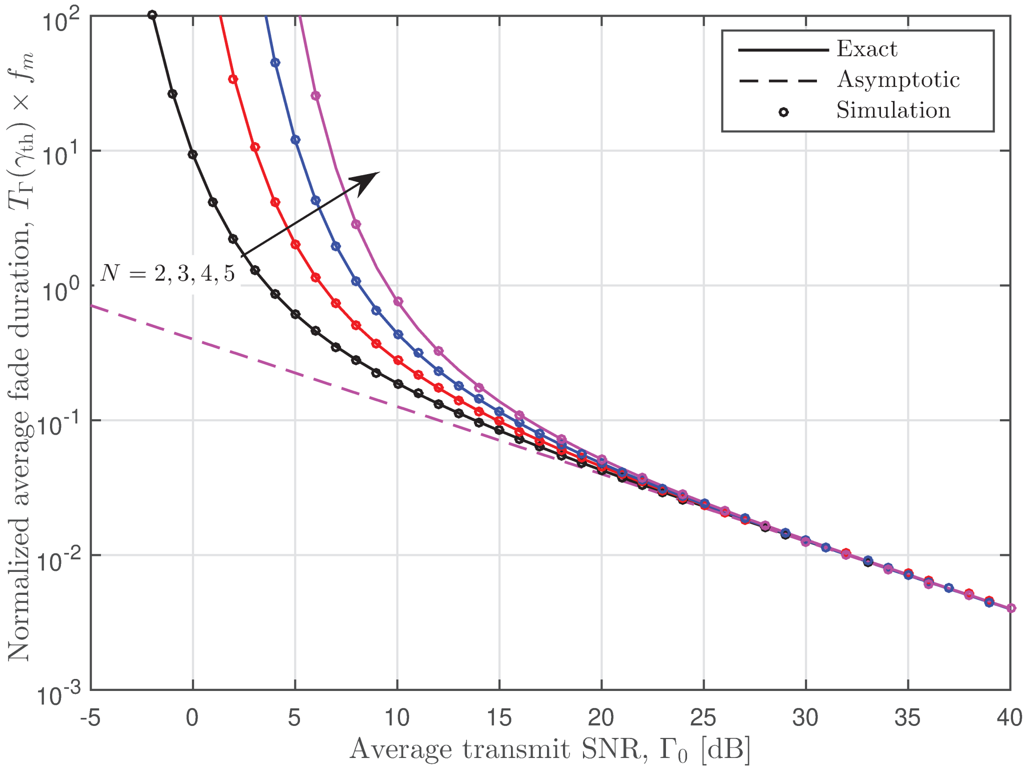

Figure 3.3 – Average fade duration of variable-gain amplify-and-forward relaying with multiple hops: $\Omega_1 = \cdots = \Omega_N = 1$ and $\gamma_{\text{th}} = 0$ dB.

3.5 Conclusions

In this work, we investigated the second-order statistics of a variable-gain amplify-and-forward relaying system containing an arbitrary number of hops subject to Rayleigh fading. We obtained analytical expressions for the LCR and AFD based on exact and asymptotic approaches. As a byproduct, we also obtained a novel asymptotic expression for the outage probability. In particular, our closed-form asymptotic results revealed that the diversity gain is independent of the number of hops, namely, 1/2 for both LCR and AFD, and unity for the outage probability. In turn, the coding gains proved quite different. In the special case where all hops are identically distributed, the coding gain of the LCR is inversely proportional to the squared number of hops, while that of the AFD remains unchanged. All our formulas were corroborated by Monte Carlo simulations.

The derived solutions for LCR and AFD can be used to assist in the design of key system parameters such as packet length, symbol rate, and transmission time interval. A next step is to explore channel models other than Rayleigh fading, thereby allowing for various degrees of freedom.

Appendix 3.A Proof of Theorem 3.1

In this section, we describe the procedure to determine the LCR for the exact end-to-end SNR using Brennan's method.

Note from Rice's formula in (3.6) that the joint PDF of the SNR and its time derivative is required. By rewriting this joint PDF, (3.6) can be expressed as

$$N_{\Gamma_e}\left(\gamma_{\text{th}}\right) = \int_0^\infty \dot{\gamma}_e f_{\dot{\Gamma}_e|\Gamma_e}\left(\dot{\gamma}_e|\,\gamma_{\text{th}}\right) f_{\Gamma_e}\left(\gamma_{\text{th}}\right)\, d\dot{\gamma}_e, \tag{3.18}$$

where $f_{\dot{\Gamma}_e|\Gamma_e}\left(\cdot|\cdot\right)$ is the conditional PDF of $\dot{\Gamma}_e$ given Γ_e and $f_{\Gamma_e}\left(\cdot\right)$ is the PDF of Γ_e. To our best knowledge, there is no closed-form solution for $f_{\Gamma_e}\left(\cdot\right)$. Thus, we rely on Brennan's geometric approach to determine this PDF. In [26], Brennan proposed a method to calculate the PDF of a generic sum of N non-negative, arbitrarily correlated, arbitrarily distributed RVs. In this work, we consider the sum

$$Z = \sum_{n=1}^{N} Z_n, \tag{3.19}$$

where $\{Z_n\}_{n=1}^{N}$ are non-negative, independent RVs. From Brennan's results, the PDF of Z is given by [26], [30]

$$f_Z\left(z\right) = \int_0^z \int_0^{z-z_N} \cdots \int_0^{z-\sum_{n=3}^{N} z_n} f_{Z_1}\left(z - \sum_{n=2}^{N} z_n\right) \prod_{n=2}^{N} f_{Z_n}\left(z_n\right) dz_2 \ldots dz_{N-1} dz_N. \tag{3.20}$$

To apply (3.20) to our problem, we need to rewrite the exact SNR given in (3.2) so as to fit the format in (3.19). This can be accomplished by restating (3.2) as [27]

$$\left(1 + \Gamma_e^{-1}\right) = \prod_{n=1}^{N} \left(1 + \Gamma_n^{-1}\right) \tag{3.21}$$

and by taking the natural logarithm on both sides of the equation, so that

$$Z \triangleq \ln\left(1 + \Gamma_e^{-1}\right) \tag{3.22}$$

$$Z_n \triangleq \ln\left(1 + \Gamma_n^{-1}\right). \tag{3.23}$$

After simple transformations of variables, we obtain the mappings that matter to us:

$$f_{\Gamma_e}\left(\gamma_e\right) = \frac{1}{\gamma_e\left(\gamma_e+1\right)} f_Z\left(\ln\left(1+\gamma_e^{-1}\right)\right) \tag{3.24}$$

$$f_{Z_n}\left(z_n\right) = \frac{e^{z_n}}{\left(e^{z_n}-1\right)^2} f_{\Gamma_n}\left(\frac{1}{e^{z_n}-1}\right). \tag{3.25}$$

The PDF of Γ_e is obtained by replacing (3.4), (3.24), and (3.25) into (3.20), yielding

$$f_{\Gamma_e}(\gamma_e) = \int_0^{\ln\left(1+\frac{1}{\gamma_e}\right)} \int_0^{\ln\left(1+\frac{1}{\gamma_e}\right)-z_N} \cdots \int_0^{\ln\left(1+\frac{1}{\gamma_e}\right)-\sum_{n=3}^{N} z_n} \times \frac{e^{-\frac{\gamma_e}{\Gamma_0\Omega_1\left[-\gamma_e+(1+\gamma_e)e^{-\sum_{n=2}^{N} z_n}\right]}-\sum_{n=2}^{N}\frac{1}{\Gamma_0\Omega_n(e^{z_n}-1)}-2z_n}}{\Gamma_0^N\left(1+\gamma_e-\gamma_e e^{\sum_{n=2}^{N} z_n}\right)^2 \Omega_1 \prod_{n=2}^{N}\Omega_n\left(e^{z_n}-1\right)^2} dz_2 \ldots dz_{N-1} dz_N. \quad (3.26)$$

Now, to find the conditional PDF $f_{\dot{\Gamma}_e|\Gamma_e}(\cdot|\cdot)$ also required in (3.18), we first determine the time derivative of Γ_e:

$$\dot{\Gamma}_e = \frac{\partial \Gamma_e}{\partial t} = \frac{\partial}{\partial t}\left[\prod_{n=1}^{N}\left(1+\Gamma_n^{-1}\right)-1\right]^{-1} = -\frac{\frac{\partial}{\partial t}\left[\prod_{n=1}^{N}\left(1+\Gamma_n^{-1}\right)-1\right]}{\left[\prod_{n=1}^{N}\left(1+\Gamma_n^{-1}\right)-1\right]^2}$$
$$= \frac{\prod_{n=1}^{N}\left(1+\Gamma_n^{-1}\right)\sum_{n=1}^{N}\frac{\dot{\Gamma}_n}{\Gamma_n^2\left(1+\Gamma_n^{-1}\right)}}{\left[\prod_{n=1}^{N}\left(1+\Gamma_n^{-1}\right)-1\right]^2}. \quad (3.27)$$

By substituting $\dot{\Gamma}_n = \partial\Gamma_n/\partial t = 2\sqrt{\Gamma_0}\sqrt{\Gamma_n}\dot{\alpha}_n$ (from (3.3)), $\Gamma_n = (e^{Z_n}-1)^{-1}$ (from (3.23)), and $Z_1 = \ln(1+\Gamma_e^{-1}) - \sum_{n=2}^{N} Z_n$ (from (3.20) and (3.22)) into (3.27), we obtain

$$\dot{\Gamma}_e = 2\sqrt{\Gamma_0\Gamma_e}\left\{e^{-\sum_{n=2}^{N}\frac{Z_n}{2}}\left[1+\Gamma_e\left(1-e^{\sum_{n=2}^{N} Z_n}\right)\right]^{\frac{3}{2}}\dot{\alpha}_1 + \sqrt{\Gamma_e}\left(1+\Gamma_e\right)\sum_{n=2}^{N} e^{-Z_n}\left(e^{Z_n}-1\right)^{\frac{3}{2}}\dot{\alpha}_n\right\}. \quad (3.28)$$

Thus, it becomes apparent that $\dot{\Gamma}_e$ is a weighted sum of $\{\dot{\alpha}_n\}_{n=1}^{N}$, which are zero-mean Gaussian RVs with variances given by (3.5). As a result, conditioned on Γ_e and $Z_2, \ldots, Z_N$, $\dot{\Gamma}_e$ is also a Gaussian RV with zero mean and variance computed from (3.28) as

$$\sigma^2_{\dot{\Gamma}_e|\Gamma_e,Z_2,\ldots,Z_N} = 4\Gamma_0\gamma_e\left\{e^{-\sum_{n=2}^{N} z_n}\left[1+\gamma_e\left(1-e^{\sum_{n=2}^{N} z_n}\right)\right]^3\sigma^2_{\dot{\alpha}_1} + \gamma_e(1+\gamma_e)^2\sum_{n=2}^{N} e^{-2z_n}\left(e^{z_n}-1\right)^3\sigma^2_{\dot{\alpha}_n}\right\}. \quad (3.29)$$

By exploiting the Gaussianity of $f_{\dot{\Gamma}_e|\Gamma_e,Z_2,\ldots,Z_N}(\cdot|\cdot,\ldots,\cdot)$, we can compute the integral in (3.18) with respect to $\dot{\gamma}_e$, thus yielding [31]

$$\int_0^\infty \dot{\gamma}_e f_{\dot{\Gamma}_e|\Gamma_e,Z_2,\ldots,Z_N}(\cdot|\cdot,\ldots,\cdot)\, d\dot{\gamma}_e = \sqrt{\frac{\sigma^2_{\dot{\Gamma}_e|\Gamma_e,Z_2,\ldots,Z_N}}{2\pi}}. \quad (3.30)$$

By substituting (3.26) and (3.30) into (3.18), an exact analytical expression for the LCR is obtained as in (3.9), which completes the proof.

Appendix 3.B Proof of Theorem 3.3

In the high-SNR regime ($\Gamma_0 \to \infty$), the exact end-to-end SNR in (3.2) can be approximated by [5]

$$\begin{aligned} \Gamma &\triangleq \left[\sum_{n=1}^{N} \Gamma_n^{-1}\right]^{-1} \\ &\geq \Gamma_e \\ &\simeq \Gamma_e, \quad \text{for} \quad \Gamma_0 \to \infty. \end{aligned} \tag{3.31}$$

Using an approach similar to that in Appendix 3.A, the LCR for the approximate end-to-end SNR given in (3.31) can be rewritten as

$$N_\Gamma\left(\gamma_{\text{th}}\right) = \int_0^\infty \dot{\gamma}\, f_{\dot{\Gamma}|\Gamma}\left(\dot{\gamma}|\gamma_{\text{th}}\right) f_\Gamma\left(\gamma_{\text{th}}\right)\, d\dot{\gamma}, \tag{3.32}$$

where $f_{\dot{\Gamma}|\Gamma}\left(\cdot|\cdot\right)$ is the PDF of $\dot{\Gamma}$ conditioned on Γ, and $f_\Gamma\left(\cdot\right)$ is the PDF of Γ. Accordingly, to express (3.31) as in (3.19), we introduce the relationships

$$Z \triangleq \Gamma^{-1} \tag{3.33}$$

$$Z_n \triangleq \Gamma_n^{-1}, \tag{3.34}$$

with the respective PDFs being related as $f_\Gamma\left(\gamma\right) = \gamma^{-2} f_Z\left(\gamma^{-1}\right)$ and $f_{Z_n}\left(z_n\right) = z_n^{-2} f_{\Gamma_n}\left(z_n^{-1}\right)$. Using this and (3.4) into (3.20), the PDF of Γ is then obtained as

$$\begin{aligned} f_\Gamma\left(\gamma\right) =& \frac{1}{\Gamma_0^N} \int_0^{\frac{1}{\gamma}} \int_0^{\frac{1}{\gamma}-z_N} \cdots \int_0^{\frac{1}{\gamma}-\sum_{n=2}^{N} z_n} \\ &\times \frac{\exp\left[-\frac{\gamma}{\Omega_1 \Gamma_0 \left(1-\gamma \sum_{n=2}^{N} z_n\right)} - \sum_{n=2}^{N} \frac{1}{\Gamma_0 \Omega_n z_n}\right]}{\left(1-\gamma\sum_{n=2}^{N} z_n\right)^2 \Omega_1 \prod_{n=2}^{N} z_n^2 \Omega_n} dz_2 \dots dz_{N-1} dz_N. \end{aligned} \tag{3.35}$$

Now, to obtain $f_{\dot{\Gamma}|\Gamma}\left(\cdot|\cdot\right)$, we first determine the time derivative of Γ from (3.31) as

$$\begin{aligned} \dot{\Gamma} = \frac{\partial \Gamma}{\partial t} &= \frac{\sum_{n=1}^{N} \Gamma_n^{-2} \dot{\Gamma}_n}{\left(\sum_{n=1}^{N} \Gamma_n^{-1}\right)^2} = \frac{2\sqrt{\Gamma_0} \sum_{n=1}^{N} Z_n^{\frac{3}{2}} \dot{\alpha}_n}{\left(\sum_{n=1}^{N} Z_n\right)^2} \\ &= 2\sqrt{\Gamma_0 \Gamma}\left[\left(1-\Gamma\sum_{n=2}^{N} Z_n\right)^{\frac{3}{2}} \dot{\alpha}_1 + \Gamma^{\frac{3}{2}} \sum_{n=2}^{N} Z_n^{\frac{3}{2}} \dot{\alpha}_n\right], \end{aligned} \tag{3.36}$$

since $\dot{\Gamma}_n = \partial \Gamma_n / \partial t = 2\sqrt{\Gamma_0}\sqrt{\Gamma_n}\dot{\alpha}_n$ (from (3.3)), $\Gamma_n = Z_n^{-1}$, and $Z_1 = \Gamma^{-1} - \sum_{n=2}^{N} Z_n$ (from (3.20)). Similarly to (3.29), it follows that $f_{\dot{\Gamma}|\Gamma, Z_2 \dots, Z_N}\left(\cdot|\cdot, \dots, \cdot\right)$ also corresponds to the PDF of a Gaussian random variable with zero mean and variance given by

$$\sigma^2_{\dot{\Gamma}|\Gamma, Z_2, \dots, Z_N} = 4\Gamma_0 \gamma\left[\left(1-\gamma\sum_{n=2}^{N} z_n\right)^3 \sigma_{\dot{\alpha}_1} + \gamma^3 \sum_{n=2}^{N} z_n^3 \sigma_{\dot{\alpha}_n}\right]. \tag{3.37}$$

By integrating (3.32) with respect to $\dot{\gamma}$ while taking into account the Gaussianity of $f_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N}(\cdot|\cdot,\ldots,\cdot)$, we have [31]

$$\int_0^{\infty} \dot{\gamma}\, f_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N}(\cdot|\cdot,\ldots,\cdot)\, d\dot{\gamma} = \sqrt{\frac{\sigma^2_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N}}{2\pi}}. \tag{3.38}$$

Then, by applying (3.35) and (3.38) into (3.32), we obtain an approximate analytical expression for the LCR of a multi-hop VG-AF relaying system over Rayleigh fading as

$$\begin{aligned} N_{\Gamma}(\gamma_{\text{th}}) =& \frac{\sqrt{2\pi\gamma_{\text{th}}}}{\Gamma_0^{N-1/2}} \int_0^{\frac{1}{\gamma_{\text{th}}}} \int_0^{\frac{1}{\gamma_{\text{th}}}-z_N} \cdots \int_0^{\frac{1}{\gamma_{\text{th}}}-\sum_{n=3}^{N} z_n} \\ &\times \sqrt{f_{m,1}^2 \Omega_1 \left(1-\gamma_{\text{th}}\sum_{n=2}^{N} z_n\right)^3 + \gamma_{\text{th}}^3 \sum_{n=2}^{N} f_{m,n}^2 \Omega_n z_n^3} \\ &\times \frac{\exp\left[-\frac{\gamma_{\text{th}}}{\Omega_1\Gamma_0\left(1-\gamma_{\text{th}}\sum_{n=2}^{N} z_n\right)} - \sum_{n=2}^{N}\frac{1}{\Gamma_0\Omega_n z_n}\right]}{\left(1-\gamma_{\text{th}}\sum_{n=2}^{N} z_n\right)^2 \Omega_1 \prod_{n=2}^{N} z_n^2 \Omega_n} dz_2\ldots dz_{N-1} dz_N. \end{aligned} \tag{3.39}$$

In order to calculate the AFD of Γ as in (3.7), we need to find the associated outage probability. By substituting (3.35) into (3.8), we obtain an approximate analytical expression for the outage probability of a multi-hop VG-AF relaying system over Rayleigh fading as

$$\begin{aligned} P_{\Gamma}(\gamma_{\text{th}}) =& \frac{1}{\Gamma_0^N} \int_0^{\frac{1}{\gamma_{\text{th}}}} \int_0^{\frac{1}{\gamma}} \int_0^{\frac{1}{\gamma}-z_N} \cdots \int_0^{\frac{1}{\gamma}-\sum_{n=2}^{N} z_n} \frac{\exp\left[-\frac{\gamma}{\Omega_1\Gamma_0\left(1-\gamma\sum_{n=2}^{N} z_n\right)} - \sum_{n=2}^{N}\frac{1}{\Gamma_0\Omega_n z_n}\right]}{\left(1-\gamma\sum_{n=2}^{N} z_n\right)^2 \Omega_1 \prod_{n=2}^{N} z_n^2\Omega_n} \\ &\times dz_2\ldots dz_{N-1} dz_N d\gamma. \end{aligned} \tag{3.40}$$

The final step is to derive asymptotic representations for (3.39), (3.40), and the corresponding AFD as $\Gamma_0 \to \infty$. In such cases, a popular approach is to replace the exponential function in the integrand by its Maclaurin series expansion, drop the terms beyond the second one (i.e., $\exp(-x_i) \simeq 1 - x_i$), and simplify the integral. But this approach alone does not work in our case; the resulting integrals turn out to diverge, since their integrands contain poles on each integration limit. To overcome this, we split the integration interval into two parts with only one pole per interval, and then some mathematical manipulations are made to ensure the integral will converge when the exponential function is approximated. But applying this method in (3.39) and (3.40) directly for an arbitrary number of hops (N) proves quite involved. Rather, we apply the method for two and three hops first, and then, building on these results, we generalize them for any number of hops. Next, to gain insight into the general solution, we start with the two-hop scenario.

The LCR for the dual-hop case is obtained by setting $N = 2$ in (3.39), yielding

$$N_\Gamma(\gamma_{\text{th}}) = \frac{\sqrt{2\pi}\sqrt{\gamma_{\text{th}}}}{\Omega_1\Omega_2\Gamma_0^{3/2}} \int_0^{\frac{1}{\gamma_{\text{th}}}} \frac{\sqrt{(1 - z_2\gamma_{\text{th}})^3\Omega_1 f_{m,1}^2 + (z_2\gamma_{\text{th}})^3\Omega_2 f_{m,2}^2}}{(1 - z_2\gamma_{\text{th}})^2 z_2^2} \times \exp\left[-\frac{\gamma_{\text{th}}}{(1 - z_2\gamma_{\text{th}})\Gamma_0\Omega_1} - \frac{1}{z_2\Gamma_0\Omega_2}\right] dz_2. \tag{3.41}$$

A high-SNR asymptotic expression for (3.41) cannot be obtained by directly using the Maclaurin representation of the exponential function and dropping the terms beyond the first, since the resulting integral will diverge due to the poles on $z_2 = 0$ and $z_2 = 1/\gamma_{\text{th}}$. To overcome this, as mentioned before, we split the integration interval into two parts with only one pole per interval, and then a change of variable is made to ensure the integral will converge when the exponential function is approximated. Namely, after the change of variable $u \triangleq z_2\gamma_{\text{th}}$, (3.41) can be split as

$$N_\Gamma(\gamma_{\text{th}}) = \frac{\sqrt{2\pi}\gamma_{\text{th}}^{3/2}}{\Omega_1\Omega_2\Gamma_0^{3/2}} (I_1 + I_2), \tag{3.42}$$

where

$$I_1 \triangleq \int_0^{1/2} \frac{\sqrt{(1-u)^3\Omega_1 f_{m,1}^2 + u^3\Omega_2 f_{m,2}^2}}{(1-u)^2 u^2} \exp\left[-\frac{\gamma_{\text{th}}}{\Gamma_0\Omega_1(1-u)} - \frac{\gamma_{\text{th}}}{\Gamma_0\Omega_2 u}\right] du \tag{3.43}$$

$$I_2 \triangleq \int_{1/2}^{1} \frac{\sqrt{(1-u)^3\Omega_1 f_{m,1}^2 + u^3\Omega_2 f_{m,2}^2}}{(1-u)^2 u^2} \exp\left[-\frac{\gamma_{\text{th}}}{\Gamma_0\Omega_1(1-u)} - \frac{\gamma_{\text{th}}}{\Gamma_0\Omega_2 u}\right] du. \tag{3.44}$$

After two other changes, $v \triangleq \frac{(\gamma_{\text{th}} - 2u\gamma_{\text{th}})}{(\Gamma_0\Omega_2 u)}$ in (3.43) and $w \triangleq \frac{(-\gamma_{\text{th}} + 2\gamma_{\text{th}} u)}{(\Gamma_0\Omega_1 - \Gamma_0\Omega_1 u)}$ in (3.44), we obtain

$$I_1 = \frac{\Gamma_0\Omega_2}{\gamma_{\text{th}}} \int_0^\infty \frac{(2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^2}{(\gamma_{\text{th}} + v\Gamma_0\Omega_2)^2} \sqrt{\frac{(\gamma_{\text{th}} + v\Gamma_0\Omega_2)^3\Omega_1 f_{m,1}^2}{(2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^3} + \frac{\gamma_{\text{th}}^3\Omega_2 f_{m,2}^2}{(2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^3}} \times \exp\left[-v - \frac{2\gamma_{\text{th}}}{\Gamma_0\Omega_2} - \frac{2\gamma_{\text{th}}^2 + v\gamma_{\text{th}}\Gamma_0\Omega_2}{\Gamma_0\Omega_1(\gamma_{\text{th}} + v\Gamma_0\Omega_2)}\right] dv \tag{3.45}$$

$$I_2 = \frac{\Gamma_0\Omega_1}{\gamma_{\text{th}}} \int_0^\infty \frac{(2\gamma_{\text{th}} + w\Gamma_0\Omega_1)^2}{(\gamma_{\text{th}} + w\Gamma_0\Omega_1)^2} \sqrt{\frac{\gamma_{\text{th}}^3\Omega_1 f_{m,1}^2}{(2\gamma_{\text{th}} + w\Gamma_0\Omega_1)^3} + \frac{(\gamma_{\text{th}} + w\Gamma_0\Omega_1)^3\Omega_2 f_{m,2}^2}{(2\gamma_{\text{th}} + w\Gamma_0\Omega_1)^3}} \times \exp\left[-w - \frac{2\gamma_{\text{th}}}{\Gamma_0\Omega_1} - \frac{2\gamma_{\text{th}}^2 + w\gamma_{\text{th}}\Gamma_0\Omega_1}{\Gamma_0\Omega_2(\gamma_{\text{th}} + w\Gamma_0\Omega_1)}\right] dw. \tag{3.46}$$

Since $1/\Gamma_0 \to 0$ as $\Gamma_0 \to \infty$, we can apply the Maclaurin series to the whole integrand in (3.45) and (3.46), taking only the first term to find the asymptotic behavior:

$$I_1 \simeq \frac{\Omega_2\sqrt{\Omega_1 f_{m,1}^2}\Gamma_0}{\gamma_{\text{th}}} \int_0^\infty \exp[-v]\, dv = \frac{\Omega_2\sqrt{\Omega_1 f_{m,1}^2}\Gamma_0}{\gamma_{\text{th}}} \tag{3.47}$$

$$I_2 \simeq \frac{\Omega_1\sqrt{\Omega_2 f_{m,2}^2}\Gamma_0}{\gamma_{\text{th}}} \int_0^\infty \exp[-w]\, dw = \frac{\Omega_1\sqrt{\Omega_2 f_{m,2}^2}\Gamma_0}{\gamma_{\text{th}}}. \tag{3.48}$$

The substitution of (3.47) and (3.48) into (3.42) yields a high-SNR asymptotic expression for the LCR of a dual hop VG-AF relaying system:

$$N_{\Gamma_e}(\gamma_{\rm th}) \simeq \left[\frac{1}{2\pi\gamma_{\rm th}\left(\frac{f_{m,1}}{\sqrt{\Omega_1}}+\frac{f_{m,2}}{\sqrt{\Omega_2}}\right)^2}\Gamma_0\right]^{-1/2}. \quad (3.49)$$

The procedure described in (3.41)–(3.49) can also be applied for the outage probability. For $N = 2$, we eventually obtain

$$P_{\Gamma_e}(\gamma_{\rm th}) \simeq \left[\frac{1}{\gamma_{\rm th}\left(\frac{1}{\Omega_1}+\frac{1}{\Omega_2}\right)}\Gamma_0\right]^{-1}. \quad (3.50)$$

Although lenghty and tedious, the derivation of the asymptotes for $N = 3$ follows a similar rationale as the one detailed here for $N = 2$. For brevity, we shall omit the steps. These are the final expressions:

$$N_{\Gamma_e}(\gamma_{\rm th}) \simeq \left[\frac{1}{2\pi\gamma_{\rm th}\left(\frac{f_{m,1}}{\sqrt{\Omega_1}}+\frac{f_{m,2}}{\sqrt{\Omega_2}}+\frac{f_{m,3}}{\sqrt{\Omega_3}}\right)^2}\Gamma_0\right]^{-1/2} \quad (3.51)$$

$$P_{\Gamma_e}(\gamma_{\rm th}) \simeq \left[\frac{1}{\gamma_{\rm th}\left(\frac{1}{\Omega_1}+\frac{1}{\Omega_2}+\frac{1}{\Omega_3}\right)}\Gamma_0\right]^{-1}. \quad (3.52)$$

Building on the results in (3.49)–(3.52), we arrive at the general solution given in (3.12)–(3.14).

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4 Impact of Channel Diversity on the Second-Order Statistics of Multi-Hop Transparent Relaying

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Abstract

In this paper, we investigate the high-SNR behavior of multi-hop variable-gain amplify-and-forward relaying systems undergoing Nakagami-m fading. In particular, we derive simple and closed-form high-SNR expressions for the level crossing rate and average channel duration, whereby the impact of the channel diversity on the second-order statistics is shown straightforwardly. The derived expressions reveal that the system's high-SNR performance is ruled by the hop enduring the most severe fading, corresponding to the lowest fading parameter m^*. More specifically, we found that the diversity gain of the level crossing rate is $m^* - 1/2$, whereas that of the average fading channel remains unaffected by the channel diversity, being equal to 1/2.

4.1 Introduction

Over the past two decades, relay-based communication has been established as a key technology for coverage extension and reliability improvement of current and next-generation wireless networks, in which the mobility of terminals is an inherent feature. In this context, an in-depth characterization of the dynamic nature of the wireless relay channel can be accomplished by analyzing its second-order statistics, such as level crossing rate (LCR) and average fade duration (AFD). These two metrics are very useful in mobile communication systems by relating the time rate of change of the received signal to the signal level and velocity of the mobile [1]. In particular, LCR and AFD is especially important for the analysis of burst error statistics, the choice of suitable channel coding

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schemes, the transmission time interval, and the optimization of the interleaver size[1], [2].

Despite their practical relevance, few works have investigated second-order statistics for multi-hop relaying networks [2]–[5]. Among them, only [2] and [5] analyze the LCR and AFD with channel diversity (by using the Nakagami-m fading model), for the fixed-gain (FG) variant of the amplify-and-forward (AF) protocol. Besides, those works disregard the accumulated noise at the destination by either analyzing only the end-to-end channel gain (i.e., a cascaded product channel) [2] or approximating the resulting noise at the destination by a Gaussian term (thus neglecting the impact of the channel gains throughout the multiple hops) [5]. Although [2] and [5] are remarkable contributions for multi-hop FG-AF relaying systems, their exact or approximate closed-form solutions for the LCR and AFD bring little or no insights in how the performance is affected by the channel diversity (characterized by the Nakagami-m index). Furthermore, the analysis of the LCR and AFD for the variable-gain (VG) variant of the AF protocol with multiple hops, as well the impact of the channel diversity on those statistics, is still open for investigation. In this work, we aim to shed light on the second-order statistics of multi-hop VG-AF relaying systems under Nakagami-m fading by deriving simple closed-form solutions at high signal-to-noise ratio (SNR). To do so, we consider the end-to-end SNR (which includes the accumulated noise). Our solutions provide clear insights into how the channel parameters, node mobility, and number of hops impact the diversity gain and coding gain, thus being helpful for the system design and analysis.

4.2 System Model

Consider a multi-hop AF relaying system composed of N hops, as depicted in Fig. 4.1, in which the communication process between the source S and the destination D occurs through $N-1$ VG-AF relays $\{R_n\}_{n=1}^{N-1}$. There is no direct link between S and D. Each relay R_n receives the information signal coming from the preceding relay R_{n-1} (or from S, in the case of R_1), amplifies the signal by a factor of G_n, and then forwards it to the next node. This process is carried on hop by hop, up to the destination D. All terminals, including the source, are assumed to transmit information with same power P_{T}. The amplification factor at the nth relay is given by $G_n^2 = \left(\alpha_n^2 + \Gamma_0^{-1}\right)^{-1}$ [6, eq. (9)], where α_n is the channel coefficient of the nth hop, and $\Gamma_0 \triangleq P_{\mathrm{T}}/N_0$ is the transmit SNR at the source and relays, with N_0 being the mean power of the additive white Gaussian noise at the relays and destination. In such a case, the exact end-to-end SNR is obtained as $\Gamma_e = [\prod_{n=1}^{N}(1+\Gamma_n^{-1}) - 1]^{-1}$, where $\Gamma_n = \Gamma_0\alpha_n^2$ is the received SNR at the nth hop [7].

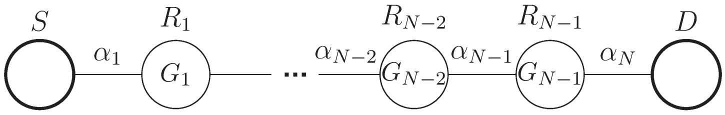

Figure 4.1 – Variable-gain amplify-and-forward relaying system with multiple non-identically distributed hops.

A widely used upper bound on Γ_e is [7]

$$\Gamma \triangleq \left[\sum_{n=1}^{N} \frac{1}{\Gamma_n}\right]^{-1} \simeq \Gamma_e, \quad \text{for} \quad \Gamma_0 \to \infty, \tag{4.1}$$

where "$\simeq$" denotes asymptotic equivalence, indicating that Γ increasingly approaches Γ_e at high SNR. The upper bound in (4.1) serves as a benchmark for practical transparent relaying systems and will provide a basis for the high-SNR analysis attained in the next section.

We assume that the channel coefficients $\{\alpha_n\}_{n=1}^{N}$ of the hops are independent and non-necessarily identically Nakagami-m random variables (RV). Hence, each received SNR Γ_n is distributed according to a gamma distribution given by $f_{\Gamma_n}(\gamma_n) = \frac{m_n^{m_n}\gamma_n^{m_n-1}}{\bar{\Gamma}_n^{m_n}\Gamma(m_n)} \exp\left[-\frac{m_n\gamma_n}{\bar{\Gamma}_n}\right]$, where $m_n > 0.5$ is the Nakagami-m fading parameter, $\Gamma(\cdot)$ is the gamma function, $\bar{\Gamma}_n = \mathrm{E}[\Gamma_n] = \Gamma_0\Omega_n$ is the average received SNR at the nth hop, and $\Omega_n = \mathrm{E}[\alpha_n^2]$ is the corresponding average channel power gain, with $\mathrm{E}[\cdot]$ denoting statistical expectation. Finally, considering an isotropic propagation environment, it is known that the time derivative of the channel amplitude $\dot{\alpha}_n$ is independent of α_n itself, and follows a zero-mean Gaussian distribution with variance $\sigma_{\dot{\alpha}_n}^2 = \pi^2 f_{m,n}^2 \Omega_n / m_n$, where $f_{m,n}$ denotes the maximum Doppler frequency shift at the nth hop. This shift is proportional to the relative speed between the communicating nodes.

4.3 Second-Order Statistics

4.3.1 Preliminaries

A system outage occurs whenever the end-to-end SNR Γ_e drops below a certain threshold level γ_{th}. The LCR $N_{\Gamma_e}(\gamma_{\mathrm{th}})$ gives the average temporal rate at which outage events take place, which can be obtained from Rice's formula [1]

$$N_{\Gamma_e}(\gamma_{\mathrm{th}}) = \int_0^{\infty} \dot{\gamma}\, f_{\Gamma_e,\dot{\Gamma}_e}(\gamma_{\mathrm{th}}, \dot{\gamma})\; d\dot{\gamma} \tag{4.2}$$

in terms of the joint probability density function (PDF) $f_{\Gamma_e,\dot{\Gamma}_e}(\cdot,\cdot)$ of Γ_e and its time derivative $\dot{\Gamma}_e$. The AFD $T_{\Gamma_e}(\gamma_{\mathrm{th}})$, in turn, gives the average duration of an outage event, being obtained as [1]

$$T_{\Gamma_e}(\gamma_{\mathrm{th}}) = \frac{P_{\Gamma_e}(\gamma_{\mathrm{th}})}{N_{\Gamma_e}(\gamma_{\mathrm{th}})}, \tag{4.3}$$

where

$$P_{\Gamma_e}(\gamma_{\text{th}}) = \int_0^{\gamma_{\text{th}}} f_{\Gamma_e}(\gamma)\, d\gamma \tag{4.4}$$

is the outage probability, with $f_{\Gamma_e}(\cdot)$ denoting the PDF of Γ_e.

4.3.2 High-SNR Analysis

It is worth noting that there is no closed-form exact solution for either the first- or second-order statistics of VG-AF relaying systems. Regarding the outage probability, the lowest complexity solution is a two-fold integral-form expressions provided in [8] for an arbitrary number of hops, when the channels undergo Rayleigh, Rice, or Nakagami-m fading. Regarding the LCR and AFD, a two-fold integral-form solution was provided in [9] considering the existence of a direct link between source and destination, but for a dual-hop system only. In light of the mathematical complexity associated with VG-AF relaying, and aiming at better insights into the impact of each channel parameter on the system performance, we develop a high-SNR analysis from which simple closed-form expressions are attained for the LCR, AFD, and, as a byproduct, outage probability. Our solutions apply to an arbitrary number of hops and any fading parameter $m_n > 0.5$.

In the high-SNR regime ($\Gamma_0 \to \infty$), each investigated metric can be expressed asymptotically as $k_{\Gamma_e}(\gamma_{\text{th}}) \simeq (c_k \Gamma_0)^{-d_k}$, $k \in \{P, N, T\}$, where d_k is called the diversity gain, and c_k is called the coding gain [1]. In the following theorem, we provide simple closed-form high-SNR expressions for the LCR, AFD and outage probability of a VG-AF relaying system with multiple hops operating under Nakagami-m fading:

Theorem 4.1 *At the high SNR regime, the LCR, AFD, and outage probability of a multi-hop VG-AF relaying system operating under Nakagami-m fading are given, respectively, as*

$$N_{\Gamma_e}(\gamma_{th}) \simeq \left[\frac{1}{m^* \gamma_{\text{th}}} \left(\frac{\sqrt{2\pi}}{\Gamma(m^*)} \sum_{\substack{n=1 \\ m_n = m^*}}^{N} f_{m,n} \Omega_n^{\frac{1}{2}-m^*} \right)^{\frac{1}{1/2-m^*}} \Gamma_0 \right]^{-\left(m^* - \frac{1}{2}\right)}, \tag{4.5}$$

$$T_{\Gamma_e}(\gamma_{th}) \simeq \left[\frac{2\pi\, m^*}{\gamma_{\text{th}}} \left(\frac{\sum_{\substack{n=1 \\ m_n = m^*}}^{N} \Omega_n^{-m^*}}{\sum_{\substack{n=1 \\ m_n = m^*}}^{N} f_{m,n} \Omega_n^{1/2-m^*}} \right)^{-2} \Gamma_0 \right]^{-\frac{1}{2}}, \tag{4.6}$$

$$P_{\Gamma_e}(\gamma_{th}) \simeq \left[\frac{1}{m^* \gamma_{\text{th}}} \left(\frac{1}{\Gamma(1+m^*)} \sum_{\substack{n=1 \\ m_n = m^*}}^{N} \Omega_n^{-m^*} \right)^{-\frac{1}{m^*}} \Gamma_0 \right]^{-m^*}. \tag{4.7}$$

where $m^* = \min_{n \in \{1,\ldots,N\}} m_n$.

Proof See Appendix 4.A.

Corollary 4.1 *Assuming a homogeneous scenario in which* $\{m_n\}_{n=1}^N \triangleq m$, $\{\Omega_n\}_{n=1}^N \triangleq \Omega$, *and* $\{f_{m,n}\}_{n=1}^N \triangleq f_m$, (4.5)–(4.7) *simplify, respectively, to*

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{\Omega}{m\gamma_{\text{th}}}\left(\frac{\sqrt{2\pi}Nf_m}{\Gamma(m)}\right)^{\frac{1}{1/2-m}}\Gamma_0\right]^{-(m-1/2)}, \tag{4.8}$$

$$T_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left(\frac{2\pi f_m^2 m\Omega}{\gamma_{\text{th}}}\Gamma_0\right)^{-1/2}, \tag{4.9}$$

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \left[\frac{\Omega}{m\gamma_{\text{th}}}\left(\frac{N}{\Gamma(1+m)}\right)^{-\frac{1}{m}}\Gamma_0\right]^{-m}. \tag{4.10}$$

4.3.3 Remarks

The asymptotic expressions in (4.5)–(4.7) reveal how the diversity and coding gains of each investigated metric are influenced by the fading parameters $\{m_n\}_{n=1}^N$, average channel power gains $\{\Omega_n\}_{n=1}^N$, and maximum Doppler shifts $\{f_{m,n}\}_{n=1}^N$, which are related to the fading severity, channel quality, and node mobility at each hop, respectively. In general, note from (4.5)–(4.7) that, at high SNR, the system performance is a function of the lowest fading parameter (m^*), which corresponds to the hop with the severest fading. In particular, note that the diversity gain of the LCR and outage probability is determined by m^*, while the diversity gain of the AFD remains constant, irrespective of any system parameter indeed. Regarding the coding gains, we can notice that the higher the values of m^* (less fading severity), the lower the values of c_P (more likely fades) and c_N (more frequent fades), and the higher the values of c_T (shorter fades). Note also that the higher the values of Ω_n (better channel quality), the higher the values of c_P, c_N, and c_T, with the smaller values of Ω_n (weaker channels) governing these coding gain values. Furthermore, notice that the higher the values of $\{f_{m,n}\}_{n=1}^N$ (higher mobility of the nodes), the smaller the values of c_N (more frequent fades) and the higher the values of c_T (shorter fades). As expected, the Doppler shifts (a dynamic channel feature) have no impact on the outage probability (a static metric).

The asymptotic expressions in (4.8)–(4.10) give us more insights on one more system parameter, the number of hops N. In the homogeneous scenario, we can note that the coding gain of both LCR and outage probability decreases with the number of hops (more frequent and more likely fades). On the other hand, the coding gain of the AFD remains unchanged.

4.4 Numerical Results

In this section, focusing on the second-order statistics, the asymptotic expressions derived in (4.5) and (4.6) for the LCR and AFD are evaluated for some sam-

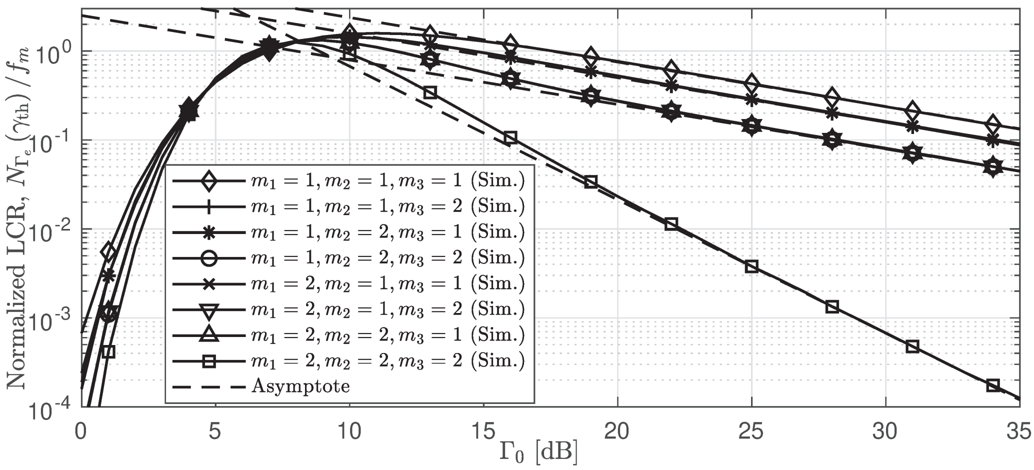

Figure 4.2 – Normalized level crossing rate for a VG-AF relaying system with three hops: $\Omega_1 = \cdots = \Omega_5 = 1$ and $\gamma_{\text{th}} = 0$ dB. Solid lines correspond to the simulation for the exact end-to-end SNR.

ple scenarios. Monte Carlo simulations are also performed to validate our analysis, for which the exact end-to-end SNR Γ_e is considered. Bringing our attention to the impact of the fading severity over the system performance, we vary the fading parameters at each hop as $m_n \in \{1, 2\}$, while setting $N = 3$, $\gamma_{\text{th}} = 1$, $\Omega_1 = \cdots = \Omega_N = 1$, and $f_{m,1} = \cdots = f_{m,N} = f_m$, for illustration.

Figs. 4.2 and 4.3 show the normalized LCR and normalized AFD versus the average transmit SNR, respectively. In all the cases, note how the asymptotic curves (dashed lines) are accurate with respect to the exact ones (solid lines) at medium to high SNR. Note from Fig. 4.2 that the lowest fading parameter (m^*) governs the diversity gain of the LCR, as remarked in Section 4.3.3. Note also that the coding gain of the LCR decreases as m^* vary from 1 to 2, while it increases as the number of hops with $m_n = m^*$ diminishes. In turn, we can observe from Fig. 4.3 that the diversity gain of the AFD remains the same regardless of the values of fading severity at each hop, while its coding gain only increases when m^* increases from 1 to 2.

4.5 Conclusions

In this work, we provided a high-SNR analysis of a multi-hop variable-gain amplify-and-forward relaying system under Nakagami fading. In particular, the impact of the channel diversity (characterized by the Nakagami-m fading parameter) on the second-order statistics is assessed through simple high-SNR closed-form expressions derived for the LCR and AFD. These asymptotic expressions revealed that the diversity gain of the LCR is governed by the hop with the severest fading, and the diversity gain of the AFD is constant and equal to 1/2.

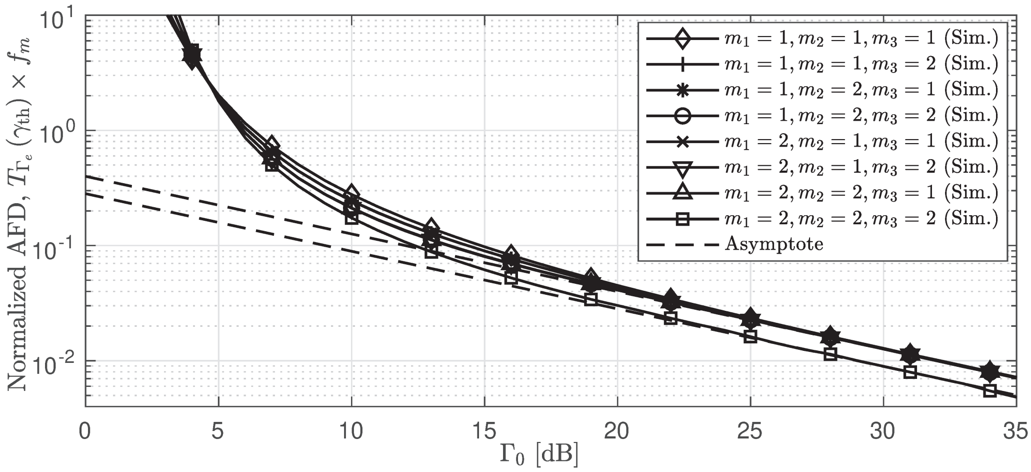

Figure 4.3 – Normalized average fade duration for a VG-AF relaying system with three hops: $\Omega_1 = \cdots = \Omega_5 = 1$ and $\gamma_{\text{th}} = 0$ dB. Solid lines correspond to the simulation for the exact end-to-end SNR.

Appendix 4.A Proof of Theorem 4.1

To calculate the LCR of Γ, we start by rewriting (4.2) as

$$N_\Gamma(\gamma_{\text{th}}) = \int_0^\infty \dot{\gamma}\, f_{\dot{\Gamma}|\Gamma}(\dot{\gamma}|\gamma_{\text{th}})\, f_\Gamma(\gamma_{\text{th}})\, d\dot{\gamma}, \tag{4.11}$$

where $f_{\dot{\Gamma}|\Gamma}(\cdot|\cdot)$ is the PDF of $\dot{\Gamma}$ conditioned on Γ, and $f_\Gamma(\cdot)$ is the PDF of Γ. The latter can be obtained as follows. In [10], Brennan proposed a method to calculate the PDF of a generic sum of N non-negative, arbitrarily correlated, arbitrarily distributed RVs. In our case, let us consider the sum

$$Z = \sum_{n=1}^{N} Z_n, \tag{4.12}$$

where $\{Z_n\}_{n=1}^N$ are non-negative, independent RVs. From Brennan's results, the PDF of Z is given by [10]

$$f_Z(z) = \int_0^z \int_0^{z-z_N} \cdots \int_0^{z-\sum_{n=3}^N z_n} f_{Z_1}\left(z - \sum_{n=2}^N z_n\right) \prod_{n=2}^N f_{Z_n}(z_n)\, dz_2 \ldots dz_{N-1} dz_N. \tag{4.13}$$

To express (4.1) as in (4.12), we introduce the relationships $Z \triangleq \Gamma^{-1}$ and $Z_n \triangleq \Gamma_n^{-1}$ with the respective PDFs being related as $f_\Gamma(\gamma) = \gamma^{-2} f_Z(\gamma^{-1})$ and $f_{Z_n}(z_n) = z_n^{-2} f_{\Gamma_n}(z_n^{-1})$. Using this into (4.13), the PDF of Γ is then obtained as

$$\begin{aligned} f_\Gamma(\gamma) =& \frac{1}{\gamma^2} \int_0^{\frac{1}{\gamma}} \int_0^{\frac{1}{\gamma}-z_N} \cdots \int_0^{\frac{1}{\gamma}-\sum_{n=3}^N z_n} \frac{1}{\left(\frac{1}{\gamma} - \sum_{n=2}^N z_n\right)^2} f_{\Gamma_1}\left(\frac{1}{\frac{1}{\gamma} - \sum_{n=2}^N z_n}\right) \\ &\times \prod_{n=2}^N \frac{1}{z_n^2} f_{\Gamma_n}\left(\frac{1}{z_n}\right) dz_2 \ldots dz_{N-1} dz_N. \end{aligned} \tag{4.14}$$

Now, to obtain $f_{\dot{\Gamma}|\Gamma}(\cdot|\cdot)$, we first determine from (4.1) the time derivative of Γ:

$$\dot{\Gamma} = \frac{\partial \Gamma}{\partial t} = \frac{\sum_{n=1}^N \frac{1}{\Gamma_n^2} \dot{\Gamma}_n}{\left(\sum_{n=1}^N \frac{1}{\Gamma_n}\right)^2} = \frac{2\sqrt{\Gamma_0} \sum_{n=1}^N Z_n^{3/2} \dot{\alpha}_n}{\left(\sum_{n=1}^N Z_n\right)^2}, \tag{4.15}$$

since $\dot{\Gamma}_n = \partial\Gamma_n/\partial t = 2\sqrt{\Gamma_0}\sqrt{\Gamma_n}\dot{\alpha}_n$ and $\Gamma_n = Z_n^{-1}$. Hence, it becomes apparent that $\dot{\Gamma}$ is a weighted sum of $\{\dot{\alpha}_n\}_{n=1}^N$, zero-mean Gaussian RVs with variances given by $\sigma^2_{\dot{\alpha}_n} = \pi^2 f_{m,n}^2 \Omega_n/m_n$. The weights depend on $\{Z_n\}_{n=1}^N$. As a result, conditioned on Γ and $Z_2,\ldots,Z_N$, $\dot{\Gamma}$ is also a Gaussian RV, with zero mean and variance computed from (4.15) as

$$\sigma^2_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N} = 4\Gamma_0\gamma_{\text{th}}\left[\left(1-\gamma_{\text{th}}\sum_{n=2}^{N} z_n\right)^3 \sigma_{\dot{\alpha}_1} + \gamma_{\text{th}}^3 \sum_{n=2}^{N} z_n^3 \sigma_{\dot{\alpha}_n}\right]. \quad (4.16)$$

By integrating (4.11) with respect to $\dot{\gamma}$ while taking into account the Gaussianity of $f_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N}(\cdot|\cdot,\ldots,\cdot)$, we have [11]

$$\int_0^\infty \dot{\gamma}\, f_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N}(\cdot|\cdot,\ldots,\cdot)\, d\dot{\gamma} = \sqrt{\frac{\sigma^2_{\dot{\Gamma}|\Gamma,Z_2,\ldots,Z_N}}{2\pi}}. \quad (4.17)$$

Then, by applying (4.14) and (4.17) into (4.11), we obtain an approximate analytical expression for the LCR of a multi-hop VG-AF relaying system over Nakagami-m fading as

$$\begin{aligned} N_\Gamma(\gamma_{\text{th}}) =& \frac{\sqrt{2\pi}\gamma_{\text{th}}^{m_1-1/2}}{\Gamma_0^{-1/2+\sum_{n=1}^N m_n}} \prod_{n=1}^{N} \frac{m_n^{m_n}}{\Omega_n^{m_n}\Gamma(m_n)} \int_0^{\frac{1}{\gamma_{\text{th}}}} \int_0^{\frac{1}{\gamma_{\text{th}}}-z_N} \cdots \int_0^{\frac{1}{\gamma_{\text{th}}}-\sum_{n=3}^N z_n} \\ &\times \sqrt{\frac{f_{m,1}^2\Omega_1}{m_1}\left(1-\gamma_{\text{th}}\sum_{n=2}^N z_n\right)^3 + \gamma_{\text{th}}^3\sum_{n=2}^N \frac{f_{m,n}^2\Omega_n z_n^3}{m_n}} \\ &\times \frac{\exp\left[-\frac{m_1\gamma_{\text{th}}}{\Omega_1\Gamma_0\left(1-\gamma_{\text{th}}\sum_{n=2}^N z_n\right)} - \sum_{n=2}^N \frac{m_n}{\Gamma_0\Omega_n z_n}\right]}{\left(1-\gamma_{\text{th}}\sum_{n=2}^N z_n\right)^{m_1+1}\prod_{n=2}^N z_n^{m_n+1}} dz_2\ldots dz_{N-1}dz_N. \end{aligned} \quad (4.18)$$

To calculate the AFD of Γ as in (4.3), we need to find the associated outage probability. By substituting (4.14) into (4.4), we obtain an approximate analytical expression for the outage probability of a multi-hop VG-AF relaying system over Nakagami-m fading as

$$\begin{aligned} P_\Gamma(\gamma_{\text{th}}) =& \frac{\gamma^{m_1-1}}{\Gamma_0^{\sum_{n=1}^N m_n}} \prod_{n=1}^{N} \frac{m_n^{m_n}}{\Omega_n^{m_n}\Gamma(m_n)} \int_0^{\frac{1}{\gamma_{\text{th}}}} \int_0^{\frac{1}{\gamma}} \int_0^{\frac{1}{\gamma}-z_N} \cdots \int_0^{\frac{1}{\gamma}-\sum_{n=3}^N z_n} \\ &\times \frac{\exp\left[-\frac{m_1\gamma}{\Omega_1\Gamma_0\left(1-\gamma\sum_{n=2}^N z_n\right)} - \sum_{n=2}^N \frac{m_n}{\Gamma_0\Omega_n z_n}\right]}{\left(1-\gamma\sum_{n=2}^N z_n\right)^{m_1+1}\prod_{n=2}^N z_n^{m_n+1}} dz_2\ldots dz_{N-1}dz_N d\gamma. \end{aligned} \quad (4.19)$$

Then, by replacing (4.18) and (4.19) into (4.3), we obtain a corresponding expression for the AFD.

The final step is to derive asymptotic representations for (4.18), (4.19), and the corresponding AFD as $\Gamma_0 \to \infty$. In such cases, a popular approach is to replace the exponential function in the integrands by its Maclaurin series expansion, to drop the terms beyond the second one (i.e., $\exp(-x_i) \simeq 1 - x_i$), and to solve and simplify the integral.

But this approach alone does not work in our case; the resulting integrals turn out to diverge, since their integrands contain poles on each integration limit. To overcome this, we split the integration interval into two parts containing a single pole each, and then we perform certain mathematical manipulations to ensure the integral will converge when the exponential function is approximated.

But applying the above method to (4.18) and (4.19) proves quite involved if done directly for an arbitrary number of hops N. Rather, we apply the method for two and three hops first, and then, building on these results, we generalize them by induction for any number of hops. Next, due to the lack of space, we present the derivation steps only for the LCR under two hops. The same rationale can be also applied for the outage probability, as well as for a larger number of hops. Finally, using (4.3), those results can be combined into corresponding AFD expressions.

The LCR for the dual-hop case is obtained by setting $N = 2$ in (4.18), yielding

$$\begin{aligned} N_\Gamma(\gamma_{\text{th}}) =& \frac{\sqrt{2\pi}\gamma_{\text{th}}^{m_1-1/2} m_1^{m_1-1/2} m_2^{m_2-1/2}}{\Gamma_0^{-1/2+m_1+m_2}\Omega_1^{m_1}\Omega_2^{m_2}\Gamma(m_1)\Gamma(m_2)} \int_0^{\frac{1}{\gamma_{\text{th}}}} \\ &\times \frac{\sqrt{(1-z_2\gamma_{\text{th}})^3 m_2\Omega_1 f_{m,1}^2 + (z_2\gamma_{\text{th}})^3 m_1\Omega_2 f_{m,2}^2}}{(1-z_2\gamma_{\text{th}})^{1+m_2} z_2^{1+m_1}} \\ &\times \exp\left[-\frac{m_1\gamma_{\text{th}}}{(1-z_2\gamma_{\text{th}})\Gamma_0\Omega_1} - \frac{m_2}{z_2\Gamma_0\Omega_2}\right] dz_2. \end{aligned} \quad (4.20)$$

A high-SNR asymptotic expression for (4.20) cannot be obtained by directly using the Maclaurin representation of the exponential function and dropping the terms beyond the first, since the resulting integral will diverge due to the poles on $z_2 = 0$ and $z_2 = 1/\gamma_{\text{th}}$. To overcome this, as mentioned before, we split the integration interval into two parts with only one pole each, and then a change of variable is made to ensure the integral will converge when the exponential function is approximated. Specifically, after the change of variables $u \triangleq z_2\gamma_{\text{th}}$, (4.20) can be split as

$$N_\Gamma(\gamma_{\text{th}}) = \frac{\sqrt{2\pi}\gamma_{\text{th}}^{m_1+m_2-1/2} m_1^{m_1-1/2} m_2^{m_2-1/2}}{\Gamma_0^{-1/2+m_1+m_2}\Omega_1^{m_1}\Omega_2^{m_2}\Gamma(m_1)\Gamma(m_2)} (I_1 + I_2), \quad (4.21)$$

where

$$I_1 \triangleq \int_0^{1/2} \frac{\sqrt{(1-u)^3 m_2\Omega_1 f_{m,1}^2 + u^3 m_1\Omega_2 f_{m,2}^2}}{(1-u)^{m_1+1} u^{m_2+1}} \exp\left[-\frac{m_1\gamma_{\text{th}}}{\Gamma_0\Omega_1(1-u)} - \frac{m_2\gamma_{\text{th}}}{\Gamma_0\Omega_2 u}\right] du \quad (4.22)$$

$$I_2 \triangleq \int_{1/2}^{1} \frac{\sqrt{(1-u)^3 m_2\Omega_1 f_{m,1}^2 + u^3 m_1\Omega_2 f_{m,2}^2}}{(1-u)^{m_1+1} u^{m_2+1}} \exp\left[-\frac{m_1\gamma_{\text{th}}}{\Gamma_0\Omega_1(1-u)} - \frac{m_2\gamma_{\text{th}}}{\Gamma_0\Omega_2 u}\right] du. \quad (4.23)$$

By changing also $v \triangleq m_2\gamma_{\text{th}}(1-2u)/(\Gamma_0\Omega_2 u)$ in (4.22), and $w \triangleq m_1\gamma_{\text{th}}(1-2u)/(\Gamma_0\Omega_1 u - \Gamma_0\Omega_1)$

in (4.23), we have

$$\begin{aligned} I_1 =& \frac{\Gamma_0\Omega_2}{(m_2\gamma_{\text{th}})^{m_2}} \int_0^\infty \frac{(2m_2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^{m_1+m_2}}{(m_2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^{m_1+1}} \\ &\times \sqrt{\frac{(m_2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^3 m_2\Omega_1 f_{m,1}^2}{(2m_2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^3} + \frac{m_1 m_2^3\gamma_{\text{th}}^3\Omega_2 f_{m,2}^2}{(2m_2\gamma_{\text{th}} + v\Gamma_0\Omega_2)^3}} \\ &\times \exp\left[-v - \frac{2m_2\gamma_{\text{th}}}{\Gamma_0\Omega_2} - \frac{2m_1m_2\gamma_{\text{th}}^2 + v\gamma_{\text{th}}\Gamma_0 m_1\Omega_2}{\gamma_{\text{th}}\Gamma_0 m_2\Omega_1 + v\Gamma_0^2\Omega_1\Omega_2}\right] dv \end{aligned} \tag{4.24}$$

$$\begin{aligned} I_2 =& \frac{\Gamma_0\Omega_1}{(m_1\gamma_{\text{th}})^{m_1}} \int_0^\infty \frac{(2m_1\gamma_{\text{th}} + w\Gamma_0\Omega_1)^{m_1+m_2}}{(m_1\gamma_{\text{th}} + w\Gamma_0\Omega_1)^{m_2+1}} \\ &\times \sqrt{\frac{m_1^3 m_2\gamma_{\text{th}}^3\Omega_1 f_{m,1}^2}{(2m_1\gamma_{\text{th}} + w\Gamma_0\Omega_1)^3} + \frac{(m_1\gamma_{\text{th}} + w\Gamma_0\Omega_1)^3 m_1\Omega_2 f_{m,2}^2}{(2m_1\gamma_{\text{th}} + w\Gamma_0\Omega_1)^3}} \\ &\times \exp\left[-w - \frac{2m_1\gamma_{\text{th}}}{\Gamma_0\Omega_1} - \frac{2m_1m_2\gamma_{\text{th}}^2 + w\gamma_{\text{th}}\Gamma_0 m_2\Omega_1}{\gamma_{\text{th}}\Gamma_0 m_1\Omega_2 + w\Gamma_0^2\Omega_1\Omega_2}\right] dw. \end{aligned} \tag{4.25}$$

Since $1/\Gamma_0 \to 0$ as $\Gamma_0 \to \infty$, we can apply the Maclaurin series to the whole integrand in (4.24) and (4.25), and take only the first term to track the asymptotic behavior:

$$I_1 \simeq \sqrt{m_2\Omega_1 f_{m,1}^2}\left(\frac{\Omega_2\Gamma_0}{m_2\gamma_{\text{th}}}\right)^{m_2}\int_0^\infty \frac{\exp[-v]}{v^{1-m_2}}dv = \sqrt{m_2\Omega_1 f_{m,1}^2}\,\Gamma(m_2)\left(\frac{\Omega_2\Gamma_0}{m_2\gamma_{\text{th}}}\right)^{m_2} \tag{4.26}$$

$$I_2 \simeq \sqrt{m_1\Omega_2 f_{m,2}^2}\left(\frac{\Omega_1\Gamma_0}{m_1\gamma_{\text{th}}}\right)^{m_1}\int_0^\infty \frac{\exp[-w]}{w^{1-m_1}}dw = \sqrt{m_1\Omega_2 f_{m,2}^2}\,\Gamma(m_1)\left(\frac{\Omega_1\Gamma_0}{m_1\gamma_{\text{th}}}\right)^{m_1}. \tag{4.27}$$

Then, by substituting (4.26) and (4.27) into (4.21), we obtain a high-SNR asymptotic expression for the LCR of a dual-hop VG-AF relaying system:

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \frac{\sqrt{2\pi} f_{m,1}}{\Gamma(m_1)}\left(\frac{m_1\gamma_{\text{th}}}{\Omega_1\Gamma_0}\right)^{m_1-1/2} + \frac{\sqrt{2\pi} f_{m,2}}{\Gamma(m_2)}\left(\frac{m_2\gamma_{\text{th}}}{\Omega_2\Gamma_0}\right)^{m_2-1/2}. \tag{4.28}$$

As already mentioned, the procedure described in (4.20)–(4.28) can be also applied for the outage probability and for any number of hops. For these cases, due to space constraints, we only reproduce the final expressions. The high-SNR asymptotic expression for the outage probability with $N = 2$ is obtained as

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \frac{1}{\Gamma(1+m_1)}\left(\frac{m_1\gamma_{\text{th}}}{\Omega_1\Gamma_0}\right)^{m_1} + \frac{1}{\Gamma(1+m_2)}\left(\frac{m_2\gamma_{\text{th}}}{\Omega_2\Gamma_0}\right)^{m_2}. \tag{4.29}$$

Expressions similar to (4.28)–(4.29) can be found for $N > 2$. All in all, building on these results, and taking into account only the channels related to the lowest fading parameter (as $\Gamma_0 \to \infty$, the leading-order term is given by $\Gamma_0^{-m^*}$, where $m^* = \min_{n\in\{1,\ldots,N\}} m_n$), we arrive by induction at a general asymptotic solution given in (4.5)–(4.7).

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5 On Second-Order Statistics of Full-Duplex Relaying

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Abstract

Although the outage probability of full-duplex relaying communications has been extensively investigated, its outage rate and duration—crucial to the design of effective signaling schemes—remain an open problem. Herein we narrow this gap by analyzing a full-duplex, variable-gain transparent relaying protocol operating in a standard (Nakagami-m) fading environment. We obtain exact as well as asymptotic expressions for the outage rate and duration. Also, as a byproduct, we generalize an existing asymptotic solution for the outage probability. Our results reveal that, at high transmission power, the relay-to-destination link's strength and mobility have a negligible impact on the outage rate and duration of full-duplex relaying.

5.1 Introduction

Full-duplex (FD) relaying has drawn a great deal of attention from industry and academia, as it combines the high spectral efficiency of the FD operation with the extended coverage and improved reliability of cooperative networks [1]. Unlike the half-duplex (HD) mode, whereby the relay receives and transmits information through orthogonal channels, the FD mode enables the relay to receive and transmit information within the same time slot and the same frequency band. As a downside, the relay's transmit antenna interferes with its receive antenna—the so-called loop-interference (LI). In other words, the signal received at the FD relay consists of the desired signal (from the source) plus the LI signal (from the relay itself), which in turn consists of direct and reflected wave components [2], [3]. If no countermeasures are taken, the desired signal proves too weak compared to the LI signal, because the distance between the source and relay is usually much greater than the distance between the nearly collocated relay's antennas. To

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make the FD operation feasible, the relay must suppress the LI signal as much as possible while maintaining a strong desired signal. Fortunately, the latest advances in antenna technology and signal processing have led to satisfactory levels of LI suppression [1], [2].

The first-order statistics (e.g., outage probability and bit-error rate) of FD relaying-based networks have been extensively studied for various relaying schemes, protocols, and fading conditions [1], [4]–[6]. While important, those performance metrics can be insufficient for some essential design issues such as block length, symbol rate, transmission time interval, error-correction codes, and adaptive modulation schemes [7]. These system parameters can be addressed by second-order statistics such as the level crossing rate (LCR), which provides the rate at which the channel crosses a given threshold level, and the average fade duration (AFD), which determines the average amount of time that the channel remains below that level. To our best knowledge, the open technical literature has no results on the second-order statistics of full-duplex relaying. There is no available clue about how these statistics are affected by each system parameter in light of the LI phenomenon. Herein we help uncover this impact by deriving exact as well as simple asymptotic expressions for the LCR and AFD of a benchmark FD relaying system—dual-hop, variable-gain (VG), amplify-and-forward (AF)—operating in a typical propagation scenario, described by the Nakagami-m fading model. In passing, we also derive a general asymptotic expression for the associated outage probability, previously reported only for integers values of the fading parameter m related to the source-to-relay link. We show that, at high signal-to-noise ratio (SNR), (i) the LCR and AFD are chiefly governed by the signal-to-loop-interference power ratio at the relay and by the relative motion between the source and relay, (ii) the LCR is significantly affected by the amount of fading in the source-to-relay link, and (iii) the LCR and AFD are barely affected by the relay-to-destination link conditions.

5.2 System Model

Consider a dual-hop relaying system as depicted in Fig. 5.1. A source node S communicates with a destination node D with the assistance of a VG-AF-FD relay node R. The direct link between S and D is assumed to be inactive, e.g., due to poor end-to-end channel conditions. Since R operates in FD mode, it is equipped with two antennas (one for transmission, one for reception), while S and D are single-antenna nodes. As transmission and reception co-occur at R, some residual LI remains due to imperfections in LI cancellation techniques [5]. In Fig. 5.1, h_1, h_2, and h_L denote the complex-valued channel coefficients of the first hop, second hop, and residual LI link, respectively. The corresponding amplitudes, $\alpha_n = |h_n|$, $n \in \{1, 2, L\}$, are modeled as independent, non-

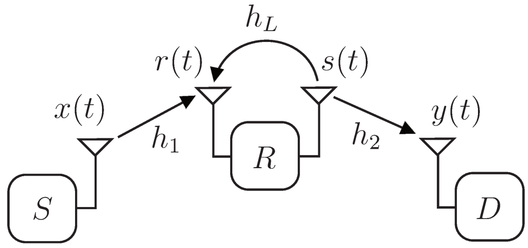


Figure 5.1 – The full-duplex transparent relaying system.

necessarily identically distributed Nakagami-m random variables (RV)[2], and all nodes undergo additive white Gaussian noise with mean power N_0.

Assuming that S and R transmit with the same power P_{T}, the received signal at R can be written as

$$r(t) = \sqrt{P_{\mathrm{T}}}h_1 x(t) + \sqrt{P_{\mathrm{T}}}h_L s(t) + n_R(t), \tag{5.1}$$

where $x(t)$ is the unit-power (normalized) source signal; $s(t) = Gr(t-\tau)$ is the signal transmitted by R, with G being the amplification factor and τ being the processing delay; and $n_R(t)$ is the noise term at R. By recursively using (5.1), $s(t)$ can be written as [5]

$$s(t) = G\sum_{j=1}^{L}\Big(G\sqrt{P_{\mathrm{T}}}h_L\Big)^{j-1}\Big[\sqrt{P_{\mathrm{T}}}h_1 x(t-j\tau)+n_R(t-j\tau)\Big], \tag{5.2}$$

where L is the length of the transmission block. To assure that the average transmit power at R is regulated as P_{T} (or, equivalently, that $s(t)$ has unit power), the amplification factor is set to [5]

$$G^2 = \Big(P_{\mathrm{T}}\alpha_1^2 + P_{\mathrm{T}}\alpha_L^2 + N_0\Big)^{-1}. \tag{5.3}$$

Then, the received signal at D can be written as $y(t) = \sqrt{P_{\mathrm{T}}}h_2 s(t) + n_D(t)$, where $n_D(t)$ is the noise term at D. After some algebraic manipulations, the end-to-end signal-to-interference-plus-noise ratio (SINR) received at D can be expressed as [10]

$$\Gamma_e = \frac{\Gamma_1\Gamma_2}{\Gamma_1 + (\Gamma_2+1)(\Gamma_L+1)}, \tag{5.4}$$

where $\Gamma_n = \Gamma_0\alpha_n^2$ denotes the received SNR at the nth link, $n \in \{1, 2, L\}$, with $\Gamma_0 \triangleq P_{\mathrm{T}}/N_0$ being the transmit SNR at S and R. The probability density function (PDF) of Γ_n is given as [8]

$$f_{\Gamma_n}(\gamma_n) = \frac{(m_n)^{m_n}\gamma_n^{m_n-1}}{\bar{\Gamma}_n^{m_n}\Gamma(m_n)}\exp\left[-\frac{m_n\gamma_n}{\bar{\Gamma}_n}\right], \tag{5.5}$$

where $m_n > 0.5$ is the Nakagami-m fading parameter (the higher its value, the less severe the fading scenario); $\Gamma(\cdot)$ is the gamma function; and $\bar{\Gamma}_n = \mathrm{E}[\Gamma_n] = \Gamma_0\Omega_n$ is

2 We selected the Nakagami-m fading model due to its mathematical simplicity, good fit to empirical data, and great flexibility to describe propagation conditions ranging from severe to moderate to light or no fading [8]. In particular, the Nakagami-m model has been used to describe the residual LI link [4], [6]. The Rayleigh and Rice models have also been used [9].

the average received SNR at the nth link, $\Omega_n \triangleq \mathrm{E}\left[\alpha_n^2\right]$ being the corresponding average channel power gain, and $\mathrm{E}\left[\cdot\right]$ denoting statistical expectation. In addition, considering an isotropic propagation environment, it is known that the time derivative $\dot{\alpha}_n$ of the channel amplitude α_n follows a zero-mean Gaussian distribution with variance given by $\sigma_{\dot{\alpha}_n}^2 = \pi^2 f_{m,n}^2 \Omega_n / m_n$, where $f_{m,n}$ denotes the maximum Doppler frequency shift in Hz at the nth link, $n \in \{1, 2, L\}$. For the source-to-relay and relay-to-destination links, the Doppler shift is proportional to the relative speed between the communicating nodes. On the other hand, for the LI link, such relative speed is nil, for the transmitter and receiver are co-located at the relay. In this case, the Doppler shift is caused by the relative motion between the relay and the surrounding environmental reflectors [3].

5.3 Second-Order Statistics

A system outage occurs whenever the end-to-end SINR Γ_e drops below a certain threshold level γ_{th}. The LCR $N_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)$ gives the average temporal rate at which outage events take place, which can be obtained from Rice's formula [8]

$$N_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right) = \int_0^{\infty} \dot{\gamma}_e \, f_{\Gamma_e, \dot{\Gamma}_e}\left(\gamma_{\mathrm{th}}, \dot{\gamma}_e\right) \, d\dot{\gamma}_e \tag{5.6}$$

in terms of the joint PDF $f_{\Gamma_e, \dot{\Gamma}_e}\left(\cdot, \cdot\right)$ of Γ_e and its time derivative $\dot{\Gamma}_e$. The AFD $T_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)$, in turn, gives the average duration of an outage event, obtained as [8]

$$T_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right) = \frac{P_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)}{N_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right)}, \tag{5.7}$$

where

$$P_{\Gamma_e}\left(\gamma_{\mathrm{th}}\right) = \int_0^{\gamma_{\mathrm{th}}} f_{\Gamma_e}\left(\gamma_e\right) d\gamma_e \tag{5.8}$$

is the outage probability, with $f_{\Gamma_e}(\cdot)$ denoting the PDF of Γ_e. Next, we elaborate on these metrics for the system at hand.

5.3.1 Exact Analysis

Theorem 5.1 *An exact expression for the LCR of a dual-hop VG-AF-FD relaying system under Nakagami-m fading is given by*

$$
\begin{aligned}
N_{\Gamma_e}(\gamma_{\text{th}}) = {} & \frac{\sqrt{2\pi} m_1^{m_1} m_2^{m_2} m_L^{m_L} \gamma_{\text{th}}^{m_1-\frac{1}{2}}}{\Gamma_0^{m_1+m_2+m_L-\frac{1}{2}} \Omega_1^{m_1} \Omega_2^{m_2} \Omega_L^{m_L} \Gamma(m_1)\Gamma(m_2)\Gamma(m_L)} \\
& \times \int_0^\infty \int_{\gamma_{\text{th}}}^\infty \frac{\gamma_2^{m_2-1}\gamma_L^{m_L-1}(\gamma_2+1)^{m_1-1}(\gamma_L+1)^{m_1-1}}{(\gamma_2-\gamma_{\text{th}})^{m_1+1}} \left[\frac{(\gamma_2+1)(\gamma_2-\gamma_{\text{th}})^3(\gamma_L+1) f_{m,1}^2 \Omega_1}{m_1} \right. \\
& \left. + \frac{\gamma_{\text{th}}\gamma_2(\gamma_{\text{th}}+1)^2(\gamma_L+1)^2 f_{m,2}^2 \Omega_2}{m_2} + \frac{\gamma_{\text{th}}\gamma_L(\gamma_2-\gamma_{\text{th}})^2(\gamma_2+1)^2 f_{m,L}^2 \Omega_L}{m_L} \right]^{\frac{1}{2}} \\
& \times e^{\left[-\frac{m_1\gamma_{\text{th}}(1+\gamma_2)(1+\gamma_L)}{\Gamma_0\Omega_1(\gamma_2-\gamma_{\text{th}})} - \frac{m_2\gamma_2}{\Gamma_0\Omega_2} - \frac{m_L\gamma_L}{\Gamma_0\Omega_L} \right]} d\gamma_2 d\gamma_L. \qquad (5.9)
\end{aligned}
$$

Proof: See Appendix 5.A.

Now, to compute the AFD in (5.7), we need the associated outage probability. Exact outage-probability solutions for VG-AF-FD relaying systems over non-identical Nakagami-m fading channels were derived in closed form in [4] and in integral form in [6]. Yet the solution in [4] was given in terms of the extended generalized bivariate Meijer-G function (EGBMGF), which is unavailable to date in most standard computing software, thus requiring ultimately a numerical evaluation. Moreover, this EGBMGF-based solution is valid only for integer values of m_1 and m_2. Therefore, here we use the integral-form solution in [6], as it allows for arbitrary real values of the referred parameters:

$$
\begin{aligned}
P_{\Gamma_e}(\gamma_{\text{th}}) = 1 - \int_0^\infty & \left[1 - \frac{m_L^{m_L}}{(\Gamma_0\Omega_L)^{m_L}\Gamma(m_1)\Gamma(m_L)} \right. \\
& \left. \times \int_0^\infty v^{m_L-1} \gamma\left(m_1, \frac{m_1\gamma_{\text{th}}(u+\gamma_{\text{th}}+1)(v+1)}{\Gamma_0\Omega_1 u} \right) e^{-\frac{v m_L}{\Gamma_0\Omega_L}} dv \right] \\
& \times \frac{m_2^{m_2}(u+\gamma_{\text{th}})^{m_2-1}}{(\Gamma_0\Omega_2)^{m_2}\Gamma(m_2)} e^{-\frac{m_2(u+\gamma_{\text{th}})}{\Gamma_0\Omega_2}} du. \qquad (5.10)
\end{aligned}
$$

Finally, a corresponding expression for the AFD is obtained by replacing (5.9) and (5.10) into (5.7).

Remark 5.1 *From* (5.9) *and* (5.10)*, note that the first- and second-order statistics of the investigated system are written in double integral form. The lack of closed-form solutions is often the case for relaying systems. As far as we know, there is no exact closed-form solution for the mathematically less intricate HD counterpart of the relaying system investigated here, even when operating in the special Rayleigh condition [7], [11]. Cases like these motivate the search for approximate closed-form results, as those that follow.*

5.3.2 Asymptotic Analysis

Although the integral-form solutions derived in the previous section are exact, they provide little if any insight about the impact of each system parameter on the transmission performance. To this end, we now derive corresponding asymptotic closed-form expressions at high SNR, i.e., as $\Gamma_0 \to \infty$, written in terms of the average channel power gains Ω_n, the fading parameters m_n, and the maximum Doppler shifts $f_{m,n}$, $n \in \{1, 2, L\}$.

Theorem 5.2 *A high-SNR asymptotic expression for the LCR of a dual hop VG-AF-FD relaying system under Nakagami-m fading is given by*

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \frac{(m_L\Omega_1)^{m_L-\frac{1}{2}}(m_1\Omega_L\gamma_{\text{th}})^{m_1-\frac{1}{2}}\Gamma\left(m_1+m_L-\frac{1}{2}\right)}{(m_L\Omega_1+m_1\Omega_L\gamma_{\text{th}})^{m_1+m_L-\frac{1}{2}}\Gamma(m_1)\Gamma(m_L)} \times\sqrt{2\pi\left(m_L\Omega_1 f_{m,1}^2+m_1\Omega_L\gamma_{\text{th}}f_{m,L}^2\right)}. \tag{5.11}$$

Proof: See Appendix 5.B.

An asymptotic expression for the outage probability was derived in [4] and [6], but only for integer values of m_1 (the fading parameter of the source-to-relay link). Next, we derive a general asymptotic solution for the outage probability.

Theorem 5.3 *A high-SNR asymptotic expression for the outage probability of a dual-hop VG-AF-FD relaying system under Nakagami-m fading is given by*

$$P_{\Gamma_e}(\gamma_{\text{th}}) \simeq \frac{\Gamma(m_1+m_L)}{m_1\Gamma(m_1)\Gamma(m_L)}\left(\frac{m_1\gamma_{\text{th}}\Omega_L}{m_L\Omega_1}\right)^{m_1} \times\ {}_2F_1\left(m_1, m_1+m_L; 1+m_1; -\frac{m_1\gamma_{\text{th}}\Omega_L}{m_L\Omega_1}\right), \tag{5.12}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function.

Proof: See Appendix 5.C.

Finally, a high-SNR asymptotic expression for the AFD can be obtained by inserting (5.11) and (5.12) into (5.7), thus yielding

$$T_{\Gamma_e}(\gamma_{\text{th}}) \simeq \frac{\Gamma(m_1+m_L)}{\Gamma\left(m_1+m_L-\frac{1}{2}\right)}\left(\frac{m_L\Omega_1+m_1\gamma_{\text{th}}\Omega_L}{m_L\Omega_1}\right)^{m_1+m_L-\frac{1}{2}} \times\frac{{}_2F_1\left(m_1, m_1+m_L; 1+m_1; -\frac{m_1\gamma_{\text{th}}\Omega_L}{m_L\Omega_1}\right)\gamma_{\text{th}}\Omega_L}{\sqrt{2\pi m_1\gamma_{\text{th}}\Omega_L\left(m_L\Omega_1 f_{m,1}^2+m_1\Omega_L\gamma_{\text{th}}f_{m,L}^2\right)}}. \tag{5.13}$$

5.3.3 Main Takeaways

In practice, by combining advanced LI cancellation schemes, such as passive, analog, and digital techniques, the relay's self-interference can be highly attenuated [2]. Thus, the average power of the residual LI link becomes much smaller than the average power of the source-to-relay channel, i.e., $\Omega_1 \gg \Omega_L$. Applying this condition to (5.11) and (5.13), we have

$$N_{\Gamma_e}(\gamma_{\text{th}}) \simeq \sqrt{2\pi} f_{m,1} \left(\frac{m_1 \Omega_L \gamma_{\text{th}}}{m_L \Omega_1} \right)^{m_1 - \frac{1}{2}} \frac{\Gamma\left(m_1 + m_L - \frac{1}{2}\right)}{\Gamma(m_1)\Gamma(m_L)} \tag{5.14}$$

$$T_{\Gamma_e}(\gamma_{\text{th}}) \simeq \sqrt{\frac{\Omega_L \gamma_{\text{th}}}{2\pi m_1 \Omega_1 m_L}} \frac{\Gamma(m_1 + m_L)\ {}_2F_1\left(m_1, m_1 + m_L; 1 + m_1; -\frac{m_1 \gamma_{\text{th}} \Omega_L}{m_L \Omega_1}\right)}{f_{m,1} \Gamma\left(m_1 + m_L - \frac{1}{2}\right)}. \tag{5.15}$$

The above formulas bring valuable insights, as follows.

Remark 5.2 *As expected for FD relaying, the system performance bears a floor at high SNR, i.e., the diversity order is zero. The relay-to-destination link (Ω_2, m_2, and $f_{m,2}$) plays no role in the high-SNR LCR and AFD, which mimics a known behavior for the outage probability [4], [6]. Regarding the source-to-relay link, the higher the channel strength (Ω_1) or the less severe the fading scenario (higher values of m_1), the less frequent and shorter the fades (lower LCR and lower AFD). It is worth noting that the first hop's fading severity has a more pronounced impact on the outage rate than on the outage duration, as can be seen from the exponent m_1 in* (5.14)*. Regarding the LI link, the more effective the interference cancellation (lower values of Ω_L), the less frequent and shorter the fades. On the other hand, it turns out that the LI link's fading severity m_L barely affects the system performance. Finally, regarding the system parameters related to the nodes' mobility, namely $f_{m,n}$, the higher the relative motion between the source and relay (higher $f_{m,1}$), the more frequent and shorter the fades. Interestingly, the relative motion between the relay and its surroundings, which affects $f_{m,L}$, has a negligible impact on the end-to-end second-order statistics (when the LI signal is highly attenuated).*

In short, the high-SNR outage rate and duration of VG-AF-FD relaying are governed by the source-to-relay channel strength (Ω_1) and mobility ($f_{m,1}$), the residual LI (Ω_L), and, in case of the outage rate, also by the source-to-relay fading severity (m_1).

5.4 Sample Plots

In this section, we provide some illustrative examples for the LCR and AFD, including Monte Carlo simulation results to corroborate our analytical derivations. For simplicity, we assume $f_{m,1} = f_{m,2} = f_{m,L} = f_m$ and, somewhat arbitrarily, we set the outage threshold to $\gamma_{\mathrm{th}} = 0$ dB.

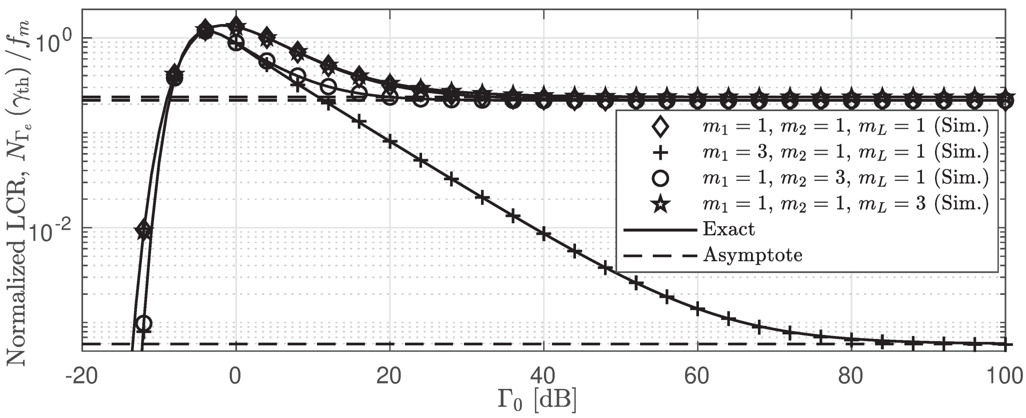
We consider two different scenarios, using fading conditions observed in practice and conservative levels of LI suppression. In Fig. 5.2, we vary the fading parameters ($m_n \in \{1, 3\}$, with $n \in \{1, 2, L\}$) while keeping the average channel gains fixed ($\Omega_1 = \Omega_2 = 10$ and $\Omega_L = 0.1$). In Fig. 5.3, we vary the average channel gains ($\Omega_1, \Omega_2 \in \{10, 100\}$ and $\Omega_L \in \{0.01, 0.1\}$) while keeping the fading parameters fixed ($m_1 = m_2 = m_L = 1$). In all the cases, note how the simple asymptotic results provided in (5.11) and (5.13) meet the complicated integral-form exact expressions provided in (5.9) and (5.10) at medium to high SNR, and how the asymptotic performance is unaffected by the relay-to-destination link. It can also be noticed that the exact analytical expressions perfectly match the simulation results.

In Fig. 5.2, we observe that the fading parameter m_L of the LI link slightly affects the system performance. On the other hand, the fading parameter m_1 of the source-to-relay link considerably affects the LCR. In turn, m_1 barely affects the AFD, except at the medium SNR, as can be seen in Fig. 5.2b. This way, the AFD performance floor at high SNR is dominated by no link's fading parameter.

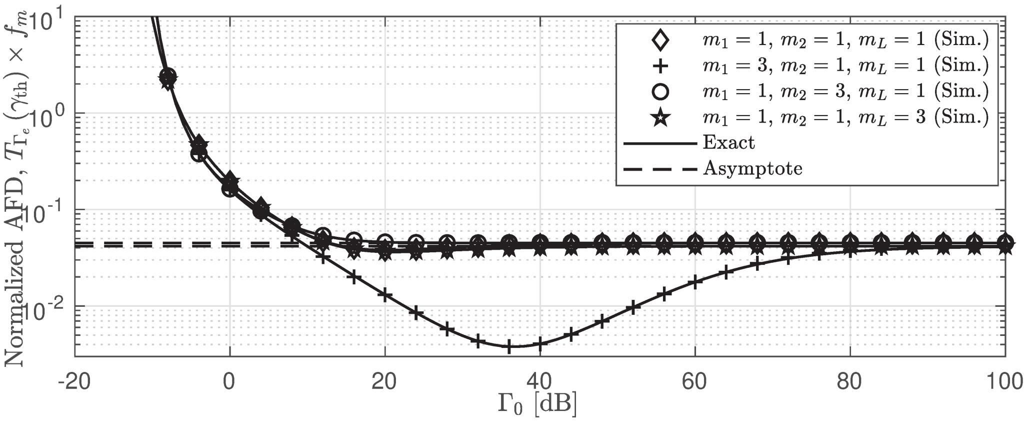
Fig. 5.3 shows that the high-SNR system performance is governed by the average channel power gains Ω_1 and Ω_L, related to the source-to-relay and LI links, respectively. Indeed, as indicated by (5.14) and (5.15), the high-SNR performance under effective LI suppression depends on Ω_1/Ω_L, i.e., on the signal-to-loop-interference average channel power ratio.

5.5 Conclusions

We investigated essential second-order statistics of a dual-hop VG-AF-FD relaying system under Nakagami-m fading. We obtained exact analytical expressions for the LCR and AFD, in a twofold integral form. More usefully, we obtained simple asymptotic closed-form expressions that well approximate the system performance at medium to high SNR, while shedding light on the influence of each system parameter. Our results revealed that, if the LI link is mitigated via modern interference-suppression techniques, then the high-SNR performance becomes dominated by the mobility of the first hop and the ratio between the average channel power gains of the first hop and LI link. For the outage rate, the first hop's fading parameter also plays a significant role.



(a) Level crossing rate.



(b) Average fade duration.

Figure 5.2 – LCR and AFD of dual-hop VG-AF-FD relaying with $\Omega_1 = \Omega_2 = 10$, $\Omega_L = 0.1$, and various fading scenarios.

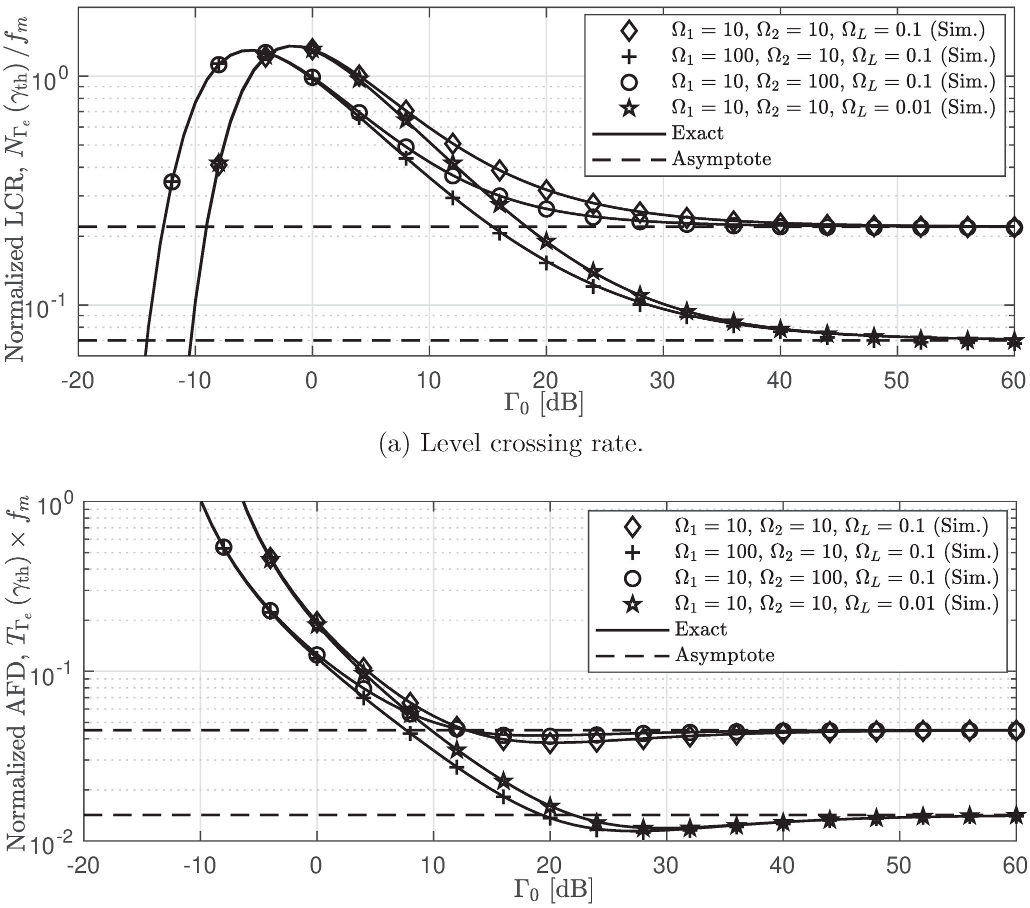
Appendix 5.A Proof of Theorem 5.1

To compute the LCR of Γ_e, we need to determine the joint PDF of Γ_e and its time derivative $\dot{\Gamma}_e$. As far as we know, there are no closed-form solutions for the marginal PDFs of Γ_e and $\dot{\Gamma}_e$. In addition, Γ_e and $\dot{\Gamma}_e$ are correlated RVs. Next, we rely on a transformation-of-variables method to determine the LCR.

Since the first hop, second hop, and LI link are assumed to be independent, we can rewrite the joint PDF in (5.6) such that the LCR is expressed as

$$\begin{aligned} N_{\Gamma_e}(\gamma_{\text{th}}) = & \int_0^\infty \int_0^\infty \int_0^\infty \dot{\gamma}_e f_{\dot{\Gamma}_e|\Gamma_e,\Gamma_2,\Gamma_L}(\dot{\gamma}_e|\,\gamma_{\text{th}},\gamma_2,\gamma_L) \\ & \times f_{\Gamma_e|\Gamma_2,\Gamma_L}(\gamma_{\text{th}}|\,\gamma_2,\gamma_L)\, f_{\Gamma_2}(\gamma_2)\, f_{\Gamma_L}(\gamma_L)\, d\dot{\gamma}_e d\gamma_2 d\gamma_L, \end{aligned} \tag{5.16}$$

where the PDF of Γ_n is given by (5.5), $n \in \{1, 2, L\}$. The conditional PDF $f_{\Gamma_e|\Gamma_2,\Gamma_L}(\cdot|\cdots)$ can be written in terms of the PDF of Γ_1. After performing a transformation of variables,



(a) Level crossing rate.

(b) Average fade duration.

Figure 5.3 – LCR and AFD of dual-hop VG-AF-FD relaying with $m_1 = m_2 = m_L = 1$ and various channel strengths.

we obtain

$$f_{\Gamma_e|\Gamma_2,\Gamma_L}(\gamma_e|\,\gamma_2,\gamma_L) = \frac{\gamma_2(m_1(\gamma_2+1)(\gamma_L+1))^{m_1}}{\gamma_e^{1-m_1}(\gamma_2-\gamma_e)^{m_1+1}\Gamma(m_1)\,\bar{\Gamma}_1^{m_1}} \times \exp\left[\frac{m_1\gamma_e(\gamma_2+1)(\gamma_L+1)}{(\gamma_e-\gamma_2)\,\bar{\Gamma}_1}\right], \quad \text{for } \gamma_2 \geq \gamma_e. \tag{5.17}$$

Now, to find the conditional PDF $f_{\dot{\Gamma}_e|\Gamma_e,\Gamma_2,\Gamma_L}(\cdot|\cdots)$ in (5.16), we first determine the time derivative of Γ_e, which can be obtained from (5.4) as

$$\dot{\Gamma}_e = \frac{2\sqrt{\Gamma_e(\Gamma_2-\Gamma_e)^3\Gamma_0}}{\Gamma_2\sqrt{(\Gamma_2+1)(\Gamma_L+1)}}\dot{\alpha}_1 + \frac{2\Gamma_e(\Gamma_e+1)\sqrt{\Gamma_0}}{\sqrt{\Gamma_2}(\Gamma_2+1)}\dot{\alpha}_2 + \frac{2\Gamma_e(\Gamma_e-\Gamma_2)\sqrt{\Gamma_L\Gamma_0}}{\Gamma_2(\Gamma_L+1)}\dot{\alpha}_L, \tag{5.18}$$

where we used $\dot{\Gamma}_n = \partial\Gamma_n/\partial t = 2\sqrt{\Gamma_0}\sqrt{\Gamma_n}\dot{\alpha}_n$. Hence, it becomes apparent that, conditioned on Γ_e, Γ_2, and Γ_L, $\dot{\Gamma}_e$ corresponds to a linear combination of three independent zero-mean Gaussian RVs—$\dot{\alpha}_1$, $\dot{\alpha}_2$, and $\dot{\alpha}_L$—with variances given by $\sigma^2_{\dot{\alpha}_n} = \pi^2 f^2_{m,n}\Omega_n/m_n$, such that

$f_{\dot{\Gamma}_e|\Gamma_e,\Gamma_2,\Gamma_L}(\cdot|\cdots)$ is a zero-mean Gaussian PDF with variance given by

$$\sigma^2_{\dot{\Gamma}_e|\Gamma_e,\Gamma_2,\Gamma_L} = \frac{4\Gamma_0\gamma_e(\gamma_2-\gamma_e)^3}{\gamma_2^2(\gamma_2+1)(\gamma_L+1)}\sigma^2_{\dot{\alpha}_1} + \frac{4\Gamma_0\gamma_e^2(\gamma_e+1)^2}{\gamma_2(\gamma_2+1)^2}\sigma^2_{\dot{\alpha}_2} + \frac{4\Gamma_0\gamma_e^2\gamma_L(\gamma_e-\gamma_2)^2}{\gamma_2^2(\gamma_L+1)^2}\sigma^2_{\dot{\alpha}_L}. \quad (5.19)$$

Hence, the innermost integral in (5.16) can be simplified as [12]

$$\int_0^\infty \dot{\gamma}_e\, f_{\dot{\Gamma}_e|\Gamma_e,\Gamma_2,\Gamma_L}(\cdot|\cdots)\, d\dot{\gamma}_e = \sqrt{\frac{\sigma^2_{\dot{\Gamma}_e|\Gamma_e,\Gamma_2,\Gamma_L}}{2\pi}}. \quad (5.20)$$

Finally, by substituting (5.20), (5.17), and (5.5) into (5.16), we obtain an exact analytical expression for the LCR as given in (5.9).

Appendix 5.B Proof of Theorem 5.2

Considering a high-SNR regime, i.e., $\Gamma_0 \to \infty$, the end-to-end SINR in (5.4) can be approximated as [4, Eq. (20)]

$$\Gamma_e \simeq \Gamma_a \triangleq \frac{\Gamma_1}{\Gamma_L}, \quad (5.21)$$

where "$\simeq$" denotes asymptotic equivalence, indicating that Γ_a increasingly approaches Γ_e at high SNR. The approximation in (5.21) provides the basis for the asymptotic analysis performed here and in Appendix 5.C.

The high-SNR behavior of the LCR is obtained by applying to (5.21) the same rationale described in Appendix 5.A. Thus, by using $f_{\Gamma_e,\dot{\Gamma}_e}(\cdot,\cdot) \simeq f_{\Gamma_a,\dot{\Gamma}_a}(\cdot,\cdot)$ for $\Gamma_0 \to \infty$, the LCR can be written as

$$N_{\Gamma_a}(\gamma_{\text{th}}) = \int_0^\infty \int_0^\infty \dot{\gamma}_a f_{\dot{\Gamma}_a|\Gamma_a,\Gamma_L}(\dot{\gamma}_a|\gamma_{\text{th}},\gamma_L)\, f_{\Gamma_a|\Gamma_L}(\gamma_{\text{th}}|\gamma_L)\, f_{\Gamma_L}(\gamma_L)\, d\dot{\gamma}_a d\gamma_L. \quad (5.22)$$

By applying a transformation of variables, as in (5.17), the conditional PDF $f_{\Gamma_a|\Gamma_L}(\cdot|\cdot)$ can be expressed as

$$f_{\Gamma_a|\Gamma_L}(\gamma_a|\gamma_L) = \frac{(m_1\gamma_a\gamma_L)^{m_1}}{\gamma_a\Gamma(m_1)\bar{\Gamma}_1^{m_1}} \exp\left[-\frac{m_1\gamma_a\gamma_L}{\bar{\Gamma}_1}\right]. \quad (5.23)$$

Now, to find the conditional PDF $f_{\dot{\Gamma}_a|\Gamma_a,\Gamma_L}(\cdot|\cdot,\cdot)$, we first obtain the time derivative of Γ_a as

$$\dot{\Gamma}_a = 2\sqrt{\frac{\Gamma_a\Gamma_0}{\Gamma_L}}\dot{\alpha}_1 - 2\Gamma_a\sqrt{\frac{\Gamma_0}{\Gamma_L}}\dot{\alpha}_L, \quad (5.24)$$

where we used $\dot{\Gamma}_n = 2\sqrt{\Gamma_0}\sqrt{\Gamma_n}\dot{\alpha}_n$. Since $\dot{\alpha}_1$ and $\dot{\alpha}_L$ are independent Gaussian RVs with zero mean and variance given by $\sigma^2_{\dot{\alpha}_n} = \pi^2 f^2_{m,n}\Omega_n/m_n$, the conditional PDF $f_{\dot{\Gamma}_a|\Gamma_a,\Gamma_L}(\cdot|\cdot,\cdot)$

is also Gaussian, with zero mean and variance $\sigma^2_{\dot{\Gamma}_a|\Gamma_a,\Gamma_L} = \frac{4\Gamma_0\gamma_a}{\gamma_L}\left(\sigma^2_{\dot{\alpha}_1} + \gamma_a\sigma^2_{\dot{\alpha}_L}\right)$. Thus, the innermost integral in (5.22) can be solved as

$$\int_0^\infty \dot{\gamma}_a\, f_{\dot{\Gamma}_a|\Gamma_a,\Gamma_L}\left(\cdot|\cdot,\cdot\right)\, d\dot{\gamma}_a = \sqrt{\frac{\sigma^2_{\dot{\Gamma}_a|\Gamma_a,\Gamma_L}}{2\pi}}. \tag{5.25}$$

Finally, by substituting (5.25), (5.23), and (5.5) into (5.22), and then solving the resulting integral using [13, Eq. 3.326.2], we obtain the asymptotic LCR as in (5.11).

Appendix 5.C Proof of Theorem 5.3

Using (5.21), at high SNR the outage probability becomes

$$P_{\Gamma_a}\left(\gamma_{\text{th}}\right) = \int_0^{\gamma_{\text{th}}}\int_0^\infty f_{\Gamma_a|\Gamma_L}\left(\gamma_a|\gamma_L\right) f_{\Gamma_L}\left(\gamma_L\right) d\gamma_L d\gamma_a. \tag{5.26}$$

Now, by replacing (5.5) and (5.23) into (5.26), and then solving the resulting integral with the use of [13, Eq. 3.326.2] and [13, Eq. 3.194.1], the outage probability is obtained as in (5.12).

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6 Conclusões e Trabalhos Futuros

6.1 Conclusões

Este trabalho investigou estatísticas de segunda ordem, mais especificamente a LCR e AFD, para a SNR fim-a-fim de redes cooperativas com *relays* AF de ganho variável. Dois principais cenários foram analisados: sistemas HD com múltiplos saltos e sistemas FD com dois saltos.

No primeiro cenário, foram obtidas expressões assintóticas simples e em forma fechada para a LCR e AFD em canais com distribuição Rayleigh e também em canais com distribuição Nakagami-m. Apesar de a tese apresentar as expressões exatas da LCR e da AFD apenas para canais com distribuição Rayleigh, o mesmo procedimento descrito no Apêndice 3.A pode ser diretamente utilizado para canais com distribuição Nakagami-m. Vale lembrar que a distribuição Nakagami-m reduz à distribuição Rayleigh quando o parâmetro de desvanecimento assume o valor $m = 1$. Para canais Nakagami-m, as expressões assintóticas mostram que o desempenho do sistema em alta SNR é governado pelo canal com o desvanecimento mais severo, associado ao parâmetro de desvanecimento m^* de menor valor. Em particular, o ganho de diversidade da LCR é igual a $m^* - 1/2$, enquanto que o ganho de diversidade da AFD é constante igual a $1/2$. Por outro lado, o ganho de codificação tanto da LCR quanto da AFD dependem dos diversos parâmetros do sistema, quais sejam a severidade, a potência média, e o deslocamento Doppler máximo de cada canal, assim como o número de saltos. Vale destacar que, para o caso de canais identicamente distribuídos (caso homogêneo), o ganho de codificação apenas da LCR depende do número de saltos.

No segundo cenário, considera-se que a comunicação entre a fonte e o destino acontece com o auxílio de somente um *relay* FD do tipo AF de ganho variável. O desvanecimento do canal em cada salto possui distribuição Nakagami-m. Nesse contexto, foram obtidas expressões exatas, em formato integral, e assintóticas, em forma fechada, para a LCR e a AFD da SNR fim-a-fim do sistema. Em particular, as expressões assintóticas desvendam como cada parâmetro do sistema influencia o desempenho em termos das estatísticas de segunda ordem. Curiosamente, mostra-se que nenhum parâmetro do canal entre *relay* e destino influencia o desempenho do sistema na região de alta SNR. Por outro lado, nesse mesmo regime de alta SNR, observa-se que tanto a LCR quanto a AFD são governadas pelo ganho médio de potência e pela mobilidade associados ao canal entre fonte e *relay*, pela potência média do canal LI residual e, no caso da LCR, pela severidade do desvanecimento no canal entre fonte e *relay*. Finalmente, como é de se esperar para o modo de operação full-duplex, mostra-se que a ordem de diversidade tanto da LCR quanto da AFD é zero, ou seja, ambas as métricas tendem a um patamar constante na região de alta SNR.

6.2 Trabalhos Futuros

A seguir são apresentadas propostas de pesquisa tendo como base os problemas investigados nesta tese.

1. Uma extensão imediata da análise apresentada neste trabalho seria obter as estatísticas de segunda ordem em termos da SNR fim-a-fim de redes cooperativas HD com múltiplos saltos e *relays* AF de ganho fixo. Como mencionado ao longo deste trabalho, os estudos disponíveis na literatura fazem essa análise em termos da amplitude fim-a-fim do canal, ou seja, desprezando o ruído acumulado no destino ao longo dos saltos. Dessa forma, seria possível comparar o desempenho do AF de ganho fixo com o AF de ganho variável em termos da LCF e AFD, à medida que o número de saltos varia.

2. A análise das estatísticas de segunda ordem de sistemas FD continua em aberto para várias configurações de redes cooperativas. Neste trabalho foi analisado a rede FD com *relays* AF de ganho variável e com dois saltos sob desvanecimento Nakagami-m. A lista a seguir apresenta alguns exemplos de redes cooperativas FD que ainda não tiveram suas estatísticas de segunda ordem analisadas:

    a) *relay* AF de ganho fixo [1], [2];

    b) *relay* DF [3], [4];

    c) redes com fonte, *relay* e destino operando no modo FD (FD-TWR – *full-duplex two-way relaying*) [1], [5], [6];

    d) redes com múltiplos saltos [7], [8]; e

    e) redes com seleção de *relays* [9]–[13].

3. Outra possibilidade interessante é estender os estudos desta tese ou aqueles sugeridos nos itens anteriores para ambientes de desvanecimento mais gerais (por exemplo, $\alpha-\mu$, $\eta-\mu$, e $\kappa-\mu$), a fim de valiar o impacto de outros fatores de propagação (por exemplo, não linearidade do meio, desbalanceamento fase-quadratura e componente de visada direta) sobre as estatísticas de segunda ordem de sistemas cooperativos.

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Apêndices

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On Second-Order Statistics of Full-Duplex Relaying

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