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On the question of the energy dependence of inelasticity ☆

J. Bellandi, R.J.M. Covolan, C. Dobrigkeit, C.G.S. Costa, L.M. Mundim
Departamento de Raios Cósmicos e Cronologia, Instituto de Física "Gleb Wataghin", Unicamp, 13083 Campinas, SP, Brazil

and

J. Dias de Deus
Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais, P-1096 Lisbon Codex, Portugal

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We discuss the question of the energy dependence of inelasticity in very-high-energy hadronic interactions. From existing extensive-air-shower data, a definite conclusion cannot yet be reached. However, Glauber model realistic calculations and recent results from Tevatron suggest a slow increase of the mean inelasticity with energy.

Recent data from the Tevatron [1] show that the cross-section does not have a strong increase with energy, the tendency being of a softer dependence on energy than $\ln^2 E$. We discuss in this paper the consequence of these results for the analysis of the energy dependence of inelasticity at very high energy. From Glauber calculations of the inelastic proton–air cross-section we find the inelasticity using experimental data for nucleonic flux and EAS data for the attenuation length in the atmosphere.

The evolution of the nucleonic cosmic-ray component in the atmosphere is controlled by two physical quantities, related to high-energy strong interactions. One is the interaction nucleon–air mean free path, λ, which is inversely proportional to the inelastic cross-section. The other one is the inelasticity, k, related to the leading-particle effect in high-energy collisions and defined as the fraction of the whole invariant energy used for multiparticle production, while the remainder is carried by the leading particle. Both are, in principle, energy-dependent quantities, measurable in accelerator experiments.

The solution of the nucleonic diffusion equation in the atmosphere, when $\lambda(E)$ and $k(E)$ are functions of energy, was recently discussed by Bellandi et al. [2]. If the primary spectrum at the top of the atmosphere is assumed to be in the form of a power law

$$F_N(E, t=0) = N_0 (E/\mathrm{GeV})^{-(\gamma+1)}, \qquad (1)$$

the differential spectrum at depth t (in g/cm^2), $F_N(E, t)$, is given by

$$F_N(E, t) = N_0 E^{-(\gamma+1)} \exp[-t/\Lambda(E)], \qquad (2)$$

where $\Lambda(E)$ is the attenuation length and experimentally $N_0 = 1.737$ (cm^2 s sr GeV)$^{-1}$ and $\gamma = 1.76$ [3].

The attenuation length, $\Lambda(E)$, is related to the interaction mean free path $\lambda(E)$ by

$$\Lambda(E) = \lambda(E)/\{1 - [1 - k(E)]^\gamma\}, \qquad (3)$$

which is a generalization of the result obtained when λ and k are considered as energy-independent quantities [4].

Consequently, the study of the energy dependence of inelasticity by means of cosmic-ray data is strongly related to the study of the energy dependence of the inelastic nucleon–air cross-section. This is so because

$$\lambda(E) = 24\,100/(\sigma_{\mathrm{in}}^{\mathrm{p-air}}/\mathrm{mb})\ \mathrm{g/cm^2}. \qquad (4)$$

The inelastic proton–air cross-section may be derived from experimental data obtained in inclusive proton—nuclei reactions with accelerators, using the scaling property in the mass number A. In this way,

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Liland [5] found from experimental data on p–Be and p–C scattering that

$$\sigma_{\rm in}^{\rm p-air}=249.88 \times[1+0.004975\ln^2(E/10\ {\rm GeV})]\ ({\rm mb}) \qquad (5)$$

in the range 24 GeV $\leqslant E \leqslant$ 400 GeV, showing an increase in energy as $\ln^2(E)$. Nevertheless, recent data from the Tevatron [1], as we mentioned above, show that the cross-section has not such a strong increase with energy, the tendency being of a softer dependence in energy than $\ln^2 E$.

In order to verify this point, we calculate the inelastic nucleon–air cross-section, assuming the validity of Glauber's model [6]. In this model $\sigma_{\rm in}^{\rm p-air}$ is given by

$$\sigma_{\rm in}^{\rm p-air}=\int {\rm d}^2 b\,\{1-\exp[-\sigma_{\rm t}({\rm pN})T(b)]\}\,, \qquad (6)$$

where

$$T(b)=\int_{-\infty}^{+\infty}\rho(b,z)\,{\rm d}z\,, \qquad (7)$$

and $\rho(b, z)$ is the nuclear density distribution, normalized as

$$\int \rho(\boldsymbol{r})\,{\rm d}^3\boldsymbol{r}=A\,, \qquad (8)$$

and $\sigma_{\rm t}({\rm pN})$ is the total proton–nucleon cross section.

In the present calculations we shall use for the nuclear density the same model as has been employed for light elements in the analysis of high-energy electron scattering experiments [6,7]:

$$\rho(r)=\rho_0(1+\tfrac{4}{3}r^2/a_0^2)\exp(-r^2/a_0^2)\,. \qquad (9)$$

Using the normalization condition and the root mean square radius, $\langle r^2\rangle^{1/2}$, we can determine ρ_0 and a_0. In order to obtain the mass-number dependence, we assume the parametrization $\langle r^2\rangle^{1/2}=\beta A^{1/3}-\gamma A^{-1/3}$ and fit the experimental values of ρ_0 for light nuclei, obtaining $\beta=1.096$ and $\gamma=0.410$.

We approximate $\sigma_{\rm t}({\rm pN})$ by $\sigma_{\rm t}({\rm pp})$ and calculate $\sigma_{\rm in}^{\rm p-air}$ using the following fit for experimental data of $\sigma_{\rm t}({\rm pp})$:

$$\sigma_{\rm t}({\rm pp})=39.5s^{-0.38}+21.7s^{0.08}\ ({\rm mb})\,,$$

where we have included the recent results of the Tevatron [1]. The first term in this expression results from the interplay of secondary reggeon exchanges and the second one represents the pomeron exchange.

In fig. 1 we show the behaviour of $\sigma_{\rm in}^{\rm p-air}$ as a function of energy, calculated by Glauber's model (dashed curve) and an extrapolation to higher energies of Liland's $\sigma_{\rm in}$ (solid curve). In the energy region 10^2 GeV $\leqslant E \leqslant 10^6$ GeV both show, roughly, the same dependence with energy, but the calculated $\sigma_{\rm in}$, using Glauber's model, shows a slower increase at higher energies than Liland's one.

We also show in fig. 1a comparison with $\sigma_{\rm in}$ obtained by Linsley [8] using cosmic-ray data for the attenuation length in the region $E > 10^6$ GeV. In this calculation an inelasticity distributed uniformly between 0 and 1 with a mean value of 0.5 is assumed.

Unfortunately, the large dispersion of experimental data does not allow a definite conclusion. For instance, the experimental points of Akeno [9] agree with a $\ln^2 E$-dependence, while data from other experiments, including Fly's Eye [10], show a slower increase with energy. These results are strongly dependent on the average inelasticity.

Knowing $\sigma_{\rm in}$, we are able to determine $\lambda(E)$ by means of eq. (4). Consequently, the interaction mean free path in Liland's parametrization decreases faster with energy than the one calculated adopting Glauber's model.

The correct determination of the energy dependence of $\sigma_{\rm in}$ has then consequences in the study of the

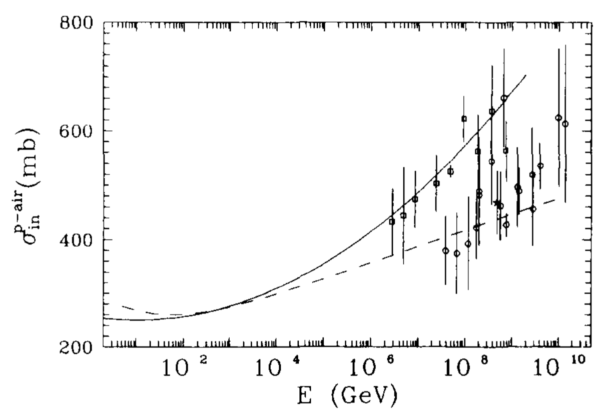

Fig. 1. Inelastic proton–air cross-section as calculated with Liland's parametrization (solid curve) and with Glauber's model (dashed curve). Experimental points: ⊖: ref. [8]; ⊟: ref. [9]; ⋆: ref. [10].

behaviour of cosmic-ray components in the atmosphere and, consequently, in the study of inelasticity.

In cosmic-ray experiments either the flux of particles is directly measured or the attenuation length, which is related to $\lambda(E)$ via the inelasticity by means of eq. (3), is obtained in an indirect way.

Consequently, the study of the energy dependence of the inelasticity can be done with cosmic-ray data. In a recent paper [11] we calculated the inelasticity from data for the nucleonic flux in the atmosphere [12,13], using eq. (2) in the energy region 10^2 GeV $\leqslant E \leqslant 10^4$ GeV and $\lambda(E)$ as parametrized by Liland. In this region and within experimental errors, the value of $k(E)$ is consistent with a constant $0.5 \leqslant k(E) \leqslant 0.6$.

In fig. 2 we present the same calculation for $k(E)$, but using $\lambda(E)$ as given by Glauber's model. In the region 10^2 GeV $\leqslant E \leqslant 10^4$ GeV there are no differences in the behaviour of $k(E)$ previously obtained, since in both cases the dependence with energy is practically the same.

In the region of higher energies, $E \leqslant 10^6$ GeV there are no experimental data for the flux of cosmic-ray components, and the analysis of $k(E)$ can only be obtained through the measurement of the attenuation length.

Starting from eq. (3), obtained in ref. [11], Wilk and Wlodarczyk [14] determined $k(E)$ in the higher-energy region, using Liland's parametrization for $\lambda(E)$. They concluded that the tendency of $k(E)$ is

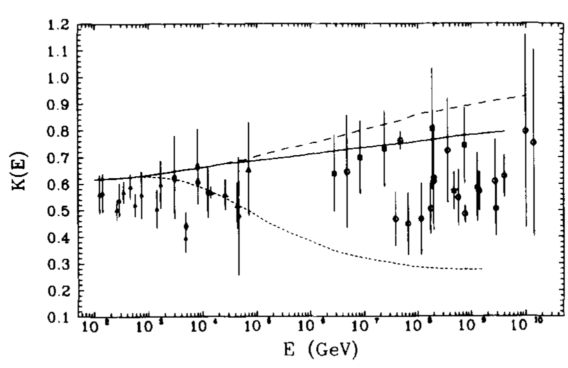

Fig. 2. Experimental determination of inelasticity from cosmic rays compared with model calculations. Experimental points: (a) $E < 10^6$ GeV are from refs. [12,13]; (b) $E > 10^6$ GeV; ⊖: ref. [8] ⊟: ref. [9]; ⋆: ref. [10]. Calculation: QCD pomeron model (dashed curve); minijet model (solid curve) and statistical model (dotted curve).

to decrease slowly with energy and argued that this result is consistent with that obtained from the statistical model. However, if the inelastic cross-section has a slower increase with energy, as suggested by the Tevatron data, this tendency does not appear.

Using $\lambda(E)$ determined from Glauber's model, we show in fig. 2 the results obtained for $k(E)$ in the region $E \geqslant 10^6$ GeV, with experimental data for the attenuation length [8]. The tendency within the limits of error bars is more in the sense of a very slow increase with energy, although also compatible with a constant mean value, as usually assumed by cosmic-ray physicists.

In this way the behaviour of inelasticity with energy is strongly correlated with the increase of the cross-section. A slower increase in σ, as recent Tevatron results seem to indicate, implies a slow increase or even constant value of the mean inelasticity.

The various models for hadronic interaction show different behaviour for the proton–proton inelasticity $k_{\rm pp}$. The statistical model of Fowler et al. [15] points to a decrease in $k_{\rm pp}$ with energy. On the contrary, the minijet model of Gaisser and Stanev [16] presents a very moderate increase of $k_{\rm pp}$ with energy and in the QCD pomeron model of Kopeliovich et al. [17] this increase is faster.

Using the multiple-scattering theory of Glauber we can estimate the mean value for $k_{\rm p\text{-}air}$, from the $\bar{k}_{\rm pp}$ determined in those three models (see ref. [16]), connected by the relation [18]

$$k_{\rm p\text{-}air} = 1 - \sum_{n=1}^{n_{\max}} P_n (1 - k_{\rm pp})^n ,$$

where P_n is the probability of n-fold collisions of the primary nucleon in the nucleus, being expressed by

$$P_n = \frac{\int {\rm d}^2 b \; P_n(b)}{\sigma_{\rm in}^{\rm p\text{-}air}} ,$$

with

$$P_n(b) = \frac{1}{n!} \left[\sigma_{\rm t}({\rm pp}) T(b)\right]^n \exp\left[-\sigma_{\rm t}({\rm pp}) T(b)\right] .$$

In fig. 2 the results obtained with different models are shown in comparison with those obtained from cosmic-ray data.

From the statistical model the calculated $k_{\rm p\text{-}air}$ shows a strong decrease with energy. In the energy region $E \leqslant 10^6$ GeV the inelasticity values are quite below the experimental results, even when compared with the results obtained by Wilk et al. [14]. With the QCD pomeron model, k_{p-air} shows a fast increase with energy, resulting in values of inelasticity above the experimental results. The minijet model leads to a slow increase with energy very much compatible with experimental data.

In conclusion, we have shown that the behaviour of inelasticity is strongly correlated with the behaviour of the inelastic cross-section. In the energy region 10^2 GeV $\leqslant E \leqslant 10^6$ GeV, where σ shows a $\ln^2 E$-dependence, the average inelasticity is compatible with a constant value, in agreement with accelerator data at the ISR region [19]. At very high energy, if the tendency of the inelastic cross-section is to increase slower than $\ln^2 E$, as the Tevatron results indicate, the inelasticity shows a slow increase with energy, being perhaps constant.

Glauber calculations and extrapolation of Liland's parametrization for σ at high energy give different results. Present EAS data are not good enough to eliminate one of the calculations. Any conclusion regarding the question of the energy dependence of inelasticity depends on the considered model for the cross-section. The hope to solve this ambiguity lies in the new generation of accelerators, and the possibility of measuring the leading-particle inclusive distribution, as it happened at the ISR. In this way, reliable information about the energy dependence of the average inelasticity, could be obtained.

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