Mode pattern dependence on the eccentricity of microstadium resonators

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Microdisk and microstadium resonators based on InGaAsP multiquantum-well laser structures were fabricated by focused ion beam employing Ga+ ion milling and polishing followed by selective chemical etching. Stadia with very good morphology and with different eccentricities were fabricated for the study of optical mode selection. Light emission was investigated by infrared microscopy and spectroscopy. The measured emission pattern and the spectra agree well with a simple model based on the summation over periodic orbits or scar modes. The dependence of the scar mode emission with the resonator eccentricity was modeled based on the difference between photon lifetime and orbital round-trip time. The mode selection dependence with the surrounding index of refraction is suggested for chemical sensing applications. © 2010 American Institute of Physics. [doi:10.1063/1.3393296]

I. INTRODUCTION

Microdisk resonators support whispering gallery modes (WGM) tightly confined near the edges of the disk. In terms of ray treatment, WGM can be seen as polygonal orbits with quasitotal internal reflection at their vertices.1–3 The high optical confinement and consequent high quality factor of these resonators is attractive because it leads to very long photon lifetimes in small volumes. A limitation of the disk resonator is its spatially isotropic emission. Also, typically very small unpractical disk radius is required for single mode emission.

The use of quasiopen stadium resonators has been suggested as a possible solution for this limitation. Stadium geometry is a distortion of the disk geometry consisting of two semicircles of radius R connected by a rectangle of length 2d. Figure 1(a) shows a schematic drawing of a stadium resonator. The solid elastic reflection wall stadium billiard has been shown to be classically strongly chaotic. Therefore all periodic trajectories are unstable. In the high quantum number limit, i.e., in the semiclassical limit, the eigenstates tend to concentrate over preferential periodic trajectories, called scars of the classical domain.4,5 These scars are surrounded by a sea of eigenstates uniformly distributed in physical space.6 The isomorphic mathematical description of a resonator and the time independent Schrödinger equation allows us to expect the same behavior for confined photons in stadium optical resonator with perfect metallic boundary, as long as photon wavelength is much smaller than the dimensions of the resonator. In principle all these photonic eigenstates, or modes, have equal probability to exist and one should not expect any preferential excitation of the scar modes. However, the real dielectric stadium resonator is a quasiopen system where photons are only partially held by the refractive index discontinuity between the resonator and the surrounding media. The open system leads to a mixing of neighboring modes and the relationship to the classical description is almost lost. Empirically, however, several authors report the observation of emission patterns corresponding to the periodic orbits or scars. This behavior may be qualitatively explained using the ray description. In fact the ray description surprisingly has been shown to be very adequate for the description of these systems.7 Using the ray description, one may affirm that most chaotic trajectories will vanish since photons will escape quickly by eventually hitting the interface at incidence angles smaller than the critical value for total internal reflection.8 Further modal selection among the periodic orbits may also be explained by the

FIG. 1. Schematic drawing of (a) stadium and (b) scars: diamond (A), bow-tie (B), double-diamond (C), Z-shaped (D), and two Fabry–Perot modes on axis (E and F).
ray treatment. Each periodic orbit or scar can be treated as an independent one-dimensional path with multireflections. The losses and consequent photon lifetime of a given mode are given by the optical linear scattering and the Fresnel reflectivity coefficient. Thus, geometry and/or external index of refraction become crucial for the establishment of a mode since it can only exist if the photon lifetime is larger than the its round-trip time. In this work we discard all the nonscar modes, the scars modes that have incidence below the critical angle as well as those too long for the photon to survive. Figure 1(b) shows some typical scars: diamond (A), bow-tie (B), double-diamond (C), Z-shaped (D), and two Fabry–Perot modes on axis (E and F). Of these scars, clearly E and F are readily discarded. The others may survive depending on the elongation of the stadium.

Several authors have reported the observation of the emission related to scar modes. Lebenthal et al.9 showed high directionality in stadium shape polymer microlasers. Shinohara et al.10 published a depth theoretical and experimental study of emission patterns in stadium-shaped microcavity lasers. Figueira and Frateschi11 presented evidence of coexistence of bow-tie and diamond scars on optically pumped Er+ doped Si resonators. Munoz et al.12 presented high emission directionality and enhanced side mode suppression with the injection of carriers forced over a diamond scar13. This last observation was attributed to the perturbation of the bow-tie scar that leads to a larger modal separation within a fixed gain bandwidth, hence, leads to higher spectral purity.

The fact that diamond and bow-tie modes are either observed separately or indirectly by beating leads us to believe that the existence of these modes depends on the exact geometry of the stadia. This dependence was used by Fang et al.8 to control lasing in stadium-shaped lasers. In this work we try to provide a more direct verification of this conjecture. A simple ray treatment description of the modal selection dependence with the stadium geometry is initially presented. Subsequently, we design, fabricate and characterize two stadia with different geometry to investigate the process of modal selection. Infrared imaging and optical spectroscopy of the emissions show evidence of both the presence of the scar modes as well as the beating between them. Moreover, good agreement with our simulation is achieved.

II. STADIUM RESONATOR DESIGN

In order to quantify the geometry of a stadium resonator, we have arbitrarily introduced the eccentricity parameter $\varepsilon$ given by:

$$\varepsilon = \frac{1}{2} \left[ \frac{(d + R)^2 - R^2}{(d + R)^2 + R^2} \right].$$

(1)

Notice that $\varepsilon$ varies from 0 to 1/2 going from a pure disk geometry disk with $d=0$ to a Fabry–Perot linear resonator with $d\gg R$.

The first step in designing the resonator consists of defining the optical modes whose photon lifetime, $\tau_p$, is greater than the orbital round-trip time, $T_{\text{round-trip}}$. This is done for different eccentricities. The round-trip time and photonic lifetime are given by Eqs. (2) and (3).

$$T_{\text{round-trip}} = \frac{L}{V},$$

(2)

$$\tau_p = \frac{Q}{\nu},$$

(3)

where, $L$ is the scar length, $V$ is the group velocity of the wave in the resonator, $Q$ is the quality factor of the resonator, and $\nu$ is the electromagnetic wave frequency.

Assuming a simple ray treatment, the quality factor can be calculated by Ref. 13 [Eq. (4)]

$$Q = \frac{\pi \cdot L \cdot n_{\text{eff}}}{\lambda \cdot \sin h \left[ q \cdot \left( \alpha_{\text{mat}} - \gamma_{\text{optical}} \right) \cdot L + \sum_{i=1}^{q} \ln \left[ r_i (\varepsilon) \right] \right]},$$

(4)

where, $\gamma_{\text{optical}}$ is the material optical modal gain, $\alpha_{\text{mat}}$ is the internal optical scattering loss, $n_{\text{eff}}$ is the effective refractive index and $\lambda$ is the electromagnetic radiation wavelength in vacuum. The term $\sum_{i=1}^{q} \ln \left[ r_i (\varepsilon) \right]$ has the sum over all $q$ scar reflections where the reflectivity $r_i (\varepsilon) = r_i^2(\varepsilon)$ depends on the eccentricity, the resonator index of refraction, and the environment index of refraction.

The photon lifetime is calculated for the transparency condition ($\gamma_{\text{optical}}=0$). The scar length $L$ [Fig. 1(b)] is easily obtained for a given scar, eccentricity $\varepsilon$ and radius $R$. A scattering loss value of $\alpha_{\text{mat}}=40 \text{ cm}^{-1}$ was used.14 The reflectivity at each one of the vertices of the scars was obtained assuming a 17% scattering loss.15 Therefore $R=83\%$ for incidence above the critical angle for total internal reflection. For angles below the critical angle, the reflectivity was calculated using the s polarization (magnetic field in the plane of incidence which is parallel to the axial direction) also subtracted by 17%. Photon lifetime and round-trip time were calculated for structures with 10 $\mu$m radius. Preliminary calculations indicate that we can neglect modes (D), (E) and (F) shown in Fig. 1 because the photon lifetime is always smaller than the round-trip time for the chosen radii. Effective refractive index in a range of 2.5 $\leq n_{\text{eff}} \leq 2.7$ was obtained assuming an In$_{0.47}$Ga$_{0.43}$As$_{0.24}$P$_{0.76}$ (300 nm)/air infinite slab waveguide. We assume a $\pm 10\%$ range due to the use of an infinite slab waveguide geometry for the calculation and by geometrical and compositional variations in the structure.

Figure 2 shows the calculated photon lifetime versus eccentricity for (a) the bow-tie scar, (b) the diamond scar, and (c) the double-diamond scar. The calculation is done for $n_{\text{eff}}=2.5$ (light dotted line), $n_{\text{eff}}=2.7$ (dark dotted line) and $n_{\text{eff}}=2.6$ (solid line). Figure 2(a) shows that the bow-tie scar is strongly supported for 0.21 $< \varepsilon < 0.40$. Figure 2(b) shows that the diamond scar is supported for $\varepsilon < 0.34$. Figure 2(c) shows that the double-diamond is marginally supported for $\varepsilon < 0.30$.

A. Fabrication

Two stadia resonators with $\varepsilon=0.19$, $\varepsilon=0.36$, and $R=10 \mu$m were fabricated. A disk resonator, $\varepsilon=0$, also with
$R=10 \mu m$, was fabricated for comparison. The epitaxial structure of the resonators consisted of a conventional InGaAsP/InP multiquantum well laser with 300 nm InGaAsP active region sandwiched by InP cladding layers, all grown on n-type (001) InP substrate. The bottom n cladding layer is doped with silicon, and the upper p cladding layer is doped with zinc. A 200 nm top p$^+$-InGaAs layer was used as the p contact layer. The first step in the fabrication process consists of a Ti/Pt/Au metallization of the p-InGaAs layer. Subsequently, Au/Ge/Ni/Au is deposited on the back of the substrate. The sample was alloyed in forming gas at 420 °C for 30 s. Cylinders with each respective resonator cross-section shape were obtained by ion milling using dual beam focused ion beam/scanning electron microscope (FIB/SEM—NEW 200 model from FEI Co.). Ga$^+$ ion current of 20 nA at 30 keV was employed for 15 min to reach a depth of 4 $\mu m$. Subsequently a smoothening process was performed also by FIB (6 min milling with an ion current of 1 nA and 30 keV). Finally $1H_2PO_4:3HCl$ (~40 s) selective wet etching was performed to remove InP, leaving suspended structures of InGaAs contact layer and InGaAsP based active region held by InP pedestals.$^15$

### III. EXPERIMENTAL RESULTS

Figure 3 shows several micrographs organized in rows (a, b, c, d, and e) and columns (i, ii, and iii). Column (i) correspond to a microdisk ($\varepsilon=0$) and columns (ii) and (iii) correspond to stadia of $\varepsilon=0.19$ and $\varepsilon=0.36$, respectively. Row (a) shows SEM micrographs of the fabricated devices. Very good wall morphology is obtained with our fabrication approach. Row (b) shows visible light optical microscope image of the devices. The dark regions are due to the shadow of the microprobe used for current injection. Row (c) shows infrared optical microscope image (Find-R-Scope $\lambda<2200$ nm). Row (d) shows a color rendered graph of row (c). The color code varies from blue to red as the intensity increases. Finally, row (e) shows a schematic drawing of the InP pedestal cross-section for each resonator. The infrared image shown in rows (c) and (d) clearly shows an isotropic emission for the disk, four stronger spots for the stadium with $\varepsilon=0.19$ and a more complex pattern with several spots for the stadium with $\varepsilon=0.36$. It is difficult to identify exactly the scar that would match this pattern. Nevertheless, it is clear that it cannot be assigned to a single scar. The fact that the pedestal has a diamond shape may mask the actual mode spatial distribution due to light scattering on its edges. However, no scattering is observed for the disk even though the disk has a pedestal of the same shape placed very close to its edge. Also, no light is observed at the sharper edge of the pedestal of the stadium with $\varepsilon=0.36$. Therefore, we believe that the infrared images provide real information of the spatial distribution of the modes.

The emission spectra were obtained using an Optical Spectrum Analyzer (HP70004A). The light emitted from the resonator was collected by a multimode optical fiber positioned next to the device. Two directions were evaluated: lateral (along the minor axis) and frontal (along the major axis).

Figure 4 shows the lateral and frontal emission spectra for an injection current density of 3.2 kA/cm$^2$ at 18 °C. All
spectra are normalized. Figure 4(a) and 4(b) shows the lateral and frontal emission spectra of the stadium with eccentricity $\varepsilon=0.19$, respectively. Figure 4(c) and 4(d) shows the lateral and frontal emission spectra of the stadium with eccentricity $\varepsilon=0.36$, respectively. The better finesse for the lateral emission is readily observed. The lateral emission free spectral range (FSR) is 15 nm for both stadia. A quality factor of $Q \sim 190$ for $\varepsilon=0.19$, at $\lambda=1450$ nm and $Q \sim 250$ for $\varepsilon=0.36$, at $\lambda=1435$ nm were estimated for the lateral emission. The quality factor was obtained from the spectrum, $Q=\Delta\lambda/\lambda$, where $\Delta\lambda$ is the full width at half maximum of the peak above the spontaneous emission background. The frontal emission has a much larger contribution from spontaneous emission. Nonuniform pumping caused by the larger distance from the pedestal may lead to photon absorption at the edge which is reemitted as spontaneous emission. Therefore, it is very difficult to resolve the scar emission in the frontal direction.

IV. ANALYSIS

The infrared image in Fig. 3, row (d) column (ii) shows higher emission intensity spots that may fit the vertices of the diamond scar. This is expected since the photon lifetime excess over the roundtrip time is largest for this scar in the case of eccentricity $\varepsilon=0.19$. Also, the diamond shaped pedestal favors the injection with high overlap with this scar. Also, the 15 nm FSR is in agreement with the scar length ($L \sim 52$, $\mu$m, $n_{eff}=2$ and 7, and $\lambda=1450$ nm). Small shoulders are also observed above 1450 nm. These shoulders may be due to other confined scars in the cavity. As predicted in Fig. 2(a), the bow-tie scar is not supported by the stadium with this eccentricity.

The emission image in Fig. 3, row (d) column (iii) for the stadium with $\varepsilon=0.36$ is much more complex. The lateral bright spots may be related to the bow-tie or the double-diamond. The spots at the edge may arise from the double-diamond or the diamond scar. Our simulation shows that the bow-tie and the double-diamond should dominate in this case. However, the 15 nm FSR cannot be fitted by any of these scars if they are exactly inscribed in the stadium. One important observation about the fabrication of this particular stadium is that unlike the smaller eccentricity stadium, the pedestal base extends very close to the lateral edge. Also, both the double-diamond and the bow-tie scars have a large portion of their orbit traveling over regions with InP (pedestal) cladding layers instead of air. Therefore, the optical feedback at the interfaces between pedestal and air regions affects the final spectrum. Experiments with cylinders without pedestals are ongoing and may clarify this point.

A simple ray tracing incoherent beating approach may be employed to describe the observed lateral spectra. The electric field amplitude $E_R$, can be obtained as an infinite summation of the round trips as

$$E_R(\lambda) = \sum_{m=1}^{\infty} E_o \exp \left( ik \cdot L \cdot m - \left[ \alpha_s - g_{optical}(\lambda) \right] \cdot L - \sum_{i=1}^{q} \ln[r_i(\varepsilon)] \right) \cdot m \right),$$

This summation is performed for each wave-vector $k$, given by $k(\lambda)=2\pi \times n_{eff}/\lambda$, where $\lambda$ is the wavelength.

If $\alpha_s - g_{optical} > 0$ for a given wavelength, this summation converges to

$$\frac{E_R(\lambda)}{E_o} = \frac{1}{1 - \exp(ik(\lambda) \cdot L - \left[\alpha_s - g_{optical}(\lambda)\right] \cdot L \cdot \sum_{i=1}^{q} \ln[r_i(\varepsilon)]}),$$

FIG. 4. Spectra measured for a stadium with $\varepsilon=0.19$: (a) lateral and (b) frontal. Spectra measured for a stadium with $\varepsilon=0.36$: (c) lateral and (d) frontal.
The phase among different scars is the only fitting parameter. Figure 5 shows the stadium with eccentricity $\varepsilon = 0.19$. The agreement between the peak position of the simulation and the measured spectra is good. However, we observe that there is some extra factor contributing to a line width enhancement. Figure 5(b) shows the spectra simulation after the internal optical loss was increased such that the simulated $Q$ matches the measured value. After this procedure the calculated spectrum agrees very well with the experimental result. Also, it becomes clear that the diamond scar dominates the emission while the bow-tie is essentially nonexistent. All measured peaks, including the small shoulders, indicated by arrows, are obtained by the calculation. Figure 5(c) shows the calculation of the different scars, the total summation and the measured spectrum for the stadium with eccentricity $\varepsilon = 0.36$ and $n_{\text{eff}} = 2.5$. Again, the internal loss has to be increased in order to match the measured $Q$. Also, the FSR does not match the experimental result. In order to obtain the correct FSR the optical length has to be reduced by a factor 1.4. We believe this is due to the effects of the multiple reflections at the air-pedestal interfaces. The result after the optical length reduction and internal loss increase is shown in Fig. 5(d). The agreement measurement is much better. Nevertheless it is still not clear the reason for the reduction in optical length. Qualitatively, we may consider that a mixture of several neighboring modes near the scars may widen the trajectory. This fact together with the nonuniform current injection and the discontinuity in the cladding layer index caused by the presence of the pedestal, may lead to an effectively smaller area for transparency. Nevertheless, it is clear the change in the emission pattern observed as the eccentricity exceeds the limit for the bow-tie mode to be supported. Therefore, the change in geometry apparently leads to scar selectivity. Because the exact eccentricity needed for a transition between scars depends on the photonic lifetime, it critically depends on the refractive index difference between the cavity and the environment. Therefore, one may design a structure that changes its spatial modal distribution/emission with a small change in the surrounding index. Since spatial mode variation can be easily detected and measured, this structure can be used for sensing.

V. CONCLUSION

Very good morphology stadium resonators were fabricated by a hybrid process involving FIB and conventional microelectronic fabrication techniques. Emission composed of incoherent beating between different scar modes was observed. The dependence of the emission with the resonator geometry is presented. A modal selection criteria based on the difference between photon lifetime and orbital round-trip time was used to explain the observed geometrical dependence. A transition from the diamond to bow-tie scar is predicted as the eccentricity increases. A transition between diamond and a complex pattern was measured in our devices. Since, this transition depends strongly on the surrounding
refractive index, it may be used for sensing, i.e., depending on the external refractive index, a different mode and emission profile may result being easily detected.

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