Influence of propagation on the coherent accumulation of excitation induced by an ultrashort pulse train

A. A. Soares
Universidade Federal de São Carlos, Campus Sorocaba, 18052-780 Sorocaba, SP, Brazil

Luís E. E. de Araujo*
Instituto de Física “Gleb Wataghin,” Universidade Estadual de Campinas, 13083-970 Campinas, SP, Brazil
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We study the effects of propagation through an extended sample of two- and three-level atoms on the coherent accumulation of excitation by an ultrashort pulse train. In the two-level case, previous pulses in the train may prepare the medium such that a later pulse may experience amplification and absorption in different positions inside the sample. For a large number of pulses, the atomic medium may be saturated by the train, and some pulses propagate without experiencing either absorption or amplification. In general, absorption of the resonant pulse frequency during propagation compromises the accumulation efficiency. In the three-level case, the coherent accumulation of excitation leads to electromagnetically induced transparency of the pulse train, and the pulses exit the medium without any distortion in their temporal profile.

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I. INTRODUCTION

It has been long known that a train of short pulses can coherently excite atomic and molecular samples. This coherent nature of multipulse excitation was first investigated by Teets and co-workers [1] who showed experimental evidence of a significant signal enhancement over single-pulse excitation of a two-photon atomic transition. In fact, in the weak excitation regime, the transition probability is proportional to the squared number of excitation pulses [2–4]. For a nondecaying two-level atom, Vitanov and Knight [2] showed theoretically that a train of equally spaced identical short pulses can induce Rabi cycling of the atomic population. Because the atomic excitation coherently accumulates from one pulse to the next, efficient excitation of the atom is possible even if individually the pulses are weak and operate in the transient regime. For decaying atoms, if the pulse repetition period is shorter than the relaxation time of the system, population may still coherently accumulate in the excited state between pulses [5,6]. The coherent-accumulation-of-excitation effects have been shown to play a role in atomic coherent-control experiments [3,4,7]. And in recent work, we showed that the coherent accumulation of excitation in three-level Λ atoms leads to an electromagnetically induced transparency of the driving pulses [8].

In molecular systems, Warren and Zewail [9] showed that a sequence of nonresonant strong pulses could dramatically increase population inversions and multiphoton pumping. Diels and Besnainou [10] showed that matching the frequency peaks, associated with the train spectrum, with the successive transition frequencies of an anharmonic ladder of vibrational levels resulted in a nearly complete population transfer to the upper state. Terahertz-rate trains of femtosecond pulses have been experimentally demonstrated to amplify selected vibrational modes of an α-perylene crystal by matching the pulse repetition rate to the desired phonon frequency [11]. Recently, Pe’er and colleagues [12] theoretically explored coherent accumulation effects to implement an efficient scheme for performing narrow-band Raman transitions between molecular vibrational levels. Another recent proposal explores the coherent accumulation in the excitation of a diatomic molecule by a train of ultrashort pulses to achieve a selective and efficient excitation of the molecule’s vibrational levels [13].

In this paper, we study such accumulation effects and show that a very short propagation through a sample of two-level atoms can have a very deleterious effect on the accumulation. We also consider the propagation of a train of ultrashort pulses through an extended collection of three-level Λ atoms. We show that, once the regime of electromagnetically induced transparency has been reached, the ultrashort pulses propagate without reshaping their temporal profile. The propagation of a pulse train has been investigated previously. Newbold and Salamo [14] studied the effects of relaxation on the propagation of a slow train (repetition periods longer than the excited state lifetime) of long pulses (pulse widths longer than the excited state lifetime). Bouchene [15] showed that the phase relation between pulses in a train is paramount in controlling dispersion in the propagation of trains with terahertz repetition rates. Sautenkov and co-workers [16] observed experimentally an increase in the transmission intensity of an optical pulse train through rubidium vapor when they matched the pulse repetition rate to a subharmonic of the hyperfine splitting of the ground state. To our knowledge, the effects of pulse propagation on the coherent accumulation of excitation by a train of ultrashort pulses have not been investigated yet.

This paper is organized as follows. Section II is dedicated to the propagation of an ultrashort pulse train through an extended two-level atomic sample, while in Sec. III we discuss the three-level atomic case. Section IV concludes the paper.

*araujo@ifi.unicamp.br
loss of generality, we take the dipole matrix element $T$ where

$$
\dot{\rho}_{ab}(z, \tau) = -0.5i\Omega(\rho_{ba} - \rho_{ab}) - (1/T_1)\rho_{aa},
$$

(1a)

$$
\dot{\rho}_{bb}(z, \tau) = -0.5i\Omega(\rho_{ab} - \rho_{ba}) + (1/T_1)\rho_{aa},
$$

(1b)

$$
\dot{\rho}_{ba}(z, \tau) = -(1/T_2)\rho_{ba} - 0.5i\Omega(\rho_{bb} - \rho_{ab}),
$$

(1c)

where $T_1$ and $T_2=2T_1$ are the population and the polarization decay rates, respectively; $\rho_{aa}$ ($\rho_{bb}$) is the excited (ground-) state population; $\rho_{ba}$ is the atomic polarization; and $\rho_{ba} + \rho_{bb}=1$ for a closed system. The electric field is expressed in terms of the instantaneous Rabi frequency $\Omega=2E_d/\hbar$, which is a function of both $z$ and $\tau$. The time variable has been replaced with the local time $\tau = \tau - z/v_x$, where $v_x$ is the group velocity of the pulse at the center of the spectrum. Without loss of generality, we take the dipole matrix element $d$ to be real and aligned parallel to the field polarization vector.

The reduced Maxwell wave equation gives, under the slowly varying envelope approximation, the modifications to the field due to the atoms,

$$
\frac{\partial}{\partial z} \Omega(z, \tau) = 2i\mu \rho_{ba}(z, \tau).
$$

(2)

In the above equation, $\mu = \omega_d N_d^2/\epsilon_0 c$ is the coupling constant between the field and the polarization, where $N$ is the number density, and $\epsilon_0$ is the permittivity of the two-level medium for nonresonant excitation.

We will consider a train of $N$ pulses at the entrance of the medium described by

$$
\Omega_{\text{total}}(0, \tau) = \sum_{m=0}^{N-1} \Omega(0, \tau - mT)e^{im\varphi},
$$

(3)

where $T$ is the repetition period of the pulse train and $\varphi$ is the phase difference between consecutive pulses. The slowly varying envelopes of the input pulses, $\Omega(0, \tau - mT)$, are identical in shape and uniformly spaced in time. The pulses are ultrashort, and their temporal width is orders of magnitudes smaller than the repetition period $T$.

We are interested in ultrashort pulse sequences where repetition rates are on the order of or larger than the population decay rate of the excited state. In this case, the atomic excitation (population and coherence) will not have enough time to completely decay in between pulses. It has been shown that the excitation then coherently accumulates from one pulse in the sequence to the next [5,6]. Because of accumulation of excitation, even if individually the pulses are weak, together they may significantly disturb the atoms.

The propagation of a single pulse [17] and that of a pulse train [15] can be described analytically in the weak excitation regime when the ground-state population does not change significantly. But when the coherent accumulation of excitation cannot be neglected, in general, a pulse in the train will find the atomic sample in a different initial condition from that found by the previous pulse. Furthermore, the initial condition for each pulse may not be homogeneous along the medium and may exhibit a $z$ dependence. A fully analytical solution to the coupled Maxwell-Bloch equations is no longer possible. We developed a semianalytical approach to solving the Maxwell-Bloch equations for an individual pulse through which the Bloch equations are solved analytically and the Maxwell propagation equation is integrated numerically. The two solutions are then combined iteratively to describe the propagation of the pulse train.

To solve the Bloch equations analytically, we rely on the fact that the individual ultrashort pulses interact with the atoms only for a very small fraction of the time $T$ between consecutive pulses in the train. The interaction time is a few orders of magnitude smaller than the pulse repetition period. We consider then two distinct time intervals. Take, for example, an arbitrary pulse in the train that starts interacting with the atoms at time $\tau_{0m}$. The first time interval to be considered, $\tau_{0m} \leq \tau < \tau_{0m} + \alpha$, is very short ($\alpha \ll T$). During this time, the atoms do not experience any significant decay, and any atomic evolution occurs only due to the pulse excitation. But by $\tau = \tau_{0m} + \alpha$, the driving pulse will have either turned off completely, or its amplitude will be significantly low that it no longer excites the atoms in any appreciable
way. Then, during the second time interval, $\tau_{0m}+a \leq \tau < \tau_{0m}+T$, the atoms experience only spontaneous decay before the next pulse arrives.

For $\tau_{0m} \leq \tau < \tau_{0m}+a$, we can neglect the relaxation terms, and Eqs. (1) are rewritten as

$$\frac{\partial}{\partial \tau} \rho_{aa}(z, \tau) = -0.5i\Omega(\rho_{ba} - \rho_{ab}), \quad (4a)$$

$$\frac{\partial}{\partial \tau} \rho_{bb}(z, \tau) = -0.5i\Omega(\rho_{ab} - \rho_{ba}), \quad (4b)$$

$$\frac{\partial}{\partial \tau} \rho_{00}(z, \tau) = -0.5i\Omega(\rho_{0b} - \rho_{b0}). \quad (4c)$$

In matrix notation,

$$\frac{\partial}{\partial \tau} \mathbf{R}(z, \tau) = -\mathbf{M}(z, \tau) \cdot \mathbf{R}(z, \tau), \quad (5)$$

where $\mathbf{R}(z, \tau) = (\rho_{aa} \rho_{bb} \rho_{ba} \rho_{ab})^T$ is a four-dimensional vector of solutions for the density-matrix elements and

$$\mathbf{M}(z, \tau) = \mathbf{\Omega}(z, \tau)\mathbf{A}, \quad (6)$$

with

$$\mathbf{A} = 0.5i \begin{pmatrix} 0 & 0 & +1 & -1 \\ 0 & 0 & -1 & +1 \\ +1 & -1 & 0 & 0 \\ -1 & +1 & 0 & 0 \end{pmatrix} \quad (7)$$

being a constant matrix.

Because the commutator $[\mathbf{M}(z, \tau), \int_{\tau_{0m}}^{\tau} \mathbf{M}(z, s) ds] = 0$, a solution to Eq. (5) is

$$\mathbf{R}(z, \tau) = \exp[-\mathbf{\Theta}(z, \tau)\mathbf{A}] \cdot \mathbf{R}(z, \tau_{0m}), \quad (8)$$

where $\mathbf{\Theta}(z, \tau) = \int_{\tau_{0m}}^{\tau} \mathbf{\Omega}(z, s) ds$ is the area of the driving pulse up to time $\tau$ evaluated at position $z$. $\mathbf{R}(z, \tau_{0m})$ gives the atomic conditions before the driving pulse starts interacting with the atoms. For the first pulse in the train ($n=0$ in Eq. (3)), $\mathbf{R}(z, 0) = (1 \ 0 \ 0 \ 0)^T$ and the dot product in Eq. (8) can be evaluated explicitly in terms of $\mathbf{\Theta}(z, \tau)$ (see the Appendix). But as the initial atomic conditions change from one pulse to the next, such an evaluation will no longer be possible for an arbitrary pulse.

Finally, for $\tau_{0m}+a \leq \tau < \tau_{0m}+T$, there is only spontaneous decay of the atoms and from Eqs. (1),

$$\rho_{aa}(z, \tau) = \rho_{aa}(z, \tau_{0m}) + a e^{-\tau T_1}, \quad (9a)$$

$$\rho_{bb}(z, \tau) = \rho_{bb}(z, \tau_{0m}) + a e^{-\tau T_1}. \quad (9b)$$

The full procedure to solve the Maxwell-Bloch equations is then: We spatially propagate a single pulse in the train by numerically solving Eq. (2) together with Eq. (8); the resulting probability density elements along the medium are then substituted into Eqs. (9), determining the initial conditions for the next pulse. The results to be presented next have been confirmed by a full numerical solution of the Maxwell-Bloch equations.

### B. Two-level system results

The individual input pulses in the excitation train are taken to be Gaussians,

$$\mathbf{\Omega}(0, \tau) = (\theta_0/\sigma)e^{-\pi(\tau/\sigma)^2}, \quad (10)$$

with $\theta_0 = \pi/8$ being the area and $\sigma = 200$ fs the width of the input pulses, respectively. The pulses have the same carrier frequency, which is resonant to the atomic transition, and the phase $\varphi$ between consecutive pulses is set to zero. The polarization decay time was set to $T_2 = 56$ ns and the pulse repetition period was set to $T = 1$ ns, such that about 28 pulses can excite the atoms in the excited state’s lifetime ($T_1$). The interaction time between the individual pulses and the atoms, during which Eq. (8) applies, is set to a very short $a = 20$ ps.

Due to the transient nature of the excitation, $T_2$ can be made arbitrarily large without affecting the basic shape of a transmitted pulse, leading to a deceivingly large conventional optical depth. So we follow Ref. [18] and define the optical depth as $\mu_2$ (with units of inverse time), instead of the conventional definition of $\mu_2 T_2$ (unitless). For example, an optical depth of $\mu_2 = 1$ (ps)$^{-1}$ corresponds to a conventional depth of $\mu_2 T_2 = 56,000$ for $T_2 = 56$ ns.

Figure 2 shows the atomic inversion $w = \rho_{aa} - \rho_{bb}$ just before excitation by a given pulse, as a function of the number of pulses at various optical depths inside the medium. At the entrance to the medium ($\mu_2 = 0$), the combined action of the pulses not only can invert the atomic population, but also induce Rabi cycling of the population [Fig. 2(a)]. After a large number of pulses, the excitation rate balances out the population decay rate, and the atomic inversion stabilizes close to zero. However, as the pulses propagate into the atomic medium, their resonant frequency is absorbed by the atoms. The rate of coherent accumulation decreases and more pulses are needed to invert the atoms [Fig. 2(b)]. Eventually, the pulses can no longer invert the atomic sample, as shown in Fig. 2(c). At a short optical depth of only $\mu_2 = 226$ (ns)$^{-1}$, even for a large number of excitation pulses, the train merely perturbs the atoms, which remain in their ground state [Fig. 2(d)].

Figure 3 shows the spatial evolution of three pulses from the train (in frequency space). As the first pulse ($N = 1$) propagates into the atomic medium, its resonant frequency is removed from its spectrum. The side panel shows the population inversion as a function of optical depth. This pulse's behavior is well explained by linear dispersion theory [17]. At optical depths between 1 and 3.5 (ns)$^{-1}$, the 15th pulse finds the atoms inverted by the previous pulses and has its central frequency amplified by stimulated emission. Since the atomic inversion does not extend too deeply into the medium ($\mu_2 \approx 3.5$ (ns)$^{-1}$), the resonant frequency starts to be absorbed again by the atoms. When the 300th pulse enters the medium, it finds the atoms saturated by the previous pulses. It then propagates for a distance 20 times longer than that propagated by the first pulse before its central frequency is completely absorbed by the atoms.

Although the spectra shown in Fig. 3 for the different pulses are distinct from one another, the optical depths con-
considered here are relatively small such that the effects of propagation on the pulse’s temporal shapes are not very strong. (The distortion illustrated in the output pulse of Fig. 1 is greatly exaggerated in this respect.) In the time domain, we saw no significant difference between the temporal profiles of the different pulses in the train.

III. THREE-LEVEL ATOMIC MEDIUM

We now consider the propagation of a train of ultrashort pulses through an extended sample of three-level atoms. Figure 4 shows the atomic level configuration. The excited a state has a spontaneous lifetime of \( T_1 \) and decays to both lower b and c states at equal rates; we neglect any decay between b and c. Again, we ignore the Doppler broadening of the lines.

A. Coupled Maxwell-Bloch equations

In the rotating wave approximation, the optical Bloch equations describing the temporal evolution of the density matrix elements are

\[
\begin{align*}
\dot{N}_a &= -2\Omega_a I_a e^{i\omega_T t} - \Gamma_a N_a, \\
\dot{N}_b &= 2\Omega_a I_a e^{i\omega_T t} - \Gamma_b N_b, \\
\dot{N}_c &= 2\Omega_c I_c e^{i\omega_c t} - \Gamma_c N_c, \\
\dot{N}_d &= 2\Omega_d I_d e^{i\omega_d t} - \Gamma_d N_d,
\end{align*}
\]

FIG. 2. (Color online) Atomic population inversion as a function of the number of excitation pulses in the train at an optical depth \( \mu z \) equal to (a) 0, (b) 58, (c) 114, and (d) 226 (ms)^{-1}.

FIG. 3. (Color online) Spatial evolution of three pulses in the train: \( N=1, 15, \) and 300. The figure shows the pulses’ spectral amplitude (in arbitrary units) as a function of optical depth. The side panels show the population inversion along the medium. The pulse’s spectral frequencies are calculated relative to the atomic resonance.

FIG. 4. The model three-level atomic system: a degenerate-\( \Lambda \) atoms. The excited a state has a spontaneous lifetime of \( T_1 \) and decays to both lower b and c states at equal rates; we neglect any decay between b and c. Again, we ignore the Doppler broadening of the lines.
INFLUENCE OF PROPAGATION ON THE COHERENT ...

\[ \frac{\partial}{\partial t} \rho_{ab}(z, \tau) = (1/2T_1)\rho_{aa} + 0.5i\Omega(\rho_{ba} - \rho_{ab}), \]

\[ \frac{\partial}{\partial t} \rho_{ac}(z, \tau) = (1/2T_1)\rho_{aa} + 0.5i\Omega(\rho_{ca} - \rho_{ac}), \]

\[ \frac{\partial}{\partial t} \rho_{bc}(z, \tau) = - (1/T_1)\rho_{aa} + 0.5i\Omega[(\rho_{ab} + \rho_{cb}) - \text{c.c.}], \]

\[ \frac{\partial}{\partial t} \rho_{ab}(z, \tau) = - (1/T_2)\rho_{ab} - 0.5i\Omega(\rho_{cb} + \rho_{bb} - \rho_{ab}), \]

\[ \frac{\partial}{\partial t} \rho_{ac}(z, \tau) = - (1/T_2)\rho_{ac} + 0.5i\Omega(\rho_{cc} - \rho_{ac} - \rho_{bc}), \]

\[ \frac{\partial}{\partial t} \rho_{bc}(z, \tau) = - 0.5i\Omega(\rho_{ac} + \rho_{ba}), \]

where \( \rho_{aa} + \rho_{bb} + \rho_{cc} = 1 \). We assume that the dipole matrix elements \( d_{ij} \) between the lower and the excited states are real and equal: \( d = d_{ia} = d_{ia} \). Here, \( T_2 = 2T_1 = 56 \) ns, and the ultrashort pulses are resonant with the atomic transitions. The atoms start with population equally distributed between the lower states, but with no initial coherence between levels \( b \) and \( c \). The propagation of the ultrashort pulses is described by the reduced wave equation

\[ \frac{\partial}{\partial z} \Omega(z, \tau) = 2i\mu[\rho_{ab}(z, \tau) + \rho_{ac}(z, \tau)], \]

where the term inside the brackets is the total induced atomic polarization.

To solve the coupled Maxwell-Bloch equations, we employ a similar procedure to that used in the two-level system case. During the time they interact with the individual driving pulses, the atoms evolve according to Eq. (8), now with

\[
A = -0.5i \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & +1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & +1 \\
0 & 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & +1 \\
0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\
+1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & +1 & 0 & +1 & 0 & 0 & 0 & 0 \\
+1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & +1 & 0 & +1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

and

\[ R(z, \tau) = (\rho_{aa} \quad \rho_{bb} \quad \rho_{cc} \quad \rho_{bc} \quad \rho_{cb} \quad \rho_{ac} \quad \rho_{ca} \quad \rho_{ab} \quad \rho_{ba})^T. \]

The pulse propagation is then calculated by solving the wave equation [Eq. (12)] numerically along with Eqs. (8), (13), and (14). For the remaining time until the next pulse arrives, the atoms experience only spontaneous decay,

\[ \rho_{aa}(z, \tau) = \rho_{aa}(z, \tau_0 + a)e^{-\tau_1}, \]

\[ \rho_{bb}(z, \tau) = \rho_{bb}(z, \tau_0 + a)e^{-\tau_2T_1}, \]

\[ \rho_{cc}(z, \tau) = \rho_{cc}(z, \tau_0 + a)e^{-\tau_2T_1}. \]

This procedure is repeated iteratively with the results from interaction with one pulse being used as the initial atomic conditions for the next pulse in the train.

We again consider an input train of ultrashort Gaussian pulses described by Eq. (3). The pulse area and the width are \( \theta = \pi/15 \) and \( \sigma = 200 \) fs, respectively. The phase between consecutive pulses is \( \varphi = 0 \), chosen to maximize the accumulation rate [8]. The pulse repetition period is \( T = 2.8 \) ns, and the pulse-atom interaction time is \( a = 20 \) ps.

B. Three-level system results

The first pulse in the train finds all the atoms with a population equally distributed between the lower states, but without any atomic coherence between these levels. Due to interaction with this first pulse, a small atomic population and coherence are excited. Since the atomic system will not have enough time to completely relax before the next excitation pulse, population and coherence will accumulate in the atomic system from one pulse to the next.

Figure 5 shows the coherence \( \rho_{bc} \) between the lower states, just before the excitation, as a function of the number of driving pulses at three different positions inside the atomic medium. The behavior at the entrance of the medium was discussed in detail in Ref. [8], in which the spatial propagation of the pulses was not considered. As the number of pulses increases, the ground-state coherence builds up from zero until it reaches \( \rho_{bc} = -0.5 \). A ground-state coherence of \( -0.5 \) is associated with coherent population trapping in a dark atomic state, which does not interact with the driving pulses. As can be seen in Fig. 5, more pulses are needed to reach full coherence as we move deeper into the atomic sample. For the first few hundred pulses, full coherence is not completely established, even at \( \mu z = 0 \). These pulses have then their resonant frequency absorbed by the

FIG. 5. (Color online) Coherence between the lower atomic states as a function of the pulse number for three different positions along the medium: \( \mu z = 0 \) (dotted black line), \( \mu z = 3.2 \) (ps)\(^{-1} \) (dashed blue line), and \( \mu z = 6.4 \) (ps)\(^{-1} \) (solid red line).
atoms as they enter the extended medium. Consequently, their areas decrease with propagation. We previously showed in Ref. [8] that the number of pulses necessary to establish full coherence between the lower states depends on the pulse area: the smaller the area, the higher the number of pulses required. Therefore, the further into the medium, the more pulses are needed to drive the atomic population into a dark state.

Figure 6 shows the temporal and the spectral profiles for two pulses \( N=1 \) (left column) and \( N=2400 \) (right column). The figure shows the pulses’ spectral and temporal amplitudes (in arbitrary units) as functions of optical depth. The pulse’s spectral frequencies are calculated relative to the atomic transition. The side panels, in the top plots, show the ground-state coherence \( \rho_{bc} \) along the medium.

Figure 6 shows the temporal and the spectral profiles for two pulses \( N=1 \) and \( N=2400 \) in the train as a function of optical depth. Again, the first pulse in the train finds the atoms with zero coherence between the atomic ground states throughout the sample. Its resonant frequency is absorbed as it propagates and, consequently, its temporal profile is reshaped into an oscillatory function of time. The optical depth here is much deeper than that of Fig. 3. Further into the medium, more off-resonance frequencies are removed from the spectrum by absorption from the atoms. Hence, the width of the absorption dip appears wider than that of Fig. 3. The 2400th pulse finds the sample prepared by the previous pulses with full coherence \( \rho_{bc}=-0.5 \) for a significant optical depth \( \mu_z \approx 4.0 \ (\text{ps})^{-1} \). Up to this point, the pulse experiences an electromagnetically induced transparency: neither its amplitude nor its temporal shape is modified as it propagates through the sample. For optical depths greater than 4.5 \( (\text{ps})^{-1} \), the atoms no longer show full ground-state coherence, and they start to interact with the pulse, absorbing its resonant frequency. Figure 7 shows the temporal shape of the first and the 2400th pulses at \( \mu_z=6.4 \ (\text{ps})^{-1} \). A distinct difference between the two pulses is clear, with the 2400th pulse showing a small distortion in its profile, but not much different to that of the input Gaussian distribution.

**IV. CONCLUSIONS**

We studied the propagation of an ultrashort pulse train through extended samples of two- and three-level atoms. The effects of propagation on the coherent accumulation of excitation in these systems were investigated.

In the two-level case, the absorption of the pulse’s resonant frequency by the atoms quickly compromises the accumulation of population in the excited state. While at the entrance of the medium the pulse train may drive Rabi oscillations in the atoms, at a short optical depth of only 226 \( (\text{ns})^{-1} \), the train merely perturbs the atoms, which remain mostly in their ground state. Because different pulses in the train find the atoms under different initial conditions, the dynamics of pulse propagation may vary from one pulse in the train to the next. Some pulses will experience only absorption, while others may have their central frequency amplified by stimulated emission. Saturated absorption may also be induced by the train such that later pulses in the sequence propagate for a distance without being absorbed by the atoms.

The results presented here may be relevant to experiments that require significant excitation accumulations by the pulse

![Figure 6](image1.png)

**FIG. 6.** (Color online) Spatial evolution of two pulses in the train: \( N=1 \) (left column) and \( N=2400 \) (right column). The figure shows the pulses’ spectral and temporal amplitudes (in arbitrary units) as functions of optical depth. The pulse’s spectral frequencies are calculated relative to the atomic transition. The side panels, in the top plots, show the ground-state coherence \( \rho_{bc} \) along the medium.

![Figure 7](image2.png)

**FIG. 7.** (Color online) Pulse temporal profile at \( \mu_z=0 \) (dotted black line) and \( \mu_z=6.4 \ (\text{ps})^{-1} \) for pulses \( N=1 \) (dashed blue line) and \( N=2400 \) (solid red line).
train. In such cases, the propagation of the pulses through large optical depths should be avoided.

In the three-level case, the pulse train excites a coherence between the two lower states of the atoms, driving the population into a dark superposition state. The medium becomes transparent to the driving fields. Later pulses in the train propagate through the atomic medium with both their amplitude and temporal profile preserved.

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APPENDIX: EXPLICIT EVALUATION OF VECTOR $R(z, \tau)$ FOR THE FIRST EXCITATION PULSE

The first pulse in the train to reach the atomic sample finds all the (two-level) atoms in their ground electronic state, and Eq. (8) can be evaluated explicitly as a function of the pulse area $\theta(z, \tau)$. The matrix exponential in Eq. (8) can be shown to be given by

$$\exp\left[-\theta(z, \tau) A\right] = \frac{1}{2} \begin{pmatrix}
1 + \cos \theta & 1 - \cos \theta & -i \sin \theta & +i \sin \theta \\
1 - \cos \theta & 1 + \cos \theta & +i \sin \theta & -i \sin \theta \\
-i \sin \theta & +i \sin \theta & 1 + \cos \theta & 1 - \cos \theta \\
+i \sin \theta & -i \sin \theta & 1 - \cos \theta & 1 + \cos \theta
\end{pmatrix},$$

(A1)

with $\theta = \theta(z, \tau)$. We consider the initial atomic condition $R(z, 0) = (0 1 0 0)^T$ throughout the sample. Taking the dot product between $R(z, 0)$ and the matrix in Eq. (A1) yields

$$R(z, \tau) = \begin{pmatrix}
\rho_{aa} \\
\rho_{bb} \\
\rho_{ba} \\
\rho_{ab}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
1 - \cos \theta \\
1 + \cos \theta \\
+i \sin \theta \\
-i \sin \theta
\end{pmatrix},$$

(A2)

where $\rho_{ij}$ are functions of $z$ and $\tau$. Therefore, from Eq. (A2), the population inversion is found to be

$$w = \rho_{aa} - \rho_{bb} = -\cos \theta,$$

(A3)

in agreement with the well-known Rabi solution for excitation of a nondecaying two-level atom by a pulse of area $\theta$ [19].