Aperture effects in 2.5D Kirchhoff migration: A geometrical explanation

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ABSTRACT
Seismic images obtained by Kirchhoff time or depth migration are always accompanied by some artifacts known as migration noise, migration boundary effects, or diffraction smiles, which may severely affect the quality of the migration result. Most of these undesirable effects are caused by a limited aperture if the algorithms make no special disposition to avoid them. Strong amplitude variation along reflection events may cause similar artifacts. All of these effects can be explained mathematically by means of the method of stationary phase. However, such a purely theoretical explication is not always easily understood by applied geophysicists. A geometrical interpretation of the terms of the stationary-phase approximation in relation to the diffraction and reflection traveltime curves in the time domain can help to develop a more intuitive understanding of the migration artifacts. A simple numerical experiment for poststack (zero-offset) data indicates the problem and helps to demonstrate the effects and the methods to avoid them.

INTRODUCTION
Since the early work of Hagedoorn (1954), migration concepts have improved greatly and are now an important tool in the world of seismic imaging, either as prestack or poststack time and depth migration (e.g., Yilmaz, 2001). Hagedoorn’s original (graphical) migration scheme using surfaces of maximum convexity was later related to the wave equation and became familiar as Kirchhoff migration (Schneider, 1978). The name was chosen with regard to the Kirchhoff integral, which describes the (forward) propagation of seismic waves within a given depth model. Since the Kirchhoff integral by itself cannot be used to solve the inverse problem, i.e., to describe backward propagation, Kirchhoff migration was introduced as its adjoint operation that describes the forward propagation of the recorded wavefield in the reverse direction. This is a very good approximation to backward propagation as long as evanescent waves can be neglected.

On the basis of this more mathematical treatment of Kirchhoff migration (Schneider, 1978; Bleistein, 1987; Schleicher et al., 1993), it became possible to study its boundary effects by means of the method of stationary phase (e.g., Bleistein, 1984; Sun, 1998; Bleistein et al., 2001). However, the new analytical studies have much less geometrical appeal than Hagedoorn’s original insights. In this paper, we try to connect these analytical aspects of Kirchhoff migration to its geometrical properties to provide a better understanding of the boundary effects.

Kirchhoff migration treats each depth point \( M \) on a sufficiently dense grid like a diffraction point. In an a priori given macrovelocity model, the relevant part of the Green’s function of a point source at any single diffraction point \( M \) in the depth domain is calculated. The kinematic part of this Green’s function is the configuration-specific diffraction-traveltime surface, also called the Huygens surface.

The amplitudes of the input seismograms (or, to be more specific, of their derivatives) are stacked along the Huygens surface and assigned to \( M \). This explains why the Kirchhoff migration scheme is also called a diffraction stack. If so desired, the effect of geometrical spreading can be removed from the output amplitudes by multiplying the data during the stack with a true-amplitude weight factor calculated from the dynamic part of the Green’s function.

Ideally, the extent of the Huygens surfaces—that is, the migration aperture—should be limitless so that no contributions from the abrupt truncation of the sum occur. In practice, of course, the aperture is always limited by the region over which seismic data have been acquired. In other words, because of the finiteness of the survey area, Kirchhoff migration is always a limited-aperture migration (LAM) (Sun, 1998).

However, this is not the only reason why we have to deal with the effects of a finite migration aperture. In practical migration implementations, it is often advantageous to further restrict the
aperture, i.e., to exclude certain source and receiver positions from the computations where data have been acquired. The main reasons for such an aperture restriction are

1) the use of fewer traces in the migration summation accelerates the entire migration process,
2) a smaller operator excludes steeper dips, which helps to avoid operator aliasing (e.g., Abma et al., 1999), and
3) less summation of data away from the signal reduces the stacking of unwanted noise.

For the best possible reduction of aliasing and noise as well as the best computational efficiency, one would like to use a model-based aperture restriction, i.e., the projected Fresnel zone. This zone is obtained by a projection of the actual Fresnel zone at the reflector along the rayfield onto the measurement surface (for details, see Hubral et al. 1993). The projected Fresnel zone coincides with the minimal migration aperture that correctly recovers true amplitudes (Schleicher et al., 1997; Sun, 2000; Sun and Bancroft, 2001).

Unfortunately, it is difficult to determine the exact center and size of the Fresnel zone for each depth point before or during migration. A reasonable compromise between accuracy and practicality is to fix an aperture or a maximum migration dip. Note that a fixed aperture will image smaller maximum dips with increasing depth. On the other hand, to migrate a fixed maximum dip at all depths—e.g., in a 45° migration—requires an increasing migration aperture with depth. In regions where dips are known to be restricted, these very convenient ways reduce aliasing and improve computational efficiency at the same time. However, close to the maximum dip, these dip-restricted migration operators achieve only kinematically correct images (e.g., Schleicher et al., 1997; Sun, 1998). For true-amplitude migration, the maximum operator dip must always be chosen somewhat larger than the maximum reflector dip to be imaged.

The fact that the migration aperture is limited causes artifacts known as migration noise, boundary or aperture effects, or migration smears. We relate the mathematical explanation of the migration artifacts by means of the method of stationary phase to the 3D migration integrals. The stacking curve \( t = \tau_D(\xi, M) \) is the above-mentioned Huygens curve. The symbol \( D_1^\tau \) denotes the Hilbert transform of the time half-derivative. It is the 2.5D filter operation corresponding to the 3D time derivative needed to correctly recover the source pulse (Newman, 1975).

We assume that at least one reflection event is present in the seismic data \( U(\xi, t) \). Such an event can be described by zero-order ray theory (e.g., Červený, 2001) as

\[
U(\xi, t) = R_c B \cdot F(t - \tau_R),
\]

where \( R_c \) denotes the angle-dependent reflectivity, \( B \) symbolizes the point-source geometrical spreading factor, and \( F \) represents all other effects on the amplitude, such as source strength, source and receiver coupling, transmission loss, and attenuation in the reflector overburden. Moreover, \( F(t) \) is the analytic source waveform which is shifted to the arrival time \( \tau_R \) (reflection traveltimes). A seismic trace with several (primary) events may be described by superposition of individual seismic events of the type of equation (2).

Upon substitution of expression (2), a time-dependent version of equation (1) reads in the frequency domain as

\[
\hat{V}(M, \omega) = \frac{1}{\sqrt{2\pi}} \int_A d\xi \hat{W}_{DS}^{(2.5)}(\xi, M) D_i \frac{1}{2} \left. U(\xi, t) \right|_{t = \tau_D(\xi, M)},
\]  

where \( V(M) \) is the value assigned to one diffraction point \( M \) in the depth domain after migration and \( U(\xi, t) \) denotes the data in the time domain (seismograms). These data are assumed to consist of analytic traces formed by the actual traces recorded in the field as the real parts and their Hilbert transforms as the imaginary parts. In this way, migration by equation (1) correctly treats the phase shifts because of complex reflection coefficients (supercritical reflections) and possible caustics along the raypaths. The parameter \( \xi \) is the so-called configuration parameter and represents the trace position. Sources and receivers are grouped into pairs, whose locations are described as a function of \( \xi \). The actual form of this function depends on the measurement configuration. The migration aperture \( A \) is the area over which \( \xi \) varies to cover all source–receiver pairs used in the stack.

The factor \( W_{DS}^{(2.5)}(\xi, M) \) is a true-amplitude weight function which may (true-amplitude migration) or may not (purely kinematic migration) be included in the migration scheme. The stacking curve \( t = \tau_D(\xi, M) \) is the above-mentioned Huygens curve. The symbol \( D_1^\tau \) denotes the Hilbert transform of the time half-derivative. It is the 2.5D filter operation corresponding to the 3D time derivative needed to correctly recover the source pulse (Newman, 1975).

The method of stationary phase reads in the 2.5D case as (Martins et al., 1997)

\[
V(M) = \frac{1}{\sqrt{2\pi}} \int_A d\xi \hat{W}_{DS}^{(2.5)}(\xi, M) D_i \frac{1}{2} \left. U(\xi, t) \right|_{t = \tau_D(\xi, M)},
\]

\[
\hat{V}(M, \omega) = \frac{1}{\sqrt{2\pi}} \int_A d\xi \hat{W}_{DS}^{(2.5)}(\xi, M) D_i \frac{1}{2} \left. U(\xi, t) \right|_{t = \tau_D(\xi, M)},
\]

where \( \tau_D(\xi, M) \) is the difference between the diffraction and reflection traveltimes, i.e., \( \tau_D = \tau_0 - \tau_R \). Moreover, \( \hat{F}(\omega) \) and \( \hat{V}(M, \omega) \) denote the Fourier transforms of \( F(t) \) and \( V(M) \), respectively. The qualitative discussion involves the same arguments and leads to the same conclusions. The quantitative analysis is similar but is slightly more complicated, mainly resulting in a different amplitude behavior of the artifacts.

**2.5D KIRCHHOFF MIGRATION**

Mathematically, the Kirchhoff migration process is expressed as an integration over the recorded wavefield and reads
of analyzing the main contributions. Although in principle a high-frequency approximation, the method of stationary phase yields highly accurate predictions of the migration results in the seismic frequency range. Mathematically, the prerequisites for applying the method of stationary phase are fulfilled implicitly since we perform all calculations within the framework of zero-order ray theory, which is strictly valid only for high frequencies.

Reducing it to its basic structure, the integral in equation (3) can be written in the form

\[ I(\omega) = \int_a^b f(\xi) e^{i\omega q(\xi)} d\xi. \]  

(4)

The method of stationary phase is based on the observation that for high frequencies, i.e., for large values of \( \omega \), the factor \( e^{i\omega q(\xi)} \) oscillates very rapidly, thus covering full periods in very small intervals of \( \xi \). If \( f(\xi) \) is not itself an oscillating function, its values do not strongly vary in any such interval. Thus, the integration over a full period of \( e^{i\omega q(\xi)} \) yields approximately zero and does not contribute to the overall value of the integral. The only regions where \( e^{i\omega q(\xi)} \) does not oscillate are those where the phase function \( q(\xi) \) remains approximately constant or stationary. Mathematically, points of stationary phase are those where the phase function \( q(\xi) \) has a horizontal tangent, i.e., a vanishing derivative. Nonnegligible contributions to integral (4) are therefore to be expected from the vicinity of these points. Further contributions to integral (4) are to be expected from the boundaries of the integration interval because there the integration generally does not cover a full period of \( e^{i\omega q(\xi)} \).

To illustrate these observations, we consider the migration of zero-offset data from a simple earth model with a horizontal reflector at a depth of 1 km. For a point \( M \) at \( x = 3 \) km on the reflector and a frequency of 30 Hz, Figure 1a shows the phase \( q \) of the integrand in equation (3) as a function of \( \xi \). The dashed line in Figure 1b depicts the corresponding amplitude function \( f \). The solid line in Figure 1b shows the real part of the full integrand function. Note that this function strongly oscillates everywhere except in the vicinity of the point where the phase is stationary. It is evident that the amplitude modulation does not alter the oscillatory character of the integrand function.

Let us now discuss integral (4) in a more quantitative way. In our case, the phase function \( q \) is the difference between the diffraction and reflection traveltme curves, \( \tau_{df} \). Thus, the real part of the integrand function (Figure 1b) has zeroes at

\[ |\tau_{df}| = |\tau_D - \tau_R| = n \frac{\pi}{\omega} = n \frac{T}{2}, \]  

(5)

where \( T = 2\pi/\omega \) is the period of the monofrequency wave under consideration. Equation (5) is equivalent to the definition of the boundary of the \( n \)th Fresnel zone (e.g., Červený and Soares, 1992). Therefore, the alternating zones of negative and positive amplitude of the integrand function are physically equivalent to the Fresnel zones. To be exact, the Fresnel zone is measured along the reflector, while the integration in equation (4) is carried out along the Huygens curve. To see its influence on the integration, the Fresnel zone must be projected to the earth’s surface (Hubral et al., 1993). The true Fresnel zone in depth can be observed in a fully analogous manner in the Kirchhoff-Helmholz modeling integral.

Now consider an integration of the function \( f : \exp{i\omega q} \) from the center (where \( \tau_D = \tau_R \)) to the sides. At first, this sums up positive contributions from the first Fresnel zone, ending at the first zero in either direction. Subsequent Fresnel zones, each ending at the next zero, will add purely negative or positive contributions to integral (4). In other words, Fresnel zones with odd numbers contribute positively to the integral while Fresnel zones with even numbers contribute negatively. Because of the above observation that an integration over a full period, i.e., over two consecutive Fresnel zones, yields approximately zero, it becomes clear why the principal contribution to integral (4) will stem from the vicinity of the stationary point. Hence, an integration over only the first Fresnel zone already provides a very good approximation of the total integral. On the other hand, its full value cannot be recovered if the integration interval does not completely cover the first Fresnel zone. This observation has led to the quantification of the minimum migration aperture (Schleicher et al., 1997).

However, the above discussion holds strictly only for a monofrequency signal. For a transient, band-limited signal, one has to replace the half-period \( T/2 \) in equation (5) by some estimate \( \tau_w \) of the wavelet length.

An analysis of the migration integral (4) by means of the method of stationary phase under the assumption of a single,
simple, and isolated point of stationary phase has been carried out by Bleistein (1984) and Sun (1998). Provided that \( f \) and \( q' \) are continuous functions of \( \xi \), the analysis shows that the leading-order contributions to \( I(\omega) \) are

\[
I(\omega) \approx f(\xi^*) e^{i q(\xi^*)} \frac{2\pi}{-i q''(\xi^*)} + \frac{1}{i \omega} \left[ \frac{f(b)}{q'(b)} e^{i q(b)} - \frac{f(a)}{q'(a)} e^{i q(a)} \right],
\]

where the prime denotes the derivative with respect to \( \xi \). The point of stationary phase, defined by the condition \( f'(\xi^*) = 0 \), is denoted as \( \xi^* \). We see that the terms in equation (6) are of the order \( 1/\sqrt{\omega} \) and \( 1/\omega \), respectively. For high frequencies, the two terms in equation (6) describe the major contributions to the final migrated image.

The first term of equation (6), which stems from the stationary point \( \xi^* \), is of lower order in \( 1/\sqrt{\omega} \) than the second term. Thus, it generally represents the dominant part of the total migrated section wherever it is nonzero. This term constitutes the actual migrated image of the reflector.

The second term of equation (6) is proportional to \( 1/\omega \). It comes from the endpoints of the integration/stacking operator. This second contribution describes the main migration artifacts. Because of the higher order in \( 1/\sqrt{\omega} \), its amplitudes generally are lower than those of the reflector image. However, under certain circumstances these effects can be as strong as (or even stronger than) a reflector image. This can happen when the amplitudes of the unmigrated section at the data margins (these enter into \( f(a) \) or \( f(b) \)) are significantly larger than those at the stationary points (which enter into \( f(\xi^*) \)). The situation is the more probable the lower the frequencies contained in the unmigrated data.

Equation (6) must be modified slightly if the stationary point coincides with one of the boundaries, i.e., if \( q'(a) = 0 \) or \( q'(b) = 0 \). The corresponding boundary contribution at \( a \) or \( b \) is eliminated, and the leading term is divided by two. Note that the decay of the amplitudes across the aperture boundary is not abrupt but is in the sense of an error function (Felsen and Marcuvitz, 1973).

If there are \( N \) simple, isolated stationary points of the phase function \( q(\xi) \) in the integration interval \((a, b)\), the interval is divided into \( N \) parts where each one contains exactly one stationary point. The independent analysis of each of the separate integrals can then be carried out as before, yielding the sum of all individual stationary-point contributions. The artificially introduced integral boundaries do not contribute to the final value of \( I(\omega) \) because the corresponding boundary terms cancel each other.

Apart from the edges of the acquisition aperture and the stacking operator, abrupt amplitude or phase changes along the reflection events in the seismic data may also cause the same kind of endpoint contributions. These changes cause discontinuities in the amplitude or phase functions of integral (4), which make the integral act piecewise on the data. In other words, we may say that artificial endpoints are created which cause the additional aperture effects.

To describe these effects mathematically by means of the method of stationary phase, we suppose such discontinuities to happen at \( J \) points \( d_j (j = 1, \ldots, J) \) within the integration interval \((a, b)\). Then, the integration interval has to be further divided at all points \( d_j \). The contributions from these new integration boundaries do not cancel because of the different values of \( f, q, \) and/or \( q' \) on either side of the discontinuities. Thus, one obtains the general expression

\[
I(\omega) \simeq \sum_{n=1}^{N} f(\xi^*_n) e^{i q(\xi^*_n)} \frac{2\pi}{-i q''(\xi^*_n)} + \frac{1}{i \omega} \left[ \frac{f(b)}{q'(b)} e^{i q(b)} - \frac{f(a)}{q'(a)} e^{i q(a)} \right]
\]

\[
+ \frac{1}{i \omega} \sum_{j=1}^{J} \left[ \frac{f(d_j^+)}{q'(d_j^+)} e^{i q(d_j^+)} - \frac{f(d_j^-)}{q'(d_j^-)} e^{i q(d_j^-)} \right],
\]

for \( f(d_j^+) \) denotes the values of \( f \) at the right- and left-hand sides of the discontinuity, i.e.,

\[
f(d_j^\pm) = \lim_{\xi \to d_j^\pm} f(\xi),
\]

where we use corresponding definitions for \( q(d_j^+) \) and \( q(d_j^-) \). For expression (7) to be valid, all stationary points \( \xi^*_n \) and all discontinuity points \( d_j \) must be isolated from each other.

Migration artifacts from a limited aperture, illumination problems, attenuation, or missing traces are inherent to seismic migration, independent of the actual migration scheme used. However, artifacts from strong amplitude variations resulting from focusing effects are a consequence of Kirchhoff migration and can be largely reduced with other migration schemes such as finite-difference wave-equation migration. In addition, Kirchhoff algorithms based on ray-tracing methods do not reflect waves around obstacles; hence, these methods are more sensitive to illumination problems.

In contrast to the data boundaries, reflectors actually terminating in the earth do not provoke migration smiles. In this case, edge diffractions are present in the seismic data that are collapsed by migration into the endpoint of the reflector. Because of the diffractions, the reflection event in the data has no actual endpoint but dies off over a larger number of traces. In this way, endpoint contributions are suppressed. The latter observation already points toward a well-known way of suppressing migration artifacts: tapering. We discuss this in a later section.

For the geometrical interpretation of the migration artifacts, we need the final migration result in depth. This is obtained by multiplying equation (6) with the factors in front of the integral in equation (3), together with an inverse Fourier transform under consideration of the imaging condition \( t = 0 \).
Migration aperture effects are most easily explained by means of a simple numeric experiment for poststack data. The model consists of two half-spaces separated by a horizontal interface. The P-wave velocities in the upper and lower half-spaces are $v_{p}^{(1)} = 2$ km/s and $v_{p}^{(2)} = 3$ km/s, respectively, and the shear-wave velocities are given by $v_{s} = v_{p}/\sqrt{3}$. The density is constant in the whole model. The zero-offset (ZO) seismogram is generated by dynamic ray tracing using a zero-phase Ricker wavelet with a dominant frequency of 20 Hz, a time sampling of $\Delta t = 1$ ms, and a trace distance of $\Delta \xi = 5$ m. It is migrated with a 2.5D Kirchhoff true-amplitude depth migration scheme on a dense grid ($\Delta x = 10$ m, $\Delta z = 2$ m) using the true velocity.

For this simple model, the stacking operator is given by a hyperbola. We limit its spatial extent to a radius of 800 m with respect to the horizontal coordinate of the apex. In this way, the number of traces contributing to the stack for each depth point is 320. The migration target zone is placed at the end of the survey line to show the boundary effects. No effort is made to enhance or reduce the migration artifacts. The resulting migrated image is depicted in Figure 2. The red and blue in Figure 2 represent the positive and negative parts, respectively, of the signal as indicated in the magnified area on the right side of the figure.

By means of Figure 2, we are now going to discuss the boundary effects from a geometrical point of view, which allows us to gain a more intuitive insight. We then relate them to the discussion of the interference in integral (4) and to the result of its stationary-phase evaluation as given by equation (9). For this purpose, we discuss the position of the Huygens curves pertaining to a series of characteristic depth points $M_{1}$ to $M_{9}$.
Points on the reflector: \(M_1\)

The actual reflector (which is unknown prior to migration) is built up by depth points such as \(M_1\). The pertinent Huygens curves are tangent to the reflection traveltime curve. (In general for laterally inhomogeneous media, the tangency points do not coincide with the apices of the stacking curves.) Thus, amplitudes gathered along such curves sum up coherently and provide high stacking results that are assigned to the corresponding depth points. No boundary effects are present because the input data at the endpoints of the stacking operator, which correspond to the limits of integration \(a\) and \(b\) in equation (3), are zero. Of course, in practice there are always some endpoint contributions because of the noise inherent in the seismograms.

We now relate this physical explanation to our earlier considerations of the method of stationary phase. The stationary-phase condition \(q'(\xi^*) = 0\) is satisfied where \(\tau_D\) and \(\tau_R\) have equal dip. Thus, we identify the horizontal coordinates of the tangency points with the points of stationary phase \(\xi^*\). The value assigned to \(M_1\) is mathematically described by the first term in equation (9). The second and third (boundary) terms of the stationary-phase result (9) vanish. The reason is that the differences between the diffraction and reflection traveltimes at \(\xi = a\) and \(\xi = b\) are larger than half the wavelet length, which implies that both \(F^\prime(q(a))\) and \(F^\prime(q(b))\) are zero.

Points very close to the reflector: \(M_2\)

Points such as \(M_2\) also comprise part of the migrated image of the reflector. At the point where the diffraction traveltime curve of point \(M_2\) has the same dip as the reflection traveltime curve, the former falls within a wavelet’s length from the latter. Thus, migration acts almost as at \(M_1\), i.e., amplitudes gathered along the stacking operator coherently interfere, in this way participating in the reconstruction of the source wavelet at the reflector image. The ratio between the vertical distance of point \(M_2\) to the reflector and the shortest distance between the diffraction and reflection traveltime curves is the migration stretch factor (Tygel et al., 1994). In terms of the method of stationary phase, the contribution is still described by the first term in equation (9), which will yield nonzero contributions whenever this traveltime difference is smaller than half the wavelet length, i.e., \(|\tau_D(\xi^*) - \tau_R(\xi^*)| < \tau_w/2\).

Points on the reflector boundary: \(M_3\)

The point \(M_3\) represents the boundary of the migrated reflector image. The Huygens curve of this point is, in principle, equivalent to the one of point \(M_1\). However, since the stationary point is located directly at the margin of the ZO gather, only half of the operator is within the data volume. Thus, summing up along the stacking curve results in an amplitude value that is half of the value assigned to \(M_1\). This coincides with the stationary-phase analysis for the case when the stationary point falls on the boundary of the integration interval. As mentioned in connection with equation (6), in this situation the leading term is divided by two.

Points off the reflector: \(M_4\)

Points such as \(M_4\) represent most diffraction points within the target zone. They have Huygens curves which completely cross the reflection signal. Summing up amplitudes along such operators leads to low values as the result of destructive interference. In other words, the phase of the integrand in equation (4) is rapidly varying, so there is no leading order contribution. For the geometrical interpretation of equation (9), the point of stationary phase is to be identified with that point in the seismic section where the traveltime and Huygens curves have the same time dip. We see that at this point as well as at its endpoints, the operator lies outside the signal, i.e., \(|\tau_D(\xi^*)| > \tau_w/2\), \(|\tau_R(a)| > \tau_w/2\), and \(|\tau_R(b)| > \tau_w/2\). Therefore, all terms in equation (9) are zero.

Migration artifacts caused by the finite stacking operator: \(M_5\), \(M_6\), and \(M_7\)

For points such as \(M_5\), the endpoints of the stacking operator lie within the reflection signal. Because of the limited aperture, the stack does not sum up all the data necessary for complete destructive interference in the same way as it does for point \(M_4\). Thus, the migration output at \(M_5\) is not as low as that for \(M_4\). As a consequence, a migration artifact appears in parallel to the actual reflector. With increasing size of aperture, the effect at \(M_5\) moves away from the actual reflector and might be located outside of the target zone. Sun (1998) shows that this aperture effect completely separates from the reflector image if the aperture is larger than one Fresnel zone (see “How to Avoid Aperture Effects” following).

The relationship of these observations to the method of stationary phase is straightforward. As for points \(M_1\) to \(M_5\) the point of stationary phase corresponds to the point of equal dip of the traveltime curves \(\tau_D\) and \(\tau_R\). At this point, the stacking line is outside of the signal, i.e., \(|\tau_D(\xi^*)| > \tau_w/2\). Therefore, the first term of equation (9) yields no contribution to the migrated image. However, both endpoints of the operator lie inside the signal, i.e., \(|\tau_D(a)| < \tau_w/2\) and \(|\tau_D(b)| < \tau_w/2\). Therefore, the boundary terms of equation (9) predict a nonzero migration output at \(M_5\).

The situation at point \(M_6\) is in principle equivalent to that at point \(M_5\). However, because only one endpoint lies within the reflection signal (the other endpoint lies outside the data), the amplitude of the aperture effect at \(M_6\) is just half of that at \(M_5\). Point \(M_7\) marks the transition between the situations of points \(M_5\) and \(M_6\). The endpoint of its pertinent Huygens curve coincides with the boundary point \(P\) in the data, where the survey ends. For this reason, at \(M_7\) the migration artifact splits into two effects. Additionally to the limited operator effect described above, a limited data effect appears in the migrated traces.

Migration artifacts caused by the finite survey area: \(M_8\) and \(M_9\)

The most prominent migration artifact is the migration smile, represented by points \(M_8\) and \(M_9\). The pertinent Huygens curves cross the reflection signal exactly at the end of the survey line. In this way, the destructive interference is incomplete at one of the endpoints, leading to a nonnegligible contribution. The position of the migration smile is given by the geometrical location of all points of the type of \(M_8\) and \(M_9\) whose Huygens curves cut the border point \(P\) of the reflection signal. Because of the duality between the Huygens curve and the isochron (e.g., Tygel et al., 1995), this is the isochron of \(P\). The resulting migration artifact follows this isochron, which is a half-circle.
for our constant-velocity, zero-offset experiment as shown in Figure 3a.

Observe the inverted polarity (red is positive, blue is negative) of the artifact between points \( M_8 \) and \( M_9 \). This can be explained with the help of the symmetry of the operator. The dashed part of the Huygens curve of \( M_8 \) that is outside the data is identical to the solid part of the Huygens curve of \( M_9 \) that is inside the data. Thus, the stack at \( M_9 \) will contribute with exactly that part of the data that is missing at \( M_8 \) for complete destructive interference. The actual values of the migration results at points \( M_8 \) and \( M_9 \) depend on the form of the source wavelet as well as on the (half) derivative applied in the migration process. However, the fact that these values are complementary to each other is independent of these conditions. For a better visualization of this complementarity, we pick the peak amplitudes along both branches of the migration smile corresponding to points \( M_8 \) and \( M_9 \) and add them. We can verify in Figure 3b that the sum of amplitudes at each pair of two opposite points from the two branches indeed yields zero (except, of course, for a numerical error).

Again, we can directly relate the geometrical interpretation to the terms of the stationary-phase evaluation of the Kirchhoff migration integral. The migration outputs at \( M_8 \) and \( M_9 \) are described by the third term in equation (9). The first term yields a zero contribution since, at the stationary point, the stacking line lies outside of the reflection signal as in the case of points \( M_4, M_5, M_6, \) and \( M_7 \). The second term in equation (9), which stems from the upper integral limit \( b \), is also zero. At both \( M_8 \) and \( M_9 \), the actual contribution stems from the lower integral limit, \( a = 2500 \) m. Since the Huygens curves of both points terminate at the same position, \( f(a) \) is the same for both of them. So where is the inverted polarity? It is in the sign of the derivative \( q'(a) \), i.e., the dip of the stacking curve at the survey end. As we can easily observe in Figure 2, this sign is positive for \( M_9 \) but negative for \( M_8 \).

The method of stationary phase evaluation allows for a more quantitative analysis of the migration smile. Using equation (6) and recalling the additional factor \( \sqrt{\omega} \) in front of the integral in equation (3) (which stems from the time half-derivative in the original Kirchhoff migration integral), we see that the main contribution to the migration result will be frequency independent while the boundary effects will decay proportionally to \( 1/\sqrt{\omega} \). Figure 4 shows the amplitude of the migration output at points \( M_5 \) (circles) and \( M_8 \) (crosses) as a function of the dominant frequency of the source wavelet used in the modeling. The actually observed amplitudes follow almost exactly the predicted behavior (solid line).

**Pre-stack migration and comparison with Sun (2000)**

The reader might notice that the examples of limited aperture migration (LAM) in Sun (2000) do not distinguish between the different types of migration artifacts described here. The reason is quite simple: Sun uses a pre-stack migration example with a single shot only and, in addition, he shows only a single trace of the migration result in the center of the survey. In that case, only one artifact is visible, namely that because of the limited operator as represented by points \( M_5 \) and \( M_6 \). Of course, both aperture effects are also present in pre-stack.

![Fig. 3. Analysis of the migration smile. (a) Kinematically, it coincides with the isochron of the border point \( P \) of the data. (b) The sum of peak amplitudes of two opposite points on isochron branches [1] and [2] yields approximately zero.](image)

![Fig. 4. Frequency behavior of the boundary effects in 2.5 dimensions. The amplitude at \( M_5 \) (circles) and \( M_8 \) (crosses) decays with \( 1/\sqrt{\omega} \) as predicted by the method of stationary phase.](image)
migration, as can be seen in Figure 5. To construct this figure, the data were sorted into common offset gathers and then migrated separately. The actual migration operator was limited to a maximum aperture radius of 0.8 km around its apex. The respective migration results are displayed in planes parallel to the front face of the cube. In this way, the axis perpendicular to the front face of the cube represents the source–receiver offset. The front face itself is identical to the zero-offset migrated section shown in Figure 2. The side face of the cube is an image gather that depicts the same depth-migrated trace for every offset. Since the migration was performed in a true-amplitude sense, the picked amplitude along an event in the image gather would yield the AVO curve for the respective depth point.

As seen from Figure 5, both aperture effects vary as a function of offset. The artifact from the limited operator size shows a moveout in the image gather (the side face of the cube in Figure 5). For larger offsets, this artifact moves closer to the migrated reflection because the curvature of the operator (and thus the traveltime difference between the operator endpoints and its apex) reduces with offset. The isochron-type artifact resulting from the limited survey area (i.e., the migration smile) broadens and moves along the x-axis because the reflector illumination changes with offset. Because of this offset dependence, a postmigration stack can significantly reduce both migration artifacts. In spite of that, in complex media some strong artifacts generally remain visible in the final migrated section.

A brief outlook to boundary effects in three dimensions

In three dimensions the physical conditions that cause boundary effects are the same as in 2.5 dimensions, these being the limits of the seismic data and the stacking operator. Therefore, the migration artifacts observed in 3D Kirchhoff migration are conceptually the same as in 2.5D migration. One sees the migration smiles from the survey ends as well as the reflector shadow as a result of the limited operator size. This is confirmed by a corresponding stationary-phase analysis of the Kirchhoff migration integral, which also reveals the two leading-order contributions to be those from the stationary point(s) and the integration limits (e.g., Wapenaar, 1992; Sun, 1999).

However, in quantitative terms, the increase in dimension slightly changes the geometric situation. In three dimensions, Kirchhoff migration is realized by a 2D integral. Consequently, the stacking operator is no longer a line but a surface, and its boundary is not a point but a line. For that reason, the amplitude behavior of the artifacts can differ.

Figure 6 shows corresponding numerical results from a 3D migration. The model and all its parameters are the same as for the 2.5D experiment, with identical extension into the third dimension. Indicated is the $1/\sqrt{\omega}$ behavior (solid line) together with the amplitudes of the 3D migration artifacts at points that correspond to points $M_5$ and $M_8$ in Figure 2, here denoted in quotation marks, e.g., “$M_5$” and “$M_8$.” The amplitude of the artifact at point “$M_5$” decays with $1/\sqrt{\omega}$ (as in the 2.5D case). However, the artifact built up by points such as “$M_8$” shows almost no frequency dependence.

The observed amplitude behavior of both effects can be explained by the 2D stationary-phase evaluation of the 3D Kirchhoff migration integral. The reason for the different trends is that the main contribution at “$M_5$” comes from a stationary point of the boundary line integral (e.g., Bleistein, 1984), while at “$M_8$,” this integral is no longer of oscillatory character and thus contributes almost uniformly over all frequencies. However, it goes beyond the scope of this paper to enter into the mathematical details of 3D migration artifacts and to comment on all similarities and differences to the 2.5D situation.

HOW TO AVOID APERTURE EFFECTS

We have already indicated there is a well-known technique to reduce migration artifacts resulting from the limited migration aperture. All that has to be done is to avoid an abrupt end of the operator but to let it die off over a couple of traces, i.e., apply a taper. This must be done at two different places: first,
the input seismograms are tapered at the endpoints of the survey area. Second, the finite operator is not just truncated but is also tapered at its endpoints. In terms of the stationary-phase solution (9), the values of \( f(a) \) and \( f(b) \) are artificially set to zero. This must be done smoothly to avoid violating the underlying assumption of a slowly varying function \( f(\xi) \). Then this approach reduces the contributions of the operator endpoints and thus helps to obtain a migrated image with fewer migration artifacts.

Size of the taper region

When applying a taper, the fundamental question is over how many traces it should extend. On the one hand, the taper ought to be large enough not to violate the smoothness assumption so as to effectively suppress the artifacts. On the other hand, it should not be too large so as not to lose more information than necessary on the amplitudes at the survey ends or to stack unnecessary information at the operator ends. Sun (1998) suggests that in the same way as the stacking reduces or to stack unnecessary information at the operator ends. Therefore, we must once again compromise to avoid the aperture effects.

To get an idea about the size of the taper region, we propose the following simple criterion for zero-offset (poststack) aperture effects. The stacking region should cover the first (projected) Fresnel zone, the taper region should extend over the second (projected) Fresnel zone around the stationary point. Unfortunately, this point cannot be estimated before or during migration. Therefore, we must estimate before or during migration. Therefore, we must once again compromise to avoid the aperture effects.

To get an idea about the size of the taper region, we propose the following simple criterion for zero-offset (poststack) migration. As is well known, to kinematically migrate all reflectors at depth \( z \) up to maximum dip angle \( \theta_m \), the stacking operator may be restricted to a radius of

\[
\tau = z \tan \theta_m. \tag{11}
\]

If the same reflectors are to be migrated dynamically correctly, the radius must be increased by the size \( FZ(1) \) of the projected first Fresnel zone. As shown in the Appendix, \( FZ(1) \) is given in the frequency domain by

\[
FZ(n) = \sqrt{\frac{\nu n T}{2 \cos \theta_m} + \left( \frac{n \nu T}{4} \right)^2}, \tag{12}
\]

with \( n = 1 \), where \( \nu \) is the medium velocity and \( T \) is the period of the considered monofrequency wave. As in equation (5), the half-period \( T/2 \) must be replaced by some estimate of the wavelet length \( \tau_w \) if formula (12) is to be applied in the time domain. According to Sun (1998), the artifacts are suppressed as well as possible, while affecting the amplitudes as little as possible, when the operator is increased by \( FZ(2) \) instead of \( FZ(1) \). The additional operator extension \( FZ(2) - FZ(1) \) is the second projected Fresnel zone, over which the taper is to be applied. Of course, the formulas (11) and (12) are strictly valid for constant velocity only. For inhomogeneous media, they can only be used as a rule of thumb to get a rough idea about aperture size and taper region.

Formula (12) can also be used to obtain an estimate for the size of the end-of-survey taper. By substituting \( z = \nu \tau \cos \theta_m/2 \) and setting \( n = 1 \), the size of the taper at two-way time \( \tau \) can be estimated. If a constant taper size is preferred, \( \tau \) can be replaced by the maximum time value in the data. Correspondingly, \( z \) in equation (12) can also be replaced by the maximum depth in the desired migrated image.

Figure 7 compares the amplitudes along the reflector image for different combinations of aperture and taper sizes for the zero-offset migrated data from the synthetic ZO section of Figure 2. When the aperture is too small, not even the amplitudes far away from the data margins are recovered correctly (dotted line), although the optimal taper is used. When the optimal (or a larger) aperture is applied, all amplitude problems are restricted to the data margins. For too small a taper, the survey-end artifact is not completely removed (dashed line). Too large a taper destroys the amplitudes where they can be retrieved from the data (dash-dotted line). The optimal taper size is the one that eliminates all artifacts but recovers the amplitudes as close to the margins as possible (solid line).

The taper function used for the migration examples shown here is a two-sided Hanning window for both the operator and the end-of-survey taper. For comparison, we also tested a two-sided triangular window. The shapes of these functions are depicted in Sun (1998, 2000) for two and three dimensions. Both types of taper functions yield nearly identical results. The optimal values for the aperture and taper sizes were calculated by means of equation (12) with \( z = 1 \) km, \( \nu = 2 \) km/s, \( \tau_w = 50 \) ms, and \( \theta_m = 0^\circ \), resulting in \( FZ(1) = 320 \) m and \( FZ(2) = 458 \) m. To test the effects of too small or too large aperture and/or taper sizes, the stacking region \( FZ(1) \) and the taper region \( FZ(2) - FZ(1) \) were halved or doubled.

**Real data example**

Figure 8 demonstrates the effect of tapering applied to the input data and the stacking operator for a marine data set from offshore British Columbia, Canada (Scheidhauer et al., 1999; Rohr et al., 2000). The image was obtained by a poststack depth migration of 626 traces with a common midpoint (CMP) spacing of \( \Delta x = 12.5 \) m and time sampling of \( \Delta t = 4 \) ms.

Figure 8a shows the migrated reflector image when stacked with a dip-limited 10° migration operator using the optimal aperture of one estimated projected Fresnel zone according to equation (12) without applying a taper. Both the migration artifacts from the limited operator and survey area are
present, as indicated by arrows. We immediately recognize the artifacts from the limited survey aperture that occur at the left and right margins of the picture, marked by (1). The artifact marked by (2) is the reflector shadow from the limited migration operator. It is parallel to the actual reflection events and may lead to a misinterpretation of the final migration result. Remember that although only the reflector shadow of the sea bottom is clearly visible in Figure 8a, similar artifacts parallel to all other reflector images are also present, even if hidden by other reflectors. Since the migration was performed in a true-amplitude sense, the amplitudes of the migrated reflection events are expected to be proportional to the zero-offset reflection coefficient. The hidden reflector shadows, however, may compromise the amplitudes.

Figure 8b shows the same migrated reflector image with the optimal taper according to equation (12) applied. Both artifacts are almost completely eliminated without affecting the amplitude of the reflector image. Note also that the hidden reflector shadows are eliminated.

General remarks on tapering

With respect to the application of a taper in migration, we algorithmically agree but conceptually disagree with Sun (1998, 2000). As opposed to him, we do not think the taper function should be conceived of as a part of the weight function for the following reasons. First, in kinematic Kirchhoff migration schemes there exist no true-amplitude weight functions. However, taper functions are still required to obtain a high-quality migration result with reduced artifacts. Second, two taper functions need to be applied. One serves to avoid the aperture effect of the limited survey area. This taper is completely independent of any weight function and is applied directly to the input data before migration. The second taper is applied to the operator during migration and may be implemented as part of the weight function. We stress once more that the disagreement is rather conceptual than technical. We prefer to think of the true-amplitude weight and the taper functions as different concepts, even though they may be combined in practice to speed up the algorithm.

CONCLUSIONS

Artifacts known in Kirchhoff migration as migration noise, migration boundary effects, or diffraction smiles can be explained mathematically by means of the method of stationary phase. We have provided a more intuitive explanation of these effects by discussing the constructive and destructive interference of the stack in simple geometric situations. This helps to relate the terms of the stationary-phase approximation with the actually observed migration artifacts. For practical applications, one must distinguish between two principal types of artifacts: (1) boundary effects from a limited survey aperture and (2) artifacts from a limited migration operator. Both types of artifacts are mathematically equivalent and can be explained by means of the boundary terms that result from the stationary-phase analysis of the migration integral. As predicted by the method of stationary phase, the principal migration artifacts in 2.5 dimensions exhibit a $1/\sqrt{\omega}$ decay as compared to the reflector image.

Based on our geometric analysis, we had a closer look at a well-known way to avoid the aperture effects: tapering. The most important question with respect to tapering is how to determine the taper region. Too small a region will not suppress the effects, while too large a region will destroy more information than necessary. We have shown that the ideal taper region is closely connected to the minimum aperture. Schleicher et al. (1997) have derived the minimum aperture for a dynamically correct migration to be the first projected Fresnel zone (Hubral et al., 1993) around the specular point. Sun (1998) has demonstrated that the same minimum aperture of the size of the first projected Fresnel zone is sufficient to separate the operator-end effect from the desired image. We have confirmed both.
observations numerically. Moreover, to get rid of the operator-end effect, a taper region of the size of the second projected Fresnel zone should be added to the operator. In principle, the projected Fresnel zone(s) can be determined during migration, even in inhomogeneous media, from dynamic ray quantities. However, to accelerate the process, it is often useful to fix the operator size beforehand. As we have demonstrated with a real data example, the constant-velocity formula helps provide an idea of an adequate aperture and taper region.

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APPENDIX A

PROJECTED FRESNEL ZONE

In this appendix, we derive expression (12) for the projected Fresnel zone in the zero-offset configuration, assuming a planar reflector with dip \( \theta \) and constant background velocity \( v \) (Figure A-1). The projected Fresnel zone is defined as the projection of the true Fresnel zone in depth along neighboring reflection rays onto the earth’s surface (Hubral et al., 1993). In other words, the projected Fresnel zone ends where the rays reflected at the boundaries of the true Fresnel zone reach the earth’s surface.

We start from the definition of the Fresnel zone, equation (5). At \( \xi^* \), the reflection traveltime \( \tau_R \) of the normal ray reflected at \( M \) (see Figure A-1) is

\[
\tau_R = \frac{2}{v} \sqrt{r^2 + z^2}, \tag{A-1}
\]

where \( r \) is given by equation (11). The diffraction traveltime of a neighboring reflector point \( \tilde{M}(n) \), also measured at \( \xi^* \), is

\[
\tau_D = \frac{2}{v} \sqrt{r^2 + z^2 + \ell(n)^2}, \tag{A-2}
\]

where \( \ell(n) \) is the distance between \( M \) and \( \tilde{M}(n) \). Substituting these two expressions for \( \tau_R \) and \( \tau_D \) in equation (5) and

\[
\tau = \begin{cases} 
\frac{2}{v} \sqrt{r^2 + z^2} & \text{if } \xi \leq \xi^*, \\
\frac{2}{v} \sqrt{r^2 + z^2 + \ell(n)^2} & \text{if } \xi > \xi^*.
\end{cases}
\]

FIG. A-1. Construction of the projected Fresnel zone.
solving for $\ell(n)$, one finds

$$\ell(n) = \sqrt{\left(\frac{vnT}{4}\right)^2 + \frac{vnT}{2}\sqrt{r^2 + z^2}} = \sqrt{\left(\frac{vnT}{4}\right)^2 + \frac{vnTz}{2\cos \theta}}.$$

(A-3)

This is the size of the true Fresnel zone at the reflector in depth. To obtain the size of the projected Fresnel zone, we still have to project this distance onto the earth’s surface along neighboring normal rays (dashed rays, Figure A-1). Since these rays are parallel, the projection provides an additional division by $\cos \theta$, thus yielding formula (12).