Hydrostatic pressure effects on electron states in GaAs–(Ga,Al)As double quantum rings

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Here we address a theoretical analysis of the effects of applied hydrostatic pressure on electron states in concentric GaAs–(Ga,Al)As double quantum rings, under axial magnetic fields. Emphasis is put on the dependence of such effects on the system geometry confinement described within a hard potential model and following an effective-mass approximation. The energy of the ground and excited electronic states were found to decrease with the applied hydrostatic pressure, due mainly to an effective reduction in the barrier potential confinement. Also, while the increase in the magnetic field opens the electron states degeneracy with different angular momenta, the increase in the applied hydrostatic pressure does not alter significantly the energy of these states. For both symmetric and asymmetric double quantum rings, one found that the electron-heavy hole transition energies augment with the applied hydrostatic pressure, mainly due to the increase in the GaAs gap.

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I. INTRODUCTION

Searching for possibilities of storage and sending information is an actual challenge in science with many implications in real life. In the semiconducting context different channels have been explored, for instance, by increasing the carrier quantum confinement by going from bulk to two (quantum well—QW), one (quantum well wire—QWW), and zero (quantum dot—QD) dimensional nanostructures. Other semiconducting structures named quantum rings (QRs) have also been grown by several methods, such as self-assembly techniques. Besides the quantum confinement, the use of probes such as applied electric and magnetic fields, inclusion of impurities, dressing the structures by lasers, and applied hydrostatic pressure opens further possibilities to create these channels and to tune resonant states to send information. QRs and QDs are then nature candidates to investigation, and extensive literature may be found regarding the electronic properties, excitonic, and external field effects.1–3 Of course, the interesting possibility of exhibiting Aharonov–Bohm phenomena opens even more the attractive route of investigations on QR systems under magnetic fields.4–7

Besides the single QR system, GaAs–GaAlAs double quantum rings (DQRs) are also grown, using techniques such as the droplet epitaxy, reported on the microphotoluminescence measurements by Mano et al.8,9 Electron-heavy-hole optical transitions in absence of magnetic fields, are determined in these experiments, while some theoretical approach adopting parabolic confinement potentials and exact diagonalization techniques were used when magnetic fields are considered.10 Also, we have previously reported11 a calculation on the electron-heavy and light hole transition energies in concentric GaAs–Ga0.7Al0.3As DQRs, for null pressure, motivated by the experimental data8 on such systems. Rather than using a parabolic potential10 as used previously to describe the particle (electron and hole) confinements, we adopt a hard potential, which is more suitable to describe the experimental potential profile of the ring grown by Mano et al.8

Hydrostatic pressure applied on a bulk material, as expected, would change the lattice parameter of the crystal, modifying its band structure, and leading to a number of physical responses of the system. Examples of such changes are the variations in the energy band gap, the carrier effective mass, the electron and hole g-factors, and consequently, on optical and transport phenomena. Also known is the fact that in GaAs–(Ga,Al)As QW systems, an applied hydrostatic pressure modifies the electronic band structure, leading to changes on the electron and hole energy states causing direct and indirect electron-hole transitions.12–14 Here we concentrate on the same GaAs–Ga0.7Al0.3As DQRs heterostructure studied before,11 and analyze the effects of an applied hydrostatic pressure on the electron energies.

II. THEORY

A concentric GaAs/Ga0.7Al0.3As DQR is considered, composed of internal and external rings with lengths $L_A = \rho_B - \rho_A$, $L_C = \rho_D - \rho_C$, and a Ga0.7Al0.3As barrier with $L_B = \rho_C - \rho_B$, between both rings; $\rho_A$ being the ring radii. The DQR is subjected to a magnetic field perpendicular to the ring plane. The Hamiltonian of a carrier in the effective mass-approximation, with a confinement potential $V(\rho, z)$ in the radial and $z$-direction and under an applied magnetic field is written as,

$$ H = \frac{1}{2m_e}(\mathbf{p} + e\mathbf{A}/c)^2 + V(\rho, z), $$

in concentric GaAs–Ga0.7Al0.3As DQRs, for null pressure.
with $m^*$ being the carrier effective mass and $\mathbf{A}$ the vector potential of the magnetic field. The Schrodinger equation for those structures is usually written in cylindrical coordinates. For a uniform magnetic field $\mathbf{B} = B \mathbf{e}_z$, one has $A_p = A_z = 0$ and $A_\varphi = 1/2B\rho$.

Under the effects of a hydrostatic pressure, a crossover between the $\Gamma$ and $X$ conduction-band minima takes places as the pressure increases. Two critical pressure values, $P_1 \approx 10$ kbar and $P_2 \approx 30$ kbar, are found to be fundamental in the description of the electron confining potential, corresponding to crossings between the bands of barrier and well materials [see details in Ref. 14]. Taking into account the pressure probe, the finite radial confinement potential is given by $V(\rho) = V(x, P)$ for $\rho < \rho_a$, $\rho_b < \rho < \rho_c$, and for $\rho > \rho_p$, and zero for $\rho_1 < \rho < \rho_b$ and $\rho_c < \rho < \rho_p$. The dependence of the potential with the Al concentration $x$ and the pressure $P$ is given by

$$V_c(x, P) = \alpha[l(E_g^1(x, P) - E_g^1(0, P) + \beta \Sigma_{\Gamma \chi}(x, P)]$$

with $\beta = 0$ if $0 < P \leq P_1$ and $\beta = 1$ if $P_1 < P \leq P_2$, and the pressure dependent strength coefficient $\Sigma_{\Gamma \chi}(x, P) = x \Sigma_{\Delta}(P - P_1)/P$, where $\Sigma_0$ is an adjustable parameter, chosen equal to 250 meV, to fit with experiment data. In the expression, $r$ denotes the fraction of the difference between the Ga$_1-x$Al$_x$As and GaAs energy gaps at the $\Gamma$ point, within the so-called direct gap regime. In the $z$-direction the rings are modeled by a square well of width $H$, $V(z) = V(x, P)$ for $|z| \geq H/2$, and zero elsewhere.

The effects of the applied hydrostatic pressure on the ring thick and radii are given by

$$H(P) = H_0[1 - (S_{11} + 2S_{12})P],$$

$$\rho(P) = \rho_0[1 - 2(S_{11} + 2S_{12})P]^{1/2},$$

where $H_0$ and $\rho_0$ are the ring height and radii in the absence of pressure, respectively, and $S_{11}$ and $S_{12}$ are the compliance constant of the GaAs.

Electrical and optical properties of a large number of semiconductor systems have been calculated by using $k \times p$ perturbation theory. How the conduction band energy and the electron effective mass at the $\Gamma$ and $X$ point in the reciprocal lattice change with an applied hydrostatic pressure and the Al concentration $x$, in GaAs/Ga$_1-x$Al$_x$As QWs, has already been established. Here we follow the same procedure used previously to define the electron effective mass, $m$,

$$m = m_0 \left[1 + \frac{\Pi^2}{3} \left(\frac{2}{E^1_g} + \frac{1}{E^3_g + \Delta_m}\right) + \delta_m \right]^{-1},$$

where the $\Gamma$-point energy of the conduction band is measured with respect to the top of the valence band $E^3_g = E(\Gamma^3_g) - E(\Pi^2_g)$, for which the dependence on the aluminum concentration and pressure is given by $E(x, P) = a + b x + c x^2 + \alpha(x) P$, with $a$, $b$, and $c$ being energy parameters, and $\alpha(x)$ a rate expressing energy/pressure, discussed in the literature and collected in Ref. 18. In the mass equation, $\Pi^2$ denotes the square of the interband matrix elements describing the coupling between the $s$ states of the $\Gamma^3$ conduction band and the hybrid valence states corresponding to the $\Gamma^5$ states, composed of $\Gamma^5_g$ and $\Gamma^5_u$ energies. The split off valence gap, $\Delta_m$, is given by $E(\Pi^5_g) - E(\Pi^5_u)$. The mass variation given by $\delta_m$ depends on $x$ according to $\delta(x) = \delta_0 + \delta_1 x + \delta_2 x^2$, where the relative $\delta_{0,1,2}$ values are $-3.935, 0.488$, and $4.938$, chosen in order to fit the experimental results corresponding to the effective mass at null pressure.

In this simple model, the same Hamiltonian (1) is used for electrons and holes (light and heavy), considering the respective effective masses and charge signs. Of course, a better description should take into account the anisotropy of the semiconductor mass at the top of the valence band and more complex phenomena such as interlevel mixing, which surely would alter the way the energy gap shifts with the applied pressure. For null pressure, the electron GaAs effective mass is taken as $0.067 m_0$, and the light and heavy hole effective masses as $0.082 m_0$ and $0.51 m_0$, respectively. The Hamiltonian is easily solved for electrons and holes by means of standard calculations involving here linear combination of confluent hypergeometric functions for the radial solution $w_{m,n}(x) = A F_n(a, b, x) + B U_n(a, b, x)$, with $x = m^2 w^2 / 2h$, $a = |l|/2 + 1/2 + 1/2$, and $b = |l| + 1$, $l$ being the momentum angular component. In what follows we have considered the conduction and valence band edges as the reference levels for the electron and hole energies, respectively. The transition energies, $E_{ij}$, are

$$E_{ij}(x, P) = E_{ij}^{GaAs}(x, P) + E_{ij}^{\Gamma}(x, P) + E_{ij}^{\Gamma}(x, 0),$$

with $E_{ij}^{GaAs}$ being the GaAs band gap energy (1.5194 eV) and $E_{ij}^{\Gamma}$ and $E_{ij}^{\Gamma}$ being the electron and hole energies, respectively, regarding to the corresponding band edges.

With the purpose of investigating the effects of the geometrical details of the double ring nanostructures on the energy spectra, we consider different widths values for the internal and external rings, $L_A$ and $L_C$, respectively, and for the barrier width, $L_B$. Others parameters have been chosen accordingly to the values reported on the self-assembly rings grown by the epitaxial method. For all studied DQRs, we have used fixed values for the internal ring radius, $\rho_A = 20$ nm, and ring thick, $H = 4$ nm, in the $z$-direction.

### III. RESULTS AND DISCUSSION

Results for the electron ground-state (gs) energy of GaAs-Ga$_0.3$Al$_{0.7}$As DQRs as a function of the outer ring (width $L_C$), for different values of the applied hydrostatic pressure and a fixed barrier width equal to 5 nm, are shown in Fig. 1. The main effect of a finite pressure on the DQR system is to reduce the gs energy, for all considered ring widths. The pressure promotes an effective reduction on the barrier potential and a renormalization on the ring lengths, predicted by Eqs. (3) and (4). Similar effect has already been reported on other confined semiconductor low dimensional systems under pressure. Depending on the geometric details of the nanostructured rings a large amount of energy reduction may be achieved. The general trend found for null pressure, in which the gs energy decreases only after a critical value of the external ring width ($L_C = L_A$), is preserved for all considered pressure values. As discussed previously, for $L_C \leq L_A$, the electron is con-
The dependence of the electron gs energy in a GaAs–Ga$_{0.7}$Al$_{0.3}$As DQR on the applied hydrostatic pressure is shown in Fig. 2(a), for different values of an axial magnetic field. The electron energy behaves like the potential confinements with the applied hydrostatic pressure and increases with the applied magnetic field due to the induced extrageometric confinement. The same energy decreasing behavior with the applied hydrostatic pressure is found for different DQR structures in the absence of magnetic fields. In order to investigate the role played by the pressure on symmetric and asymmetric double rings, we calculate the gs energy for the symmetric $L_C=L_A=10$ nm and asymmetric $L_C=15$ nm and $L_A=10$ nm DQRs. The results displayed in Fig. 2(b) show that the symmetric structure presents always the higher energy as compared to the asymmetric case, no matter the applied pressure value. Considering that for non-null magnetic field the gs quantum numbers vary with the field intensity, one presents in Fig. 2(c) the energy dependence on the applied hydrostatic pressure for the $n=0$ state and different angular momentum $l$, for $B=5$ T. Again the remarkable energy reduction with the applied hydrostatic pressure may be attributed to the variation in the confining barrier potential height induced by the pressure. In both low and high pressure regimes, the energy differences between the $l=0$ and $l=-1$ to $-4$ results seem to be maintained, indicating there is not an enhancement of the degeneracy lift as the pressure is increased.

To highlight the modifications on the DQR energies induced by the coupling of the two considered probes, magnetic field and hydrostatic pressure, we present in Fig. 3 results for the electronic energy spectra as the magnetic filed increases, for $P=10$ and $20$ kbar. A symmetric GaAs–Ga$_{0.7}$Al$_{0.3}$As DQR, with $L_C=L_A=10$ nm, and $L_B=5$ nm was chosen. Besides the large variation on the energy range depending on the pressure value, the general trend corresponding to null pressure is preserved even with respect to the relative energy positions corresponding to each one of the considered angular momenta. In that sense the degeneracy lift is completely governed by the magnetic field.

The hydrostatic pressure dependence of the electron heavy-hole transition energy, corresponding to the gs, is shown in Fig. 4(a) for the cases of symmetric ($L_C=10$ nm) and asymmetric ($L_C=15$ nm) DQRs. Higher transition energies are found for the symmetric structure compared to the asymmetric one, independent of the barrier width. Despite of the effective decreasing of the barrier height with the pressure, it is the GaAs band gap increasing which determines the increasing of the electron-heavy hole (e-hh) transition energy. Although up to now there are no reports on experimental work pointing out this trend, we expect this result may be verified in future data measurements. The e-hh transition energy as a function of the outer ring width displayed in Fig. 4(b) behaves essentially like the energy of the electron gs, as shown in Fig. 1. Actually, the e-hh transitions are quite insensitive to the outer ring width up to a ring geometry

![Fig. 1](image1.png)

![Fig. 2](image2.png)
hydrostatic pressures. This is an interesting fact to be taken into account in future technological applications.

Summing up, the present study on GaAs–(Ga,Al)As DQRs illustrates some possibilities of changing the electron energy spectra and e-hh transition energies, by applying external probes such as hydrostatic pressures and magnetic fields, and by controlling the geometric aspects of the DQR. It was possible to identify that the energies of the ground and excited electron states decrease with the applied hydrostatic pressure, due mainly to an effective reduction in the barrier potential confinement, as it was previously reported for other heterostructures. On the other hand, we found that while the increase in the magnetic field opens the degeneracy of the electron states with different angular momenta, the applied hydrostatic pressure does not make a robust effect on the states. The e-hh transition energy is higher for the symmetric than for the asymmetric geometry, irrespective of the in-between barrier width. With the applied hydrostatic pressure the e-hh transition energies augment for both symmetric and asymmetric DQRs, a result that is basically commanded by the GaAs gap increasing. With the actual experimental progress and facilities, we do believe that complex semiconducting systems such those DQRs studied here may be probed also under hydrostatic pressure as done previously with DQW heterostructures. The simple theoretical analysis performed here may then be used to provide an understanding of future data measurements.

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FIG. 3. (Color online) Electron energy spectra for a GaAs–GaAlAs DQR ($L_c=L_a=10$ nm and $L_B=5$ nm) as a function of the applied magnetic field, for $P=10$ and 20 kbar.

FIG. 4. (Color online) e-hh transition energies ($g_s$) in a GaAs–GaAlAs DQRs for null magnetic field, as a function of the (a) applied hydrostatic pressure and for two different structures with $L_c=10$ nm (symmetric case) and $L_c=15$ nm (asymmetric case) and barrier width equal to 1 and 5 nm, for the two cases, and the (b) outer ring width ($L_c$), for $L_B=5$ nm. Different applied hydrostatic pressure values are considered with different symbols in (b). The GaAs band gap energy is also shown (dashed line) in Fig. 4(a).