I. INTRODUCTION

Interest in doped transition metal oxides, a longstanding problem of interest, has been renewed by discovery of non-Fermi-liquid behavior observed in the normal state of the high-\(T_C\) cuprates, as well as certain other transition metal and rare-earth compounds.\(^1\) In most cases, the non-Fermi-liquid anomalies are observed on doping the Mott-Hubbard insulator, a process that is achieved by carrier doping. In real systems, carrier doping is brought about by chemical substitution, a process that simultaneously introduces disorder on some length scale. Given this, the electrons “feel” different local environments, and it is physically meaningful to speak only of disorder averaged quantities. Calculations carried out on models such as the pure Hubbard model represent a homogeneous system, and so fail to capture fully the effects of doping. This suggests that a proper description of such doped materials should include the combined effects of correlations and disorder on an equal footing.

Actually, the one-band Hubbard model with static, diagonal disorder has already been the subject of a few studies.\(^2,3\) Janiš et al.\(^2\) have studied the effects of site-diagonal disorder on the stability of the half-filled Mott-Hubbard antiferromagnet in detail, using the QMC algorithm to “solve” the \(d = \infty\) Hubbard model. Dobrosavljevic et al.\(^3\) have considered the role of off-diagonal disorder in local moment formation in disordered metals, while Sarma et al.\(^4\) investigated the effects of disorder in the \(d = \infty\) Hubbard model in a simple way to study the single particle (SP) spectral function; they have also applied their approach to compute the angle-integrated photoemission spectra in SrTiO\(_3\).\(^4\), and have demonstrated that the experimental features are reproduced quite well. However, their approach involves treating the effects of disorder by a simple weighting procedure\(^4\) that is reminiscent of the earlier known virtual crystal approximation (VCA) for alloys.\(^5\) More recently, Muto\(^6\) has used the same treatment for incorporating disorder in the Hubbard model in \(d = \infty\). It is known that the VCA is valid for small impurity concentration, and does not interpolate correctly between the weak- and the strong-scattering limits, where one has to deal with resonant scattering. In the latter case, a \(T\)-matrix approach is more suitable. Indeed, it is known from studies of noninteracting, disordered systems that the so-called coherent-potential approximation (CPA) solves the Anderson disorder problem exactly in \(d = \infty\).\(^6\) The effect of disorder in the paramagnetic, strongly correlated FL metallic phase of the \(d = \infty\) Hubbard model within a framework valid for all parameter regimes (disorder strength, interaction strength and filling) has not yet been considered.

A naive analysis might lead one to conclude that, at least at low energies, and in \(d = \infty\), disorder would have qualitatively the same effects in a free (\(U = 0\)) system and in an interacting Fermi system on a lattice. One might argue that local correlations act to renormalize the bare energy scales, keeping the Fermi-liquid correspondence intact, and disorder would then have qualitatively the same effects as in the free (\(U = 0\)) case. This reasoning, however, is not obvious for reasons pointed out in Ref. 2. With strong, local correlations, a \(M\)-\(I\) transition is possible in a pure Hubbard model. It is known that this transition is characterized by a transfer of dynamical spectral weight over large energy scales, a feature that is completely absent in noninteracting models. In the \(d = \infty\) Hubbard model, for, e.g., spectral weight is transferred over large energy scales as the temperature is lowered through the lattice Kondo scale \(T_{\text{K}}^{\text{lat}}\),\(^6\) giving rise to nontrivial effects in thermodynamics and in dc and ac transport. This makes emergence of qualitatively new behaviors (which have no analogs in the free system) possible, and hence a consistent framework to study correlations and disorder on an equal footing is in order.

In this work, we aim to study the combined effects of correlations and static, site-diagonal disorder in the quantum paramagnetic phase of the \(d = \infty\) Hubbard model. The questions we want to answer are as follows.

1. Is there an instability, or a crossover, of the strongly correlated Fermi-liquid metal to an incoherent metal as the disorder is increased, and, if so, what strength of disorder is required?
What is the character of this incoherent metallic state, if it exists?

What is the critical disorder strength required to drive this metallic state insulating? (notice that this is more appropriately a disorder driven insulator following Ref. 2).

What is the effect of the above scenario (if it exists) on the dynamical responses, e.g., on the optical conductivity, electronic Raman line shape, etc?

In this work, we aim to provide answers to these questions within the dynamical mean field theory (DMFT) $(d = \infty)$ of strongly correlated fermionic systems. In the next section, we set up the formalism to study the combined effects of correlations and disorder exactly in $d = \infty$. This is followed by a detailed discussion of the role of site-diagonal disorder in a strongly correlated FL metal, and on the nature of the incoherent metallic state at intermediate disorder strength. In particular, the qualitatively different results we obtain (in comparison to Ref. 4) are emphasized, and the differences are shown to arise from the fact that the CPA, in contrast to VCA, treats the weak- and strong-scattering limits equally well. Lastly, as an application, we describe how the formalism presented here can be fruitfully applied to study the interplay of correlations (competition between atomic and itinerant tendency) and static, diagonal disorder on the ac conductivity, and the electronic Raman line shape of doped transition metal compounds.

II. MODEL AND SELFCONSISTENT FORMALISM

We start with the one-band Hubbard model (HM) with static, site-diagonal disorder on a hypercubic lattice in $d = \infty$, 

$$
H = -t \sum_{\langle i,j \rangle, \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \text{H.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{i\sigma} v_i n_{i\sigma}
$$

(1)

where the disorder potentials $v_i$ are specified by a given probability distribution. We restrict ourselves to microscopic disorder, and so work with a binary-alloy distribution for disorder; explicitly,

$$
P(v_i) = (1 - x) \delta(v_i) + x \delta(v_i - v).
$$

(2)

In other words, upon doping, a fraction $x$ of the sites have an additional local potential $v$ for an electron (or hole) hopping onto that site. In this paper, we consider only discrete disorder. The case of a continuous distribution of disorder potentials is a separate problem. We plan to investigate this separately in detail. However, we make some qualitative remarks about the physics in this situation later. Given the above, the carriers experience different local environments in the course of their hopping, and the physically sensible object is the disorder-averaged local Green function (GF) $(G_{ii}(\omega))_c$, where $\langle \cdots \rangle_c$ means a disorder average. In $d = \infty$, all interesting dynamical information is entirely contained in $\langle G_{ii}(\omega) \rangle_c$, the computation of which is the central aim of this section.

As stated above, the problem of interacting, disordered requires a simultaneous consideration of both on an equal footing. To see how one proceeds, notice that the model, Eq. (1) is exactly soluble in $d = \infty$ by CPA when $U = 0$, and by the iterated perturbation theory (IPT), e.g., when $v = 0$. To treat disorder and interactions simultaneously requires a suitable extension, either from the $U = 0$ limit, where the disorder problem is first solved exactly, and the interactions are then considered, using the CPA DOS as an input to the interacting model, or the disorder-free Hubbard model is first “solved exactly” by IPT in $d = \infty$, and the IPT GF serves as an input to the CPA procedure. In what follows, we take the latter route. In principle, both ways of solution should yield the same final result in a fully self-consistent treatment of interactions and disorder on the same footing. We have checked that both methods of solution indeed yield identical results when a fully self-consistent procedure (described below using the latter route) is employed. In what follows, we describe the details of the fully self-consistent procedure that we have developed for this purpose.

We start by writing down the local effective action for the impurity model with site-diagonal disorder:

$$
S_{\text{eff}}[\tau] = -\sum_{\sigma} \int_0^\beta d\tau \int_0^\beta d\tau' c_{i\sigma}^\dagger(\tau) g_{\alpha}^{-1}(\tau, \tau') c_{i\sigma}(\tau') + U \int_0^\beta n_{i\uparrow}(\tau) n_{i\downarrow}(\tau),
$$

(3)

where the Weiss function, describing the dynamics on the rest of the lattice is given by

$$
g_{\sigma}^{-1}(\tau, \tau') = \delta(\tau - \tau')(-v_i - \partial_\tau)
$$

$$
-\tau^2 \int dv_i P(v_i) G_{i\sigma}(v_i, \tau, \tau')
$$

(4)

with the integral over $v_i$, representing the disorder average. In the above, $G_{ii}$ is the fully interacting local Green function of the lattice model. To solve the effective problem, one has to first solve the interacting problem, followed by a disorder average, and link these up in a proper self-consistent way. In this work, the interaction part is treated via selfconsistent iterated perturbation theory (IPT) away from half filling. We used this technique as it yields the correct Fermi-liquid metallic state, and gives results in very good agreement with numerical studies in the quantum paramagnetic regime, which we consider here. The interplay between local correlations and static disorder is studied by a combination of the IPT with the CPA, as described below.

Step 1. The disorder-free Hubbard model, Eq. (1), is “solved” in $d = \infty$ by iterated perturbation theory at and off half filling. This maps the lattice problem onto an effective one-body problem describing the propagation of renormalized fermion quasiparticles in a complex effective medium, as described by a local self-energy $\Sigma(\omega)$. This procedure is known to yield the correct FL behavior in the quantum paramagnetic phase of the $d = \infty$ Hubbard model, and gives results in good agreement from those obtained from exact diagonalization studies. The effective one-body Hamiltonian now is formally written as
\[ H_{\text{eff}} = \sum_k \left[ \epsilon_k + \Sigma(\omega) \right] c_{k\sigma}^\dagger c_{k\sigma} + \sum_{i,k,\sigma} t_k \left( c_{k\sigma}^\dagger c_{i\sigma} + \text{H.c.} \right) \]

To treat the effects of disorder, we follow the philosophy of the CPA, which entails removing one site from the effective medium, and replacing it with the actual local potential \( V_i \). The effective local potential that scatters the electrons (holes) is now \( V_i = \Sigma(\omega) \), with \( \Sigma(\omega) \) to be determined self-consistently from the CPA condition. This condition requires that the additional scattering produced by the effective potential vanish on the average; this is equivalent to the requirement that the configuration-averaged \( T \) matrix vanish, \( \langle T_{ij}[\Sigma(\omega)] \rangle_i = 0 \).

The updated GF is then fed back into the IPT routine from the zero-order iteration that involves steps (1).

The complete magneto-optical response of the DHM in the quantum parametall state can also be readily computed in a way identical to that used for the pure HM.\(^6\)

\[ \sigma_{xx}(i\omega) = \frac{1}{i\omega} \int \rho_0(\epsilon) \sum_{i\nu} G(\epsilon, i\nu) G(\epsilon, i\omega + i\nu). \]

Moreover, in the same approximation, the Raman intensity lineshape in the \( B_{1g} \) channel is simply related to the optical conductivity as \(^8\)

\[ I_{xx}(\omega) = \frac{\omega}{1 - e^{-\beta\omega}} \text{Re} \int \sigma_{xx}(\omega). \]

The complete magneto-optical response of the DHM in the quantum parametall state can also be readily computed in a way identical to that used for the pure HM.\(^9\)

**III. RESULTS**

We now proceed to discuss our results. In our calculations, we employed a Gaussian unperturbed DOS,\(^5\) and \( U/D = 3.0 \) (\( D \) is the free bandwidth). We used the IPT off \( n = 1 \) as a reliable approximation to treat the correlation part exactly in \( d = \infty \). The IPT gives results in excellent agreement with exact diagonalization studies,\(^5\) and we have checked that all desirable \( FL \) properties are reproduced in our numerics. We restricted ourselves to the paramagnetic metallic phase throughout. We varied the disorder strength in the range \( 0 \leq \nu \leq U \) at fixed \( n \), and repeated our calculations for each \( n = (1 - \chi) = 1.0, 0.9, 0.8 \) to get a clear picture of the combined effects of (correlations + disorder) and band filling.

Figure 1 shows the result of the calculation for the local DOS of the DHM in \( d = \infty \). For \( \nu = 0 \), the DOS shows all the characteristic features of the strongly correlated FL metal (also shown up in the self-energy, the real and imaginary parts of the SP self-energy for the disordered Hubbard model in \( d = \infty \).
The dotted line in Fig. 1 shows the local spectral density and the self-energy for small \( v = U/4 \) disorder strength. As expected, the central FL peak is broadened by disorder, and \( \text{Im} \Sigma(\omega) = -C - D \omega^2 \) acquires a finite value at the Fermi surface due to disorder scattering. Notice that the height of the central peak has decreased from its noninteracting value for \( v = U/4 \), and so Luttinger’s theorem is not satisfied (notice that the Luttinger sum rule, fixing the value of the chemical potential, is still satisfied, but Luttinger’s theorem breaks down, in the sense that a sharp Fermi surface no longer exists). This is the consequence of the \( k \)-space degeneracy arising from the introduction of disorder; the Luttinger theorem in its strict form is valid only for a translationally invariant system. Nevertheless, the self-energy shows a behavior characteristic of a dirty Fermi liquid. We notice that an analogous situation would arise in a treatment of the disorder in the Born approximation (or its self-consistent versions), and refer to this metallic state as a disordered Fermi liquid. At \( v = U/3 \), however, one sees the emergence of a qualitatively new behavior in Fig. 1 (see middle-sized dashed line); a pseudogap develops in a continuous way in the local DOS, and \( \Sigma(\omega) \) has a qualitatively new behavior (compared to what is expected from a local Fermi liquid picture). \( \text{Im} \Sigma(\omega) \) has a minimum at \( \mu \), and \( \text{Re} \Sigma(\omega) \) has a positive slope, invalidating the quasiparticle picture. We refer to this phase as the incoherent, disordered metallic phase. With increasing disorder strength, the pseudogap in the DOS deepens, and the shallow minimum in \( \text{Im} \Sigma(\omega) \) develops into a sharp peak at \( \mu \) (Fig. 1 with the long dashed line). We interpret this feature as a precursor of the disordered insulating phase, which sets in for larger \( v = U \). Finally, at \( v = U \), a clear gap develops at \( \mu \), and the incoherent metallic state undergoes a continuous transition to a disorder-driven insulator. Similar trends are observed off \( n = 1 \), as shown in Fig. 2 for \( n = (1 - x) = 0.9 \).

The quasiparticle residue \( Z(\mu) \) calculated as \( Z(\mu) = 1/[1 - F(\mu)] \), with \( F(\mu) = d[\text{Re} \Sigma(\omega)]/d\omega \big|_{\omega = \mu} \), illustrates the FL-disordered, incoherent metal “transition” in a clearer way (Fig. 1). With \( U/D = 3.0, n = 1 \), the Fermi liquid is stable up to \( v/U = (v/U)_c \approx 0.31 \). For \( v > v_c \), we see that \( Z(\mu) > 1 \), or even negative, invalidating the very concept of coherent FL quasiparticles. A similar behavior of \( Z(v) \) is obtained within the CPA treatment of the noninteracting, disordered model (see appendix). This illustrates how the FL metal is destroyed continuously with increasing disorder before the insulating phase is approached around \( v/U = 1 \). Thus, for small values of \( v \), the “FL metal” survives; for intermediate \( v \), we have shown that the metallic state is a disordered, incoherent metal. This metallic state is unstable to a disorder-driven insulator beyond a certain \( v = v_c \). Notice that, in contrast to the case of the pure Hubbard model, the disorder-driven insulator is not characterized by the vanishing of an effective lattice coherence (Kondo) scale simply because the combined effects of strong correlation and disorder induced strong scattering prevent the developing of the quasicoherent energy scale already in the metallic state (see the results for the local DOS and the self energy in Fig. 1).

These results are very different from those obtained in the treatment of the noninteracting, disordered model using CPA. Indeed, in this case, the disordered metal phase develops for any value of the disorder. To see this, we use the CPA eqns for the self-energy with \( U = 0 \), whereby \( G(\omega) = G_c(\omega) \). The CPA yields

\[
\Sigma(\omega) = x_U + \frac{v^2(1-x)}{\omega - v(1-x) + iD^2 \pi \rho_0(\mu)}
\]

as shown in the Appendix. With a positive definite \( \rho_0 \), \( \text{Re} \Sigma(\omega) \) has a positive slope near \( \mu \) (see the Appendix), and so the metallic state is always incoherent. The above illustrates the importance of treating both correlations and disorder on an equal footing, and shows that the actual behavior in such systems is quite different from those with only interactions (no disorder), or those with only disorder (no interactions).

The above results are also quite different from those obtained from a VCA treatment of disorder.\(^4\) To see this, we have repeated the calculation carried out by Mutou, and have observed that the incoherent, non-FL pseudogap metal phase never sets in, no matter how strong the disorder. This is a direct consequence of the fact that the VCA cannot treat resonant scattering correctly (intermediate \( v \)) while it is known that the CPA works equally well for all values of disorder.\(^2\) In fact, our results (Figs. 1, 2) are completely consistent with those of Mutou\(^4\) when \( v \) is small (\( v = U/4 \)), in which case, the CPA as well as the VCA give results in accordance with a perturbative treatment of disorder.

In this work, we have not considered the effect of a continuous distribution of disorder \( P(v_i) \). Quite generally, for continuous, singular disorder distribution, one expects the disorder averaged propagator \( \langle G(\omega) \rangle = \int_0^\infty P(v_i) G(v_i, \omega) \rangle \) to have an explicit non-FL form, with an anomalous power-law fall-off with an exponent related to that of the function \( P(v_i) \). The interesting consequences of this behavior merit a separate, more detailed consideration that is deferred to future work.
FIG. 3. Optical conductivity of the disordered Hubbard model in $d = \infty$ as a function of disorder strength for two different band fillings $n = 0.9, 0.8$.

**IV. TRANSPORT PROPERTIES OF THE DHM**

Knowledge of the full disorder Green function is a sufficient input for a controlled computation of transport properties in $d = \infty$. This is because vertex corrections drop out in the Bethe-Salpeter equation for the two-particle propagator, e.g., for the conductivity, and the corresponding susceptibility is just the convolution of the full GF’s. This means, as is known, that the $d = \infty$ formalism cannot be used to access Anderson localization effects, but should be a valid approximation in situations where the incoherent metal state results from strong resonant scattering induced by disorder. Using the formulas given in the Introduction, we have computed the optical conductivity and the electronic Raman scattering line shape for the DHM.

The changes in the local spectral density as a function of $v$ and filling for a given $U/D$ are reflected in the optical spectra. Figure 3 shows the calculated optical conductivity for $U/D = 3.0$, $n = 0.9, 0.8$ and for different disorder strengths. The solid line corresponds to the pure Hubbard model; we see that all expected features are reproduced in good agreement with earlier work. In particular, the “Drude” peak at $\omega = \mu$, the mid-IR peak around $U/2$, which corresponds to transitions between the lower-Hubbard band and the central FL resonance, and the high-energy peak corresponding to transitions between the Hubbard bands, are all reproduced well. As expected from the evolution of the DOS, small disorder, $v = U/4$, broadens the “Drude” peak, and smoothens out the mid-IR and high-energy features, while preserving a FL response (dotted line). With $v = U/3$, however, all semblance of the Drude peak has disappeared completely, and the pseudogap formation in the DOS is clearly reflected in $\sigma(\omega)$. The dashed line ($v = U/3$) therefore represents the optical response characteristic of a strongly disordered incoherent metal (since the DOS at $\mu$ is finite). Notice that the mid-IR feature is progressively broadened and shifted to higher energies with increasing $v$ as seen for $v = U/2$ (long dashed line). Finally, the onset of the disorder-dominated insulating phase is clearly reflected in the appearance of a threshold feature in the optical response. This scenario is only slightly modified as a function of band filling, $n = (1 - x)$, as shown in Fig. 3.

It is known that the optical response of the Hubbard model shows up the dramatic transfer of spectral weight from high- to low energy on hole doping, a characteristic of strongly correlated systems. A very interesting, related observation is that the $\sigma_{xx}(\omega)$ curves for different hole doping cross at a single point $\omega_c$, the so-called *isosbectic* point. Similar features are observed for the DHM as a function of hole-doping, as seen in Fig. 4. We observe that the $\sigma_{xx}(\omega)$ curves now seem to cross at two points as a function of doping. This seems to be true for all disorder strengths, and clearly warrants a closer examination. Heuristically, we can understand the crossing of the curves as arising from the high- and low-frequency behavior of the charge susceptibility; a microscopic understanding of the two crossing points is, however, a harder task.

The qualitatively different behavior of the optical response for varying disorder strengths can be understood in terms of the coherent (incoherent) part of the low-frequency spectral density. At zero or weak disorder, the Green function still is that characteristic of a Fermi liquid (real or complex pole structure), and $\sigma_{xx}(\omega)$ shows a “Drude” peak at $\omega = \mu$. With increasing disorder, one is in the resonant scattering regime, all semblance of the quasiparticle behavior is destroyed, and the GF is characterized by a branch-cut behavior. The difference in the spectra as a function of disorder is then a consequence of the fact that the response, which is caused by the action of the current operator (which has non-zero matrix elements between lower-Hubbard band states) does not create well-defined elementary excitations as $v$ is increased. Within the $d = \infty$ ideas used here, the coherent response is the manifestation of the collective band Kondo...
effect for zero or weak disorder, while the anomalous response for larger $v$ is a consequence of the suppression of this coherence scale by disorder-induced strong scattering.

In $d = 5'$, the electronic Raman scattering lineshape in the $B_{1g}$ channel can be directly obtained from the optical conductivity, as remarked above. In light of the above discussion, we expect a sharp electron-hole peak, characteristic of coherent particle-hole response in a FL, for $v = 0$, and a disorder induced broadening of this feature for small $v = U/4$. For larger $v$, we expect a completely incoherent line shape characteristic of the disordered pseudogap metal state ($v = U/3, U/2$). This is indeed what we observe in our calculations (Fig. 5) and is related to the fact that the collective particle-hole excitations are overdamped by disorder-induced resonant scattering in this regime. The evolution of the Raman spectrum with increasing disorder is then understood as follows. The stress tensor whose fluctuations are measured in the Raman line shape, which connects the eigenstates of the Hubbard Hamiltonian, creates well defined elementary excitations at low energy in a FL, giving a sharp peak at low energy, while in the disordered metal regime, the Green function has a branch cut, and the action of the stress tensor does not create well-defined elementary excitations, resulting in a continuum response at low energy.

The transfer of spectral weight with doping is also revealed in the Raman line shapes (Fig. 6), and is understood directly in terms of the evolution of $\alpha_{\varepsilon \tau}(\omega)$ as a function of doping. In Fig. 6, we show the Raman line shapes as a function of doping for two different disorder strengths, $v = U/4$ and $v = U/3$. In spite of the different nature of the metallic state at these two disorder values, the isosbestic point in $I_{\varepsilon \tau}(\omega)$ is clearly revealed, seemingly independent of details of the low-energy spectra.

V. CONCLUSION

In conclusion, we have considered the role of static, site-diagonal disorder in the quantum paramagnetic phase of the $d = 5'$ Hubbard model. We have devised a formalism that captures the interplay of dynamical local correlations inherent in the Hubbard model and doping-induced disorder on an equal footing. This is achieved by a proper extension of the IPT off half filling, which has been shown to work well in the paramagnetic phase of the $d = 5'$ Hubbard model, by combining it (self-consistently) with the CPA, which is known to yield the exact solution of the Anderson disorder problem in $d = 5'$. We have shown that the FL metal is stable against small disorder ($v = U/4$), and that strong repeated scattering effects at higher disorder values destroys low-energy coherence, characteristic of a disorder-dominated metal. At a critical $v$, this phase becomes unstable to an insulating phase. We have also considered the effects of the interplay between correlations in the FL metallic state and disorder in $d = 5'$ on the optical and Raman response, and have shown how low-energy coherence (incoherence) is manifested in the dynamical charge response of correlated, disordered systems in this limit.

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APPENDIX

In this Appendix, we review briefly the CPA in the interaction-free case ($U = 0$). The CPA essentially involves (i) replacing the actual disorder problem by an effective one-body problem, where the electrons now move in a dynamical (complex) effective medium. (ii) One now removes this
complex potential \[\Sigma (\omega)\] from one local site \(i\) and replaces it with the actual local potential \(v_i\). The additional effective potential seen by an electron is now \(V_i = v_i - \Sigma (\omega)\). For a binary alloy disorder \(P(v_i) = (1-x)\delta(v_i) + x\delta(v_i-v)\), one needs to average additionally over disorder. The site-diagonal \(T\) matrix corresponding to this additional effective scattering potential \(V_i\), after configuration averaging, is set to zero in the CPA condition.

\[
\frac{(1-x)[-\Sigma (\omega)]}{1+\Sigma (\omega)G(\omega)} + \frac{x[v - \Sigma (\omega)]}{1-V - \Sigma (\omega)G(\omega)} = 0. \quad (A1)
\]

Solving for the self-energy, \(\Sigma (\omega)\), using \(G^{-1}(\omega) = \omega - \Sigma (\omega) - A(\omega)\) gives the Eqs. (7) and (8) used in the text.

Next, we show that the unusual feature of the \(Z(\omega)\) vs \(\omega\) (Fig. 1) can be understood analytically in the CPA as well. In fact, as we show below, the CPA in the case with \(U = 0\) always leads to \(Z(\omega) \geq 1\), showing that the metallic state is always anomalous. Choosing the unperturbed DOS to be a Lorentzian with half width \(D\), one can show that \(A(\omega) = -iD\). The self-consistent equations now reduce to a single algebraic equation for the self-energy, giving exactly Eq. (11) in the text with \(\rho_0(\mu) = 1/D\pi\). The resulting real part of the self-energy has a positive slope at low energy, giving \(Z(\omega) = [1 - (\omega/2D)^2]^{-1}\), which is never FL-like.

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4 D.D. Sarma, S. R. Barman, H. Kajueter, and G. Kotliar, Europhys. Lett. 36, 307 (1996); see also T. Mutou, Phys. Rev. B 60, 2268 (1999). Mutou uses the IPT at half-filling to solve the correlation part, and the VCA to include the effects of static disorder. In contrast, we have used the IPT off half-filling and the CPA (which works well for all impurity concentrations and disorder strengths) to include the interplay between correlations and disorder away from \(n = 1\). This is more physical, since chemical substitution, which creates disorder, also introduces carriers, taking the system away from half filling.
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