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Citation: Physics of Plasmas (1994-present) **5**, 167 (1998); doi: 10.1063/1.872714
View online: http://dx.doi.org/10.1063/1.872714
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/5/1?ver=pdfcov
Published by the AIP Publishing

Drift-ballooning modes in the presence of charged dust impurities in a nonuniform rotating magnetoplasma

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(Received 1 October 1997; accepted 7 October 1997)

The linear and nonlinear properties of drift-ballooning modes in the presence of an equilibrium electric field and stationary charged dust grains are examined. It is found that the presence of these two contribute to the stability of the ballooning mode. Furthermore, the nonlinear coupling between finite amplitude drift-ballooning modes gives rise to different types of coherent vortex structures, which can affect the transport properties of an inhomogeneous magnetized plasma. The relevance of the investigation to laboratory and astrophysical plasmas is discussed. © *1998 American Institute of Physics.* [S1070-664X(98)03001-8]

I. INTRODUCTION

It is well known[1] that the ballooning mode is driven when the ratio between the gravitational acceleration and the density gradient scale length (which are opposite to each other) is larger than the square of the Alfvén transit frequency ($k_z v_A$, where k_z is the component of the wave vector parallel to the external magnetic field direction and v_A the Alfvén velocity) along the curved magnetic field lines. Thus, the ballooning mode is the electromagnetic interchange mode and appears in the bad curvature regions. The local and nonlocal properties of ballooning modes have been derived from the magnetohydrodynamic (MHD) equations when the temperature gradient is absent. However, in a nonuniform plasma with equilibrium density and temperature gradients, the MHD description fails and one has to resort to either the two-fluid or a gyro-kinetic description. It turns out that the consideration of the diamagnetic and finite Larmor radius (FLR) radius effects provide the possibility of coupling ballooning modes with drift-like waves. Studies of coupled drift-ballooning modes are of great importance for understanding the plasma confinement in tokamaks, as well as for the understanding of magnetospheric substorms.

The linear theory of drift-ballooning modes without and with magnetic shear is fully understood.[1] The negative magnetic shear tends to stabilize the mode. However, in reality, only finite amplitude drift-ballooning modes are of interest. Accordingly, in the past, several authors[2–5] derived the nonlinear mode coupling equations for finite amplitude drift-ballooning modes in an electron-ion plasma containing equilibrium density, temperature and magnetic field inhomogeneities. The nonlinear mode coupling equations have been employed to study energy cascading and self-organization in drift-ballooning mode turbulence. Specifically, it has been found[3–5] that stationary solutions of the drift-ballooning mode coupling equations can be represented in terms of the dipolar vortex. The latter constitute a new turbulent state which can affect the transport properties of fusion plasmas.

However, in tokamaks there exist radial electric fields and charged dust impurities. The latter can come from the sputtering of the wall surfaces during plasma-wall interactions in low-temperature tokamak edges. The radial electric field introduces an equilibrium plasma rotation, whereas the charged dust impurities, which are micron-sized charged particulates of solid matter, can give rise to dust-convective cells[6] in a nonuniform multi-component magnetoplasma. Charged dust grains are also ubiquitous in many astrophysical and space environments and they are supposed to introduce new types of collective interactions.[7]

In this paper, we study linear as well nonlinear properties of low-frequency (in comparison with the ion gyrofrequency) drift-ballooning modes in a nonuniform rotating dusty magnetoplasma. The latter contains both the equilibrium density and magnetic field gradients. It is found that the plasma rotation as well as charged dust grains contribute toward the stability of drift-ballooning modes. On the other hand, the nonlinear interaction among finite amplitude drift-ballooning modes can cause self-organization in the form of the vortex street as well as a dipolar vortex whose profiles are significantly different from those reported earlier.[3–5]

The manuscript is organized in the following fashion. In Sec. II, we derive the relevant nonlinear equations for drift-ballooning modes by employing the multi-fluid description in the presence of radial electric fields and stationary charged dust grains. Section III examines analytically the local dispersion relation in several interesting limiting cases. The localized nonlinear solutions of weakly interacting finite amplitude drift-ballooning modes are presented in Sec. IV. Section V contains a brief summary and possible applications of our investigation to laboratory and astrophysical plasmas.

II. DERIVATION OF THE NONLINEAR EQUATIONS

We consider a nonuniform magnetized plasma whose constituents are electrons, ions, and negatively charged dust grains. At equilibrium, the divergence of the equilibrium plasma current is zero and the overall quasineutrality condition reads

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$$n_{i0}=n_{e0}+Z_d n_{d0}, \tag{1}$$

where n_{j0} is the unperturbed number density of the particle species j (j equals e for the electrons, i for the ions, and d for the charged dust impurities), and Z_d stands for number of charges residing on the negatively charged dust grain surface. The equilibrium density and the dust charge number are supposed to be inhomogeneous. The dust grains, which are regarded as point charges, are assumed to be spherical. The intergrain spacing and the dust sizes are taken to be much smaller than the characteristic scale lengths (viz. the gyroradii, the plasma Debye length, etc.) of the plasma. Furthermore, there are a sufficient number of dust grains in the Debye sphere so that the collective effects, as described below, are in tact. Finally, the plasma equilibrium is governed by

$$\nabla\left(p_0+\frac{B_0^2}{8\pi}\right)=\rho_d\mathbf{g}\equiv-\rho_d\nabla U, \tag{2}$$

where p_0 is the total plasma pressure, B_0 the strength of the nonuniform external magnetic field $B_0(r)\hat{\mathbf{z}}$, $\rho_d=m_d n_d$ the mass density of the dust grains, $m_d(n_d)$ the mass (the number density) of the dust particulates, and $\mathbf{g}=\hat{\mathbf{r}}g(r)\equiv-\nabla_r U(r)$ the gravity expressing the curvature of the equilibrium magnetic field lines. Here, $\hat{\mathbf{r}}(\hat{\mathbf{z}})$ is the unit vector along the radial (z) axis.

The presence of the equilibrium radial electric field $E_r(r)\hat{\mathbf{r}}$ drives the plasma which rotates with the constant angular velocity $\mathbf{\Omega}=-(cE_r/rB_0)\hat{\theta}$, which implies that $E_r(r)=-B_0\Omega r/c$.

We now study the stability of the above equilibrium against intermediate frequency (viz. $\omega_{pd},\omega_{cd}\ll\omega\ll\omega_{ci}$, where $\omega_{pd}(\omega_{cd})$ is the dust plasma (gyro) frequency and ω_{ci} the ion gyrofrequency) electromagnetic waves. Thus, in the presence of the latter, the electron and ion fluid velocity perturbations are given by

$$\mathbf{v}_e=\mathbf{v}_{EB}+\mathbf{v}_{De}+v_{ez}\mathbf{B}_\perp/B_0+v_{ez}\hat{\mathbf{z}}\equiv\mathbf{v}_{e\perp}+v_{ez}\hat{\mathbf{z}}, \tag{3}$$

and

$$\mathbf{v}_i=\mathbf{v}_{EB}+\mathbf{v}_{Di}+\mathbf{v}_{ip}+\mathbf{v}_g, \tag{4}$$

where $\mathbf{v}_{EB}=c\hat{\mathbf{z}}\times\nabla\phi/B_0$, $\mathbf{v}_{Dj}=(cT_j/q_jB_0n_j)\hat{\mathbf{z}}\times\nabla n_j$, and $\mathbf{v}_{ip}=-(c/B_0\omega_{ci})[\partial_t+\nu_i+(\mathbf{v}_{Di}+\mathbf{v}_{EB})\cdot\nabla+\mathbf{v}_R\cdot\nabla+\mathbf{v}_g\cdot\nabla]\times\nabla_\perp\phi$, $\mathbf{v}_g=(g/\omega_{ci})\ \hat{\boldsymbol{\theta}}$ and $\mathbf{v}_R=c\mathbf{E}_r\times\hat{\mathbf{z}}/B_0$ are the $\mathbf{E}\times\mathbf{B}_0$ drift, the diamagnetic drift velocity, the ion polarization drift velocity, the gravitational drift velocity, and the rotational plasma drift velocity due to the constant radial electric field, respectively. The temperature of the plasma species j is denoted by T_j and $q_e(q_i)=-e(e)$, where e is the magnitude of the electron charge. The ion collision frequency is denoted by ν_i. Furthermore, $\mathbf{E}=-\nabla\phi-c^{-1}\partial_t A_z\hat{\mathbf{z}}$ is the electric field vector; $\phi(A_z)$ the electrostatic (z component of the vector) potential, and $\mathbf{B}_\perp=\nabla A_z\times\hat{\mathbf{z}}$ the perpendicular component of the wave magnetic field. The compressional magnetic field perturbation has been ignored in view of the low-β $(=8\pi p_0/B_0^2\ll1)$ plasma.

The z component of Ampére's law is used here to relate the parallel component of the electron fluid velocity v_{ez} with the parallel (to $\hat{\mathbf{z}}$) vector potential A_z. We have

$$v_{ez}\approx(c/4\pi n_{e0}e)\nabla_\perp^2 A_z, \tag{5}$$

where $\nabla_\perp^2$ is the two-dimensional Laplacian in the $(r-\theta)$ plane.

The wave dynamics in our multi-component plasma is governed by a set of equations consisting of the electron continuity equation

$$\partial_t n_e+\nabla_\perp\cdot(n_e\mathbf{v}_{e\perp})+\partial_z(n_e v_{ez})=0\ , \tag{6}$$

the parallel component of the electron momentum equation

$$(\partial_t+\nu_e+\mathbf{v}_{EB}\cdot\nabla_\perp+v_{ez}\partial_z)v_{ez}=-\frac{e}{m_e}\left[E_z+\frac{1}{c}(\mathbf{v}_e\times\mathbf{B}_\perp)\cdot\hat{\mathbf{z}}\right]-\frac{T_e}{m_e n_e}\partial_z n_e, \tag{7}$$

Poisson's equation

$$\nabla^2\phi=4\pi e(n_e-n_i+Z_d n_d), \tag{8}$$

and an equation which comes from the difference of the electron and ion continuity equations, namely,

$$\partial_t(n_e-n_i)+\nabla\cdot(n_e\mathbf{v}_e-n_i\mathbf{v}_i)=0\ , \tag{9}$$

where the electron collision frequency is denoted by ν_e. Equations (3) to (9) form a complete set.

In the presence of electromagnetic fields, the perpendicular component of the plasma current density is

$$\mathbf{J}_\perp=\frac{Z_d n_d ec}{B_0}\hat{\mathbf{z}}\times\nabla\phi+\frac{m_i n_i c^2}{B_0^2}\mathscr{L}_t\nabla_\perp\phi+\frac{cT_e}{B_0}\hat{\mathbf{z}}\times\nabla(n_{e1}+\sigma n_{i1})+\frac{m_i n_i c}{B_0}\mathbf{g}(r)\times\hat{\mathbf{z}}, \tag{10}$$

where $\mathscr{L}_t=\partial_t+\nu_i+(\mathbf{v}_{EB}+\mathbf{v}_{Di}+\mathbf{v}_g+\mathbf{v}_R)\cdot\nabla$, $\sigma=T_i/T_e$, and the ion motion parallel to $\hat{\mathbf{z}}$ has been neglected. The latter implies that the coupling between drift-ballooning and ion-acoustic modes has been discarded. We note that the first term on the right-hand side of (10) arises because the $\mathbf{E}\times\mathbf{B}_0$ current remains finite when charged dust grains are present. The divergence of this current is nonzero when $Z_d n_d/B_0$ is nonuniform. Furthermore, the second term comes from the perpendicular ion inertia including the ion-dust drag, whereas the third and fourth terms are the contributions of the diamagnetic current and the current produced by the gravitational acceleration of the ion fluid due to the curved magnetic field lines, respectively.

Expanding $n_j=n_{j0}+n_{j1}$, where $n_{j1}\ll n_{j0}$, we can express (6)–(9) in the rotating frame moving within the bulk plasma where the electric field is transformed out and only the perturbation fields are left. We have for the electron continuity equation

$$\left(\partial_t+\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla\right)n_{e1}-\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_{e0}\cdot\nabla\phi+\frac{c}{4\pi e}\hat{\mathbf{b}}\cdot\nabla\nabla_\perp^2 A_z=0\ , \tag{11}$$

the parallel component of the electron momentum equation

$$\left(\partial_t+\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla\right)(1-\lambda_e^2\nabla_\perp^2)A_z-\eta_e\nabla_\perp^2A_z$$

$$+\mathbf{v}_{De0}\cdot\nabla A_z+c\partial_z\phi-\frac{cT_e}{en_{e0}}\hat{\mathbf{b}}\cdot\nabla n_{e1}=0\ , \tag{12}$$

and the ion vorticity equation

$$\left[\partial_t+\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla+\nu_i+(\mathbf{v}_{Di0}+\mathbf{v}_g)\cdot\nabla\right]\nabla_\perp^2\phi$$

$$+\nabla\cdot[(\mathbf{v}_{Di1}\cdot\nabla)\nabla_\perp\phi]+\frac{Z_dn_{d0}\omega_{ci}}{n_{i0}}\hat{\mathbf{z}}\times\nabla\ \ln(Z_dn_{d0})$$

$$\cdot\nabla\phi+\frac{2\Omega}{n_{i0}}\hat{\mathbf{z}}\times\nabla(n_{i0}+n_{e1})\cdot\nabla\phi$$

$$+\frac{B_0}{cn_{i0}}\hat{\mathbf{z}}\cdot\left[\nabla n_{e1}\times\nabla\left(U-\frac{r^2}{2}\Omega^2\right)\right]+\frac{v_A^2}{c}\hat{\mathbf{b}}\cdot\nabla\nabla_\perp^2A_z=0, \tag{13}$$

where $\hat{\mathbf{b}}\cdot\nabla=\partial_z-B_0^{-1}\hat{\mathbf{z}}\times\nabla A_z\cdot\nabla$, $v_{EB}\cdot\nabla\gg v_{ez}\partial_z$, $\lambda_e=c/\omega_{pe}$ is the collisionless electron skin depth, $\omega_{pe}=(4\pi n_{e0}e^2/m_e)^{1/2}$ the electron plasma frequency, $\eta_e=\nu_e\lambda_e^2$ represents the plasma resistivity, and $v_A=B_0/(4\pi\rho_i)^{1/2}$ the Alfvén velocity, with $\rho_i=m_in_{i0}$ being the ion mass density. Furthermore, we have denoted $\mathbf{v}_{Dj0}=(cT_j/q_jB_0n_{j0})\hat{\mathbf{z}}\times\nabla n_{j0}$ and $\mathbf{v}_{Di1}=(cT_i/eB_0n_{i0})\hat{\mathbf{z}}\times\nabla n_{e1}$. We have assumed that the scale length of the equilibrium magnetic field inhomogeneity is much larger than that of the equilibrium density gradient. Finally, we note that in deriving (11) and (13) we have imposed the quasi-neutrality condition for the perturbation (viz. $n_{i1}=n_{e1}$), which is valid for a dense plasma in which the ion plasma frequency is much larger than the ion gyrofrequency. Equations (11)–(13) are the desired equations for the study of ballooning modes in the presence of a constant electric field and stationary charged dust impurities. In the absence of the latter, we have $n_{e0}=n_{i0}$ and the divergence of the current associated with the dust grains identically vanishes. Then, our system of equations are identical to those of Liu and Horton[8] and Shukla and Bharuthram.[9]

The origin of various nonlinear terms in (11)–(13) is obvious. The nonlinear terms in the electron continuity equation come from the $\mathbf{E}\times\mathbf{B}_0$ convection of the perturbed electron number density and the coupling of the parallel electron flow with the magnetic field perturbation. The nonlinearity retained in (12) arises from the nonlinear parallel electron inertia and the parallel component of the nonlinear Lorentz force, whereas (13) contains the nonlinear ion polarization drift as well as the divergence of the parallel nonlinear electron current involving the parallel electron flow and the perturbed magnetic field.

Finally, we also present the nonlinear equations which govern the dynamics of low-frequency electrostatic waves including the parallel electron motion in a nonuniform rotating dusty plasma. We have

$$\left(\partial_t+\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla\right)n_{e1}-\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_{e0}\cdot\nabla\phi+n_{e0}\partial_zv_{ez}=0, \tag{14}$$

$$\left(\partial_t+\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla\right)v_{ez}=\frac{e}{m_e}\partial_z\left(\phi-\frac{T_en_{e1}}{en_{e0}}\right), \tag{15}$$

and

$$\left[\partial_t+\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla+(\mathbf{v}_{Di0}+\mathbf{v}_g)\cdot\nabla\right]\nabla_\perp^2\phi$$

$$+\frac{Z_dn_{d0}\omega_{ci}}{n_{i0}}\hat{\mathbf{z}}\times\nabla\ \ln(Z_dn_{d0})\cdot\nabla\phi$$

$$+\frac{2\Omega}{n_{i0}}\hat{\mathbf{z}}\times\nabla(n_{i0}+n_{e1})\cdot\nabla\phi+\frac{B_0}{cn_{i0}}\hat{\mathbf{z}}$$

$$\times\nabla n_{e1}\cdot\nabla\left(U-\frac{r^2\Omega^2}{2}\right)$$

$$+\frac{\sigma D_B}{n_{i0}}\nabla\cdot[(\hat{\mathbf{z}}\times\nabla n_{e1})\cdot\nabla_\perp\phi]+\frac{4\pi en_{e0}v_A^2}{c^2}\partial_zv_{ez}=0, \tag{16}$$

where $D_B=cT_e/eB_0$. Equations (14)–(16) are the generalization of Horton *et al.*[10] to include the effects of the parallel electron dynamics and charged dust impurities.

III. LOCAL DISPERSION RELATION

In this section, we examine the linear properties of drift-ballooning modes in multi-species dusty plasmas by neglecting the nonlinear terms in (11)–(13). For this, purpose, we derive the local dispersion relation which is valid when the wavelength of the oscillation is much smaller than the scale length of the density and magnetic field gradients. Supposing that all the perturbed quantities are proportional to $\exp[-i(\omega t-m\theta-k_zz)]$, we Fourier transform (11)–(13) and obtain

$$\omega n_{e1}=-\frac{c}{B_0}k_\perp\frac{\partial n_{e0}}{\partial r}\phi-\frac{ck_zk_\perp^2}{4\pi e}A_z, \tag{17}$$

$$[(1+b_e)\omega+i\eta_ek_\perp^2-\omega_{e*}]A_z=ck_z\left(\phi-\frac{T_en_{e1}}{en_{e0}}\right), \tag{18}$$

and

$$(\omega+i\nu_i-\omega_{i*}-\omega_g-\omega_{dc}-\omega_0)\phi$$

$$-\frac{B_0}{n_{i0}ck_\perp}\frac{\partial(U-r^2\Omega^2/2)}{\partial r}n_{e1}-\frac{v_A^2}{c}k_zA_z=0, \tag{19}$$

where $b_e=k_\perp^2\lambda_e^2,\omega_{j*}=k_\perp(cT_j/q_jB_0)\partial\ \ln n_{j0}/\partial r$, $\omega_g=(g/\omega_{ci})k_\perp$, $\omega_0=-(2\Omega K_i/k_\perp)$, $\omega_{dc}=-Z_dn_{d0}\omega_{ci}K_d/k_\perp n_{i0}$, $K_d=\partial\ \ln(Z_dn_{d0})/\partial r$, and $K_i=\partial\ \ln n_{i0}/\partial r$. Here, $k_\perp=m/r$ and $k_z\sim 1/qR;r(R)$ being the minor (major) radius of a torus, and m the azimuthal mode number.

Combining (17)–(19), we obtain

$$\left[\omega^2+i\nu_i\omega-\Omega_s\omega-\frac{(g+r_0\Omega^2)n_{e0}K_e}{n_{i0}}\right]$$
$$\times[(1+b_e)\omega^2+i\eta_e k_\perp^2\omega-\omega\omega_{e*}-k_z^2v_A^2k_\perp^2\rho_s^2]$$
$$=k_z^2v_A^2(\omega-\omega_{e*})\left[\omega+\frac{k_\perp(g+\Omega^2r_0)}{\omega_{ci}}\right], \quad (20)$$

where $\Omega_s=\omega_{i*}+\omega_g+\omega_{dc}+\omega_0$, $K_e=\partial\ln n_{e0}/\partial r$, $\rho_s=c_s/\omega_{ci}$ is the ion Larmor radius at the electron temperature, $c_s=(T_e/m_i)^{1/2}$ the ion sound velocity, and r_0 a reference point. Equation (20), which is a fourth order polynomial in ω and contains the coupled magnetic-drift-Alfvén-ballooning and drift-convective cell modes, is the most general dispersion relation for low-frequency electromagnetic waves in the presence of charged dust impurities in a nonuniform rotating magnetoplasma. The electrostatic result follows from (20) in the limit when b_e is much larger than unity. In the following, we discuss some limiting cases.

First, we consider the flute mode for which $k_z=0$. Here, (20) gives the damped magnetic electron drift mode, given by $(1+b_e)\omega^2+i\eta_e k_\perp^2\omega-\omega\omega_{e*}=0$, and a dispersion relation governing the RT-mode in a dissipative rotating plasma, viz.

$$\omega^2+i\nu_i\omega-\Omega_s\omega=\frac{(g+\Omega^2r_0)n_{e0}K_e}{n_{i0}}. \quad (21)$$

Equation (21) predicts an oscillatory instability for $K_e<0$. Two cases are of interest. First, for $\omega\gg\nu_i$, the threshold condition for the instability is

$$(g+\Omega^2r_0)/L_n>(n_{i0}/n_{e0})\Omega_s^2, \quad (22)$$

where $L_n^{-1}=|K_e|$. The maximum growth rate of that instability is $|(g+\Omega^2r_0)n_{e0}K_e/n_{i0}|^{1/2}$; the latter exhibits that the growth rate of the Rayleigh–Taylor (RT) instability is reduced by a factor of $(n_{e0}/n_{i0})^{1/2}$ when charged dust impurities are present.

Second, for $\omega,\Omega_s<\nu_i$, (21) gives

$$\omega=-i\frac{(g+\Omega^2r_0)n_{e0}K_e}{n_{i0}\nu_i}, \quad (23)$$

which predicts the instability of a collisional RT mode for $K_e<0$.

Next, we discuss the properties of drift-ballooning modes on the basis of (20). We consider several limiting cases. First, in the absence of dissipation (viz. $\nu_i=0$ and $\eta_e=0$) we obtain for $\omega>\omega_{e*},k_z^2v_A^2k_\perp^2\rho_s^2/(1+b_e),b_e\ll1$ and $k_\perp(g+\Omega^2r_0)/\omega\omega_{ci}\ll1$, the dispersion relation for the ballooning mode

$$\omega^2-\Omega_s\omega=\frac{k_z^2v_A^2}{(1+b_e)}+(g+\Omega^2r_0)\frac{n_{e0}K_e}{n_{i0}}, \quad (24)$$

which shows that for $\omega>\Omega_s$ the ballooning mode is also stabilized by the presence of charged dust grains. The threshold condition for the ballooning instability is $(g+\Omega^2r_0)/L_n>n_{i0}k_z^2v_A^2/n_{e0}$. Second, for a uniform plasma without the gravity and rotation, we obtain for $\omega\gg\nu_i$ from (20),

$$\omega^2+i\Gamma_m\omega=\omega_{SA}^2, \quad (25)$$

when $\Gamma_m=\eta_e\lambda_\perp^2/(1+b_e)$ and $\omega_{SA}^2=k_z^2v_A^2(1+k_\perp^2\rho_s^2)/(1+b_e)$. For $|\omega|\gg\Gamma_m$, (25) gives the frequency of the shear Alfvén waves. Finally, in the absence of charged dust grains in a plasma without rotation (20) agrees with the dispersion relation presented by Hasegawa and Wakatani[2] and Shukla.[3]

IV. LOCALIZED NONLINEAR SOLUTIONS

Here, we consider the nonlinear localized solutions of (11)–(13) in the region far away from the center of the rotating plasma column, by neglecting dissipative effects. Specifically, we seek translating helical vortex solutions [8–14] in a new frame of reference $\xi=y+\alpha z-ut$, where $y=r(\theta-\theta_0),r=r_0+x$, and α and u are constants; and $(r_0,\theta_0,0)$ are the cylindrical coordinates of a reference point inside the localized region under consideration. In the stationary frame, (11)–(13) can then be written as

$$\mathscr{L}_\phi n_{e1}+\frac{cn_{e0}}{uB_0}K_e\partial_\xi\phi-\frac{c\alpha}{4\pi eu}\mathscr{L}_A\nabla_\perp^2A_z=0, \quad (26)$$

$$\mathscr{L}_\phi(1-\lambda_e^2\nabla_\perp^2)A_z+\frac{D_BK_e}{u}\partial_\xi A_z-\frac{c\alpha}{u}\partial_\xi\phi$$
$$+\frac{cT_{e0}\alpha}{en_{e0}u}\mathscr{L}_An_{e1}=0, \quad (27)$$

and

$$\mathscr{L}_\phi\nabla_\perp^2\phi-\frac{D_B\sigma}{n_{i0}u}\boldsymbol{\nabla}\cdot[(\hat{\mathbf{z}}\times\boldsymbol{\nabla}n_{e1}\cdot\boldsymbol{\nabla})\boldsymbol{\nabla}_\perp\phi]$$
$$-\frac{(V_*+V_g)}{u}\partial_\xi\boldsymbol{\nabla}_\perp^2\phi$$
$$-\left(\frac{Z_dn_{d0}}{n_{i0}u}\omega_{ci}K_d+\frac{2\Omega}{u}K_i\right)\partial_\xi\phi-\frac{v_A^2\alpha}{cu}\mathscr{L}_A\nabla_\perp^2A_z$$
$$-\frac{B_0}{n_{i0}cu}(g+r_0\Omega^2)\partial_\xi n_{e1}-\frac{2\Omega}{n_{i0}u}\hat{\mathbf{z}}\times\boldsymbol{\nabla}n_{e1}\cdot\boldsymbol{\nabla}\phi=0, \quad (28)$$

where $\mathscr{L}_\phi=\partial_\xi+(c/uB_0)(\partial_\xi\phi\partial_x-\partial_x\phi\partial_\xi)$, $\mathscr{L}_A=\partial_\xi-(1/\alpha B_0)(\partial_\xi A_z\partial_x-\partial_xA_z\partial_\xi)$, $V_*=\sigma D_BK_i$, and $V_g=g/\omega_{ci}$.

Equations (26)–(28) are highly nonlinear partial differential equations having complicated types of vector nonlinearities.[11–13] It is a formidable task to solve them analytically. However, some approximate solutions can be obtained in limiting cases which are discussed below.

We note that Liu and Horton[8] have presented the intrinsic vortex solutions of (26)–(28) by neglecting the linear and nonlinear parallel electron inertial forces, when the dust grains are absent. Thus, the $(\omega_{ci}K_dZ_dn_{d0}/n_{i0})\partial_\xi\phi$ term in (28) and the $\lambda_e\boldsymbol{\nabla}_\perp^2A_z$ term in (27) are neglected in Ref. 8. In the following, we discuss the dipolar vortex solution of (26)–(28) in the limit when the velocity u of the vortex is much larger than the electron and ion diamagnetic drift velocities. Accordingly, (27) is satisfied by

$$(1-\lambda_e^2\nabla_\perp^2)A_z=\frac{c\alpha}{u}\phi. \quad (29)$$

Substituting (29) into (26) we obtain

$$\mathscr{L}_\phi\left[n_{e1}+\frac{cn_{e0}K_e}{uB_0}\phi-\frac{c\alpha}{4\pi eu\lambda_e^2}\left(A_z-\frac{c\alpha}{u}\phi\right)\right]=0. \quad (30)$$

On using (29) we can now can rewrite (28) as

$$\mathscr{L}'_\phi\left[\nabla_\perp^2\phi+\left(\frac{v_A^2\alpha^2}{uu_0\lambda_e^2}-\frac{\omega_{ci}K_dZ_dn_{d0}}{n_{i0}u_0}-\frac{2\Omega K_i}{u_0}\right)\phi-\frac{B_0(g+\Omega^2r_0)}{n_{i0}cu_0}n_{e1}-\frac{v_A^2\alpha}{cu_0\lambda_e^2}A_z\right]=0\,, \quad (31)$$

where $u_0=u-V_g$, $g+\Omega^2r_0=2\Omega u_0$ and $\mathscr{L}'_\phi=\partial_\xi+[c/(u_0B_0)][\partial_\xi\phi\partial_x-\partial_x\phi\partial_\xi]$.

Eliminating n_{e1} from (30) and (31) we have

$$\mathscr{L}'_\phi\left[\nabla_\perp^2\phi+\left(\frac{2\Omega n_{e0}K_e}{un_{i0}}-\frac{\omega_{ci}Z_dn_{d0}K_d}{n_{i0}u_0}-\frac{2\Omega K_i}{u_0}+\frac{v_A^2\alpha^2}{\lambda_e^2uu_0}+\frac{2\Omega B_0c\alpha^2}{4\pi eu^2n_{i0}\lambda_e^2}\right)\phi-\left(\frac{v_A^2\alpha}{c\lambda_e^2u_0}+\frac{2\Omega\alpha B_0}{4\pi eun_{i0}\lambda_e^2}\right)A_z\right]=0. \quad (32)$$

To eliminate A_z from (32), we operate the equation by $(1-\lambda_e^2\nabla_\perp^2)$ and then use $(1-\lambda_e^2\nabla_\perp^2)A_z$ from equation (29). As a result, we obtain

$$(\nabla_\perp^4+\chi_1\nabla_\perp^2+\chi_2)\phi+\chi_3\frac{uB_0}{c}x=0\,. \quad (33)$$

Here $\lambda_e^2\chi_1=-(1+\lambda_e^2F)+\lambda_e^2[v_A^2\alpha^2/(uu_0\lambda_e^2)+2\Omega B_0c\alpha^2/(4\pi eu^2n_{i0}\lambda_e^2)-\chi_0]$, $\lambda_e^2\chi_2=F+\chi_0$, $\lambda_e^2\chi_3=-F$ and $\chi_0=\omega_{ci}Z_dn_{d0}K_d/(u_0n_{i0})+2\Omega K_i/u_0-2\Omega n_{e0}K_e/(un_{i0})$.

Equation (33) admits spatially bounded dipolar vortex solutions.[8,9,14,15] In the outer region $(r>R)$, a typical solution can be expressed as

$$\phi_0=[C_1K_1(s_1r)+C_2K_1(s_2r)]\cos\,\theta, \quad (34)$$

where C_1 and C_2 are constants and $s_{1,2}^2=[-\chi_1\pm(\chi_1^2-4\chi_2)^{1/2}]/2$ for $\chi_1<0$ and $\chi_1^2>4\chi_2>0$. Here χ_1 and χ_2 are with $F=0$. On the other hand, in the inner region $(r<R)$, equation (33) has a solution

$$\phi_i=\left[C_3J_1(s_3r)+C_4I_1(s_4r)-\frac{\chi_3uB_0}{c\chi_2}r\right]\cos\,\theta, \quad (35)$$

where C_3 and C_4 are constants. Here $s_{3,4}\equiv[(\chi_1^2-4\chi_2)^{1/2}\pm\chi_1]/2$.

The constants C_1, C_2, C_3, C_4 and F can be determined by matching the inner and outer solutions of ϕ and A_z and the higher derivatives $\nabla\phi$, $\nabla_\perp^2\phi$, $\nabla_\perp A_z$ and $\nabla_\perp^2A_z$ at the vortex interface $r=R$. This exercise has been carried out by Mikhailovskii *et al.*[14] and Liu and Horton,[15] and explicit expressions for the various constants had been found.

We have also examined the stationary nonlinear solutions of (14)–(16), which can be cast in the form

$$\mathscr{L}_\phi n_{e1}+\frac{cn_{e0}K_e}{B_0u}\partial_\xi\phi-\frac{n_{e0}\alpha}{u}\partial_\xi v_{ez}=0, \quad (36)$$

$$\mathscr{L}_\phi v_{ez}=-\frac{e\alpha}{m_eu}\partial_\xi\left(\phi-\frac{T_en_{e1}}{en_{e0}}\right), \quad (37)$$

and

$$\mathscr{L}_\phi\nabla_\perp^2\phi-\frac{D_B\sigma}{n_{i0}u}\nabla\cdot[(\hat{\mathbf{z}}\times\nabla n_{e1}\cdot\nabla)\nabla_\perp\phi]-\frac{(V_*+V_g)}{u}\partial_\xi\nabla_\perp^2\phi-\left(\frac{Z_dn_{d0}}{n_{i0}u}\omega_{ci}K_d+\frac{2\Omega}{u}K_i\right)\partial_\xi\phi-\frac{2\Omega}{n_{i0}u}\hat{\mathbf{z}}\times\nabla n_{e1}\cdot\nabla\phi-\frac{B_0}{un_{i0}c}(g+r_0\Omega^2)\partial_\xi n_{e1}-\frac{4\pi n_{e0}ev_A^2\alpha}{uc^2}\partial_\xi v_{ez}=0. \quad (38)$$

For $u^2\gg\alpha^2v_{te}^2$, (37) gives $v_{ez}\approx-(e\alpha/um_e)\phi$ so that (36) can be rewritten as

$$n_{e1}=-\frac{en_{e0}}{m_eu^2}\left(\alpha^2+\frac{K_eu}{\omega_{ce}}\right)\phi\equiv\lambda_1\phi, \quad (39)$$

which can be substituted into (38) to obtain

$$\mathscr{L}_\phi\nabla_\perp^2\phi-\frac{D_B\sigma\lambda_1}{n_{i0}u}J(\phi,\nabla_\perp^2\phi)-\frac{(V_*+V_g)}{u}\partial_\xi\nabla_\perp^2\phi-\left(\frac{Z_dn_{d0}}{n_{i0}u}\omega_{ci}K_d+\frac{2\Omega}{u}K_i\right)\partial_\xi\phi-\frac{B_0\lambda_1}{n_{i0}c}(g+r_0\Omega^2)\partial_\xi\phi+\frac{\omega_{pe}^2\alpha^2v_A^2}{c^2u^2}\partial_\xi\phi=0, \quad (40)$$

where $\omega_{ce}(=eB_0/m_ec)$ is the electron gyrofrequency and $J(\phi,\nabla_\perp^2\phi)=\partial_x\phi\partial_\xi\nabla_\perp^2\phi-\partial_\xi\nabla_\perp^2\phi\partial_x\phi$.

Equation (40) admits both the dipolar vortex as well as vortex street solutions.[12] The latter arise when

$$\left(\frac{Z_dn_{d0}}{n_{i0}u}\omega_{ci}K_d+\frac{2\Omega}{u}K_i\right)+\frac{B_0\lambda_1}{n_{i0}c}(g+r_0\Omega^2)-\frac{v_A^2\alpha^2}{u^2\lambda_e^2}=0, \quad (41)$$

in which case (40) is satisfied by the ansatz

$$\nabla_\perp^2\phi=\frac{4\phi_sK_s}{a_s^2}\exp\left[-\frac{2}{\phi_s}\left(\phi-\frac{u_0B_0}{\mu c}x\right)\right], \quad (42)$$

where $\mu=1+D_B\sigma\lambda_1B_0/(n_{i0}c)$ and ϕ_s, K_s, and a_s are arbitrary constants. The solution of (42) is given by[13,16]

$$\phi=\frac{u_0B_0}{\mu_sc}x+\phi_s\,\ln[2\,\cosh(K_sx)+2(1-a_s^{-2})\,\cos(K_s\xi)], \quad (43)$$

which resembles the Kelvin–Stuart ''cat eyes'' for $a_s^2>1$. On the other hand, when the left-hand side of (41) is larger than zero, (40) admits a dipolar vortex, the profiles of which are similar to that of Larichev and Reznik[12] who reported for the first time the stationary nonlinear solutions of the Charney equation.

V. SUMMARY

In this paper, we have investigated the linear and nonlinear properties of drift-ballooning modes in the presence of a radial electric field and stationary charged dust impurities in nonuniform multicomponent plasmas that have equilibrium density and magnetic field inhomogeneities. It is found that the presence of charged dust grains and radial electric fields introduce a finite divergence of the particle flux involving the $\mathbf{E}\times\mathbf{B}_0$ drift and $Z_d n_{d0}$ as well as the angular plasma rotation, respectively. The appearance of these two new effects contributes to the drift-ballooning mode stability. Furthermore, we have shown that finite amplitude drift-ballooning modes can nonlinearly interact among themselves, the evolution of which is governed by a set of nonlinear partial differential equations in which the vector nonlinearities arising from the nonlinear ion polarization drift, the $\mathbf{E}\times B_0$ convection of the perturbed densities, and the coupling of the parallel electron flow with the perturbed magnetic field play a significant role. The mode coupling equations can be employed to investigate the energy cascading within the ballooning mode spectrum. On the other hand, as an illustration, it has been shown that the newly derived nonlinear ballooning mode equations can be cast in the form of an inhomogeneous second and fourth order differential equations which admit vortex street and double vortex solutions. The latter may constitute a new turbulent state, and an ensemble of randomly distributed ballooning vortices may cause stochastic cross-field transport of charged particles. However, a detailed calculation of the vortex induced transport would require the knowledge of vortex spectra which should be determined by means of statistical methods.

The present paper assumes constant charge on the dust grains and neglects the source and sink terms involving the charging collisions. This is justified so long the characteristic time scales of the fluctuations are shorter than the charging time. On the other hand, when the two time scales are comparable we expect the damping (growth) of the disturbances owing to the dust charge perturbation (ionization/recombination).

In conclusion, we stress that the results of our investigation should be useful in understanding the properties of low-frequency electromagnetic turbulence in nonuniform magnetically confined fusion as well as astrophysical plasmas. The latter are common in cometary tails and in interstellar clouds where a significant fraction of charged dust component is present in the background of an electron-ion plasma.

ACKNOWLEDGMENTS

Two of us (A.M.M. and G.M.) would like to thank the Alexander von Humboldt (AvH) Foundation for financial support and Professor K. Elsässer and Professor M. Y. Yu for their hospitality at Ruhr-Universität Bochum. R.T.F. acknowledges Fundação Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and the Deutscher Akademischer Austauschdienst (DAAD) for a fellowship. This research was also partially supported by the Deutsche Forschungsmeinschaft through the Sonderforschungsbereich ''Physikalische Grundlagen der Niedertemperaturplasmen'' and by the Quaid-i-Azam Univeristy Research Fund & PSF Project No. C-QU/Phy (89).

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