Migration velocity analysis by double path-integral migration

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ABSTRACT

The idea of path-integral imaging is to sum over the migrated images obtained for a set of migration velocity models. Velocities where common-image gathers align horizontally are stationary, thus favoring these images in the overall stack. The overall image forms with no knowledge of the true velocity model. However, the velocity information associated with the final image can be determined in the process. By executing the path-integral imaging twice and weighting one of the stacks with the velocity value, the stationary velocities that produce the final image can then be extracted by a division of the two images. The velocity extraction, interpolation, and smoothing can be done fully automatically, without the need for human interpretation or other intervention. A numerical example demonstrated that quantitative information about the migration velocity model can be determined by double path-integral migration. The so-obtained velocity model can then be used as a starting model for subsequent velocity analysis tools like migration velocity analysis or tomographic methods.

INTRODUCTION

The quality of seismic images of the earth’s interior is strongly dependent on the available velocity model. Keydar (2004) and Landa (2004) propose a path-integral approach to seismic imaging to overcome this dependency on the knowledge of a velocity model. The idea of path-integral imaging is to sum over the migrated images obtained for a set of migration velocity models. Those velocities where common-image gathers align horizontally are stationary, thus favoring these images in the overall stack. The overall image forms with no need to know the true velocity model. This new imaging approach resembles, in a certain way, the principles of the Feynman path integral (Feynman and Hibbs, 1965). The imaging approach allows replacement of the complex optimization problem of estimating an ad-equate migration velocity model by integration along all possible trajectories, that is, by a summation over the images for all possible migration velocity models.

The path-summation method has recently enjoyed renewed attention in seismics. It was used for obtaining approximate waveform solutions to the scalar wave equation (Schlottmann, 1999; Lomax, 1999). The path-summation method constructs an approximate wavefield by summation over the contributions of elementary signals propagated along a representative sample of all possible paths between the source and observation points.

Keydar (2004) applies the technique to inversion by homeomorphic imaging, which is based on an NMO-correction formula represented as a function of certain wavefront parameters (radii of curvature and emergence angle), similar to the common-reflection-surface (CRS) method (see, e.g., Hertweck et al., 2007). By changing the wavefront parameters, the NMO time correction curve changes its position. Instead of determining optimal stacking parameters, Keydar (2004) proposes to sum along all possible NMO time curves. Because of stationarity principles, the prevailing contributions are from only those time curves that are nearly in phase. Contributions from summing along the remaining NMO curves cancel each other because the phase oscillates rapidly between positive and negative values.

Landa (2004) extends the idea to time migration. By analogy to the use of Feynman’s path integrals in waveform modeling, he proposed to obtain the subsurface seismic image by a summation of seismic signals over a representative sample of all possible paths/trajectories between a source and observation point. In his approach, the velocity model is assumed to be unknown and the summation trajectories are defined in the time (data) domain rather than in the depth (model) domain. For zero-offset (poststack) migration, the path-summation imaging consists of a summation of seismic prestack data along all possible stacking hyperbolas instead of only along a subset, corresponding to highest coherency criteria (e.g., semblance) in the conventional zero-offset imaging (stack, multifocusing, CRS). For full prestack time migration (PSTM), path-sum-mation imaging consists of a summation of elementary signals over all possible diffraction curves instead of only along a subset, corre-
sponding to the chosen migration velocity. The constructive and destructive interference of the elementary signals contributed by each path/trajectory produces an image that converges toward the correct one obtained by a migration using the correct velocity. Landa et al. (2005) and Shtivelman and Keydar (2005) present the first applications of path-summation imaging in depth migration. Anikiev et al. (2007) use a similar idea to locate seismic events in an unknown velocity field.

Landa et al. (2006) discuss path-summation imaging in more conceptual and theoretical detail. They stress that there are three essential conditions for path-summation imaging to be successful.

1) The argument of the path integral is chosen adequately.
2) The integration is carried out over a representative sample of all possible trajectories.
3) A properly designed weight function is applied in the multipath summation.

The first condition takes care that the path-integral description actually does what it is supposed to do, i.e., enhance stationary contributions and cancel anything else. The second condition guarantees the completeness of the model space, making sure that interference actually can do its job, constructively enhancing desired features of the image and destructively reducing undesired ones. The third condition makes up for deficiencies in the second one because no matter how fine we sample the complete continuous model space, there is no way of covering it completely. Keydar et al. (2008) discuss a successful weight function.

The beauty of the multipath summation method is that it eliminates the need to construct a migration velocity model before imaging. The multipath stack itself takes care of enhancing the true image as the only one that interferes constructively with images from slightly perturbed models. However, this very beauty turns into a drawback when the actual velocity model associated with the resulting image is needed, as is the case in many seismic applications. In this paper, we show how the multipath summation can be modified to extract a meaningful velocity model together with the final image. By executing the path-integral imaging twice and weighting one of the stacks with the used velocity value, the stationary velocities that produce the final image can then be extracted by a division of the two images. A numerical example demonstrates that information about the migration velocity can be extracted successfully from path-integral migration. A related idea using the weight functions of path-integral imaging for velocity estimation was recently presented by Sh-tivelman and Keydar (2008).

MULTIPATH-SUMMATION TIME MIGRATION

In the notation of Landa et al. (2006), the multipath-summation time-migration operator can be written as

\[ V_W(x) = \int d\alpha \int d\xi \int dt U(t, \xi) \delta(t - t_\delta(\xi, x, \alpha)), \]

where \( V_W \) is the resulting time-migrated image at an image point with coordinates \( x = (x, \tau) \), \( x \) being lateral distance and \( \tau \) vertical time. In integral 1, \( U(t, \xi) \) denotes a seismic trace at coordinate \( \xi \) in the seismic data, and \( t_\delta(\xi, x, \alpha) \) is a set of stacking surfaces corresponding to a set of possible velocity models \( \alpha \). Note that generally, \( \alpha \) is a set of parameters defining the particular velocity entering into the path-integral. For simplicity, we assume \( \alpha \) to directly represent the migration velocity. Note also that \( \alpha \) can be a function of the position \( x \) of the image point, i.e., \( \alpha = \alpha(x) \) because integral 1 is carried out for each image point \( x \) independently. The integration is weighted by a weight function \( w(x, \alpha) \), which is designed to attenuate contributions from unlikely trajectories and emphasize contributions from trajectories close to the optimal one. There are several possible choices for \( w(x, \alpha) \). We opt for an exponential weight function of the form

\[ w(x, \alpha) = \exp[-P(x, \alpha)/\sigma^2], \]

where \( P(x, \alpha) \) is the squared average of the absolute value of the local event slopes in the common-image gather (CIG) at \( x \). The local event slopes are estimated using corrected least-square plane-wave filters as described in Schleicher et al. (2009). Parameter \( \sigma \) adjusts the half-width of the Gaussian bell function away from the desired events with \( P = 0 \). In our implementation, we chose \( \sigma = 0.1 \Delta \tau / \Delta x \), where \( \Delta \tau \) and \( \Delta x \) are the temporal and spatial sampling intervals. Because weight 2 is a real-valued exponential weight function, the path integral in our implementation is of the Einstein-Smoluchovsky type (Landa et al., 2006). This choice of the weight function guarantees condition 3 is satisfied.

According to Laplace’s method (see, e.g., Erdélyi, 1956), integrals of the form of equation 1 with an exponential weight of the type of equation 2 have their main contribution from the stationary value \( \alpha_0 \) at which the function in the exponent has its maximum value. Clearly, in our case, the maximum value is reached at \( P = 0 \). Hence, a single, isolated stationary value \( \alpha_0 \) corresponds to the best possible migration velocity in the sense of flattened image gathers. Using Laplace’s method, integral 1 can be asymptotically evaluated to yield

\[ V_W(x) \approx \sqrt{\frac{2 \pi \sigma^2}{P''(\alpha_0)} Q_0(x; \alpha_0)}, \]

where \( P''(\alpha_0) \) denotes the second derivative of the squared local slope mean \( P \) as a function of the varying migration velocity \( \alpha \). Moreover, \( Q_0(x, \alpha_0) \) denotes the desired migration result with the stationary migration velocity \( \alpha_0 \) (see also Landa et al. 2006), viz.,

\[ Q_0(x, \alpha_0) = \int d\xi \int dt U(t, \xi) \delta(t - t_\delta(\xi, x, \alpha_0)). \]

Equation 3 justifies the claim that the result of a multipath summation produces a migrated image. In fact, the result of multipath summation is directly proportional to the desired migration result.

Double multipath summation

The observation that the summation result is proportional to the desired image has an important consequence. It implies that the use of a slightly modified weight function,

\[ \tilde{w}(x, \alpha) = \alpha \exp(-P(x, \alpha)/\sigma^2), \]

will lead to a slightly modified migration result,

\[ \tilde{V}_W(x) = \alpha_0 \sqrt{\frac{2 \pi \sigma^2}{P''(\alpha_0)} Q_0(x; \alpha_0)}. \]

In other words, results 3 and 6 differ only by a constant factor; this factor is the stationary migration velocity at \( x \).
This readily suggests that, wherever the image $V^n(x)$ is nonzero, the migration velocity can be extracted from such a procedure by simply dividing the two migration results 3 and 6, i.e.,

$$\alpha_0(x) = \frac{\bar{V}^n(x)}{V^n(x)}$$  \hspace{1cm} (7)

In fact, this idea of extracting quantities from multiple stacks has been previously discussed in the framework of Kirchhoff migration (Bleistein, 1987; Tygel et al., 1993).

Of course, because the value in the denominator will vanish away from actual reflector images, division by zero must be avoided. Many techniques exist for dividing by quantities that may be close to zero. Possible ideas for avoiding division by zero include the addition of a stabilization parameter to the denominator or masking the division so that it is carried out only at points where the denominator exceeds a certain threshold value.

Keep in mind that there may be points in the medium where the same or different reflections become horizontal in the image gather at different velocities. At such points, there may exist multiple stationary values $\alpha_0$ or no stationary value at all. In this case, the double-stack technique will not be able to extract any of these velocities correctly but will provide some weighted mean. Thus, in complex media, the resulting velocity model should be considered as an initial model for more sophisticated velocity analysis tools.

**NUMERICAL EXAMPLES**

We have applied the above technique of velocity model building to the Marmousoft data (Billette et al., 2003). These data were constructed by Born modeling in a smoothed version of the Marmousi model. The true Marmousoft velocity model is depicted in Figure 1.

We carried out a multipath-summation time migration using constant migration velocities between 1.4 km/s and 4.2 km/s in intervals of 25 m/s. This velocity sampling is sufficiently dense to satisfy condition 2 from the introduction. The resulting stacked migrated image is shown in Figure 2. The multipath-summation approach produces a very nice image that exhibits the main structures of the Marmousi model, even though the central part of the image is not perfectly recovered. This is caused by the intrinsic limitations of time migration rather than those of multipath summation. In this region, the image gathers could not be flattened by time migration. Thus, there is no stationary point in integral 1, i.e., no constructive interference to form an image.

Simultaneously, we carried out a second multipath-summation time migration using the same velocity values. It differed from the first one only by the use of the migration velocity as an additional weight factor in the stack. The resulting migrated image is shown in Figure 3. It looks quite similar to the unweighted stack result. The only difference is the increasing amplitudes with depth in comparison to Figure 2, indicating the increasing velocities that the amplitudes of Figure 3 carry. As indicated by the color bar, the migrated amplitudes are in the range of seismic velocities.

The division of the images of Figures 2 and 3 results in a migration velocity model. Figure 4 shows the result when the division is stabilized by adding a fraction of the maximum amplitude to the denominator. The overall trend of the velocity is nicely recovered, thus indicating that the velocity extraction by double multipath summation can actually work. However, the velocity model is rather unstable, with many image points where unreliable and wrong velocities have been extracted. These velocities already indicate the necessity to postprocess this velocity model in order to extract only the meaningful velocities.

The most obvious way to evaluate the quality of a time-migration velocity model is to use it for time migration. As shown in Figure 5, the velocity model obtained with the stabilized division (with a rather small $\varepsilon \times \text{sgn } V^n$ added to the denominator of equation 7) does not lead to an acceptable migrated image. The bad image quality indicates that the stabilization produces ungeologically low velocities.

![Figure 1. Marmousoft velocity model.](image1)

![Figure 2. Result of multipath-summation time imaging.](image2)

![Figure 3. Result of multipath-summation time imaging with additional velocity weight.](image3)
time migration cannot deal with, which are attributed to many locations in the model. Tests with different values for the stabilization parameter did not help to improve the image.

Because nongeological values must not be allowed in the final velocity model, an obvious idea is to simply avoid division where the absolute value of the denominator is too small (below 0.5% of the peak amplitude in a 250 ms window) and discard velocities from outside the range of velocities that were actually used for the multi-path migration. At all these image points, the velocity is set to zero. Figure 6 shows the result of such a masked division, where zero is attributed to the velocity model wherever the denominator is too small to allow for a division or where unacceptable velocity values result from the division. This eliminates the incorrect velocity values but replaces them by zeroes, thus creating the need for velocity interpolation.

This clearly can be seen from the resulting time migration using this masked velocity model (see Figure 7). In this case, the migration was actually carried out only for those grid points where the velocity is different from zero. The migrated image nicely focuses the reflectors in the less complex areas; however, this creates some holes in the more complex parts.

A simple fill of the missing velocity values by the nearest nonzero neighbor leads to the velocity model shown in Figure 8. This velocity model no longer contains any zeros, but it still is not smooth enough to be acceptable as a time migration velocity model. This can be confirmed from the resulting time-migrated image (Figure 9). The holes in the image have been filled, leading to a complete image. However, the lack of focus in the central part of the image indicates that there is still room for further improvement.

Because a time-migration velocity model theoretically consists of rms velocities, it is supposed to be smooth. Rather than smoothing the model in Figure 8, we chose to directly smooth the masked model of Figure 6, testing two kinds of smoothing techniques.

First, we applied moving-average smoothing using a window in which zero values of the velocity were ignored. It turned out that passing a smaller window several times yields a more reliable result than passing a larger window only once. Figure 10 depicts the resulting smooth velocity model after four passes of a smoothing window...
of 25 traces by 17 time samples. Note that the resulting velocity model closely resembles the model constructed with image-wave propagation in the image gather (Schleicher et al., 2008).

The smoothed model considerably increases the image quality in the complex bottom and center parts (Figure 11). While the image is still not perfect in this region, this problem should be attributed to the general limitations of time migration in geologically complex areas rather than taking it as an indication of a poor velocity model.

Another way of obtaining a reasonably smooth time-migration velocity model from the masked velocities in Figure 6 is by B-splines interpolation. In this technique, B-splines coefficients on a regular grid are estimated by regularized least squares using all the available velocity information, neglecting the gaps. The resulting velocity model for a moderate regularization is shown in Figure 12. The velocity model is similar to the one obtained with moving-average smoothing (Figure 10). The same applies to the time-migrated image (Figure 13). It is hard to spot significant differences between the two migrated images in Figures 11 and 13. Most of the slight differences that do exist occur in the center part of the model, where the
geology is so complicated that time migration realistically cannot be expected to correctly position the reflectors. Note that the time-migrated images in Figures 11 and 13 are very similar to the multipath image of Figure 2, indicating that the extracted velocity model is consistent with the multipath method.

Common-image gathers allow for a more detailed evaluation of the quality of the migration velocity model. Figure 14 shows six common-image gathers at positions 3000 m to 8000 m at every 1000 m in the moving-average smoothed model of Figure 10. These gathers are nicely flattened in the more regular parts of the model. The method only has difficulties in flattening the gathers in the central part of the model, where the geologic complexity is effectively prohibitive for any kind of time migration. For comparison, Figure 15 shows the corresponding image gathers as obtained with the B-splines model of Figure 12. Even in the image gathers, it is very difficult to see differences between the two results.

**Implementational aspects**

Let us now discuss some implementational aspects of double multipath migration. First of all, the computational cost of double multipath migration is only slightly higher than for a single multipath migration. All that is needed is the multiplication of the migrated image by the present velocity, a summation into a second, velocity-weighted image, and a division of the final results at each point in the image. The computationally most expensive part, the time migration for each of the chosen velocities, is done only once. The computational cost of a single multipath migration is, of course, \( N_v \) times the cost of a single time migration, where \( N_v \) is the number of velocities used. Note, however, that these time migrations are completely independent of each other, making the process fully parallelizable.

The memory requirements of double multipath migration are also only slightly larger than those of a single multipath migration. The full prestack migrated volume needs to be saved only once. All addi-

![Figure 13](image13.png)

Figure 13. Time migration using velocities extracted by masked division plus B-splines smoothing.

![Figure 14](image14.png)

Figure 14. Common-image gathers from time migration using velocities extracted by masked division plus moving-average smoothing: (a) \( x = 3000 \) m, (b) \( x = 4000 \) m, (c) \( x = 5000 \) m, (d) \( x = 6000 \) m, (e) \( x = 7000 \) m, (f) \( x = 8000 \) m.

![Figure 15](image15.png)

Figure 15. Common-image gathers from time migration using velocities extracted by masked division plus B-splines smoothing: (a) \( x = 3000 \) m, (b) \( x = 4000 \) m, (c) \( x = 5000 \) m, (d) \( x = 6000 \) m, (e) \( x = 7000 \) m, (f) \( x = 8000 \) m.
tional fields needed for double multipath migration have the dimensions of the final stacked image. The memory requirements of single multipath migration are of the order of a conventional prestack migration.

The total cost of the proposed velocity analysis is just the one of double multipath migration. The velocity extraction, interpolation, and smoothing can be done fully automatically, without the need of human interpretation or other intervention. This makes it highly advantageous over conventional velocity-analysis techniques which strongly rely on human interaction.

CONCLUSIONS

The idea of path-integral imaging is to sum over the migrated images obtained for a set of migration velocities. Those velocities where common-image gathers align horizontally are stationary, thus favoring these images in the overall stack. Other CIGs cancel each other in the final stack. An exponential weight function using the event slopes in the CIGs helps enhance the constructive interference and reduce undesired events that might not be completely canceled by destructive interference.

Evaluation of the resulting path integral with Laplace’s method demonstrates that the resulting image is proportional to the image that would be obtained with the correct velocity model. By executing the path-integral imaging a second time with a modified weight function including the migration velocity as an additional factor, an additional image is obtained in which the amplitudes are proportional to the stationary values of the migration velocity. Thus, these stationary velocities that produce the final image can then be extracted by a division of the two images. We have demonstrated with a numerical example that meaningful information about the migration velocity can be extracted from such a double path-integral migration.

Because multipath-summation imaging does not rely on any kind of interpretation, this technique allows for the fully automated construction of a first time-migrated image together with a first time-migration velocity model. This model can then be used as a starting model for subsequent velocity analysis tools like migration velocity analysis or tomographic methods.

It is to be stressed that the proposed velocity extraction technique does not compromise the velocity-independent philosophy. The foremost result of multipath migration continues to be the stacked velocity-independent image. However, with just a few extra operations, a velocity model is obtained as an automatic by-product of the method. While one single velocity model might not be representative of all possible good models, in practice one usually needs to select a single model to proceed with analysis and interpretation. We have shown that the automatic velocity model obtained from multipath imaging is consistent with the resulting image. Moreover, the uncertainty of this model can be assessed using velocity spectra that can be calculated from the image gathers for the full range of velocities.

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