Measurement of superluminal optical tunneling times in double-barrier photonic band gaps

S. Longhi and P. Laporta
Istituto Nazionale per la Fisica della Materia, Dipartimento di Fisica, Politecnico di Milano, Piazza Leonardo da Vinci 32, I-20133
Milan, Italy

M. Belmonte
Corning-Optical Technologies Italia S.p.A., Viale Sarca 222, 20126 Milan, Italy

E. Recami
Facoltà di Ingegneria, Università Statale di Bergamo, Dalmine, Bergamo, Italy;
INFN—Sezione di Milano, Milan, Italy;
and CCS, UNICAMP, Campinas, São Paulo, Brazil
(Received 23 October 2001; published 2 April 2002)

Tunneling of optical pulses at 1.5 \( \mu \)m wavelength through double-barrier periodic fiber Bragg gratings is experimentally investigated in this paper. Tunneling time measurements as a function of the barrier distance show that, far from resonances of the structure, the transit time is paradoxically short—implying superluminal propagation—and almost independent of the barrier distance. This result is in agreement with theoretical predictions based on phase-time analysis and provides, in the optical context, an experimental evidence of the analogous phenomenon in quantum mechanics of nonresonant superluminal tunneling of particles across two successive potential barriers.

DOI: 10.1103/PhysRevE.65.046610 PACS number(s): 42.50.-p, 03.65.Xp, 42.70.Qs

I. INTRODUCTION

Tunneling of a particle through a potential barrier is one of the most intriguing phenomena in quantum mechanics that continues to attract a great attention both theoretically and experimentally. The reason thereof stems from the fact that some questions related to the dynamics of tunneling, such as, the definition and measure of tunneling transit times [1–3], have not got a general acceptance yet. In addition, tunneling hides some amazing time-domain phenomena, the most notable one being superluminal propagation (for related experiments see, e.g., [4,5] and references therein). In case of opaque barriers, it is known that the tunneling time becomes independent of the barrier width (Hartman effect [6]; see also [2]) and can become so short to imply apparent superluminal motion, which has been the subject of a lively debate in recent years [4,5,7,8]. Since tunneling time measurements for electrons are usually difficult to achieve and of uncertain interpretation, experimental validation of the Hartman effect and direct measurements of superluminal tunneling times have been successfully obtained for the closely related problem of tunneling of photons through photonic barriers in a series of famous experiments performed at either microwave [9,10] or optical wavelengths [11–13]. In particular, in Refs. [11,12] one-dimensional photonic band gaps were used as photonic barriers, realizing the optical analog of electron Bragg scattering in the Kröning-Penney model of solid-state physics. A different, but related, issue is that of particle tunneling through a double-barrier (DB) potential structure, such as, electron tunneling in semiconductor superlattice structures. In this case, the resonant behavior of tunneling escape versus energy [14] is a clear manifestation of the wave nature of electrons and is of major importance for ultrahigh-speed resonant-tunneling devices [15]. Exploiting the analogy between electron and photon tunneling [16,17], resonant-tunneling phenomena have been also studied and observed in connection with microwave propagation in undersized waveguides and in periodic layered structures [8,9,18], and general relations have been derived between the traversal time at resonances and the lifetime of the resonant states [18]. Besides realizing resonance tunneling, it has been theoretically recognized that DB structures are also of interest to study off-resonance tunneling times [8,9,18,19]. In this case, for opaque barriers it turns out that the transit time to traverse the DB structure is independent not only of the barrier width, but even of the length of the intermediate (classically allowed) region that separates the barriers. So far, measurements of superluminal tunneling in DB structures have been performed by Nimtz and coworkers in a series of microwave transmission experiments [9]; however, no experimental study on off-resonant tunneling times in DB photonic structures at optical wavelengths has been reported yet. In recent works [20,21], some of the present authors have shown that fiber Bragg gratings (FBGs) can provide versatile tools for the study of tunneling phenomena. Besides their potential relevance in applications to optical communications, the use of FBGs as photonic barriers is very appealing from an experimental point of view because the tunneling times in FBG structures fall in the tens of picoseconds time scale, which can be easily and precisely detected by standard optoelectronic means.

In this work we report on the measurement of tunneling delay times in DB photonic structures at the 1.5 \( \mu \)m wavelength of optical communications. Our results represent an extension at optical wavelengths of similar experimental achievements previously reported with microwaves [9] and provide a clear experimental evidence that, for opaque barriers, the traversal time is independent of barrier distance
II. OPTICAL TUNNELING IN A DB FBG: BASIC EQUATIONS AND QUANTUM-MECHANICAL ANALOGY

We consider tunneling of optical pulses through a DB photonic structure achieved in a monomode optical fiber by writing onto it two periodic Bragg gratings, each of width $L_0$, separated by a distance $L$, which realize a weak modulation of the refractive index $n$ along the fiber axis $z$ according to $n(z) = n_0[1 + 2V(z)\cos(2\pi z/\Lambda)]$, where $n_0$ is the average refractive index of the structure, $\Lambda$ is the Bragg modulation period, and $V(z)$ is profiled to simulate a symmetric rectangular DB structure, i.e., $V(z) = V_0$ constant for $0 < z < L_0$ and for $L + L_0 < z < L + 2L_0$, and $V(z) = 0$ otherwise (see Fig. 1). For such a structure, Bragg scattering of counterpropagating waves at a frequency $\omega$ close to the Bragg resonance $\omega_B = c_0 \pi/\Lambda(n_0\Lambda)$ ($c_0$ is the speed of light in vacuum) occurs in the grating regions, whereas multiple wave interference between the two barriers leads to Fabry-Perot resonances in the transmission spectrum. The tunneling problem in the DB FBG structure bears a close connection to that of nonrelativistic electrons through a symmetric rectangular DB potential, which has been widely investigated in literature (see, for instance, [14,22]). The analogy is summarized in Table I, where the basic equations and the expressions for barrier transmission and group delay are given in the two cases [17,23]. In the electromagnetic case, a monochromatic field $E(z,t)$ at an optical frequency $\omega$ close to the Bragg frequency $\omega_B$ propagating inside the fiber can be written as a superposition of counterpropagating waves, $E(z,t) = u(z)\exp(-i\omega t + ik_Bz) + v(z)\exp(-i\omega t - ik_Bz) + \text{c.c.}$, where $k_B = \pi/\Lambda$ is the Bragg wave number; for a small index modulation ($V_0 \ll 1$), the envelopes $u,v$ of counterpropagating waves satisfy the following coupled-mode equations [24]:

$$\frac{du}{dz} = i\delta u + ik_B V(z) u,$$

$$\frac{dv}{dz} = -i\delta v - ik_B V(z) v,$$

where $\delta = k - k_B = n_0(\omega - \omega_B)/c_0$ is the detuning parameter between wave number $k = n_0\omega/c_0$ of counterpropagating waves and Bragg wave number $k_B$. The wave envelopes $u$ and $v$ are oscillatory (propagative) in the region $L_0 < z < L_0 + L$, whereas they are exponential (evanescent) inside the gratings when $|\delta| < k_B V_0$. The spectral transmission of the structure, given by $t(\omega) = \{u(L)/u(0)\}_{|\omega|<\omega_B}$, can be analytically determined by standard transfer matrix methods [24]. As an estimate of the tunneling time for a wave packet crossing the structure, we use the group delay (or phase time) as calculated by the method of stationary phase, which is given by $\tau = \text{Im}[-\omega]\partial \ln T/\partial \omega$, where $T = |t|^2$ is the transmission coefficient.

![FIG. 1. Schematic of tunneling through a rectangular DB photonic structure.](image)

![FIG. 2. Spectral power transmission (left) and group delay (right) for a DB FBG structure for $L_0 = 8.5$ mm, $L = 42$ mm, $V_0 = 0.9 \times 10^{-4}$, $n_0 = 1.452$, and $\omega_B = 1.261 \times 10^{13}$ rad/s. Upper and lower figures refer to measured and predicted spectral curves, respectively.](image)
quency detuning $\nu = (\omega - \omega_B)/(2\pi)$, computed for one of the DB structures used in the experiments, is shown in Fig. 2. Notice that, far from the sharp Fabry-Perot resonances, the group delay is shorter than that for free propagation from input to output planes, implying superluminal propagation. At the center of the band gap ($\delta=0$), simple analytical expressions for the power transmission and group delay can be derived and read

$$T = \frac{1}{\cosh^2(2k_B V_0 L_0)},$$

$$\tau = \tau_1 + \tau_2,$$

where

$$\tau_1 = \frac{n_0}{c_0 k_B V_0} \tanh(2k_B V_0 L_0) = \sqrt{1 - T} \frac{n_0}{c_0 k_B V_0}$$

$$\tau_2 = \frac{n_0 L}{c_0 \cosh(2k_B V_0 L_0)} = \sqrt{1 - T} \frac{n_0 L}{c_0}.$$

Equations (3) and (4) clearly show that two distinct contributions are involved in the expression for the group delay. The former term $\tau_1$ is independent of the barrier separation, and coincides with the tunneling time of a single barrier of width $2L_0$. For an opaque barrier ($L_0 V_0 k_B \gg 1$), $\tau_1$ becomes independent of barrier width and saturates to the value $\tau_1 \sim n_0/(c_0 k_B V_0)$ (Hartman effect). Conversely, the latter contribution $\tau_2$ is always shorter than the free-propagation time over a length $L$ and goes to zero for an opaque barrier, implying that the tunneling time becomes independent of the barrier distance (generalized Hartman effect). Similar results are obtained for off-resonance tunneling of a nonrelativistic electron through a rectangular DB potential $V(z)$ assuming that the incident wave packet has a below-barrier mean energy $E$ half the barrier width $V_0$; the corresponding expressions for barrier transmission and group delay in this case are given in Table I (for details, see, e.g., [22]). The tunneling through a DB FBG structure can hence be used as an experimental verifiable model for the quantum-mechanical case.

III. TUNNELING TIME MEASUREMENTS

We performed a series of tunneling time measurements through DB FBG structures operating at around 1.5 $\mu$m in order to assess the independence of the peak pulse transit times with barrier distance $L$. The FBGs used for the experiments were manufactured by using standard writing techniques, with an exposure time to UV laser beam and phase mask length such as to realize a grating with sharp fall-off edges of length $L_0 \approx 8.5$ mm and with a refractive index modulation $V_0 \approx 0.9 \times 10^{-4}$. For such a refractive index modulation, a minimum power transmission $T \approx 0.8\%$ at antiresonance is achieved for a DB structure, which is low enough to get the opaque barrier limit, but yet large enough to perform time delay measurements at reasonable power levels. The period of the phase mask was chosen to achieve Bragg resonance at around 1550 nm wavelength. Five different DB structures were realized with grating separation $L$ of 18, 27, 35, 42, and 47 mm. For such structures, both transmission spectra and group delays were measured using a phase-shift technique [26] with a spectral resolution of $\approx 2$ pm; an example of measured transmission spectrum and group delay versus frequency for the 42-mm separation DB FBG is shown in Fig. 2. Notice that, according to the theoretical curves shown in the same figure, far from the Fabry-Perot resonances the group delay is superluminal, with expected time advancements of the order of 240–250 ps. Notice also that the sharp Fabry-Perot resonances are not fully resolved in the experimental curves due to bandwidth limitations ($\approx 2$ pm) of the measurement apparatus.

Direct time-domain measurements of tunneling delay times were performed in transmission experiments using probing optical pulse with $\approx 1.3$ ns duration, corresponding to a spectral pulse bandwidth, which is less than the frequency separation of Fabry-Perot resonances for all the five DB structures. Since both transmission and group delay are slowly varying functions of frequency far from Fabry-Perot resonances (see Fig. 2), pulse advancement with a weak pulse-shape distortion is thus expected for off-resonance pulse transmission. The experimental setup for delay time measurements is shown in Fig. 3. A pulse train, at a repetition frequency $f_m = 300$ MHz, was generated by external modulation of a single-frequency continuous-wave tunable laser diode (Santec mod. ECL-200/210), equipped with both a coarse and a fine (thermal) tuning control of frequency emission with a resolution of $\approx 100$ MHz. The fiber-coupled $\approx 10$-mW output power emitted by the laser diode was amplified using a high-power erbium-doped fiber amplifier (IPG Mod. EAD-2-PM; EDFA1 in Fig. 3), and then sent to a LiNbO$_3$-based Mach-Zehnder modulator, sinuously driven at a frequency $f_m = 300$ MHz by a low-noise radiofrequency (rf) synthesizer. The bias point of the modulator and the rf modulation power level were chosen to generate a train with a pulse duration (full width at half-maximum) of $\approx 1.3$ ns; the measured average output power of the pulse train available for the transmission experiments was...
time-domain by a fast sampling oscilloscope comparable. The transmitted pulse train was detected in the antiresonances of the structure or outside the stop band, were the laser emission tuned either at Fabry-Perot resonances or put optical signal at a constant level (1550 nm). The pulse train was sent to the DB FBG through a three-port optical circulator that enables both transmitted and reflected signals to be simultaneously detected. The signal was first measured by the Fabry-Perot interferometer and recorded on a digital oscilloscope by disconnecting the laser diode from the input port of EDFA1 and sending to the DB FBG the broadband amplified spontaneous emission signal which was detected by monitoring the reflected signal, available at port 3 of the optical circulator, using both an optical spectrum analyzer (Anritsu Mod. MS9710B) with a resolution of 0.07 nm, and a plane-plane scanning Fabry-Perot interferometer (Burleigh Model RC1101R) with a free-spectral range of ~50 GHz and a measured finesse of ~180, which permits to resolve the Fabry-Perot resonances of the DB FBG structures. The reflectivity spectrum of the DB FBG was first measured by the Fabry-Perot interferometer and recorded on a digital oscilloscope by disconnecting the laser diode from the input port of EDFA1 and sending to the DB FBG the broadband amplified spontaneous emission signal of the optical amplifier. This trace is then used as a reference to tune the pulse spectrum at the center of the off-resonance plateau between the two central Fabry-Perot resonances of the recorded DB FBG structure. Figure 4 shows a typical trace [curve (1)], averaged over 64 acquisitions, of the tunneled optical pulses under off-resonance tuning condition, as measured on the sampling oscilloscope, for the 42-mm separation DB FBG structure, and compared to the corresponding trace [curve (2)] recorded when the laser was detuned apart by ~200 GHz, i.e., far away from the stop band of the DB FBG structure. A comparison of the two traces clearly shows that tunneled pulses are almost undistorted with a peak pulse advancement of ~248 ps; repeated measurements showed that the measured pulse peak advancement is accurate within ±15 ps, the main uncertainty in the measurement being determined by the achievement of the optimal tuning condition. We checked that propagation through EDFA2 does not introduce any appreciable pulse distortion nor any measurable time delay dependence on the amplification level. Time delay measurements were repeated for the five DB FBG structures, and the experimental results are summarized in Fig. 5 and compared with the theoretical predictions of tunneling time as given by Eqs. (3) and (4). The dashed line in the figure shows the theoretical transit time, from input (z = 0) to output (z = L + 2L0) planes, versus barrier separation L for pulses tuned far away from the band gap of FBG; in this case, the transit time is given merely by the time spent by a pulse traveling along the fiber for a distance L + 2L0 with a velocity c0/n0. The solid line is, in turn, the expected transit time for off-resonance tunneling of pulses, according to the phase-time analysis [see Eqs. (3) and (4)], which shows that the transit time does not substantially increase as the barrier separation is increased (generalized Hartman effect). The points in the figure are obtained by subtracting to the dashed curve the measured pulse peak advancements for the five DB FBGs, thus providing an experimental estimate of the tunneling transit time. Notice that, within the experimental errors, the agreement between the measured and predicted transit times is rather satisfactory. For each of the five DB FBG structures, the measured transit times are superluminal; it is remarkable that, for the longest barrier separation used, the transit time leads to a superluminal velocity of about 5c0, the largest one measured in tunneling experiments at optical wavelengths [27].

These paradoxically small transit times can be qualitatively explained as the result of two simultaneous effects that are a signature of the wave nature of the tunneling processes. On the one hand, following Refs. [2,11,12], peak pulse advancement occurs at each of the two barriers as a result of a reshaping phenomenon in which the trailing edge of the pulse is preferentially attenuated than the leading one; on the other hand, the independence of the tunneling time on the barrier distance can be explained, following Ref. [19], as an

![Figure 4](image1.png)  
**FIG. 4.** Pulse traces recorded on the sampling oscilloscope corresponding to the transmitted pulse for off-resonance tunneling (curve 1) and reference pulse propagating outside the stop band of the structure (curve 2) for the 42-mm separation DB FBG.

![Figure 5](image2.png)  
**FIG. 5.** Off-resonance tunneling time versus barrier separation L for a rectangular symmetric DB FBG structure. The solid line is the theoretical prediction based on group delay calculations (Table I); dots are the experimental points as obtained by time delay measurements; the dashed curve is the transit time from input (z = 0) to output (z = L + 2L0) planes for a pulse tuned far away from the stopband of the FBGs.
effective “acceleration” of the forward traveling waves in the intermediate classically allowed region that arises in consequence of the destructive interference between the two barriers. Though the observation of superluminal tunneling times does not constitute any violation of the naive Einstein’s causality [28], it is nevertheless remarkable that the reshaping phenomenon that occurs inside the FBG structure leads to an undistorted replica, albeit attenuated, of the original pulse shape.

IV. CONCLUSIONS

In this paper off-resonant tunneling of optical pulses has been experimentally investigated in fiber Bragg photonic barriers. Tunneling time measurements have been shown to be in good agreement with theoretical predictions based on phase-time analysis and have confirmed that, for opaque barriers, the tunneling time is independent not only of the barrier width, as previously shown in Ref. [12], but even of the barrier separation. Our results extend to the optical region previous experimental achievements performed with microwaves [9], and may be of interest in the field of optical tunneling and the related issue of superluminal propagation. We also envisage that the experimental demonstration of superluminal off-resonance tunneling in FBG structures may be of potential interest in optoelectronic applications whenever a precise control of the group delay of an optical signal is required (see, e.g., [29]).

ACKNOWLEDGMENTS

The authors acknowledge M. Marano for his help in the experimental measurements, and F. Fontana and G. Salesi for many useful discussions.

[14] See, for example, E. Merzbaecher, Quantum Mechanics (Wiley, New York, 1970), Chaps. 6 and 7.
[23] The analogy between tunneling of electrons and photons in superlattice structures and closed-form solutions for the tunneling times have been recently reported, in the general case, in P. Pereyra, Phys. Rev. Lett. 84, 1772 (2000). The derivation of photon tunneling times presented here follows a different and simpler approach, based on coupled-mode equations for Bragg scattering, which is suited for FBG structures.
[25] Among others, the phase time has been shown to be consistent with tunneling time measurements in previous experiments on optical tunneling through photonic bandgaps [11,12].
[27] We mention that superluminal group velocities as large as ~10c0 were observed by G. Nimtz and co-workers, but in microwave transmission experiments.
[28] We remark that the front velocity of a discontinuous step-wise signal that propagates through the DB FBG structure is always equal to c0, so that there is no violation of Einstein’s causality.
in our experimental findings. This circumstance follows technically from the analytic properties of spectral transmission $t(\omega)$ and has been discussed in Ref. [21]. It should nevertheless be pointed out that the existence and the physical relevance of a "true" discontinuous wave front, invoked to prove that superluminal signal propagation is not possible, has been the subject of controversial debate; for a general discussion on this point we refer the reader to Ref. [4] and to G. Nimtz, Eur. Phys. J. B 7, 523 (1999).