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Faculdade de Ciências Aplicadas

TAMIRES MARTINS CORRÊA

Analysis and Modeling of a Cutting and Scheduling Industrial Problem

Análise e Modelagem de um Problema Industrial de Corte de Estoque e Programação

Limeira

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Dissertação apresentada à Faculdade de Ciências Aplicadas da Universidade Estadual de Campinas como parte dos requisitos exigidos para a obtenção do título de Mestre em Engenharia de Produção e de Manufatura, na área de Pesquisa Operacional e Gestão de Processos.

Dissertation presented to the Institute of Mathematics, Statistics and Scientific Computing of the University of Campinas in partial fulfillment of the requirements for the degree of Master in Production and Manufacturing Engineering, in the area of Operational Research and Process Management.

Supervisor: Profª. Drª. Priscila Cristina Berbert Rampazzo

Este exemplar corresponde à versão final da Dissertação defendida pela aluna TAMIRES MARTINS CORRÊA e orientada pela Profª. Drª. Priscila Cristina Berbert Rampazzo.

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Resumo

Os problemas de corte de estoque e sequenciamento de produção representam um desafio na fabricação e logística modernas. Esses problemas envolvem determinar a maneira mais eficiente de cortar matérias-primas em pedaços menores para atender demandas específicas e, ao mesmo tempo, minimizar desperdícios, tempo de preparação e atrasos. Esta dissertação baseia-se num problema real de produção de uma indústria farmacêutica, em particular na produção de bobinas para *blisters*. Um modelo é proposto e diferentes metodologias foram exploradas para abordá-lo. A pesquisa se dedica a fornecer uma resolução eficaz usando uma Heurística FFD (*First Fit Decreasing*) e um Algoritmo Genético alinhado aos desafios únicos enfrentados pelos tomadores de decisão e às complexidades do ambiente de produção.

Palavras-chave: Problemas de corte de estoque. Problemas de sequenciamento de produção. Minimização de resíduos. Fabricação. Eficiência. Heurística FFD. Algoritmo genético.

Abstract

Cutting Stock and Scheduling Problems represent a challenge in modern manufacturing and logistics. These problems involve determining the most efficient way to cut raw materials into smaller pieces to meet specific demands while minimizing waste, setup time and delays. This dissertation is based on a real world manufacturing problem in the pharmaceutical industry, in particular to produce coils for blister packs. A model is proposed and different methodologies were explored to address it comprehensively. The research is dedicated to delivering an effective resolution using a FFD (First Fit Decreasing) Heuristic and a Genetic Algorithm aligned with the unique challenges faced by decision-makers and the intricacies of the production environment.

Keywords: Cutting Stock Problems. Scheduling Problems. Waste minimization. Manufacturing. Efficiency. FFD Heuristic. Genetic Algorithm.

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List of abbreviations and acronyms

CAM	Computer Aided Manufacturing
CSP	Cutting Stock Problem
EOQ	Economic Order Quantity
EDD	Earliest Due Date
FFD	First Fit Decreasing
GA	Genetic Algorithm
GRH	Greedy Rounding Heuristic
ICSSP	Integrated Cutting Stock and Scheduling Problem
ICSP	Industrial Cutting and Scheduling Problem
ILP	Integer Linear Programming
LSP	Lot Sizing Problem
MILP	Mixed Integer Linear Programming
MLCR	Multiple Level Constrained Resources
MLUR	Multiple Level Unconstrained Resources
MST	Minimum Setup Time
OR	Operations Research
PE	Polyethylene
PET	Polyester
PP	Polypropylene
PS	Polystyrene
PVC	Polyvinyl Chloride
PVDC	Polyvinylidene Chloride
SLCR	Single Level Constrained Resources
SLUR	Single Level Unconstrained Resources
1DCSP	One Dimensional Cutting Stock Problem

Contents

Introduction

Operations research is a powerful and versatile research area that plays a pivotal role in various sectors by improving decision-making processes, resource utilization and overall efficiency. Its importance continues to grow as organizations seek to adapt to complex, data-driven environments and make better-informed choices. The founders of Operations Research in World War II not only solved problems under difficult conditions but also succeeded in articulated what they had done. A simple definition of OR is "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control" (LITTLE, 1991). We live today in a world that technological and social change are rapid. The current production processes are increasingly complex and extensive. This evolution in the way products must be manufactured, the demands of regulatory agencies and the pressure for competitive prices in the market, lead industries to seek methods that eliminate unnecessary losses and internalize production stages. This environment creates a lot of opportunities for analysis and models in OR area.

In industrial context, problems involving the sequencing of production and cutting of materials are extremely relevant for the manufacture of raw materials. However, to solve problems of this type, a high degree of understanding of the process and related variables is necessary, as well as sensitivity in the use of algorithms and techniques to model the problem. A cutting stock problem is a combinatorial optimization challenge commonly encountered in manufacturing and inventory management. The objective is to find the most efficient way to cut raw materials, such as rolls of paper, metal sheets, or lumber, into smaller pieces to meet a set of demand requirements while minimizing waste. Scheduling problems refers to a task of organizing and arranging activities, tasks, or resources in a specific order or sequence, typically within a constrained environment. These constraints could involve time limitations, resource availability, precedence relationships (where one task must happen before another), or other logistical considerations. The aim of solving a scheduling problem is to optimize the allocation of resources (such as time, machinery, or personnel) to complete tasks efficiently, meet deadlines, minimize costs, or maximize productivity. Various types of scheduling problems exist across different fields, including project management, manufacturing, transportation, and computer science, each with its unique set of constraints and objectives. Many researches around this topic proposes different approaches to solve it, that will be described and different references will be presented in next chapters. An integration of this two kind of problems is called "Integrated Lot Sizing Problems" which involves optimizing production and inventory decisions simultaneously, taking into account multiple items or products within a single system. The primary objective is to determine the optimal production quantities, schedules, and inventory levels for a range of items, considering factors like setup costs, holding costs, and capacity constraints.

Motivated by this scenario and challenge, this project aims to propose a model for a cutting stock problem integrated with a lot sizing problem. At the end of the study, the expected output is to develop an adequate model, defining and analyzing the best production sequencing with the lowest material losses, which can serve as a basis for application in different production processes. Indeed, this dissertation aims to seamlessly fuse practical insights from production routines with a potent theoretical framework, offering diverse avenues for customization to decision-makers. In essence, it will allow the analysis of different programming horizons and the inclusion of specific parameters related to cutting machines.

Objectives

Challenges related to material cutting are frequently faced within the packaging industry. From a business perspective, the packaging industry is progressively evolving into a vital and integrated element for success in the marketplace. The main end use sectors are food and beverage, followed by health care and cosmetics; and main materials used are paper & board, plastics, metal, glass and wood (OLSMATS; KAIVO-OJA, 2014).

Specifically about plastic films, today we have many different polymers being used: PE (polyethylene), PP (polypropylene), PET (polyester), and PS (polystyrene). They are commonly used in monolayer format, but they are also used in multilayers films produced by coextrusion and (or) lamination processes. These films are broadly used for chemicals, consumer goods and pharmaceutical products. Some characteristics as lightweight, non-corrosive and the ability to preserve products for a long period, make them a popular choice and highly applicable for packaging (WAGNER; MARKS, 2016). The implementation of FDA (Food and Drug Administration, federal agency) regulations in the US has increased the sales of blister packs, a type of package that usually use plastic films sealed with paper or aluminium, over the years. This increment has been remarked all over the world, considering the growing incidence of diseases and the concern for health and wellness, in other words, the need for pharmaceuticals.

This dissertation is based on a real manufacturing problem in a pharmaceutical industry, in particular to produce coils for blister packs. The challenge is to obtain a model that can reflect the specifics of this process, from the cutting patterns of large coils to the most suitable programming window. Furthermore, the goal is also to contribute to existing research by introducing an additional approach and problem-solving method for this type of issue, showcasing a real scenario where theory and concepts previously studied by other authors align with this work.

Dissertation structure

This dissertation is organized as follows:

- Capter 1: Theoretical References;
- Capter 2: Integrated Cutting Stock and Scheduling Problem Description;
- Capter 3: Resolution Methodology for Packaging ICSSP;
- Capter 4: Experiments and Results;
- Capter 5: Conclusion and Future Perspectives.

The first chapter provides a theoretical approach to the problem classes separately, with bibliographic references and explanations of key concepts. In the second chapter, the integrated problem and major works in this field are discussed, where the problem's complexity may become apparent. The reader will also gain a better understanding of all the factors considered for solving the packaging problem, including practical aspects of production machines, enabling a comprehensive analysis of the problem's complexity. The third chapter provides a better understanding of the resolution methodology. Finally, results are presented in the forth chapter, and a conclusion addressing the most significant findings, as well as future prospects, is discussed.

1 Theoretical References

1.1 Cutting stock problems (CSP)

The Cutting Stock Problem (CSP) can be described as how to cut out pieces from stock material with minimum loss. Economic utilization of resource material is not only the interest of the industrialist but also of the world (CHENG; FEIRING; CHENG, 1994). Also called a trim-loss problem, they can be categorized by dimension. A one-dimensional problem is one in which only one dimension is significant. Coils, metal tubes, and film rolls are considered one-dimensional problems (Figure 1), while for wooden boards, metal sheets, and fabrics, the problems are called two-dimensional where the two orthogonal directions are significant in the determination of a solution (HINXMAN, 1980). Another probable situation less frequently occurs is the three-dimensional problem. In many real situations, this kind of issue can be standardized as one or two dimensions or can be called a loading problem. A review of solution methods and computational experiments for loading problems can be noticed in Silva, Oliveira e Wäscher (2016). In general, cutting machines receive large pieces of standard sizes cut into smaller pieces of different sizes. In order to avoid unnecessary losses in this cutting process and to maximize the number of items cut in large pieces, an optimization problem arises.

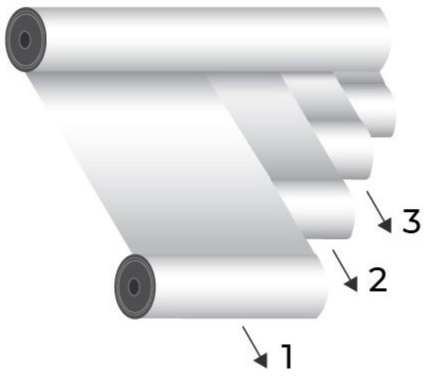

Figure 1 – One-dimensional cut (example)

An important consideration in modeling a cutting stock problem is the transition from input materials (production factors) to the desired output materials (products), as well as the technology involved in this process (DYCKHOFF et al., 1985). Hence, three categories of restrictions result from the model's quantitative properties:

- Input restrictions: limited quantities available and inbound sizes;

- Cutting operation restrictions: the amount of technology is limited by cutting parameters;
- Output restrictions: quantity of output material is restricted due to internal orders or demand.

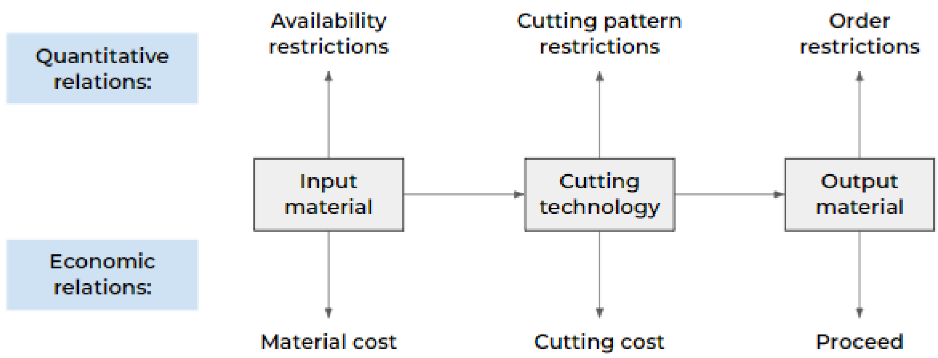

Figure 2 – Structure of the trim loss problem as input/output model. (DYCKHOFF et al., 1985)

Figure 2 provides a general framework for this restriction classification. Restrictions regarding values, such as budgets or minimal profit contributions, can also be considered but are not common in the literature.

Regarding the difficulty in modeling cutting stock problems and their economic importance, many review papers and special editions were published to correlate current models and, if applicable, their industry background. The most important papers are those of Morabito, Arenales e Yanasse (2009), Gomes et al. (2016), Parmar, Prajapati e Dabhi (2014), Cherri et al. (2014), Melega, de Araujo e Jans (2018).

Gilmore and Gomory (GILMORE; GOMORY, 1961) were pioneers in solving cutting stock problems. The Gilmore-Gomory method generates cuts through an iterative process that uses the relaxed fractional solution of the integer linear problem to generate cuts that eliminate fractional solutions. The process is repeated until the entire solution is found. The next pattern to enter the Linear Programming basis could be found by solving an associated knapsack problem. With that, solving the trim loss minimization was possible without first enumerating every feasible cutting pattern. This approach is very important, it can solve one-dimensional problems but also two-dimensions or three-dimensions, considering that many feasible patterns may exist when narrow widths are to be slit from a wide stock roll (HAESSLER; SWEENEY, 1991). After this proposal, other methods and techniques were developed in combinatorial optimization and integer linear programming.

1.1.1 One-dimensional cutting stock problem

From a large width material, or L, the cutting process will produce m item types with sizes $l_1, l_2, ..., l_m$ in varying quantities $b_1, b_2, ..., b_m$.

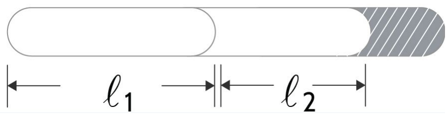

Figure 3 – Cutting pattern

In order to ensure that the process will have the lowest possible losses, it is necessary to combine the different widths of items (Figure 3), also taking into account the required quantities b_i of each of them. A combination of widths is called *cutting pattern.*

To every cutting pattern j, one-dimensional vectors are named $a_j = [a_{1j}, a_{2j}, a_{mj}]$. Thus, a vector $\alpha = (\alpha_1\ \alpha_2\ ...\ \alpha_m)^T$ represents a cutting pattern if and only if the equation below is satisfied:

$$l_1\alpha_1 + l_2\alpha_2 + ... + l_m\alpha_m \leqslant L$$
$$\alpha_1 \geqslant 0, \alpha_2 \geqslant 0, ..., \alpha_m \geqslant 0 \text{ and integers.}$$

Cutting problems that involve an integral condition, like the previous statement, can be solved with the mathematical model proposed by Kantorovich in 1939. He was the first to develop a solution for this type of problem. Subsequently, Gilmore e Gomory (1961) developed a model and a solution method with a more practical approach involving linear programming and variable rounding techniques that will be the basis for this work.

The solution method consists of minimizing the number of cut coils or minimizing losses, considering the cutting patterns. The demands of the items to be cut will represent the constraints. Consider the Figure 5 to exemplify the cutting problem.

A typical one-dimensional cutting stock problem can be defined as explained below. There are, available in stock, a quantity of objects of length equal to L and a set of items, with known demand b_i and lengths l_i, $i = 1, ..., m$. Item lengths cannot be longer than L. The problem is to produce the items from the objects available in stock, meeting the defined demand. Overproduction is prohibited, and all objects must be cut completely (no return to stock).

Although the primary objective of the one-dimensional cutting stock problem is to minimize the total length of used stock rolls, reducing interruptions in the cutting process has become an important factor in some applications (MATSUMOTO; UMETANI; NAGAMOCHI, 2011). In general, the ability to optimize the use of resources, minimize waste, and maximize efficiency makes these problems a valuable tool in various industries.

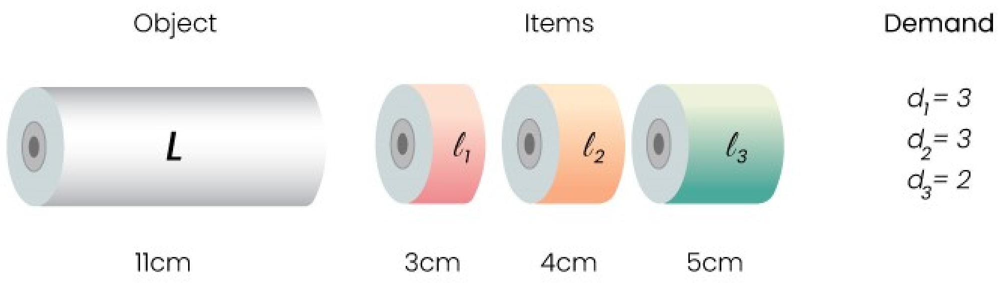

Figure 4 – One-dimensional cutting stock problem

1.1.1.1 Heuristics for the One-dimensional Cutting Problems

The majority of publications in the field of cutting stock problems deals with real-world applications from various industries. It is not common to treat general methodological issues, even with some questions to be answered across the generation of integer solutions. It's not known an exact algorithm that solves medium-size problems instances to optimally. Thus industrial integer cutting stock problems are usually dealt with through heuristics.

According to Poldi e Arenales (2006), one way to solve cutting stock problems is to apply exhaustive repetition heuristics, in other words is building a good cutting pattern and use it as many times as possible. It is called constructive heuristics. The other way is to use a residual approach, where the relaxed problem is solved by column generation, approximate its fractional solution by an integer solution, leaving a problem with less demand, called residual. Gilmore e Gomory (1961) were the first to propose this approach, but they did not present studies in this regard, they only suggested that the approximation should be for the lower integer and the residual problem solved by some ad hoc method. This has been the most used approach in practice.

1.1.1.2 Residual Heuristics

The residual heuristics procedure involves initiating a search for an integer solution to a problem using rounding techniques applied to the relaxed solution. This method allows for the identification of promising integer solutions by leveraging the information obtained from the relaxed solution. By iteratively refining the solution based on the residuals, the residual heuristics procedure can efficiently converge to a high-quality integer solution.

Wäscher e Gau (1996) proposed a residual heuristics that consists in rounding the components of vector $x = (x_1 , x_2 ,..., x_n)$ downwards and solving the residual problem by replacing the demand vector b in the following problem (with $A = [a_1 a_2 ... a_3]$ the matrix organized by columns, $a_j = (\alpha_{1j}\ \alpha_{2j}\ ...\ \alpha_{mj})^T$ represents a cutting pattern) by the residual demand: $r_i = b_i - \sum_{j=1}^{n} a_{ij}\lfloor x_j \rfloor$, $i = 1,..., m$.

$$\text{Minimize} \quad \sum_{j \in J} x_j \tag{1.1}$$

$$\text{subject to} \quad \sum_{j \in J} \alpha_{ij} x_j = b_i \tag{1.2}$$

$$\sum_{i=1}^{m} \alpha_{ij} l_i \leqslant L \tag{1.3}$$

$$x_j \in \mathbb{Z}_+ \tag{1.4}$$

If another fractional solution is encountered during the residual heuristics procedure, the process of rounding down to the nearest integer is repeated to generate a new residual demand. This process continues until all frequencies are reduced to zero. If any items still have non-zero demands, a final residual problem is solved using a suitable heuristic that provides an integer solution to the problem.

An alternative residual approach, proposed by Poldi e Arenales (2010), involves solving a relaxed cutting stock problem at each iteration. The solution vector is then sorted in descending order, and the first frequency is rounded up to the nearest integer. The feasibility of the resulting pattern is then tested to ensure that there are no overproductions of any items. If over productions exist, the frequency is reduced by one unit until they are eliminated, and the process continues to the next frequency pattern. Once the last cutting pattern is examined, the demand and stock are updated, and the residual problem is solved. This rounding procedure is repeated until all demand is supplied. Since at least one cutting pattern is accepted into the solution every time a relaxed problem is solved, the Greedy Rounding Heuristic (GRH) guarantees that every demand will be satisfied in a finite number of iterations.

The basic procedure for Residual Heuristics is presented in the Algorithm 1.

The Greedy Rounding Heuristic (GRH) ensures that every demand will be met in a finite number of iterations. Two alternative versions of the heuristic, GRH2 and GRH3, prioritize the solution vector x based on different criteria. GRH2 sorts the solution vector x in order of patterns with the least amount of waste. This approach aims to minimize the amount of unused material and optimize the use of resources. On the other hand, GRH3 prioritizes patterns in which the solution vector x has the largest fractional part. This approach aims to make the best use of limited resources by maximizing the production of items in each iteration. Overall, these variations of the GRH provide different benefits depending on the specific goals of the problem at hand (CERQUEIRA; AGUIAR; MARQUES, 2021).

Algorithm 1 – Greedy Rounding Heuristic

$b^0 = b$, k = 0;
while $b^k \neq 0$ **do**
  Solve the relaxed problem: (1.1)-(1.4) ;
  Obtain $x^k = (x_1^k, ..., x_m^k)$ and the basic matrix $B^k = [a_1^k, ..., a_m^k]$;
  a_i^k, $i = 1, ..., m$, are the cutting patterns for the solution of problem-k;
  Order the cutting patterns so that: $x_1^k \geqslant ... \geqslant x_m^k$;
  Determine a feasible solution;
  if $(\lceil x_1^k \rceil, 0, ..., 0) is feasible$ **then**
    $y_i^k = \lceil x_1^k \rceil$ **end**
    else
      $y_1^k = \lceil x_1^k \rceil - 1$
      for $i = 2 \dots m$ **do**
        $y_i^k = \lceil x_1^k \rceil$
        $y^k = (y_1^k, \dots, y_{i-1}^k, y_i^k, 0, \dots, 0)$;
      end
    end
    while $B^k y^k > b^k$ *(y^k is not a feasible solution)* **do**
      $y_i^k = y_i^k - 1$;
    end
    Keep B^k *e* y^k;
    $b^{k+1} = b^k - B\, y^k$;
    $k = k + 1$;
  end

1.1.1.3 Constructive Heuristics

One alternative approach to finding an integer solution for a one-dimensional cutting problem is to use constructive heuristics, which involve constructing an efficient cutting pattern and using it repeatedly without producing any excess material. During each iteration of this procedure, the demand for items is updated and the process is repeated until all demand is satisfied. This results in an integer solution for the problem that optimizes the use of resources and minimizes waste.

Constructive heuristics are a valuable tool for solving one-dimensional cutting problems because they provide a practical and efficient method for generating high-quality integer solutions. By constructing cutting patterns that maximize the utilization of resources and minimize waste, these heuristics can help businesses and manufacturers reduce costs and increase profitability.

1.1.1.4 FFD Heuristic

First-Fit algorithms have been extensively explored in numerous studies. Pioneering research in this domain includes works by Ullman (1971), Johnson (1973), Xia e

Tan (2010), and Johnson et al. (1974). A basic definition of First-Fit-Decreasing (FFD) heuristic involves placing the largest item in a cutting pattern as many times as possible until its demand has been met or it no longer fits. The process then continues with the second largest item and so on, until a cutting pattern is constructed using the shortest length item. The second largest item is then placed and so on. When the last item (shortest length) is examined, a cutting pattern is constructed.

To ensure that the heuristic is residual, the demand is updated after each pattern is created, and the process is repeated until all demand has been fulfilled. This approach was chosen for this particular problem because it enables the machine to start production with fewer cutting blades, which can prolong the useful life of the equipment. Additionally, it allows for the efficient processing of larger items with higher added value. Residuals, unlike constructive heuristics, have lower losses and were also chosen to optimize the cutting process as a whole. (LUCIANO; REINALDO, 2005)

The complete algorithm for the FFD Heuristic is presented in Algorithm 2:

Algorithm 2 – FFD Heuristic

```
Sort the items in non-increasing order of size: l_1 ⩾ l_2 ⩾ ... ⩾ l_m;
Let r_i be the residual demand of the item i ∈ I, I = {1, ..., m};
At first: r_i = b_i, ∀ i ∈ I;
k = 1 (First cutting pattern);
STOP = False (logic variable that indicates a non-null demand);
while STOP = FALSE do
    Rest = L;
    α_ik = 0, ∀i ∈ I;
    i = 1 (start by putting the first item in the pattern);
    while i ⩾ m and Rest ⩾ l_i do
        α_ik = min { Rest / l_i , r_i } ;
        Rest = Rest - α_ik l_i ;
        r_i = r_i - α_ik;
        i = i + 1;
    end
    if r_i = 0 ∀ i ∈ I then
        STOP = TRUE;
    end
    else
        k = k + 1;
    end
end
```

1.1.1.5 Greedy Heuristic

The Greedy Heuristic shares the same basic approach as the FFD Heuristic, but with a slight variation. While the FFD Heuristic prioritizes the largest items to construct the pattern, in the Greedy Heuristic, the pattern is determined by solving the restricted knapsack problem. The Greedy Heuristic can more effectively optimize the use of available resources and achieve better results.

1.1.2 Variants of Cutting stock problems

Common variants of cutting stock problems include:

- Two-Dimensional Cutting Problem: The materials available for cutting have two dimensions (width and length), and the items to be cut can have different dimensions.
- Three-Dimensional Cutting Problem: Similar to two-dimensional, the items and materials for cutting have three dimensions (length, width, and height).
- Multi-Stage Cutting Problem: It involves more than one cutting step where items are initially cut into intermediate pieces, and then these intermediate pieces are cut to meet the final demand.
- Irregular Cutting Problem: Considers irregular cutting patterns, allowing for non-uniform and more complex cuts to reduce waste.

A two-dimensional problem can be described as a set of rectangular pieces from a single rectangular stock sheet, minimizing the waste that can appear in various production processes (OLIVEIRA; FERREIRA, 1990). Several applications highlight how two-dimensional cutting problems are crucial in various industries, contributing to operational efficiency, cost reduction, and sustainable use of materials.

According to Gilmore e Gomory (1961), this kind of problem can be stated as follows: given a rectangular stock sheet R of dimensions L x W; a set of p distinct types of smaller rectangular pieces is to be cut from the stock sheet. This set is denoted by $S = \{(l_i,\ w_1),\ (l_2{,}w_2),\ ..,\ (l_p,\ w_p)\}$. The objective of the two-dimensional problem is:

$$Maximize \sum_{i=1}^{p} x_i v_i \tag{1.5}$$

The basic formulation of the two-dimensional cutting stock problem is described as: such that there exists a series of cuts on R, such that x_i are pieces of type i (i=1,..p) can be cut from R, $x_i \geqslant 0$.

Where v_i is the value associated with each of the rectangles to be cut, and x_i is the number of pieces of type i, which are cut. Many constraints can be imposed upon the basic formulation. One of the most frequently used is to restrict the number of pieces of any one type of rectangle in the set S. This constraint can be stated mathematically as: $x_i \geqslant d_i$, $i = 1, ..., p$. Where d_i is the maximum number of pieces of type i that may be cut from the stock sheet R, the addition of this last equation converts the *unconstrained* cutting stock problem to a *constrained* problem.

Besides that, another restriction for cutting problems is that we may use only guillotine cuts for each object. That represents cuts parallel to one of the object's sides and go from one side to the opposite one, and then they are called two-dimensional guillotine cutting problems. Another usual restriction for these problems is staged cuts. A k-staged cutting is a sequence of at most k stages of cuts, each stage of which is a set of parallel guillotine cuts performed on the objects obtained in the previous stage. The cuts in each stage must be orthogonal to those in the previous stage; with that, we assume the cuts are infinitely thin (CINTRA et al., 2008).

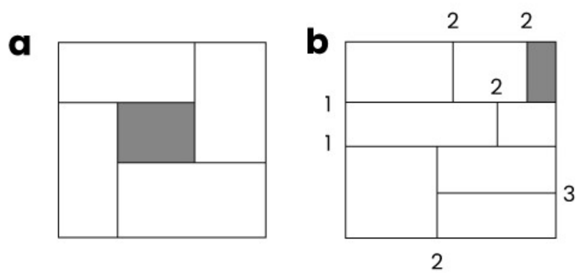

Figure 5 – Two dimensional cutting problems (CINTRA et al., 2008).

Variants of cutting problems have applications in different contexts, playing a crucial role in optimizing the use of materials in various industries. For example, the One-Dimensional Cutting Problem is fundamental in the paper industry, where paper rolls of fixed lengths need to be cut to meet the demand for sheets of different sizes. The Two-Dimensional Cutting Problem finds application in the furniture industry, where wooden boards of different dimensions must be cut to produce different pieces of furniture. In sectors such as the textile industry, the Irregular Cut Problem is crucial to optimize the cutting of complex fabric patterns, minimizing material waste. Furthermore, the Multistage Cutting Problem is commonly employed in the manufacturing industry, where intermediate parts are cut before passing through subsequent stages of production. These applications highlight the versatility and importance of cutting problems in operational efficiency and cost reduction across various industries.

1.2 Scheduling Problems

Sequencing and production planning are crucial activities in both manufacturing and service industries. Pioneering studies at the beginning of the century, led by figures such as Henry Gantt, brought prominence to this subject. Scheduling, a critical decision-making process employed across various manufacturing and service industries, revolves around efficiently allocating resources to tasks within specified time frames, aiming to optimize one or more objectives. Resources and tasks can manifest in diverse forms, ranging from machines in a workshop and runways at an airport to crews at a construction site and processing units in a computing environment. Tasks may encompass production operations, airport take-offs and landings, construction project stages, or computer program executions. With each task possessing priority levels, earliest starting times, and due dates, objectives can vary, such as minimizing completion time or reducing the number of tasks completed after their respective due dates. (PINEDO, 2012a)

1.2.1 Classification of Scheduling Problems

According to (ARENALES et al., 2015), the primary decisions at the operational level involve task assignment to machines ("jobs") and scheduling of tasks on each machine, i.e., determining the sequence of task processing and the start and end times of each task. The goal is to achieve maximum system efficiency. A schedule is essentially an assignment of one or more time intervals to one or more machines for each job. This allocation of tasks to specific time slots and machines can be effectively represented through Gantt charts (6.

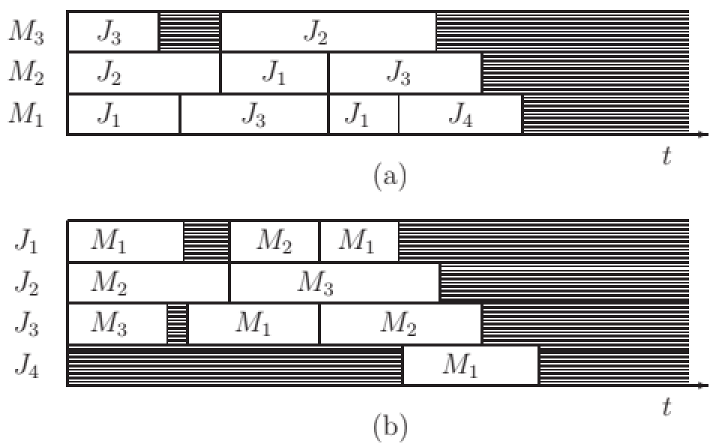

Figure 6 – Machine-oriented and job-oriented Gantt charts.

(BRUCKER, 2004)

The description outlines a job J_i, which comprises a series of operations O_{i1},...,O_{in_i}. Each operation O_{ij} has a processing requirement p_{ij}. If a job J_i has only one operation ($n_i = 1$), J_i is identified with O_{i1}, and the processing requirement is denoted as p_i. Additionally, a release date r_i may be specified for when the first operation becomes available for processing.

Each operation O_{ij} is associated with a set of machines $\mu_{ij} \subset \{M_1,...,M_m\}$, where O_{ij} can be processed on any machine within μ_{ij}. **Dedicated machines** have $_{ij}$ as one-element sets, while **parallel machines** have μ_{ij} equal to the set of all machines. The general case covers flexible manufacturing problems with machines equipped with different tools, allowing an operation to be processed on any machine with the appropriate tool. This scenario is termed scheduling problems with **multi-purpose machines** (MPM).

There's also the possibility that all machines in μ_{ij} are used simultaneously by O_{ij} throughout the entire processing period. Such scheduling problems are referred to as **multiprocessor task scheduling problems**. (BRUCKER, 2004)

Lastly, a cost function $f_i(t)$ is utilized to quantify the completion cost of J_i at time t. This cost function can be defined with the inclusion of a due date d_i and a weight w_i. In a general context, all data points, such as p_i, p_{ij}, r_i, d_i, and w_i, are considered to be integers. A feasible schedule ensures no overlapping time intervals on the same machine, no overlapping intervals for the same job, and compliance with specific problem-related constraints. An optimal schedule, on the other hand, minimizes a designated optimality criterion. (BRUCKER, 2004)

As stated by (PINEDO, 2012b), classes of scheduling problem can be represented as a triple $\alpha \mid \beta \mid \gamma$. The α field describes the manufacturing environment and contains only one entry. In this field, one can mention problems with a single machine, two machines in parallel, "Flow Shop" (where sequencing of steps on machines is necessary), or "Job Shop" (where it is necessary to go through steps or machines more than once). The β field, in turn, contains details of processing characteristics and constraints and may have a single or multiple entries. Finally, the γ field describes the objective to be minimized and usually contains a single entry.

1.2.2 Single machine

According to (BAKER, 1974), this type of problem is considered highly significant for various reasons, as it serves as a foundation for developing techniques to address more complex issues.

Consider the problem of a single machine with n tasks to be processed, assuming that all tasks can start at time zero and that there are no interruptions in task processing. The following integer and non-negative parameters are taken into account:

p_i = processing time of task i

d_i = deadline for task i

M = a large number

The following decision variables need to be defined:

$$
\begin{aligned}
C_i &= \text{completion time of task } i \\
x_i &= \text{start time of task } i \\
y_{ik} &= \begin{cases} 1, & \text{if task } i \text{ immediately precedes task } j \\ 0, & \text{otherwise} \end{cases}
\end{aligned}
$$

Additionally, depending on the optimization objective, we can define:

T_i = max $\{C_i$ - d_i, $0\}$ = delay in task i

E_i = max $\{d_i$ - C_i, $0\}$ = advancement or lead time of task i

L_i = C_i - d_i = *lateness* of task i

From these parameters and variables, various optimization problems are formulated. Examples of possible objectives include:

- Minimization of makespan
- Minimization of the maximum delay
- Minimization of the sum of delays and advancements
- Minimization of overdue tasks
- Minimization of the sum of delays

Once the problem's objective is defined, it is necessary to model the problem while considering the appropriate constraints. A possible mathematical model of the problem $1|p_i|C_{max}$, which minimizes the makespan, is presented below.

$$
\begin{aligned}
\text{minimize} \quad & C_{max} && (1.6)\\
\text{subject to:} \quad & C_{max} \geqslant x_i + p_i \quad \forall i = 1, ..., n && (1.7)\\
& x_i + p_i \leqslant x_k + M(1 - y_{ik}) \quad \forall i = 1, ..., n, \quad k > i && (1.8)\\
& x_k + p_k \leqslant x_i + M y_{ik} \quad \forall i = 1, ..., n, \quad k > i && (1.9)\\
& C_{max} \geqslant 0 && (1.10)\\
& x_i \geqslant 0 \quad \forall i = 1, ..., n && (1.11)\\
& y_{ik} \in \{0, 1\} \quad \forall i = 1, ..., n, \quad k > i && (1.12)
\end{aligned}
$$

The optimal solution will result in the sequencing of task processing that minimizes the maximum completion time of any task.

2 Integrated Cutting Stock and Scheduling Problem Description

This dissertation is based on a real manufacturing problem in pharmaceutical industry, in particular to produce coils for blister packs. It is an integrated cutting stock and scheduling problem. Blister packaging, contrary to bottles, keeps each tablet or capsule hermetically sealed in its own bubble (Figure 7). Drugs that are not taken remain in the original package and are fully protected against external conditions.(PILCHIK, 2000)

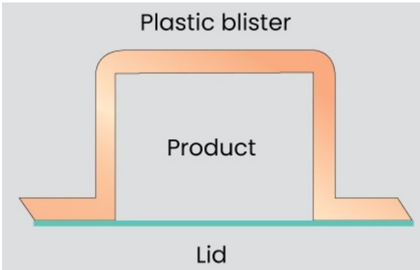

Figure 7 – Basic configuration of blister packaging.

A typical procedure for blister packaging assembly (Figure 15) involves a forming web thermoform with a web (plastic material) into blister cavities, load blister with product, place lidding material over it and heat-seal the package. An essential part of a standard blister machine is the unwinding station, where coil plastic (forming films) are at a rate that corresponds to the speed of the entire machine.

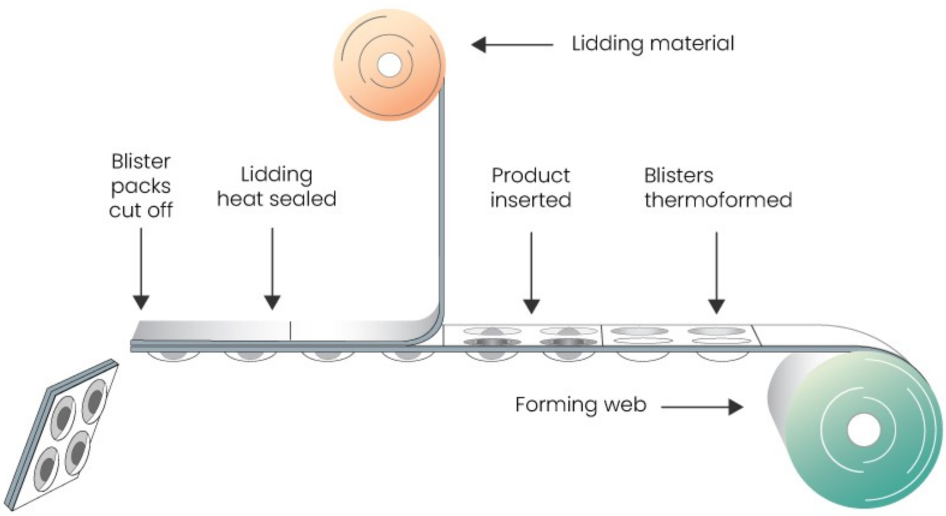

Figure 8 – Blister packaging standard procedure.

The coils for lidding material and forming web can be bought in different sizes, adjusted to be put in blister machine. In this case is important to have an ongoing supply chain control with the supplier, since a wide variety of coil types (composition, color, thickness) is necessary to deliver product specifications.

In order to reduce costs and delivery issues with suppliers, an option that has become common is to add a new manufacturing process to blister packaging. Even though it is not the main activity of the pharmaceutical industry, it is opportune to buy coils with large widths to cut them in an appropriate machines, delivering materials for blister machines. This internal new process will also be able to support changes in production scheduling quickly, without having to carry out new negotiations with an external supplier.

However, this new process also brings a new problem that comes down to deciding how to cut the large coils in order to avoid waste and attend the production demand on time.

In Section 2.1, we present the leading works in the literature with themes related to this research. These works inspired the proposed approaches. Section 2.2 presents the proposed mathematical model (adapted from the work of Lemos (2020)) for the Integrated Cutting Stock and Scheduling Problem (ICSSP) in the pharmaceutical packaging process. Section **??** details the approaches used to treat ICSSP, emphasizing the proposed Genetic Algorithm.

2.1 Literature Review

In various industries, there are numerous complex challenges that require extensive research to find viable solutions. A study conducted by Le Hesran et al. (2019) identified keywords related to waste minimization and scheduling. These keywords were combined to create 12 combinations and a literature search using the Web of Science was conducted. Over 2,000 articles were screened to match the study's scope. Additional research involved examining references and citations in selected papers and focusing on specific scheduling problems related to waste reduction. A total of 70 papers were selected, categorized by publication year and fields such as operational research, chemistry, and sustainable production. The trend (Figure 9) showed an increase in publications after 2007. The majority of articles were from operational research journals (38), followed by chemistry journals (15) and sustainable production (9). The interdisciplinary nature of the field made it challenging to connect articles, emphasizing the need for a comprehensive review.

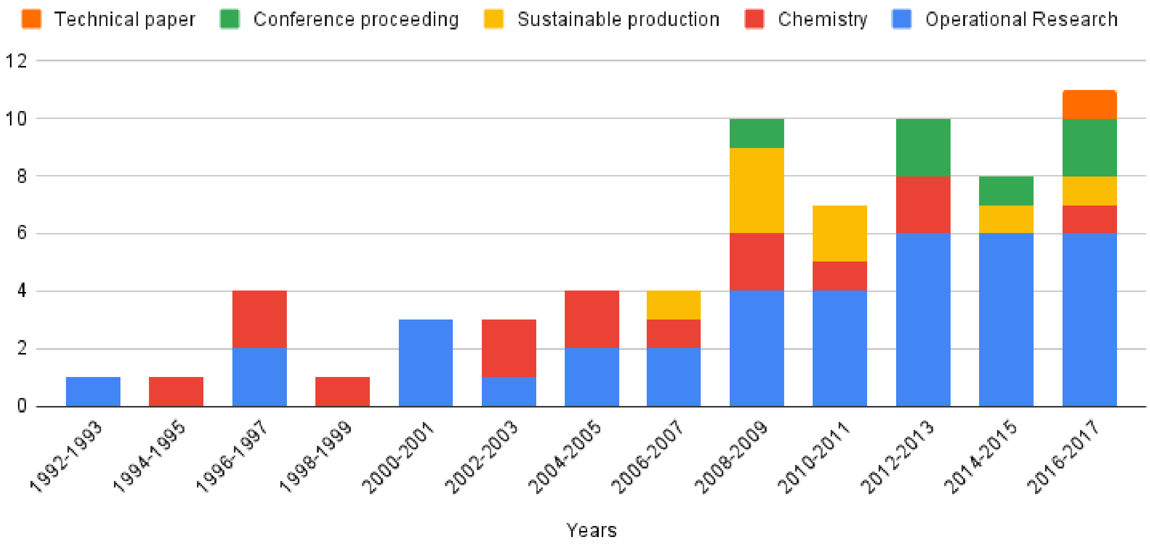

Figure 9 – Graph with papers categorized by publication year and fields.

Figure 10 presents the top 30 keywords in the works used as references for our research. The keywords were listed and counted, forming a list of occurrences. We defined a network structure; this structure represents each keyword by a node, and the size associated with each node is proportional to the number of occurrences of the keyword in the list; edges link keywords that appear together in the same document; each edge has a weight associated with the frequency with which two keywords are cited together, considering all documents. We use the Leiden Algorithm (TRAAG; WALTMAN; ECK, 2019) to group keywords and identify clusters in the network. This algorithm applies an optimization technique aimed at maximizing the network's modularity, measured through the connection strength between its nodes, to divide it into modules. The works that make up this bibliographical review or were part of the scope and development of this research are mainly divided into the main clusters: (i) cutting stock, lot sizing, column generation, heuristics, production planning, integrated problems, multiperiod, integrated problem, lagrangian relaxation; (ii) scheduling, multiobjective optimization, batch process, waste minimization, design, engineering economics, environmental impact reduction, multipurpose batch plants; (iii) cutting, integer programming, manufacturing, optimization, production, one-dimensional, typology.

The table 1 relates the main references and their respective topics that served as the basis for this research.

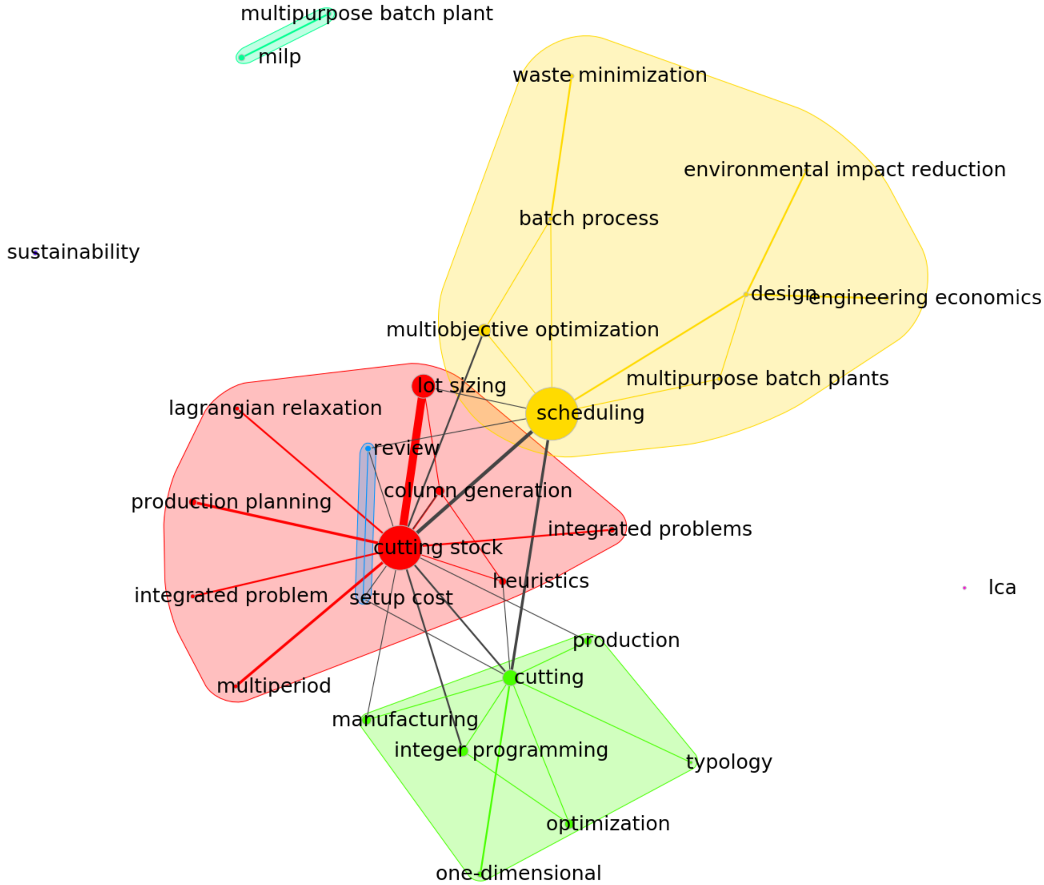

Figure 10 – Keyword cloud - literature review.

Recent developments in Cutting Stock Problems (CSP) literature emphasize the importance of considering production scheduling holistically, moving beyond just optimizing cutting patterns. This has given rise to new CSP variants, such as those involving pattern reduction or usable leftovers. These variants not only focus on minimizing trim loss through efficient patterns, but also consider how these patterns impact production scheduling. For instance, using the most efficient patterns may necessitate frequent pattern changes to meet demand, resulting in longer setup times and increased costs, potentially negating the benefits of reduced trim loss. The cutting stock problem (CSP) and its appropriate heuristics has been discussed by Poldi e Arenales (2009). The paper focuses on the one-dimensional multiple stock size cutting problem, dealing with varying stock lengths and limited quantities. The CSP is often formulated as an integer linear optimization problem, but its complexity arises from the vast number of potential cutting patterns. To tackle this, various techniques have been proposed, such as relaxation of integrality constraints, column generation and rounding procedures. The authors cite previous work, introduces various heuristic and column generation-based approaches, provides implementation recommendations and reports computational experiments.

In their work of CSP with setup considerations, Harjunkoski, Westerlund e Pörn (1999) delved into the realm of one-dimensional CSP (1DCSP) within the paper conversion industry. Their research encompassed the formulation of various objective functions, each focusing on distinct criteria such as pattern count, pattern change frequency, total waste, makespan, energy consumption and overproduction. They meticulously compared outcomes across these diverse objectives, even introducing a hybrid objective function aimed at simultaneously minimizing total waste and energy consumption. Their study underscored the significance of these hybrid functions, emphasizing that a profound understanding of the underlying processes, albeit necessitating further investigation, plays a pivotal role in enhancing result quality.

Similarly, Schilling e Georgiadis (2002) explored the 1DCSP, considering costs as an additional dimension. In their investigation, they crafted a comprehensive objective function that encompassed not only profit and setup costs but also a noteworthy inclusion - waste disposal costs. To tackle this complex problem, they introduced a Mixed Integer Linear Programming (MILP) model, highlighting how the incorporation of changeover and waste disposal expenses heightened the problem's complexity. Still considering setup in CSP, Kolen e Spieksma (2000) also studied the 1DCSP, focusing on trim loss reduction and minimizing pattern counts. They also accounted for two types of jobs, one allowing for controlled over or underproduction and the other adhering strictly to demand requirements. To solve this multi-objective problem, they devised a Branch and Bound (B&B) algorithm capable of generating a set of Pareto-optimal solutions. Lastly, Araujo, Poldi e Smith (2014) and Golfeto, Moretti e Neto (2009) used genetic algorithms (GA) for the 1DCSP. The first tackled the bi-objective optimization problem involving the minimization of both

Reference Title	**Authors and Year**	**Theme**
Heuristics for the one-dimensional cutting stock problem with limited multiple stock lengths	Poldi e Arenales (2009)	1D - Cutting stock problem
Numerical and environmental considerations on a complex industrial mixed integer non-linear programming (MINLP) problem	Harjunkoski, Westerlund e Pörn (1999)	1-dimensional Cutting stock problem
An algorithm for the determination of optimal cutting patterns	Schilling e Georgiadis (2002)	1DCSP + Costs
Solving a bi-criterion cutting stock problem with open-ended demand: A case study	Kolen e Spieksma (2000)	1DCSP + trim loss and patterns reduction
A genetic algorithm for the one-dimensional cutting stock problem with setups	Araujo, Poldi e Smith (2014)	1DCSP + Genetic Algorithm
A genetic symbiotic algorithm applied to the one-dimensional cutting stock problem	Golfeto, Moretti e Neto (2009)	1DCSP + Genetic Algorithm
Multi-Job Cutting Stock Problem with Due Dates and Release Dates	Li (1996)	Cutting Stock and Scheduling Problem
A cutting stock and scheduling problem in the copper industry	Hendry, Fok e Shek (1996)	ICSP + setup
A coupling cutting stock-lot sizing problem in the paper industry	Poltroniere et al. (2007)	Cutting stock and lot sizing problem
An integrated cutting stock and sequencing problem	Yanasse e Lamosa (2007)	Cutting Stock and Scheduling Problem
A Model-Based Heuristic for the Combined Cutting Stock and Scheduling Problem	Braga et al. (2015)	Cutting Stock and Scheduling Problem
Mathematical model and solution approaches for integrated lot-sizing, scheduling and cutting stock problems	Melega, Araujo e Morabito (2020)	Integrated lot sizing, scheduling

Table 1 – References

the number of patterns used and the trim loss incurred. With the GA, they were able to generate a set of solutions that were not dominated by one another. Compared to alternative methods employed to solve both real-life and randomly generated instances, it delivered impressive quality results without compromising computational efficiency. And the second, Golfeto et. al, with GA also had successfull results. They constructed a Pareto front that illustrated the trade-offs between trim loss and the number of setups. Furthermore, they proposed the possibility of parallel processing to enhance the computational speed of their genetic algorithm.

Scheduling problems in the realm of CSP and ICSP, particularly research focused on waste minimization, have been explored in the context of production processes that are less specialized. The ICSSP consists in determining the set of feasible cutting patterns that covers the demand of the items, and the time instants when each of these patterns must be cut such that the waste and tardiness is minimized.

One of the first contributions in the field of Cutting Stock and Scheduling Problems is the research of Li (1996). The author addresses a two-dimensional cutting stock problem involving rolls with varying lengths and widths. Jobs consist of sets of items with different sizes and defined release and due dates. The study introduces various integer programming models associating cutting patterns with time periods. Additionally, the author presents both Linear Programming (LP) and non-LP-based heuristics for generating feasible cutting patterns and schedules. The effectiveness of these heuristics is assessed using small instances where an optimal solution can be computed within a reasonable time frame. Similar to other models in the literature, authors formulation is not universally exact, depending on the length of time periods, as an optimal solution may not be optimal for the overall problem.

Poltroniere et al. (2007) paper proposes a coupled modeling and heuristic method to optimize machine scheduling and cutting stock sizing in the paper industry, minimizing waste and setup costs while maintaining production demand. Similarly, the research of Yanasse e Lamosa (2007) introduces an integrated problem presented as an integer linear programming model, aiming to attain an optimal solution for the cutting stock problem, subject to specific pattern sequencing constraints. The approach adopted utilizes a Lagrangian method, wherein the dual problem is resolved through a modified subgradient method. Additionally, the paper introduces a heuristic for the integrated problem.

In Aktin e Özdemir (2009), they handle an Industrial Cutting and Scheduling Problem (ICSP) using a two stage approach. Firstly, a heuristic generates cutting patterns to meet demand while minimizing trim loss. Then, an Integer Linear Programming (ILP) model is employed to find a cutting plan that minimizes an aggregated cost function, including material, setup and lateness costs. This approach is successfully implemented in a coronary stent manufacturing company, providing efficient full cutting plans and patterns.

Silva e Carvalho (2014) delve into a 2-dimensional Industrial Cutting and Scheduling Problem (ICSP) that allows for potential storage of leftover materials. They aim to minimize an objective function that encompasses waste, material, operational and storage costs. Two Integer Linear Programming (ILP) models, based on prior research are proposed along with two heuristic methods. Notably, the ILP models are capable of obtaining exact solutions even for large instances.

In Braga et al. (2015), the authors explored a variant of the cutting stock problem that involves scheduling cutting operations over time, combining the standard objective of minimizing raw material usage with a scheduling component penalizing tardiness. Tardiness is incurred when the final instance of an item is cut after its specified due date. In the research was presented a novel pseudo-polynomial network flow model for the combined cutting stock and scheduling problem. It was used a modified version of this model, aggregating consecutive time intervals, to develop a heuristic solution procedure. This adapted formulation, coupled with a time assignment procedure, produced effective feasible solutions for the problem.

Poldi e Araujo (2016) considered a multi-period one-dimensional ICSP, with objectives centered on minimizing trim loss and inventory costs for both raw materials and finished products over a set of production periods. They introduced an arc flow formulation complemented by a heuristic procedure, based on Carvalho (1999) work. Instances were solved with varying weights assigned to holding costs, demonstrating efficient computation times even for large instances. Additionally, their approach requires fewer patterns compared to the classical approach.

A recent research from Melega, Araujo e Morabito (2020) addresses a complex two-stage integrated problem that involves lot-sizing, scheduling, and cutting stock, incorporating sequence-dependent setup times and costs. In the initial production stage, a cutting machine is utilized to cut large objects into smaller pieces, necessitating the generation and sequencing of cutting patterns to formulate a comprehensive cutting plan. The cut pieces from the first stage are then utilized in the second production stage to assemble final products, which are scheduled to align with client demands. The authors propose solution methods based on a price-and-branch approach, incorporating a column generation procedure and decomposition solution approaches to address the integer problem. Through a computational study using randomly generated data, the authors analyze the impact of these solution approaches on the integrated problem. Additionally, they compare the performance and advantages of the integrated approach with an empirical simulation of the common sequential practice.

2.1.1 Major references

Two important research studies significantly contributed to the foundation of this thesis. The first of these studies is cited from Silva e Arenales (2006), their work focuses on the integration of lot sizing and cutting stock problems particularly applied in the paper industry. The aim is to minimize costs associated with production, setup, inventory of materials and losses during the cutting problems. The challenge of this integration lies in the complexity of both problems. Lot sizing involves parallel machines with limited capacities and losses in machine setup. The cutting stock problem is complicated due to a large number of variables and integer constraints.

To address these issues, various heuristic methods have been proposed, including relaxing constraints, using the simplex method with column generation and heuristic rounding of solutions. Additionally, attention was drawn to the independence of cutting decisions across different time periods due to variables related to final item inventory, referred to as the "multiperiod cutting stock problem". Two heuristic methods were implemented to solve the multi period cutting problem. The first approach tackles the problem batch by batch, while the second considers item inventory variables to optimize cutting. The second approach performed better in reducing losses. Other heuristics based on problem decomposition were suggested, including the Lot-Cut and Cut-Lot heuristics. They were compared and showed satisfactory results, solving approximately 90% of the generated examples. The research emphasizes the need to review the parameters of the integrated model‘s objective function and explore integer constraints in the stock cutting problem. It also mentions the potential application of rolling horizon strategies for dynamic problems.

Lemos (2020) is another key study used in this thesis. This study addresses three different problems: cutting stock and production scheduling, cutting stock with saw cycle minimization, and cutting stock with alternative manufacturing modes. The first problem and its solution served as the primary inspiration for the situation discussed in this thesis. The driving force behind Lemos (2020)'s work stems from the supply of materials to assembly lines, where even minor delays can significantly impede production. Their research addresses a pertinent example, where they explore the cutting of tubes using band saws or circular saw machines to craft trusses forming the structural components of agricultural aircraft. The scheduling of cutting patterns depends on their processing time, which may or may not vary depending on their configuration, depending on the process. Another important aspect is that while the formation of cutting patterns aims to minimize the use of raw materials, their sequence is intended to meet order delivery dates. Lemos (2020) mentions that their resolution requires a different delay configuration from what has been shown in the literature so far, necessitating approaches that can handle this peculiarity in the choice of generated patterns. The mathematical formulation described below aims to fulfill the demand for all items while respecting the constraints of the problem and the non-overlapping production time of lots, thereby minimizing the total cost of weighted delays and raw material usage.

In the model (2.1)-(2.6) proposed by Lemos (2020), the objective function (Equation 2.1) seeks to minimize the sum of two components of the total cost. The first component pertains to the sum of weighted delays for the NI (quantity of items or cut coils) to be processed. The second cost component is the sum of raw material costs for the various cutting patterns (represented by NP) utilized. The number of productive slots available is represented by NK and the number of items is a. The variable X_{pk} represents the number of patterns of type p cut in the sequence k of the programming. So, the constraint set (2.2) ensures that the total demand is met for all items i, as it enforces that the sum of all production for this item in the chosen patterns is equal to or greater than the demand b_i.

Model:

$$\text{minimize} \quad \sum_{i=1}^{NI} w_i T_i + \theta \sum_{p=1}^{NP} \sum_{k=1}^{NK} X_{pk} \tag{2.1}$$

$$\text{subject to:} \quad \sum_{p=1}^{NP} \sum_{k=1}^{NK} a_{ip} X_{pk} \geqslant b_i \qquad i = 1, ..., NI \tag{2.2}$$

$$T_i \geqslant \sum_{p=1}^{NP} \sum_{q=1}^{k} t_p x_{pq} - d_i - M_i\ Z_{ik} \qquad i = 1, ..., NI \qquad k = 2, ..., NK-1 \tag{2.3}$$

$$Z_{ik} \leqslant \left(\sum_{p=1}^{NP} \sum_{q=1}^{k-1} a_{ip} X_{pq}\right)/b_i \qquad i = 2, ..., NI \qquad k = 1, ..., NK \tag{2.4}$$

$$X_{pk} \geqslant 0 \qquad p = 1, ..., NP \qquad k = 1, ..., NK \tag{2.5}$$

$$Z_{ik} \in \{0, 1\} \qquad i = 1, ..., NI \qquad k = 1, ..., NK \tag{2.6}$$

$$T_i \geqslant 0 \qquad i = 1, ..., NI \tag{2.7}$$

In constraint (2.3), the delay T_i is defined, bounded from below by the difference between the sum of processing times up to a certain sequence k and the delivery date of item d_i, provided that item i has not yet been completed up to sequence $k-1$ (i.e., $Z_{ik} = 0$). When $Z_{ik} = 1$, the constraint becomes inactive, as the processing time up to sequence k becomes irrelevant for calculating the delay of i, which is ensured by the parameter M_i, sufficiently large to perform this disjunction.

Lastly, constraint (2.4) the decision variable Z_{ik} is defined logically, bounded from above by the fraction of order i that has already been completed up to the scheduling sequence $k-1$. Thus, an item is considered complete in sequence k when its demand b_i has already been fully met in $k-1$, so the accumulated processing time up to k can be disregarded for the calculation of i's delay (according to constraints (2.3)). The domain of the decision variables is defined in (2.5)-(2.7).

Due to the potentially vast number of possible cutting patterns, explicit enumeration of all possible combinations is computationally impractical. The column generation method is an iterative approach that aims to add columns (in this case, cutting patterns) to the problem that have the potential to enhance its objective function. It commences with a reduced set of columns, ensuring the existence of a feasible solution to the problem. Typically, these columns consist of homogeneous cutting patterns, which include only one type of item. In a feasible solution, each column has an associated reduced cost, which represents the difference between its coefficient in the objective function and the summation of its coefficients in the constraints, multiplied by the corresponding dual values of those constraints. The problem without the complete set of columns is referred to as the "restricted master problem". In each iteration, columns with improved (negative) reduced costs are generated and incorporated into the master problem to enhance the objective function. These columns are obtained by solving a sub problem, determining the best cutting pattern associated with the current solution. This process is repeated as long as improvements in the objective function are achievable.

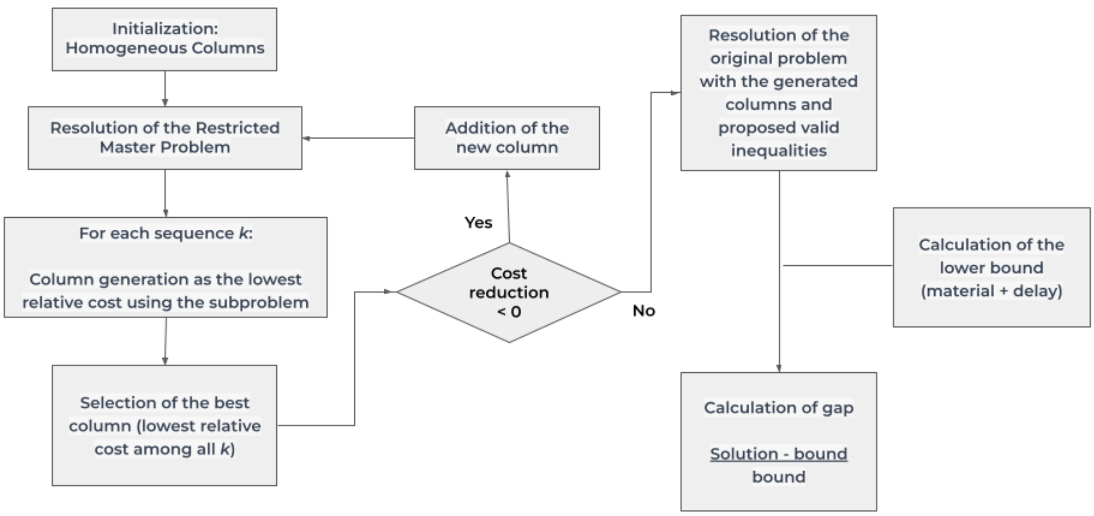

Figure 11 – Solution method schema (LEMOS, 2020)

To obtain an integer solution after the column generation process, the heuristic described by Wascher and Gau (1996) is employed. This approach utilizes the same model (2.1)-(2.7) with the integrality constraints of X_{pk} and Z_{ik} retained but is restricted to the columns comprising the relaxed optimal solution and other columns generated during the process, along with the homogeneous columns that initialize it. Although relatively straightforward, it is used in cases found in the literature where gaps relative to the optimal solution of the relaxed problem are acceptable for the specific application. Additionally, valid inequalities are proposed to reduce the space of feasible solutions without constraining the optimal solution. The aim is to enhance the solution within the given time limit. Finally, a lower bound calculation method is proposed to improve the one generated by the optimal solution of the linear relaxation of the problem, which is affected by the use of disjunctive constraints. Lower bounds are separately calculated for the cutting stock problem and the weighted delay problem and both are combined to compute the lower bound of the integrated problem. All this solution method is illustrated at Figure 11.

Taking into account the work by Lemos (2020), which highlighted the similarities between the problem addressed in their thesis and the problem under discussion in this research, the mathematical model served as a primary point of reference. As a result, this study proposed several enhancements to the existing model.

2.2 ICSSP on pharmaceutical packaging process

In summary, an Integrated Cutting Stock and Scheduling Problem (ICSSP) revolves around three primary aspects:

1. Determining the optimal cutting patterns j for producing m types of items from a large piece with a width of L, considering R, the jumbo coil trim, and, C: number of cuts of the jumbo coil;

2. Deciding how many times and which cutting patterns j should be employed on the machine within a specific timeframe to meet the demand b for items;

3. Finding the optimal sequence for the cutting patterns j, considering their processing times and task due dates, to minimize total processing times, setup times, and delivery delays.

To illustrate the connection with the pharmaceutical packaging process explained in the first chapter, consider the scheme depicted in Figure 16. In simple terms, this problem involves cutting jumbo coils into different item types, which are then used in blister machines.

Additionally, it's important to emphasize that the model will incorporate specific parameters related to jumbo rolls, machinery, and setup procedures to closely replicate real-world processes that will be further explored in the next sections.

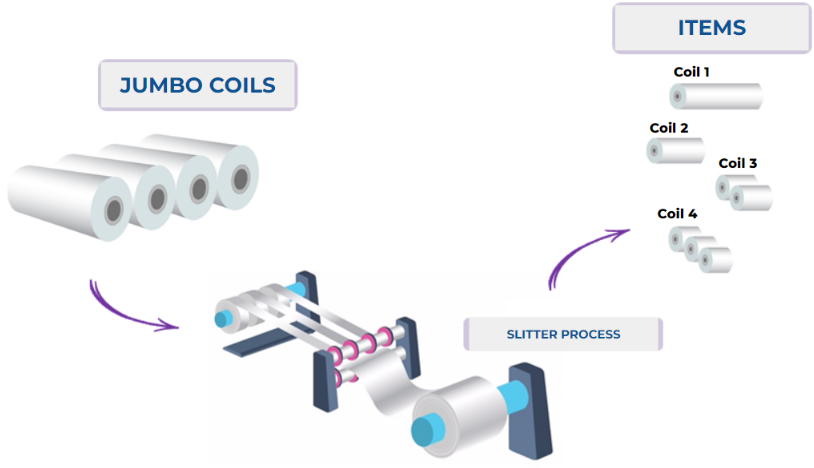

Figure 12 – ICSSP in pharmaceutical packaging process

2.2.1 Jumbo coils

Jumbo coils, also known as master coils or parent coils, are large and continuous rolls of various materials such as steel, aluminum, plastic, or paper (Figure 13). These jumbo coils serve as the primary raw material in a wide range of industries, including metal processing, packaging, construction, and manufacturing.

Figure 13 – Jumbo coils warehouse. (Image by the author)

One of the key advantages of jumbo coils is their ability to carry a substantial amount of material in a single roll, significantly reducing the need for frequent material changes during production. This feature not only improves production efficiency but also reduces downtime and material waste, contributing to cost-effectiveness and sustainable manufacturing practices.

The packaging industry also heavily relies on jumbo coils, particularly in the production of flexible packaging materials like plastic films and laminates. Jumbo coils of plastic films provide a continuous feed for packaging machines, ensuring seamless and uninterrupted packaging operations.

Moreover, jumbo coils are not only advantageous for large-scale production but also provide flexibility for customization. Below is outlined all the parameters related to jumbo coils that may interfere in cutting stock problems:

1. Material: plastic jumbos come in various materials, colors, weights and other distinct characteristics;
2. Width: typically, larger pieces have widths close to the maximum capacity supported by the cutting machine;
3. Cutting processing time: the cutting process time exhibits variation based on factors such as thickness, type of plastic material, cutting speed, and other relevant parameters;
4. Coil preparation: the *setup* time required to prepare the jumbo coil for cutting can vary depending on the type of jumbo, and it will be taken into account in this modeling.

Several global suppliers cater to the pharmaceutical industry's demand for jumbo coils, including Alloyd Brand, Nelipak Healthcare Packaging and Klockner Pentaplast. These suppliers play a crucial role in driving innovation and developing new materials that offer specific characteristics to suit different pharmaceutical products. Each type of medication destined for blister packaging requires a distinct material to ensure optimal protection and preservation.

2.2.1.1 Pharmaceutical industry jumbo materials

PVC (Polyvinyl Chloride) is a widely utilized packaging material in blister packaging, known for its transparency, cost-effectiveness, high thermomoldability, resistance to external forces and low permeability rate. It remains a popular choice, being chosen in about 80% of blister packaging cases. The thickness of PVC typically ranges from 0.2mm to 0.8mm, providing versatility for various pharmaceutical products.

However, it's important to note that PVC has some drawbacks, primarily related to its environmental impact. When burned, PVC can release toxic substances, prompting the exploration of alternative materials like Polypropylene (PP), Polyethylene terephthalate (PET) and Polystyrene (PS). Although the permeability to humidity compared to PVC restricts the use. A polymer known for its excellent barrier properties is PVDC (Polyvinylidene Chloride) and its also applied to packaging materials for pharmaceutical industries. The high clarity of PVDC films makes them suitable for transparent packaging, allowing consumers to see the product while enjoying the benefits of protection and preservation.

Aluminium is another viable material for jumbo coils used in blister packaging. It offers a superior barrier properties against humidity, oxygen and light; and is available in the market with thickness from 20 micrometers to 25 micrometers. Unlike PVC, the process for blister formation is a cold pressure process. This method demands a larger packing area and, subsequently, increases the overall cost of production. (FERREIRA, 2017)

2.2.2 Slitter machinery

Slitter machines play a crucial role in modern manufacturing industries by providing precise and efficient solutions for processing various materials. These cutting-edge machines are designed to transform wide rolls or coils of materials, such as paper, plastic, metal, or fabric, into narrower strips with exceptional accuracy and consistency.

The primary function of slitter machines is to slit or cut materials into desired widths, catering to the specific needs of different industries. In the paper and packaging industry, slitter machines are instrumental in creating various paper products, such as labels, tapes, and packaging materials, with consistent dimensions. In the metal industry, they are used to produce strips and sheets for various applications like automotive parts, electronic components, and household appliances. Similarly, in the textile industry, slitter machines facilitate the production of textile strips and ribbons used in garments and home textiles.

One of the key advantages of slitter machines is their ability to handle a wide range of materials and accommodate varying thicknesses. The machines can be equipped with different types of blades or cutting mechanisms, depending on the material being processed, ensuring precise and clean cuts without compromising material integrity.

Additionally, slitter machines offer high-speed processing capabilities, significantly increasing production efficiency and reducing lead times. With automation and advanced control systems, these machines can perform continuous slitting operations, ensuring consistent and uniform results in large-scale manufacturing environments.

Moreover, slitter machines contribute to minimizing material waste, as they are designed to optimize material utilization by maximizing the number of narrow strips obtained from a single wide roll or coil. This not only reduces production costs but also aligns with sustainable manufacturing practices.

2.2.2.1 Cutting process

The manufacturing process for the pharmaceutical packaging items entails the utilization of a slitter machine. This equipment operates by arranging jumbo coils onto the input shaft, with precise cutting patterns being established through the use of blades. Subsequently, the severed coils are meticulously wound onto the output shaft, completing the cutting cycle. Figure 14 provides a straightforward visual representation of this process.

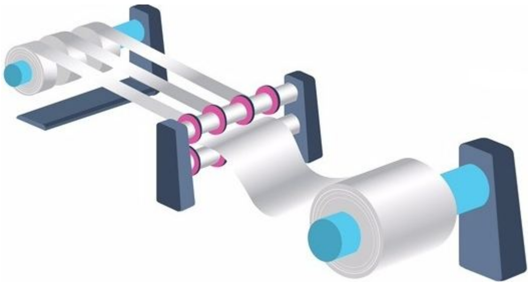

Figure 14 – Example of the structure of a slitting machine

(Image by the author)

Several characteristics can influence in the cutting process using a slitter machine. These factors will determine the quality, accuracy and efficiency of the cut and some of them can be seen below:

1. Material properties: different materials have a variation at hardness, thickness and flexibility levels. It can affect how the blades interact with the material and how "clean" the cuts are.

2. Blade sharpness and its material quality: the good condition of blades are essential. Dull or improperly maintained blades can lead to uneven cuts, jagged edges and also increase material stress.

3. Material tension: the tension at which the material is fed through the slitter machine can impact the precision of the cut. Proper tension control helps prevent material stretching, wrinkling or curling during the cutting process.

4. Cutting pressure: the pressure exerted by the cutting blades on web needs to be controlled. Low pressure can result in incomplete cuts and the opposite, high pressure, can damage the material.

5. Slitting speed: excessive speed at which the material is fed through the machine can cause material distortion. On the other hand, slower speed may lead to production bottlenecks.

6. Cutting blades configuration: the arrangement of blades (angle, overlap, position) affects the final cut.

7. Winding and unwinding: the way that the material is unwound from the supply roll and wound onto the output rolls can impact the cutting process with misalignment or wrinkles for instance.

8. Jumbo material defects: uneven thickness, defects and wrinkles can impact negatively the cutting process and overall quality.

9. Machine stability and control systems: vibrations or instability can introduce variations in the cutting process. Modern machines often incorporate automation and control systems that allow precise parameter adjustments.

10. Operator training: expertise of operational team in setting up and operating the machine, adjusting parameters and monitoring the cutting process is essential for achieving good results.

A specific parameter to guarantee the cutting quality is "trim". In the context of cutting process of jumbo rolls, "trim" refers to the excess material along the edges that is removed to ensure the final product cleanness. Occasionally, operators opt to use additional blades into the setup, factoring in the overall width of the jumbo coil prior to configuring the cutting pattern. Proper trimming is crucial for producing high-quality rolls while managing waste and maintaining efficient production. It's also worth highlighting that a significant number of the most modern cutting machines allow for material removal from already cut pieces while other coils are being produced, thanks to their multiple axes. This significantly contributes to reducing setup time because, when coils are produced from the same pattern, there is no need for any additional time to remove the already produced coils.

In a world driven by efficiency, slitter machines manufacturers play an important role to meet the demands of production. Euromac, Pasquato, Atlas and Kingsun Machiney are some of these manufacturers that combine expertise in engineering mechanics and automation to design and develop slitter machines.

2.2.3 Blister machinery

Blister machines are cutting-edge devices that have revolutionized pharmaceutical packaging, offering precise and efficient solutions for drug containment. In the pharmaceutical industry, blister packaging has become increasingly popular due to its ability to ensure product integrity, patient safety, and convenience. The versatility of blister machines allows them to accommodate a wide range of pharmaceutical products, including tablets, capsules and even liquids, making them ideal for various drug formulations.

One of the significant advantages of blister machines is their high-speed production capabilities. With automation at its core, these machines can produce blister packs at a rapid pace, meeting the demands of large-scale pharmaceutical manufacturing. This not only improves production efficiency but also reduces labor costs and human intervention, leading to standardized and consistent packaging results.

In recent years, blister machines have embraced digital advancements, incorporating smart features and connectivity. Integration with Industry 4.0 concepts allows for real-time monitoring, data analytics and predictive maintenance. This digital transformation empowers pharmaceutical manufacturers to optimize machine performance, reduce downtime, and improve overall productivity. The basic sets of a blister machine, as can be seen at Figure 15, are:

1. Bottom foil uncoiler
2. Cold forming station
3. Feeding device
4. Empty checker
5. Sealing & Embossing
6. Cover foil uncoiler
7. Cooling & Slitting
8. Draw off
9. Punching
10. Waste foil coiler
11. Discharge conveyor

The process sequence involves heating the plastic, thermofolding it into blister cavities, loading the blister with the product, placing lidding material over the blister and heat-sealing the package. After this, it's time to installing the aluminium foil, cold forming it into blister pouch and seal it on thermoformed blister to give extra protections and finally cutting into individual blisters. Several blister machines suppliers include CAM packaging machines, Uhlmann Pac-Systeme, Romaco Noack, Haicheng Pharmaceutical Machinery and Marchesini Group. Notably, these manufacturers distinguish themselves through variations in technology, features, capacity and customization options. Blister machines are available in diverse production capacities and speeds. Certain suppliers enhance their machines with dedicated vision systems, enabling real-time oversight and management. Additionally, customization extends to blister size, shape and layout options. Machine's price correspond to its specifications and the companies often extend support, training and maintenance plans to ensure optimal machine utilization.

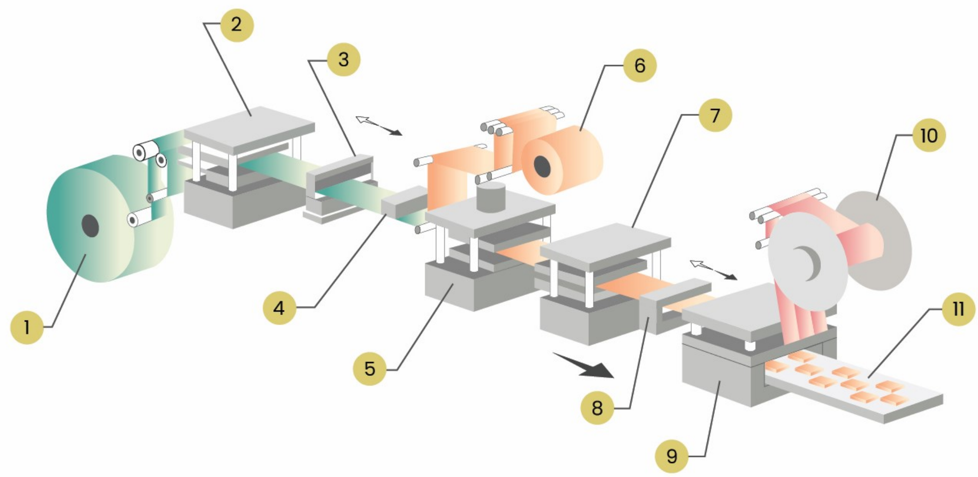

Figure 15 – Blister machine process

(Image by the author)

2.2.4 Item demand

Demand refers to the quantity of products that customers are willing to purchase within a specific period. Understanding and forecasting demand are crucial aspects of decision-making for industrial enterprises. In some industries, demand can be highly uncertain and subject. So in this cases, companies may adopt flexible manufacturing processes and responsive inventory management systems for instance.

In the pharmaceutical packaging environment, the demand to produce blisters is closely linked to the entire supply chain. The production of blisters is affected not only by immediate market needs but also by the dynamics of the supply chain.

At times, it becomes essential to maintain higher inventory levels to ensure uninterrupted production over an extended period. This precautionary measure helps safeguard against potential disruptions in the supply chain and ensures a consistent flow of blisters to meet customer demands. Furthermore, certain products may experience sudden surges in demand, necessitating prompt action. In such situations, a rapid response is vital to meet the increased demand promptly. Quick adjustments in production schedules and inventory management are necessary to cater to these unexpected spikes in product demand.

One thing important that can be configured by the decision maker using the algorithm proposed in this thesis is the time frame of the production plan. In other words, it is possible to determine the level of detail required in the production schedule, considering all relevant parameters. By adjusting the time frame, the decision maker can choose the granularity of the production plan, whether it be for a few days, weeks, or even months. This customization allows them to align the production with specific needs and available resources, for instance. For short term planning or even for long term, the decision maker will have a day-by-day breakdown of production quantities and associated setup times. This level of detail ensures precise resource allocation and quick response to changing demand or unforeseen events.

2.3 Mathematical Modeling

Jumbo coils require distinct cutting patterns. In order to guarantee maximum use of jumbo width (or minimum waste) and also considering variety of jumbos and items, an heuristic is necessary to achieve a solution with low computer requirements. One important consideration of real process situation is to produce large items first, considering the difficulties to stock and low demand. FFD heuristic is perfect to integrated model, since it elects from the largest to the narrowest. The items for blister machine will be made directly from jumbo coils (purchased materials) and its demands are assessed from market forecasts.

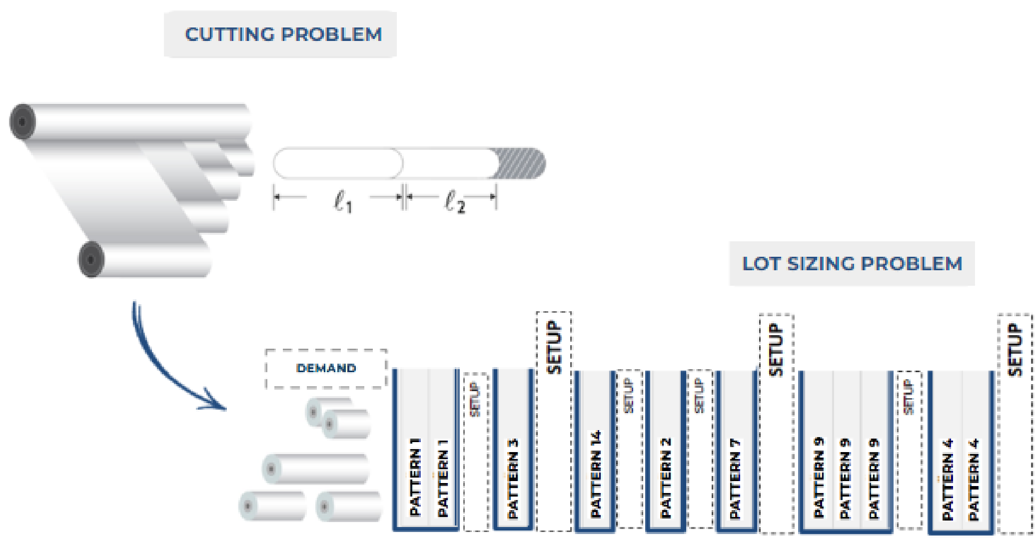

Figure 16 – ICSSP model

The ideal sequencing will be defined using the model described below based on the defined cutting patterns. The FFD heuristic generated the cutting plans in this work according to the available NT jumbo coil types.In this problem, the quantities and characteristics (types) of jumbo rolls are predefined based on the input data provided by the decision-maker. The solution must meet item demand and allocate cutting planes without temporal overlap.

Parameters:

NI: quantity of items (cut coils)

NT: number of types of large parts (jumbo coils)

NP: cutting patterns
NK: number of productive units
b_i: item demand i
d_i: item delivery date i
a_{ip}: number of items of type i in pattern p
t_p: pattern processing (cutting) time associated with the pattern p according to each type of jumbo coil
s_{pr}: setup time between cutting patterns p e r according to each type of jumbo coil
W_1: importance of the first part of the objective function (total setup programming)
W_2: importance of the second part of the objective function (total delay in deliver demand)
W_3: importance of the third part of the objective function (total number of cutting patterns)

Decision Variables:

$$
\begin{aligned}
x_{pk} &= \text{number of patterns of type } p \text{ cut in the sequence } k \text{ of the programming} \\
T_i &= \text{delay in fulfilling the demand for the item } i. \\
z_{ik} &= \begin{cases} 1, & \text{if the item } i \text{ has its demand fulfilled up to the schedule sequence } k-1; \\ 0, & \text{otherwise.} \end{cases} \\
y_{pk} &= \begin{cases} 1, & \text{if the pattern } p \text{ was cut in the sequence } k; \\ 0, & \text{otherwise.} \end{cases} \\
w_{prk} &= \begin{cases} 1, & \text{if the pattern } p \text{ was cut in the window } k \text{ and the pattern } r \\ & \text{was cut in the window } (k+1); \\ 0, & \text{itherwise.} \end{cases}
\end{aligned}
$$

Model:

$$\text{minimize} \quad z = W_1 \sum_{p=1}^{NP} \sum_{r=1}^{NP} \sum_{k=1}^{NK} w_{prk} s_{pr} + W_2 \sum_{i=1}^{NI} T_i + W_3 \sum_{p=1}^{NP} \sum_{k=1}^{NK} x_{pk} \tag{2.8}$$

$$\text{subject to:} \quad \sum_{p=1}^{NP} \sum_{k=1}^{NK} a_{ip} x_{pk} \geqslant b_i \qquad i = 1, ..., NI \tag{2.9}$$

$$z_{ik} \leqslant (\sum_{p=1}^{NP} \sum_{q=1}^{k-1} a_{ip} x_{pq})/b_i \qquad i = 2, ..., NI \qquad k = 1, ..., NK \tag{2.10}$$

$$\sum_{k=1}^{NK} z_{ik} \geqslant 1 \qquad i = 1, ..., NI \tag{2.11}$$

$$T_i \geqslant \sum_{p=1}^{NP} \sum_{q=1}^{k} t_p x_{pq} + \sum_{p=1}^{NP} \sum_{r=1}^{NP} w_{pr(k-1)} s_{pr} - d_i - M_2 \ z_{i(k+1)} \qquad i = 1, ..., NI \qquad k = 2, ..., NK-1 \tag{2.12}$$

$$\sum_{p=1}^{NP} y_{pk} \leqslant 1 \qquad k = 1, ..., NK \tag{2.13}$$

$$x_{pk} \leqslant M_1 \ y_{pk} \qquad p = 1, ..., NP \qquad k = 1, ..., NK \tag{2.14}$$

$$w_{prk} \geqslant y_{pk} + y_{r(k+1)} - 1 \qquad p = 1, ..., NP \qquad r = 1, ..., NP \qquad k = 1, ..., NK \tag{2.15}$$

$$T_i \geqslant 0 \qquad i = 1, ..., NI$$

$$x_{pk} \geqslant 0 \qquad p = 1, ..., NP \qquad k = 1, ..., NK$$

$$z_{ik} \in \{0, 1\} \qquad i = 1, ..., NI \qquad k = 1, ..., NK$$

$$y_{pk} \in \{0, 1\} \qquad p = 1, ..., NP \qquad k = 1, ..., NK$$

$$w_{prk} \in \{0, 1\} \quad p = 1, ..., NP \qquad r = 1, ..., NP \qquad k = 1, ..., NK$$

The model proposed in this paper is an extension of the model put forth by Lemos (2020). In the model by Lemos (2020), the objective function comprised the minimization of two components: the sum of weighted delays and the sum of raw material costs for various cutting patterns used. In the approach presented in this study, the objective function (Equation (2.8)) minimizes the sum of three weighted components: total setup time minimization, total delay minimization and total cutting pattern count minimization.

Constraint (2.9) ensures that the total demand (b_i) will be met for all items i, through the sum of all productions of this item in the selected cutting patterns. In constraints (2.10) and (2.11), the decision variable z_{ik} is upper-bounded by the fraction of the demand for item i met up to sequence $k-1$. Thus, an item whose demand is fully met in sequence k, has $z_{ik} = 1$. In constraint (2.12), it's defined the delay T_i, which is lower-bounded by the different between the sum of processing times and setup times up to a certain sequence k and the delivery date d_i of item i, provided that i has not yet its demand met up to sequence k, or in other words, $z_{i(k+1)} = 0$. If $z_{i(k+1)} = 1$, this constraint becomes inactive due to the parameter M_2, which is sufficiently large to enforce this disjunction. Constraints (2.9) and (2.10) are the same as in the work by Lemos (2020). Constraint (2.12) was inspired by the same work, incorporating setup time.

Constraints (2.13) and (2.14) ensure that $y_{pk} = 1$ if pattern p was cut in sequence k, and in a given sequence k, at most one type of pattern van be cut. However, the same type of pattern can be cut multiple times in the same sequence. Constraint (2.15) establishes that $s_{prk} = 1$ if there will be a setup between cutting patterns p and r, with pattern p being cut in sequence k and the pattern r being cut in sequence $k + 1$.

An important feature of this model is to allow different values to be assigned to W_1 or W_2, assigning greater weight to minimize the total setup time or to minimize delays in meeting demand. This allows flexibility for solving the problem, from a practical point of view, and different values can be applied depending on the objective and production context.

2.3.1 Difficulties in the proposed model

In the model proposed by Lemos (2020), empty production sequences were allowed. Empirical tests with the model proposed and adapted for this work revealed a challenge: the definition of the parameter NK. A very large NK value allowed for several empty production sequences because in this way setup times were not computed (they were only computed between consecutive sequences). In Chapter 5, the strategy considered for defining the parameter NK is discussed. The difficult in modeling in a way that accurately accounted for setup times prompted the proposal of alternative approaches, presented in the following section.

3 Resolution Methodology for Packaging IC-SSP

The search for innovative solutions to address the challenge of NP-hard problems is a constant in optimization research. Often, the initial mathematical model faces significant difficulties when confronted with instances of considerable size, leading to excessive demands on computational time.

In this context, it is worth analyzing and comparing these alternative approaches with the exact method proposed earlier. In doing so, it is crucial to consider the dynamics of factory scheduling, specifically the planning horizon. While the exact method can offer precision and optimal guarantees, its scalability can become a practical obstacle when dealing with a large volume of data or an extensive planning horizon.

On the other hand, alternate approaches can bring innovation to the resolution process, prioritizing computational efficiency over absolute accuracy. They can be particularly useful when seeking timely solutions, considering the constantly evolving demands and constraints of a factory.

Ultimately, the choice between these approaches should be carefully weighted, taking into account the nature of the problem, the size of the instance, time constraints and specific optimization objectives in the factory. The pursuit of a balance between precision and efficiency is essential to achieve practical and viable solutions in complex industrial environments.

3.1 FFD + Solver

Computational tools that solve mathematical programming problems have an algebraic modeling language to write the objective function to be optimized and the constraints considered in the model. After implementing the model, the program is executed by a solver. There are several commercial packages and free packages available to solve entire linear programming problems. In general, they differ in the methods they implement and the types of problems they can solve. In this work, we used the commercial solver CPLEX[1]

The FFD heuristic generates the cutting plans necessary to meet the demand, and the solver CPLEX solves the model presented in Section 2.3. Repeated cutting plans, generated by the FFD heuristic to meet demand, are considered once as input to the

[1] IBM ILOG CPLEX Optimization Studio, academic version.

optimization model, and the solver calculates how many times each cutting plan will be used.

3.2 FFD + Genetic Algorithm

Considering this integrated lot-sizing and cutting stock problem is NP-hard, using alternative approaches, such as Genetic Algorithm, is more appropriate. The FFD heuristic generates the cutting plans necessary to meet the demand, and the Genetic Algorithm (GA) generates the sequence in which the machine must cut.

Genetic Algorithms are methods that simulate, through algorithms, the processes of natural evolution aiming, mainly, to solve optimization problems (BARCELLOS, 2000). According to Cluitmans (1992), Genetic Algorithms try to find the optimum or at least guarantee a quality solution of a given set according to a given cost function. The idea behind genetic algorithms is quite simple. First, a set of genotypes is created randomly. This set is called the population. In the population, the cost function (also called fitness) is calculated. An individual in the population (genotype) is said to be fit when it has a higher cost than most other genotypes in the population (assuming it is a maximization problem).

A new population is created based on the appropriate genotypes from the previous population. The genotypes in the new population are created by making small changes to fit the genotypes of the old (mutating) population or by combining parts of different genotypes suitable for a new genotype (crossover). The fittest genotypes from the old population are copied into the new population as well. By repeatedly creating new populations, the average fitness of the genotypes will get better for each new population. The best genotype in the last generated population is the solution that the genetic algorithm offers.

There are many advantages of genetic algorithms over traditional optimization algorithms. Two most notable are: the ability of dealing with complex problems and parallelism. Genetic algorithms can deal with various types of optimization, whether the objective (fitness) function is stationary or non-stationary (change with time), linear or nonlinear, continuous or discontinuous, or with random noise. Because multiple off-springs in a population act like independent agents, the population (or any subgroup) can explore the search space in many directions simultaneously. This feature makes it ideal to parallel the algorithms for implementation. Different parameters and even different groups of encoded strings can be manipulated at the same time.

However, Genetic Algorithms also have some disadvantages. The formulation of fitness function, the use of population size, the choice of the important parameters such as the rate of mutation and crossover, and the selection criteria of the new population should be carried out carefully. Any inappropriate choice will make it difficult for the algorithm to converge or it will simply produce meaningless results. Despite these drawbacks, genetic algorithms remain one of the most widely used optimization algorithms in modern and complex optimization (YANG, 2021).

3.3 Proposal and implementation details

In GA, repeated cutting plans generated by the FFD heuristic to meet demand are permitted and form the set $\{1, 2, ..., NP\}$.

An individual is encoded using a vector of integers of size NP. Each position in this vector can take non-repeating values from the set $\{1, 2, ..., NP\}$. This value defines the processing order of cutting patterns (Figure 17). In this figure, the first cutting pattern to be processed is 3, followed successively by patterns 7, 6, 4, 9, and so on.

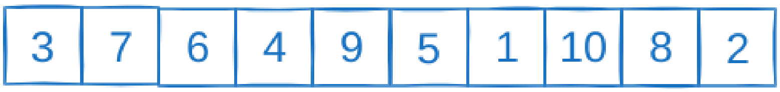

Figure 17 – GA: individual coding.

The Algorithm 3 starts with input of the instance parameters: the number of items, the number of jumbo coils, the quantity demanded of each item, the deadline for delivery of the demand, the width of each item, the width of the jumbo coil, the refill which must be considered in the cutting pattern, the processing time of cutting patterns associated with a specific type of jumbo coil (P_j) and the setup time between two types of jumbo coil (S_{jl}), followed by the programming window and the number of hours per day with the machine in production. We also specify the population size, number of generations, and crossover and mutation probabilities established for the GA.

The result of the FDD + GA Algorithm for ILSCSP is the machine configuration (as shown in the Tables 4.2.2.1, 4.2.2.1, 4.2.2.1).

The FFD heuristic is called and returns the NP cutting patterns created to meet the demand. The next steps refer to the association of processing times with each cutting pattern generated and setup times between all cutting patterns, according to the jumbo coils.

The population of size TP is initialized as follows:

- An individual is created by ordering subgroups of cutting patterns; subgroups aggregate patterns associated with the same type of jumbo coil; we order the subgroups SG_1, SG_2, ... SG_{NT}, considering that between SG_1 and SG_2 there is the shortest setup time, and we continue like this successively. The objective of this procedure is to insert a priori knowledge into the population: ordered blocks that carry information that can speed up the search process.

- Part of the individuals is generated considering subgroups of cutting patterns originating from jumbo coils of the same type. By doing this, we introduce a priori information into the initial population that can help minimize setup time. As the number of combinations in the ordering of these subgroups can be huge, depending on the size of the instance, some combinations were inserted into the initial population randomly.

- The third and largest part of the population is initialized randomly.

The initial population is decoded by calculating the processing start time for each pattern. In this way, we can calculate the fitness (adaptation function) associated with each individual.

Then, the iterative and evolutionary process begins with selection, reproduction with genetic inheritance (crossover), the introduction of random variations (mutation), and the promotion of competition to form a new population. One-point crossover and mutation based on position exchange between two patterns were implemented.

Algorithm 3 – ILSCSP - FDD + GA Algorithm

Data: NI, NT, b_i, d_i, l_i, L_j, R, C, P_j, S_{jl}, $Prog$, H, TP, G, PC, PM, W_1, W_2
Result: Machine configuration
a_{ip}, NP = Create the cutting patterns with FFD heuristic (NI, NT, b_i, l_i, L_j, R, C);
t_p, s_{pr} = Association of processing times and setup times (a_{ip}, NP, NT, P_j, S_{jl});
pop = Population initialization (TP, NP, NT, S_{jl});
X_p = Population decoding (TP, pop, NP, t_p, s_{pr});
$fitness$ = Population evaluation (X_p, NP, NI, W_1, W_2, d_i, s_{pr});
for $g = 1 \dots G$ **do**
  $parents$ = Binary tournament(TP, pop, $fitness$);
  $offspring$ = Crossover ($parents$, TP, NP, PC);
  Mutation ($offspring$, TP, NP, PM);
  XO_p = Population decoding (TP, $offspring$, NP, t_p, s_{pr});
  $fitnessO$ = Population evaluation (XO_p, NP, NI, W_1, W_2, d_i, s_{pr});
  $mixed = pop \cup offspring$;
  pop = update population($mixed$, TP);
end
Presentation of results and visualization ;

The algorithm was implemented in Python language, IDE Jupyter Notebook (open-source web application).

4 Experiments and Results

Considering the specificity of the pharmaceutical packaging industry, the author used data based on the real problem to evaluate the algorithm's effectiveness. It was also created custom instances, designed to simulate strategic scenarios and allowing the solution to be rigorously tested under varying conditions. This approach not only demonstrated the adaptability of the solution but also provided valuable insights into potential improvements and further developments in the field.

4.1 Instances

The decision to initiate producing internally the packaging materials (coils) stemmed from the imperative to minimize costs and decreasing reliance on external suppliers. Recognizing that the primary focus of a pharmaceutical industry lies outside packaging, and consequently lacking the essential proficiency in this domain, a deliberate evaluation led to the initiation of manufacturing packaging coils internally.

Each coil will have a weight of 30 kilograms, a specification imposed by the unwinding constraint of the blister machine forming material. Consequently, the initial premise involved calculating the processing time for each material, factoring in this weight restriction. While certain parameters from the machine manufacturer, such as machine speed and slitter pressure, were taken into account, it's important to note that these specifications are rooted in a stabilized process. Given the start-up phase and the need to acclimate to this new process, process times were estimated to facilitate the learning curve (table 2).

Material	Process time (minutes)
PVC	2
PVDC	4

Table 2 – Process time for each material

The packaging coils consist of PVC and PVDC materials. Although aluminum was also utilized in the packaging process, it was omitted from the initial consideration due to its thickness and challenging processability. Additionally, sourcing jumbo aluminum rolls from the market proved challenging, necessitating the importation from global suppliers.

The chosen setup times (table 3) were determined by utilizing predefined parameters provided by the slitter machine manufacturer. These setup times encompass a thoughtful analysis of both the incoming material from the jumbo coil and the succeeding material scheduled for the cutting process. This approach holds true when working with the same type of material in consecutive runs. In such cases, essential adaptations and preparatory steps become imperative. theses include tasks such as transferring the coil onto the spindle or clearing out the completed items from the production line.

Jumbo coil materials (From - To)	Setup time (minutes)
PVC - PVDC	15
PVDC - PVC	15
PVC - PVC	5
PVDC - PVDC	5

Table 3 – Setup times for each material

4.1.1 Instance 0 - base scenario

The initial scenario (Appendix A) was designed for algorithm testing with a focus on a small-scale item production, incorporating all the parameters detailed in the preceding section. The primary objective is to assess the algorithm's performance and present the generated outputs, which are intended to assist the production planning decision-maker for a 5-day production horizon.

4.1.2 Instance 1 - real-world based scenario

Real-world based instance was inspired by data from a pharmaceutical industry (Appendix A). The demand (quantity of items) under consideration constituted a portion of the overall demand for blister machines. This decision was motivated by the intention to deplete the existing coil stock from other suppliers and to ensure a seamless supply chain during the initial phase of the new slitter machine's implementation.

Another important aspect to elucidate is the demand factor. It is imperative that all the coils essential for sustaining blister production be fabricated in advance, ensuring the availability of all required materials prior to beginning the complete medication line production setup. To address this, the proposed approach involves producing all coils with high demand within the initial ten days of each month, subsequently allowing for the production of the remaining materials.

4.1.3 Instance 2 - production increased

The second instance stems from the first, resulting in a 40% surge in production (table 14). Furthermore, the operational hours extend to 24 hours, with an additional shift compared to the alternate scenario (Appendix A).

4.2 Results

Testing has been completed on the operating system Linux, Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz × 8, 64Gb of RAM memory. Each instance was executed via the following approaches: 1. FFD+CPLEX (Section 3.1) and 2. FFD+GA (Section 3.2).

The same cutting patterns generated by the FFD heuristic were used in both approaches. The execution times for cutting patterns generation are presented in Table 4.2. In the same table, NP indicates the number of different cutting patterns generated by the heuristic; to meet the demand, a cutting pattern can be used more than once.

	Instance 0	**Instance 1**	**Instance 2**
FFD (s)	0.00027	0.00328	0.00442
NP total	65	1019	1421
NP without repetition	20	46	46

Table 4 – FFD results.

4.2.1 FFD + Solver

The results obtained with the CPLEX solver are presented in Table 4.2.1. We established $W_1 = 1$, $W_2 = 1$, $W_3 = 1$ as weights for the different components of the objective function (Equation 2.8). Due to the difficulty in determining the number of production sequences, NK, and aiming to minimize the number of empty time sequences and the exponential growth of the problem's variables and constraints, we chose to limit the value of $NK = NP$. In this way, the intention was to force cutting patterns of the same type to be cut together in the same time sequence, as one of the components of the objective function is related to minimizing setup time. The more production sequences are used, the more changes in cutting patterns or coil types occur, and the longer the setup time on the machine. Despite this limitation on the value of NK, the obtained solution (NK useful) indicates that not all production sequences were used.

	Instance 0	**Instance 1**	**Instance 2**
Number of variables	9680	103730	103730
Number of restrictions	9321	101706	101706
CPLEX (s)	1803.5	1800.5	1800.4
gap	0.3936	0.3301	0.3289
total delay (hours)	12.50	35.8	24.56
total setup (hours)	3.33	6.25	6.41
useful NK	19	44	44

Table 5 – FFD+Solver results, $W_1 = 1$, $W_2 = 1$, $W_3 = 1$.

4.2.2 FFD + GA

Due to the random nature of Genetic Algorithms, 10 rounds of tests were executed for each instance. Regarding the GA, we empirically set the crossover probability to $PC = 0.95$ and the mutation probability to $PM = 0.1$, with the implementation of binary tournament as the method for selecting individuals for crossover. Furthermore, for each instance, the following set of parameters (population size and number of generations) were used: $TP = 200$, $G = 400$ (Instance 0); $TP = 200$, $G = 600$ (Instance 1); $TP = 200$, $G = 600$ (Instance 2). The parameters were empirically defined by performing various tunning tests.

In this initial testing phase, we established $W_1 = 1$ and $W_2 = 1$ for the weighting of the two components considered in the GA's evaluation function: setup time and total delay in item delivery, respectively. In the GA, there was no need to consider the third component of the objective function, which minimizes the total number of cutting patterns (Equation 2.8), since the total number of cutting patterns generated by the FFD heuristic to meet the demand was considered, even if generated repeatedly. For each instance, the following total quantities of cutting patterns were generated (assuming repetitions) by the FFD heuristic: 65 (Instance 0); 1019 (Instance 1); 1019 (Instance 2).

Table 4.2.2 presents the results obtained with FFD+GA for Instance 0, considering the 10 rounds of testing. The results obtained with the metaheuristic were better than the result presented by the solver, especially regarding the total delay time. Figures 18 and 19 refer to the evolution of the fitness of the best individual in the population and the average fitness of the population over the generations in one of the execution rounds. The other rounds exhibited similar behavior. In the graph, fitness represents $W_1\cdot$ (total setup) $+W_2\cdot$ (total delay) in minutes. Further information about the solution generated by the best execution (round 9) is presented in Appendix B.

Round	**total setup (hours)**	**total delay (hours)**	**execution time (s)**
1	1.58	2.15	12.45
2	1.75	2.35	12.14
3	1.58	2.15	12.13
4	1.58	2.15	12.23
5	1.58	2.15	12.31
6	1.58	2.15	12.53
7	1.58	2.15	12.29
8	1.58	2.15	12.62
9	1.75	2.27	12.33
10	1.58	2.15	12.41
average	1.61	2.18	12.34

Table 6 – FFD+GA, Instance 0 results, $W_1 = 1$, $W_2 = 1$.

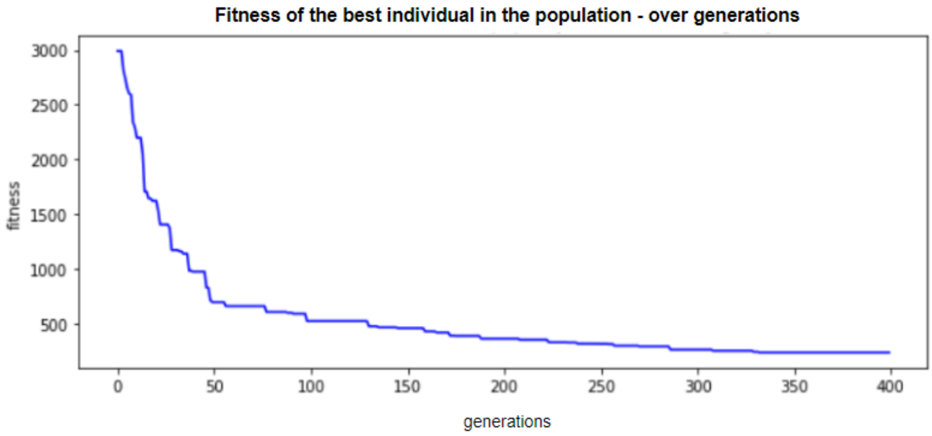

Figure 18 – Evolution of the fitness of the best individual in the population, Instance 0, FFD+GA.

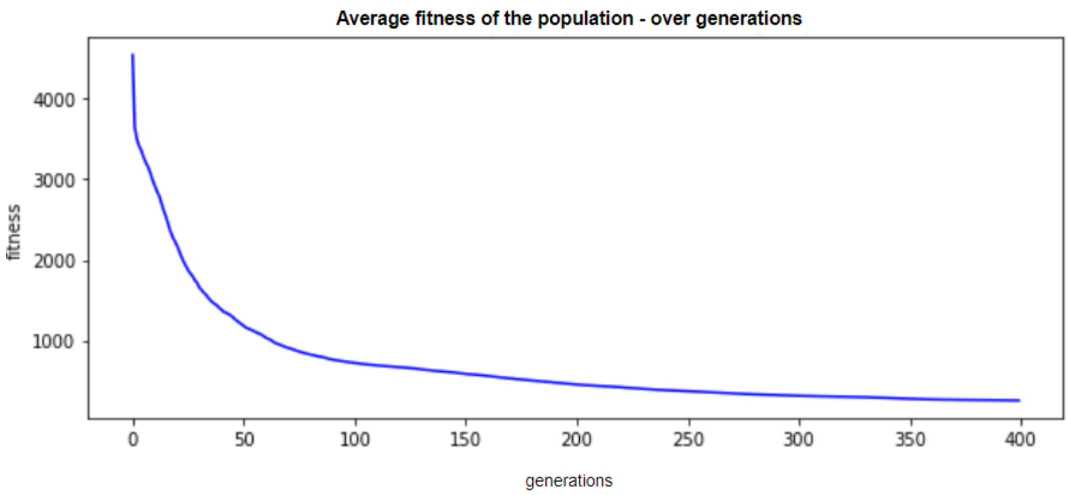

Figure 19 – Evolution of the average fitness of the population, Instance 0, FFD+GA.

Round	total setup (hours)	total delay (hours)	execution time (s)
1	5.58	16.92	745.50
2	6.42	23.15	754.03
3	5.58	18.13	749.02
4	7.25	23.57	794.51
5	5.58	21.82	722.30
6	7.08	6.08	752.99
7	6.92	13.97	746.93
8	6.08	19.97	752.27
9	6.58	27.42	747.90
10	6.42	11.80	752.27
average	6.34	18.28	751.77

Table 7 – FFD+GA, Instance 1 results, $W_1 = 1$, $W_2 = 1$.

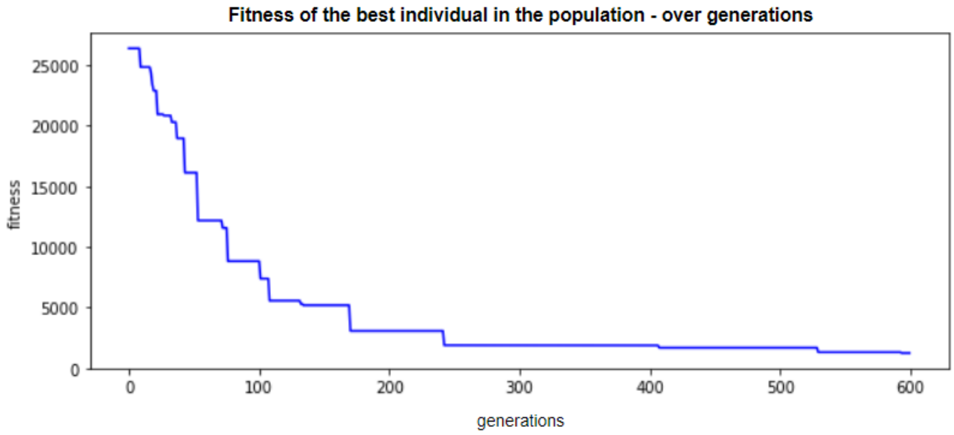

Figure 20 – Evolution of the fitness of the best individual in the population, Instance 1, FFD+GA.

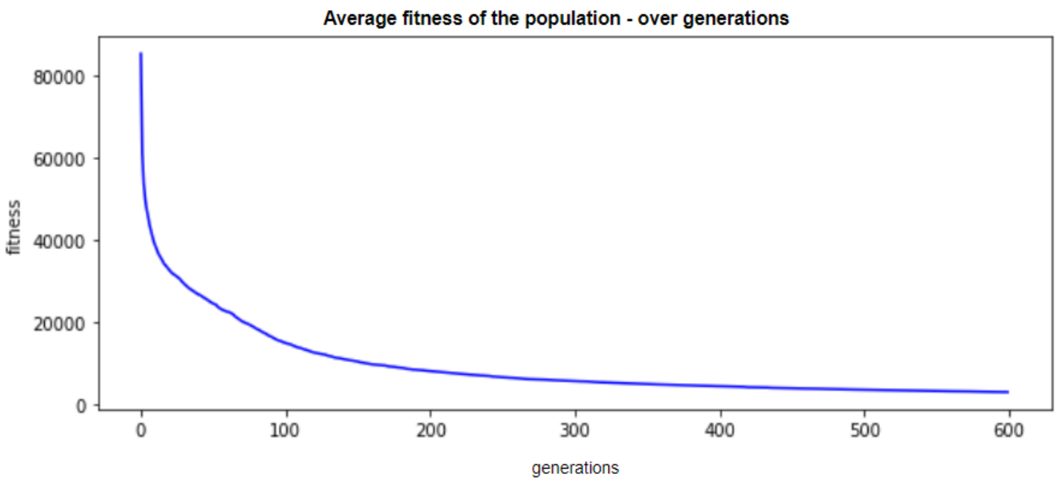

Figure 21 – Evolution of the average fitness of the population, Instance 1, FFD+GA.

Round	total setup (hours)	total delay (hours)	execution time (s)
1	6.42	0.00	1251.64
2	6.75	8.80	1255.29
3	6.75	10.68	1255.51
4	7.42	13.43	1253.24
5	6.42	15.43	1247.40
6	6.58	2.90	1251.20
7	5.75	1.37	1251.64
8	6.92	11.23	1248.73
9	5.75	0.00	1252.57
10	7.08	5.12	1247.98
average	6.58	6.89	1251.52

Table 8 – FFD+GA, Instance 2 results, $W_1 = 1$, $W_2 = 1$.

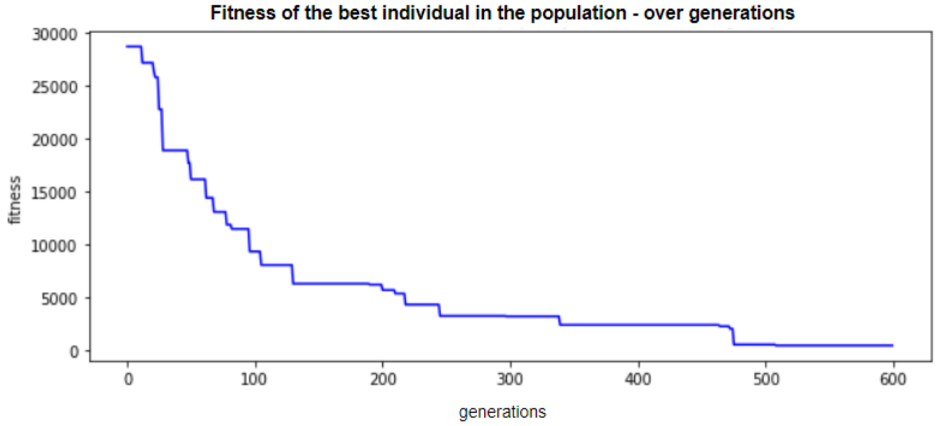

Figure 22 – Evolution of the fitness of the best individual in the population, Instance 2, FFD+GA.

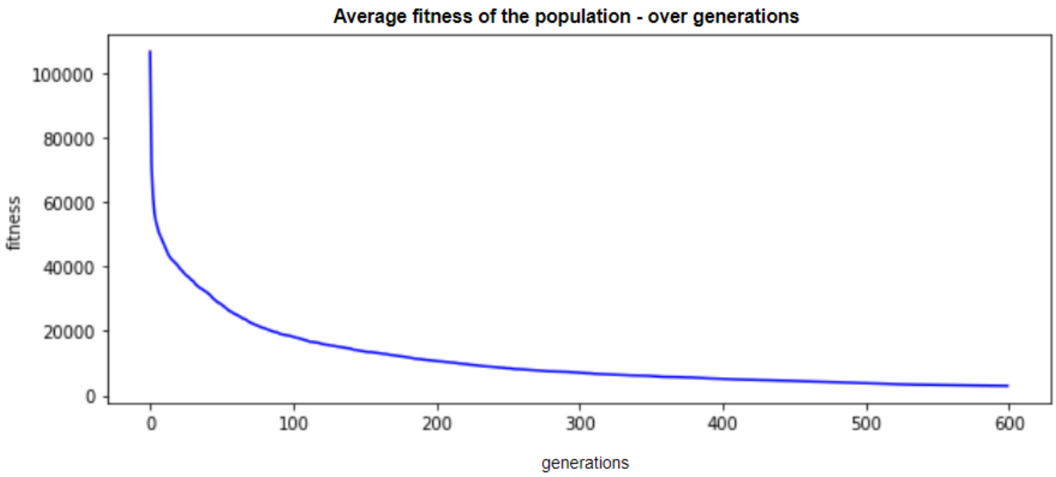

Figure 23 – Evolution of the average fitness of the population, Instance 2, FFD+GA.

In all tested instances, GA achieved a better setup time or the same order of magnitude as CPLEX, but always with lower computational time. It is worth mentioning that in all cases, the delay time obtained by GA was considerably smaller than that obtained by CPLEX. The FFD+GA proposal can be promising in solving this type of problem in a real context, applied in the pharmaceutical industry, with the requirement to present a solution in low computational time and using an open software approach.

There is still the possibility of improving these results by increasing the size of the population and the number of generations. It is also noted that at the beginning of the evolutionary process, fitness drops more quickly; this decay becomes slower towards the end, but without necessarily indicating stagnation. Implementing a local search could introduce variability and improve results at the end of the evolutionary process.

4.2.2.1 Variation of weights: W_1 and W_2

Tables 4.2.2.1, 4.2.2.1 and 4.2.2.1 present the results obtained with GA for Instance 0, considering the variation in the weights of the two portions of objective function: setup and delay. Considering only the setup minimization ($W_1 = 1$ and $W_2 = 0$, we obtain a total setup time equal to 1.58 hours. Considering only the delay minimization ($W_1 = 0$ and $W_2 = 1$), we obtain a total delay time equal to 2.15 hours. And it is precisely this combination of values that we obtain when we equally weight the two portions of the objective function.

GA seems to deal better with the weighted objective function than CPLEX. This may be explained by the need to include in the mathematical model solved by CPLEX the weighting responsible for minimizing the number of cutting patterns: $W_3 \sum_{p=1}^{NP} \sum_{k=1}^{NK} x_{pk}$, an incommensurable portion in relation to the first two, which does not happen with GA.

Regarding cutting patterns, the solution that minimizes the setup (second column of Tables 4.2.2.1, 4.2.2.1 and 4.2.2.1) tends to alternate fewer cutting configurations than they use different jumbo coils, as expected. But this makes the delay increase too much.

The variation in weights makes it an interesting approach when the decision maker wants to analyze different scenarios for machine configuration, depending on the strategies, which can vary between minimizing the setup, minimizing the delay, or a solution that includes both.

$W_1 = 1$, $W_2 = 1$	$W_1 = 1$, $W_2 = 0$	$W_1 = 0$, $W_2 = 1$
total setup (hours) = 1.58 total delay (hours) = 2.15	total setup (hours) = 1.58 total delay (hours) = 57.17	total setup (hours) = 2.75 total delay (hours) = 2.15
Configuration 1 - Jumbo coil 2 4 items with width 250	Configuration 1 - Jumbo coil 2 4 items with width 250	Configuration 1 - Jumbo coil 2 5 items with width 236
Configuration 2 - Jumbo coil 2 5 items with width 219	Configuration 2 - Jumbo coil 2 4 items with width 250	Configuration 2 - Jumbo coil 2 5 items with width 211
Configuration 3 - Jumbo coil 1 5 items with width 231	Configuration 3 - Jumbo coil 2 4 items with width 250	Configuration 3 - Jumbo coil 1 7 items with width 165
Configuration 4 - Jumbo coil 1 7 items with width 165	Configuration 4 - Jumbo coil 2 4 items with width 250	Configuration 4 - Jumbo coil 2 5 items with width 219
Configuration 5 - Jumbo coil 2 5 items with width 236	Configuration 5 - Jumbo coil 2 4 items with width 250	Configuration 5 - Jumbo coil 2 5 items with width 211
Configuration 6 - Jumbo coil 2 5 items with width 211	Configuration 6 - Jumbo coil 2 4 items with width 250	Configuration 6 - Jumbo coil 2 4 items with width 250
Configuration 7 - Jumbo coil 2 5 items with width 211	Configuration 7 - Jumbo coil 2 4 items with width 250	Configuration 7 - Jumbo coil 3 5 items with width 219
Configuration 8 - Jumbo coil 4 5 items with width 236	Configuration 8 - Jumbo coil 2 5 items with width 236	Configuration 8 - Jumbo coil 4 5 items with width 236
Configuration 9 - Jumbo coil 3 5 items with width 219	Configuration 9 - Jumbo coil 2 5 items with width 236	Configuration 9 - Jumbo coil 5 5 items with width 236
Configuration 10 - Jumbo coil 3 4 items with width 287	Configuration 10 - Jumbo coil 2 5 items with width 219	Configuration 10 - Jumbo coil 3 4 items with width 287
Configuration 11 - Jumbo coil 3 4 items with width 250 1 items with width 165	Configuration 11 - Jumbo coil 2 5 items with width 219	Configuration 11 - Jumbo coil 3 5 items with width 219
Configuration 12 - Jumbo coil 3 7 items with width 165	Configuration 12 - Jumbo coil 2 5 items with width 211	Configuration 12 - Jumbo coil 5 5 items with width 211
Configuration 13 - Jumbo coil 5 5 items with width 211	Configuration 13 - Jumbo coil 2 5 items with width 211	Configuration 13 - Jumbo coil 5 5 items with width 231
Configuration 14 - Jumbo coil 3 5 items with width 219	Configuration 14 - Jumbo coil 2 5 items with width 211	Configuration 14 - Jumbo coil 5 5 items with width 211
Configuration 15 - Jumbo coil 3 5 items with width 219	Configuration 15 - Jumbo coil 2 5 items with width 211	Configuration 15 - Jumbo coil 3 7 items with width 165
Configuration 16 - Jumbo coil 5 5 items with width 231	Configuration 16 - Jumbo coil 4 5 items with width 236	Configuration 16 - Jumbo coil 5 5 items with width 231
Configuration 17 - Jumbo coil 5 5 items with width 231	Configuration 17 - Jumbo coil 4 5 items with width 236	Configuration 17 - Jumbo coil 3 5 items with width 219
Configuration 18 - Jumbo coil 5 5 items with width 211	Configuration 18 - Jumbo coil 4 5 items with width 236	Configuration 18 - Jumbo coil 3 4 items with width 250 1 items with width 165
Configuration 19 - Jumbo coil 3 5 items with width 219	Configuration 19 - Jumbo coil 4 5 items with width 236	Configuration 19 - Jumbo coil 3 5 items with width 219
Configuration 20 - Jumbo coil 5 5 items with width 236	Configuration 20 - Jumbo coil 4 5 items with width 236	Configuration 20 - Jumbo coil 1 5 items with width 231
Configuration 21 - Jumbo coil 5 5 items with width 231	Configuration 21 - Jumbo coil 5 5 items with width 236	Configuration 21 - Jumbo coil 3 4 items with width 287
Configuration 22 - Jumbo coil 3 5 items with width 219	Configuration 22 - Jumbo coil 5 5 items with width 236	Configuration 22 - Jumbo coil 3 5 items with width 219

Table 9 – FFD+GA, Instance 0 results, variation of weights: W_1 and W_2 (part 1).

$W_1 = 1, W_2 = 1$	$W_1 = 1, W_2 = 0$	$W_1 = 0, W_2 = 1$
total setup (hours) = 1.58 total delay (hours) = 2.15	total setup (hours) = 1.58 total delay (hours) = 57.17	total setup (hours) = 2.75 total delay (hours) = 2.15
Configuration 23 - Jumbo coil 1 7 items with width 165	Configuration 23 - Jumbo coil 5 5 items with width 236	Configuration 23 - Jumbo coil 2 4 items with width 250
Configuration 24 - Jumbo coil 4 5 items with width 236	Configuration 24 - Jumbo coil 5 5 items with width 236	Configuration 24 - Jumbo coil 3 5 items with width 219
Configuration 25 - Jumbo coil 5 5 items with width 211	Configuration 25 - Jumbo coil 5 5 items with width 236	Configuration 25 - Jumbo coil 4 5 items with width 236
Configuration 26 - Jumbo coil 3 5 items with width 219	Configuration 26 - Jumbo coil 5 5 items with width 236	Configuration 26 - Jumbo coil 3 5 items with width 219
Configuration 27 - Jumbo coil 5 5 items with width 231	Configuration 27 - Jumbo coil 5 5 items with width 236	Configuration 27 - Jumbo coil 3 5 items with width 219
Configuration 28 - Jumbo coil 2 4 items with width 250	Configuration 28 - Jumbo coil 5 5 items with width 231	Configuration 28 - Jumbo coil 5 5 items with width 236
Configuration 29 - Jumbo coil 4 5 items with width 236	Configuration 29 - Jumbo coil 5 5 items with width 231	Configuration 29 - Jumbo coil 5 5 items with width 211
Configuration 30 - Jumbo coil 3 5 items with width 219	Configuration 30 - Jumbo coil 5 5 items with width 231	Configuration 30 - Jumbo coil 2 4 items with width 250
Configuration 31 - Jumbo coil 5 5 items with width 236	Configuration 31 - Jumbo coil 5 5 items with width 231	Configuration 31 - Jumbo coil 3 4 items with width 287
Configuration 32 - Jumbo coil 3 4 items with width 287	Configuration 32 - Jumbo coil 5 5 items with width 231	Configuration 32 - Jumbo coil 3 5 items with width 219
Configuration 33 - Jumbo coil 3 4 items with width 287	Configuration 33 - Jumbo coil 5 5 items with width 231	Configuration 33 - Jumbo coil 3 7 items with width 165
Configuration 34 - Jumbo coil 5 5 items with width 236	Configuration 34 - Jumbo coil 5 5 items with width 211	Configuration 34 - Jumbo coil 1 7 items with width 165
Configuration 35 - Jumbo coil 4 5 items with width 236	Configuration 35 - Jumbo coil 5 5 items with width 211	Configuration 35 - Jumbo coil 5 5 items with width 236
Configuration 36 - Jumbo coil 5 5 items with width 231	Configuration 36 - Jumbo coil 5 5 items with width 211	Configuration 36 - Jumbo coil 3 5 items with width 219
Configuration 37 - Jumbo coil 5 5 items with width 236	Configuration 37 - Jumbo coil 5 5 items with width 211	Configuration 37 - Jumbo coil 5 5 items with width 236
Configuration 38 - Jumbo coil 5 5 items with width 236	Configuration 38 - Jumbo coil 5 5 items with width 211	Configuration 38 - Jumbo coil 5 5 items with width 211
Configuration 39 - Jumbo coil 3 5 items with width 219	Configuration 39 - Jumbo coil 5 5 items with width 211	Configuration 39 - Jumbo coil 2 5 items with width 211
Configuration 40 - Jumbo coil 3 5 items with width 219	Configuration 40 - Jumbo coil 3 4 items with width 287	Configuration 40 - Jumbo coil 3 7 items with width 165
Configuration 41 - Jumbo coil 4 5 items with width 236	Configuration 41 - Jumbo coil 3 4 items with width 287	Configuration 41 - Jumbo coil 5 5 items with width 236
Configuration 42 - Jumbo coil 3 7 items with width 165	Configuration 42 - Jumbo coil 3 4 items with width 287	Configuration 42 - Jumbo coil 3 4 items with width 287
Configuration 43 - Jumbo coil 5 5 items with width 211	Configuration 43 - Jumbo coil 3 4 items with width 287	Configuration 43 - Jumbo coil 3 4 items with width 287
Configuration 44 - Jumbo coil 2 5 items with width 236	Configuration 44 - Jumbo coil 3 4 items with width 287	Configuration 44 - Jumbo coil 5 5 items with width 236

Table 10 – FFD+GA, Instance 0 results, variation of weights: W_1 and W_2 (part 2).

$W_1 = 1, W_2 = 1$	$W_1 = 1, W_2 = 0$	$W_1 = 0, W_2 = 1$
total setup (hours) = 1.58 total delay (hours) = 2.15	total setup (hours) = 1.58 total delay (hours) = 57.17	total setup (hours) = 2.75 total delay (hours) = 2.15
Configuration 45 - Jumbo coil 5 5 items with width 211	Configuration 45 - Jumbo coil 3 4 items with width 250 1 items with width 165	Configuration 45 - Jumbo coil 1 7 items with width 165
Configuration 46 - Jumbo coil 3 7 items with width 165	Configuration 46 - Jumbo coil 3 5 items with width 219	Configuration 46 - Jumbo coil 4 5 items with width 236
Configuration 47 - Jumbo coil 5 5 items with width 236	Configuration 47 - Jumbo coil 3 5 items with width 219	Configuration 47 - Jumbo coil 5 5 items with width 211
Configuration 48 - Jumbo coil 3 4 items with width 287	Configuration 48 - Jumbo coil 3 5 items with width 219	Configuration 48 - Jumbo coil 1 7 items with width 165
Configuration 49 - Jumbo coil 3 4 items with width 287	Configuration 49 - Jumbo coil 3 5 items with width 219	Configuration 49 - Jumbo coil 5 5 items with width 231
Configuration 50 - Jumbo coil 5 5 items with width 236	Configuration 50 - Jumbo coil 3 5 items with width 219	Configuration 50 - Jumbo coil 5 5 items with width 231
Configuration 51 - Jumbo coil 3 5 items with width 219	Configuration 51 - Jumbo coil 3 5 items with width 219	Configuration 51 - Jumbo coil 2 4 items with width 250
Configuration 52 - Jumbo coil 5 5 items with width 211	Configuration 52 - Jumbo coil 3 5 items with width 219	Configuration 52 - Jumbo coil 5 5 items with width 236
Configuration 53 - Jumbo coil 5 5 items with width 231	Configuration 53 - Jumbo coil 3 5 items with width 219	Configuration 53 - Jumbo coil 5 5 items with width 231
Configuration 54 - Jumbo coil 3 5 items with width 219	Configuration 54 - Jumbo coil 3 5 items with width 219	Configuration 54 - Jumbo coil 2 4 items with width 250
Configuration 55 - Jumbo coil 3 5 items with width 219	Configuration 55 - Jumbo coil 3 5 items with width 219	Configuration 55 - Jumbo coil 4 5 items with width 236
Configuration 56 - Jumbo coil 2 4 items with width 250	Configuration 56 - Jumbo coil 3 5 items with width 219	Configuration 56 - Jumbo coil 2 5 items with width 236
Configuration 57 - Jumbo coil 2 4 items with width 250	Configuration 57 - Jumbo coil 3 5 items with width 219	Configuration 57 - Jumbo coil 2 5 items with width 211
Configuration 58 - Jumbo coil 2 5 items with width 211	Configuration 58 - Jumbo coil 3 7 items with width 165	Configuration 58 - Jumbo coil 2 4 items with width 250
Configuration 59 - Jumbo coil 1 7 items with width 165	Configuration 59 - Jumbo coil 3 7 items with width 165	Configuration 59 - Jumbo coil 5 5 items with width 211
Configuration 60 - Jumbo coil 1 7 items with width 165	Configuration 60 - Jumbo coil 3 7 items with width 165	Configuration 60 - Jumbo coil 2 5 items with width 219
Configuration 61 - Jumbo coil 2 5 items with width 211	Configuration 61 - Jumbo coil 1 5 items with width 231	Configuration 61 - Jumbo coil 3 5 items with width 219
Configuration 62 - Jumbo coil 2 4 items with width 250	Configuration 62 - Jumbo coil 1 7 items with width 165	Configuration 62 - Jumbo coil 5 5 items with width 231
Configuration 63 - Jumbo coil 2 4 items with width 250	Configuration 63 - Jumbo coil 1 7 items with width 165	Configuration 63 - Jumbo coil 4 5 items with width 236
Configuration 64 - Jumbo coil 2 5 items with width 219	Configuration 64 - Jumbo coil 1 7 items with width 165	Configuration 64 - Jumbo coil 3 5 items with width 219
Configuration 65 - Jumbo coil 2 4 items with width 250	Configuration 65 - Jumbo coil 1 7 items with width 165	Configuration 65 - Jumbo coil 2 4 items with width 250

Table 11 – FFD+GA, Instance 0 results, variation of weights: W_1 and W_2 (part 3).

5 Conclusion and Future Perspectives

The main objective of this work was to provide a practical solution to a problem commonly encountered in the current manufacturing industry. The incorporation of new production processes into factories is a common situation for large companies striving for competitiveness. However, the complexity it adds to the production chain requires agility in making decisions with various variables and constraints. Therefore, any proposed solution needs to take into account the parameters and intrinsic characteristics of the production process and machines, allowing decision-makers to "customize" according to their needs. This was the primary concern of this research and the development of the solution using the FFD heuristic and Genetic Algorithm, where the user can configure various cutting and scheduling process parameters.

The class of cutting and scheduling problems has been extensively studied and researched academically. As presented in Chapter 3, various authors have proposed different ways to solve complex problems. However, considering the gap that exists in most cases between theory and practical problems, many studies do not consider the real aspects of the machines and processes involved, affecting the results. This work, on the other hand, aimed to incorporate various practical aspects to propose the best solution. An example of this is the ability to determine the amount of productive hours in the input data, allowing for different production schedules.

The model presented by Lemos (2020), which was the main reference for this thesis, brought a complex problem whose solution is based on an exact method. The use of this method is recommended for small-scale problems, but the cutting stock and scheduling problem can grow exponentially considering different instances or real industrial situations. For this reason, this work sought to use an alternative method with a Genetic Algorithm, where a satisfactory solution was found even using a large-scale instance (such as instance 2).

The computational processing times are also noteworthy when analyzing the results obtained. Comparing the results of FFD + CPLEX (average values of the rounds) with the FFD + Genetic Algorithm method, the execution time of the second method is significantly shorter. Regarding the results, the values of delay and total setup were also lower. This demonstrates that the Genetic Algorithm provides a viable and suitable solution in less time, making it more attractive to decision-makers. Another key point interesting for the decision-maker is the possibility to choose different weights for setup or delay. The mathematical model can be set up to analyze different scenarios, depending on the strategies, to minimize setup or delay, or a solution that includes both. In a production scenario where there may be situations with a lack of people or where the delivery of items may be flexible, giving this option to the decision maker is crucial for a good solution for the ICSLSP.

Future prospects for this work are extensive. Considering the importance of cutting machines and how much they can reduce costs compared to products purchased from third-party suppliers, this problem can incorporate a larger number of machines, not just one, as was addressed. It is also possible to explore other methods for the cutting problem and the pattern sequencing problem. There are other heuristics capable of dealing with this complex problem and potentially bringing even more speed and production efficiency. Finally, instances can also be approached differently. Obtaining data from large companies that perform coil cutting processes can be challenging, so using an instance generator to create scenarios for testing the solution's functionality will be a worthwhile avenue to explore.

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Appendix

APPENDIX A – Appendix A

Material	Jumbo coil type	Demand (kg/month)	Demand - items/month	Width item	D (days)
B115	PVC - 1	30	1	231	1
B126	PVC - 1	690	23	165	1
B112	PVC - 2	750	25	250	1
B114	PVC - 2	180	6	236	1
B122	PVC - 2	300	10	219	1
B124	PVC - 2	360	12	211	1
B129	PVC - 2	30	1	211	1
B110	PVC - 3	390	13	219	1
B111	PVC - 3	300	10	219	1
B118	PVC - 3	480	16	219	1
B119	PVC - 3	60	2	250	1
B125	PVC - 3	570	19	165	1
B127	PVC - 3	360	12	219	1
B128	PVC - 3	540	18	287	1
B113	PVDC - 1	750	25	236	1
B116	PVDC - 2	270	9	211	1
B117	PVDC - 2	540	18	211	1
B120	PVDC - 2	90	3	231	1
B121	PVDC - 2	1050	35	236	1
B123	PVDC - 2	690	23	231	1

Table 12 – Items parameters for instance 0 (base scenario)

Material	Jumbo coil type	Demand (kg/month)	Demand - items/month	Width item	D (days)
B111	PVC - 1	16,650	555	219	10
B121	PVC - 1	4,470	149	236	9
B132	PVC - 1	1,650	55	231	6
B137	PVC - 1	900	30	211	9
B141	PVC - 1	630	21	165	7
B110	PVC - 2	20,430	681	219	6
B112	PVC - 2	13,830	461	250	6
B115	PVC - 2	8,490	283	231	1
B116	PVC - 2	6,150	205	211	2
B126	PVC - 2	2,460	82	165	9
B117	PVC - 3	5,730	191	211	1
B120	PVC - 3	4,500	150	231	2
B125	PVC - 3	2,490	83	165	1
B128	PVC - 3	2,040	68	287	5
B130	PVC - 3	1,650	55	236	5
B139	PVC - 3	810	27	219	10
B144	PVC - 3	480	16	203	6
B143	PVC - 4	480	16	236	10
B150	PVC - 4	180	6	231	10
B155	PVC - 4	60	2	165	1
B119	PVC - 5	4,950	165	250	6
B123	PVC - 5	4,140	138	231	9
B135	PVC - 5	990	33	165	4
B140	PVC - 5	810	27	211	4
B148	PVC - 5	210	7	236	7
B151	PVC - 6	120	4	165	8
B134	PVC - 7	1,260	42	231	1
B149	PVC - 7	210	7	165	9
B152	PVC - 7	90	3	211	8
B138	PVDC - 1	810	27	211	3
B147	PVDC - 1	240	8	219	5
B127	PVDC - 2	2,280	76	219	3
B131	PVDC - 2	1,650	55	236	4
B133	PVDC - 2	1,440	48	211	10
B146	PVDC - 2	330	11	165	9
B153	PVDC - 2	90	3	231	5
B114	PVDC - 3	8,580	286	236	2
B118	PVDC - 3	5,250	175	219	4
B129	PVDC - 3	1,680	56	211	10
B136	PVDC - 3	930	31	165	4
B142	PVDC - 3	510	17	231	10
B113	PVDC - 4	10,320	344	236	9
B122	PVDC - 4	4,470	149	219	6
B124	PVDC - 4	3,900	130	211	1
B145	PVDC - 4	330	11	231	4
B154	PVDC - 4	60	2	165	1

Table 13 – Items parameters for instance 1 (real-world based)

Integrated

Material	Jumbo coil type	Demand on kg/month	Demand on items/month	Width item	D (days)
B111	PVC - 1	23,310	777	219	10
B121	PVC - 1	6,270	209	236	9
B132	PVC - 1	2,310	77	231	6
B137	PVC - 1	1,260	42	211	9
B141	PVC - 1	900	30	165	7
B110	PVC - 2	28,620	954	219	6
B112	PVC - 2	19,380	646	250	6
B115	PVC - 2	11,910	397	231	1
B116	PVC - 2	8,610	287	211	2
B126	PVC - 2	3,450	115	165	9
B117	PVC - 3	8,040	268	211	1
B120	PVC - 3	6,300	210	231	2
B125	PVC - 3	3,510	117	165	1
B128	PVC - 3	2,880	96	287	5
B130	PVC - 3	2,310	77	236	5
B139	PVC - 3	1,140	38	219	10
B144	PVC - 3	690	23	203	6
B143	PVC - 4	690	23	236	10
B150	PVC - 4	270	9	231	10
B155	PVC - 4	90	3	165	1
B119	PVC - 5	6,930	231	250	6
B123	PVC - 5	5,820	194	231	9
B135	PVC - 5	1,410	47	165	4
B140	PVC - 5	1,140	38	211	4
B148	PVC - 5	300	10	236	7
B151	PVC - 6	180	6	165	8
B134	PVC - 7	1,770	59	231	1
B149	PVC - 7	300	10	236	9
B152	PVC - 7	150	5	211	8
B138	PVDC - 1	1,140	38	211	3
B147	PVDC - 1	360	12	219	5
B127	PVDC - 2	3,210	107	219	3
B131	PVDC - 2	2,310	77	236	4
B133	PVDC - 2	2,040	68	211	10
B146	PVDC - 2	480	16	165	9
B153	PVDC - 2	150	5	231	5
B114	PVDC - 3	12,030	401	236	2
B118	PVDC - 3	7,350	245	219	4
B129	PVDC - 3	2,370	79	211	10
B136	PVDC - 3	1,320	44	165	4
B142	PVDC - 3	720	24	231	10
B113	PVDC - 4	14,460	482	236	9
B122	PVDC - 4	6,270	209	219	6
B124	PVDC - 4	5,460	182	211	1
B145	PVDC - 4	480	16	231	4
B154	PVDC - 4	90	3	165	1

Table 14 – Items parameters for instance 2 (production increased)

APPENDIX B – Appendix B

FFD+CPLEX - Instance 0
7 patterns of type 15 cut in sequence 0
2 patterns of type 4 cut in sequence 1
1 patterns of type 6 cut in sequence 2
4 patterns of type 17 cut in sequence 3
4 patterns of type 13 cut in sequence 4
4 patterns of type 9 cut in sequence 5
5 patterns of type 14 cut in sequence 6
7 patterns of type 2 cut in sequence 7
1 patterns of type 19 cut in sequence 8
5 patterns of type 16 cut in sequence 9
10 patterns of type 11 cut in sequence 10
3 patterns of type 5 cut in sequence 11
1 patterns of type 8 cut in sequence 12
4 patterns of type 7 cut in sequence 13
4 patterns of type 1 cut in sequence 14
2 patterns of type 18 cut in sequence 15
1 patterns of type 0 cut in sequence 16
2 patterns of type 3 cut in sequence 17
3 patterns of type 12 cut in sequence 18

FFD+CPLEX - Instance 1
1 patterns of type 45 cut in sequence 0
1 patterns of type 34 cut in sequence 1
136 patterns of type 6 cut in sequence 2
12 patterns of type 37 cut in sequence 3
30 patterns of type 14 cut in sequence 4
42 patterns of type 20 cut in sequence 5
11 patterns of type 10 cut in sequence 6
7 patterns of type 24 cut in sequence 7
1 patterns of type 25 cut in sequence 8
41 patterns of type 5 cut in sequence 9
17 patterns of type 11 cut in sequence 10
30 patterns of type 2 cut in sequence 11
11 patterns of type 0 cut in sequence 12
7 patterns of type 38 cut in sequence 13
1 patterns of type 19 cut in sequence 14
3 patterns of type 33 cut in sequence 15
111 patterns of type 1 cut in sequence 16
4 patterns of type 17 cut in sequence 17
16 patterns of type 32 cut in sequence 18
11 patterns of type 35 cut in sequence 19
2 patterns of type 29 cut in sequence 20
2 patterns of type 28 cut in sequence 21
6 patterns of type 3 cut in sequence 22
48 patterns of type 13 cut in sequence 23
26 patterns of type 42 cut in sequence 24
35 patterns of type 6 cut in sequence 25
1 patterns of type 27 cut in sequence 26
2 patterns of type 18 cut in sequence 27
9 patterns of type 26 cut in sequence 28
14 patterns of type 15 cut in sequence 29
6 patterns of type 21 cut in sequence 30
27 patterns of type 12 cut in sequence 31
3 patterns of type 4 cut in sequence 32
17 patterns of type 8 cut in sequence 33
2 patterns of type 23 cut in sequence 34
10 patterns of type 31 cut in sequence 35
3 patterns of type 39 cut in sequence 36
6 patterns of type 30 cut in sequence 37
69 patterns of type 41 cut in sequence 38
30 patterns of type 44 cut in sequence 39
57 patterns of type 7 cut in sequence 40
58 patterns of type 36 cut in sequence 41
3 patterns of type 43 cut in sequence 42
35 patterns of type 40 cut in sequence 43

FFD+CPLEX - Instance 2
1 patterns of type 45 cut in sequence 0
9 patterns of type 3 cut in sequence 1
4 patterns of type 43 cut in sequence 2
80 patterns of type 7 cut in sequence 3
12 patterns of type 26 cut in sequence 4
4 patterns of type 33 cut in sequence 5
81 patterns of type 36 cut in sequence 6
49 patterns of type 40 cut in sequence 7
37 patterns of type 42 cut in sequence 8
16 patterns of type 10 cut in sequence 9
58 patterns of type 20 cut in sequence 10
38 patterns of type 12 cut in sequence 11
4 patterns of type 39 cut in sequence 12
16 patterns of type 0 cut in sequence 13
3 patterns of type 29 cut in sequence 14
24 patterns of type 11 cut in sequence 15
166 patterns of type 6 cut in sequence 16
8 patterns of type 30 cut in sequence 17
73 patterns of type 6 cut in sequence 18
5 patterns of type 4 cut in sequence 19
1 patterns of type 34 cut in sequence 20
22 patterns of type 32 cut in sequence 21
97 patterns of type 41 cut in sequence 22
42 patterns of type 44 cut in sequence 23
16 patterns of type 37 cut in sequence 24
10 patterns of type 24 cut in sequence 25
14 patterns of type 31 cut in sequence 26
67 patterns of type 13 cut in sequence 27
8 patterns of type 21 cut in sequence 28
9 patterns of type 38 cut in sequence 29
1 patterns of type 25 cut in sequence 30
5 patterns of type 17 cut in sequence 31
16 patterns of type 35 cut in sequence 32
156 patterns of type 1 cut in sequence 33
42 patterns of type 2 cut in sequence 34
2 patterns of type 23 cut in sequence 35
2 patterns of type 28 cut in sequence 36
2 patterns of type 18 cut in sequence 37
58 patterns of type 5 cut in sequence 38
1 patterns of type 19 cut in sequence 39
23 patterns of type 8 cut in sequence 40
20 patterns of type 15 cut in sequence 41
42 patterns of type 14 cut in sequence 42
1 patterns of type 27 cut in sequence 43