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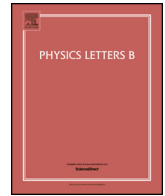
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# Neutrino primordial Planckian black holes

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## ABSTRACT

Extremal rotating black holes can be formed in the Planck energy scattering of Dirac spin parallel neutrinos in the mass state  $m_2$  (assuming  $m_1 = 0$ ), owing to the repulsive interaction between their magnetic dipoles, induced by vacuum fluctuations. Assuming that some recent results of loop-quantised Schwarzschild black holes would be also applicable for the Kerr case, we show that the resulting black hole has Planck mass and angular momentum  $\hbar$ , and that its horizon area is in the spectrum of the Loop Quantum Gravity area operator. Moreover, we argue that such black holes could be produced at the reheating, with an abundance that allows their interpretation as forming the presently observed dark matter component, provided that the energy scale at inflation is  $\approx 10^{17}$  GeV. This scale can be lower if we attribute a high chemical potential to primordial neutrinos. As extremal black holes have zero surface gravity, there is no limits on their abundance from Hawking evaporation.

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Among the candidates to dark matter, primordial or remnant black holes are promising in the senses that they only interact gravitationally, present a wide range of masses and can be described in the scope of the known standard physics [1–4]. On the other hand, there are tight limits to their present abundance [5–7], which in the case of Planck size black holes are imposed by Hawking evaporation, unless there is some mechanism that turns them stable. An interesting possibility is to consider extremal black holes, with zero surface gravity, that do not radiate. As dark matter is neutral, Reissner-Nordström black holes must be excluded, but Kerr black holes would still be good candidates. The smallest Planck scale extremal Kerr black hole has angular momentum given by the quantum of action  $\hbar$ , a Planck mass  $m_P$ , and horizon radius given by the Planck length  $\ell_P$ . Such a black hole may in principle be formed in the scattering of two parallel spin- $\frac{1}{2}$  identical particles at a centre-of-mass energy equal to  $m_P c^2$ , provided that a horizon with radius  $\ell_P$  is formed.

This is a classical reasoning, and some conditions must be fulfilled in order to validate it. First of all, the interaction between the particles should be dominant over any vacuum effect induced at so high energies. This also means that other inelastic channels would be negligible in the scattering. Secondly, quantum gravity correc-

tions to the horizon should be negligible for length scales above the Planck length, which allows the use of the classical horizon area to infer the resulting black hole mass. This is also important to assure the absence of evaporation even at these scales, as the derivation of the black hole temperature depends essentially on the geometry around its horizon.<sup>1</sup> Finally, a third condition is necessary for having a good candidate for dark matter, namely that the scattering fermions are enough abundant and energetic at the end of inflation in order to reach the required abundance of primordial black holes.

Surprisingly enough, these requirements seem to be fulfilled by Dirac neutrinos, provided they have mass, as is now universally accepted [8,9]. In the minimal extension required to include massive neutrinos in the standard model, namely the addition of right handed singlets, these particles acquire magnetic moments from vacuum fluctuations, given (in natural units) by [10]

$$\mu_\nu \approx \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2}, \quad (1)$$

where  $-e$  is the electron's charge,  $G_F$  is the Fermi constant and  $m_\nu$  is the neutrino's mass. As it has dimension of charge/mass, its higher order relative corrections are proportional to powers of

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<sup>1</sup> In the case of an extremal charged black hole, we have explicitly shown in [13] that the quantum-corrected metric corresponds indeed to an isolated horizon.

$e^2/\pi \approx 10^{-3}$ , whatever the energy scale involved, any scale dependence being absorbed by renormalisation [11]. Therefore, the dominant interaction in neutrinos scattering is given by the repulsion between the above magnetic dipoles, with interaction energy  $U = \mu_v^2/r^3$  in the case of parallel spins, where  $r$  is their mutual distance. One can now estimate the minimum total energy necessary to accommodate the two neutrinos inside a sphere of radius  $\ell_P$ . In this configuration, one needs to have  $r \leq 2\ell_P$ , implying the following minimal interaction energy,

$$M \approx \frac{9e^2 G_F^2 m_v^2}{1024\pi^4}. \quad (2)$$

Now, assuming normal ordering for the neutrinos mass states and that  $m_1 = 0$ , the lightest massive state is  $m_2 \approx 8.654 \times 10^{-3}$  eV [8,9], which leads to  $M \approx 1.001 m_P$ , meaning that neutrinos scattering can give origin to Planckian extremal Kerr black holes with  $10^{-3}$  accuracy. Such Kerr black holes have specific angular momentum  $a \approx 0.999$ , marginally above the Thorne limit  $a^* = 0.998$ .

As already mentioned, in the above estimation we have used the classical relation between the horizon radius and the black hole mass. Such analysis will be valid for Planckian scales only if quantum gravity corrections to the black hole horizon area are negligible for a Planck mass black hole. This would also allow to invoke the no-hair conjecture to assure that the neutrinos magnetic dipoles are not observable from outside once the horizon is formed. In fact, extending to the Planck scale a LQG inspired model for large Schwarzschild black holes recently proposed by Ashtekar, Olmedo and Singh [12], we have shown in a previous publication [13] that the quantum corrections to the horizon area for the spherically symmetric case are bounded above by

$$\frac{\Delta \mathcal{A}}{\mathcal{A}} = \frac{3\gamma^2}{64m^4}, \quad (3)$$

where  $\gamma$  is the Barbero-Immirzi parameter, and  $m$  is the smallest admissible black hole mass. Assuming that such bound also holds for the case of extremal Kerr black holes with  $m = m_P$ , we will have  $\Delta \mathcal{A}/\mathcal{A} \approx 10^{-3}$ , that is, the same order of magnitude of both the second-order relative correction to the neutrinos magnetic moment and the relative error found for  $M$ . Although the AOS model and (3) are indeed only valid for spherically symmetric black holes, this suggests that quantum corrections to the horizon would be negligible at Planck scale.<sup>2</sup>

This surprising coincidence between the classical and quantum areas of a Planck mass horizon also appears in the fact that we can write the classical horizon area of an extremal black hole of angular momentum  $J = 1$  as [13,14]

$$\mathcal{A} = 8\pi J = 8\pi \gamma \ell_P^2 \sum_{i=1}^4 \sqrt{j_i(j_i + 1)}, \quad (4)$$

with  $\gamma = \sqrt{3}/6$  and  $j_i = 1/2$ . This is the eigenvalues equation for the area operator of Loop Quantum Gravity [15], corresponding to a horizon pierced by four spin network lines in the  $SU(2)$  fundamental representation, provided that we identify  $\gamma$  with the Barbero-Immirzi parameter. Its value here is only 5% above the approximate value  $\gamma \approx 0.274$  found from the Bekenstein-Hawking entropy of large black holes with the Gosh-Mitra count of microstates [16,17], and it gives precisely the correct slope 1/4 for the linear leading term of the entropy vs area relation of Planck size

horizons [13]. As there is an even number of lines with same colours  $j_i$ , we can choose the punctures quantum numbers  $m_i$  in such a way to satisfy the horizon isolation condition, given by the projection constraint [18]

$$\sum_i^4 m_i = 0. \quad (5)$$

Hence, Planck energy scattering of Dirac neutrinos may lead to the formation of stable black holes of Planck mass. On the other hand, these particles form part of the relativistic content of our Universe, with an abundance that can, in principle, lead to black holes formation at the end of the inflationary period in a rate high enough to supply the presently observed amount of dark matter. In order to verify this possibility, let us estimate the required energy scale at the end of inflation.

The cross section for black hole formation in the scattering is roughly given by  $\sigma \approx \pi r_s^2$ , where  $r_s$  is the horizon radius [7]. In the case of neutrinos black holes we have  $r_s = \ell_P$ . Consider a target neutrino at the centre of this cross section. The number of Planck energy incident neutrinos orthogonally crossing the section in a given direction during an infinitesimal time interval  $dt$  is equal to their number density  $\eta_v f_P$  multiplied by the volume  $\sigma(cdt)$ . Here,  $\eta_v$  is the neutrinos number density,  $f_P$  is the fraction of Planck energy neutrinos, and we have approximated the neutrinos velocity by  $c$ . Therefore, apart from numerical factors of the order of unity, related to the isotropy of neutrinos motion and to the fact that only parallel neutrinos<sup>3</sup> in the mass state  $m_2$  form extremal black holes, the rate of black hole formation per neutrino can be estimated as

$$\frac{dN}{dt} \approx (\eta_v f_P)(\pi \ell_P^2)c. \quad (6)$$

The rate of black hole formation is found by multiplying the above rate by the neutrinos number density. If we also take into account the effect of space expansion, this leads (in Planck units) to

$$\frac{1}{a^3} \frac{d\bar{\eta}}{dt} = \frac{dN}{dt} \eta_v \approx \eta_v^2 f_P, \quad (7)$$

where  $a$  is the scale factor and  $\bar{\eta} = \eta a^3$  is the black holes comoving number density. Using  $\eta_v \approx \rho_v/T_v$ , and  $\bar{\rho} = \bar{\eta}$  for the black holes comoving mass density, we have

$$\frac{d\bar{\rho}}{dt} \approx \frac{\rho_v^2}{T_v^2} f_P a^3. \quad (8)$$

As the number of relativistic particles in the present Universe is  $\approx 10^9$  times the number of non-relativistic particles, we can treat neutrinos as a conserved reservoir, and use the relations  $\rho_v \approx T_v^4$ ,  $aT_v = T_{v0}$  and  $\rho_v \approx H^2$ , valid in a radiation dominated universe, to pass from the integration in time to an integration in the neutrinos temperature  $T_v$  (the index 0 refers to present time, and  $H = \dot{a}/a$  is the Hubble function). We have

$$dt \approx -\frac{dT_v}{T_v^3}, \quad (9)$$

and in this way we obtain a comoving density

$$\bar{\rho} \approx \left(\frac{m_P}{\ell_P^3}\right) T_{v0}^3 \int_0^{T_{\max}} f_P dT_v. \quad (10)$$

<sup>2</sup> Quantum corrections are non-negligible at trans-Planckian lengths [13], e.g. in the black holes interior, where their presence may avoid the singularity [12].

<sup>3</sup> Let us remind that we are describing the formation of black holes in quantum particles scattering, where the spins are either parallel or anti-parallel.

In this integral we have set the inferior limit to zero, as the temperature at the end of the radiation era is negligible compared to the reheating temperature  $T_{\text{max}}$ .  $T_{\nu 0} \approx 10^{-31}$  is the present neutrino temperature in Planck units, and the factor  $m_P/\ell_P^3 \approx 10^{97} \text{ kg/m}^3$  was introduced to restore dimensional units.

The fraction of neutrinos with energy above the Planck scale can be calculated as<sup>4</sup>

$$f_P(T_\nu) = \int_1^\infty \frac{p^2 dp}{e^{p/T_\nu} + 1} \times \left( \int_0^\infty \frac{p^2 dp}{e^{p/T_\nu} + 1} \right)^{-1}, \quad (11)$$

where we have set the chemical potential  $\mu$  to zero. For  $T_\nu \ll 1$ , the numerator can be written as

$$\int_1^\infty p^2 dp e^{-p/T_\nu} \approx T_\nu e^{-1/T_\nu}. \quad (12)$$

The denominator is the standard number density,

$$\eta_\nu \approx \frac{3}{2} \zeta(3) T_\nu^3, \quad (13)$$

and  $f_P$  can be written as

$$f_P \approx T_\nu^{-2} e^{-1/T_\nu}. \quad (14)$$

From (10), we have

$$\bar{\rho} \approx \left( \frac{m_P}{\ell_P^3} \right) T_{\nu 0}^3 \int_0^{T_{\text{max}}} T_\nu^{-2} e^{-\frac{1}{T_\nu}} dT_\nu. \quad (15)$$

Therefore, for having the comoving density of cold matter at the end of the radiation era, namely  $\bar{\rho} \approx 10^{-27} \text{ kg/m}^3$ , we need

$$\int_0^{T_{\text{max}}} T_\nu^{-2} e^{-\frac{1}{T_\nu}} dT_\nu = e^{-1/T_{\text{max}}} \approx 10^{-31}. \quad (16)$$

This integral is very sensitive to the value of  $T_{\text{max}}$ , which turns unimportant the numerical factors we have ignored in (15). The figure above is obtained for  $T_{\text{max}} \approx 0.014$ , that is,

$$T_{\text{max}} \approx 1.7 \times 10^{17} \text{ GeV}. \quad (17)$$

Note that the rate (8) scales as  $T_\nu e^{-1/T_\nu}$ , decreasing very fast as the neutrinos temperature drops and leaving a constant comoving density deep inside the radiation era. Like in (15), its integration gives

$$\bar{\rho}(T_\nu) \approx \left( \frac{m_P}{\ell_P^3} \right) T_{\nu 0}^3 \left( e^{-1/T_{\text{max}}} - e^{-1/T_\nu} \right). \quad (18)$$

In Fig. 1, with the help of (9), we show the resulting time evolution of the black holes comoving density. The black holes formation lasts for a few hundred Planck times.

The energy (17) is an order of magnitude higher than the upper bound predicted by the simplest single-field inflationary models

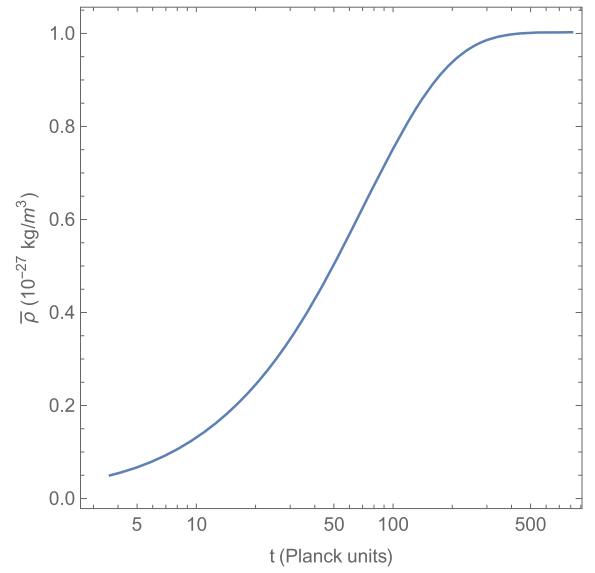


Fig. 1. Evolution of the comoving density of primordial black holes with time, for  $\mu = 0$ . We set  $t = 0$  at the reheating.

from the limits imposed by CMB observations [19] on the tensor-to-scalar ratio  $r$ . In these models, the inflaton potential can be approximated by [20,21]

$$V^{1/4} \approx 3.3 \times 10^{16} r^{1/4} \text{ GeV}. \quad (19)$$

Assuming a fast reheating, the temperature of the produced bath is given by  $V \approx (\pi/30) g_* T^4$ , where  $g_* \approx 10^2$  is the number of standard model relativistic degrees of freedom. Therefore, for  $r < 0.15$ , the temperature is bounded by  $T \lesssim 10^{16} \text{ GeV}$ . As shown in the Appendix,  $T_{\text{max}}$  can satisfy this upper bound if the neutrinos chemical potential is high enough at this scale, where neutrinos are in equilibrium with photons and other relativistic species at an extremely high density. Although at lower energies it is relatively well constrained by primordial nucleosynthesis as  $\mu/T \lesssim 0.1$  [22–24], there is no reliable hint about its value at the reheating. Note, as well, that the above upper limit can be higher in some more elaborate inflation models [25]. In particular, (19) is not valid in multi-field inflation [26,27] or in models with non-canonical kinetic term [28].

As the black holes are extremal, they do not radiate and, therefore, the limits on the primordial black holes density coming from Hawking evaporation [5–7] do not apply. Nevertheless, the strong dependence of (16) on  $T_{\text{max}}$  implies that the latter cannot differ to much from (17). For example, with only 1% difference in that value, the integral (16) changes by one order of magnitude. On one hand, once we know the neutrinos chemical potential, this determines with precision the scale of inflation if we assume that dark matter is composed by these primordial black holes. On the other hand, it turns this hypothesis easily falsifiable if we can determine that scale in some other way.

A point to be better understood is the possible accretion of matter after the black holes formation, and how it may affect the above estimation of abundance. By now, we can only remark that the comoving density of Planck energy particles decreases exponentially with the scale factor, which makes their subsequent capture very unlikely. The same can be said about possible black holes mergers, since their physical density also decreases with the expansion. Furthermore, any energy added through matter accretion or black holes merger takes them out of extremality and is fast released by evaporation. This mechanism was considered in detail in [29] for macroscopic black holes, where the authors present some conditions assuring that the near-extremal character of primordial

<sup>4</sup> Strictly speaking, only Planck energy scatterings will form extremal black holes, but the effect of more energetic neutrinos is negligible, because their number drops sharply with energy.

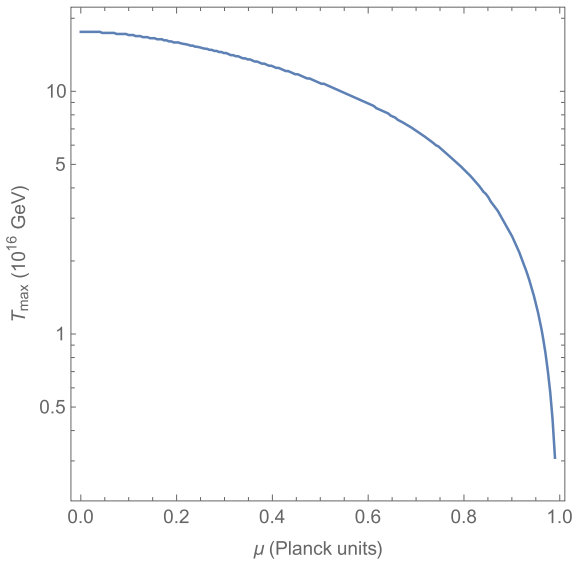


Fig. 2. Reheating temperature as a function of the neutrinos chemical potential.

black holes above the Thorne limit could be preserved. Even in the situations where the black holes can lose angular momentum more rapidly than mass, it is shown that, for masses above some limit, the black holes maintain the near-extremality, and that the higher the specific angular momentum, the smaller that mass limit. If extrapolated to Planck size black holes, this suggests that horizons with  $a$  above the Thorne limit  $a^*$  remains near-extremal. In fact, in the worse case of a Planck size Schwarzschild black hole, the Hawking temperature represents at most 4% of the black hole mass [13,30].

As a matter of fact, the present results must be taken with some caution, as to date there is no complete solution for a Kerr black hole in the context of Loop Quantum Gravity, that is, a solution that not only satisfies the area operator eigenvalues equation at the horizon but also the Hamiltonian constraint. We are basing our estimations on the area vs mass relation for an extremal horizon, a procedure corroborated by an effective quantum model of spherically symmetric black holes, but that should be deeper verified in the case of spinning horizons. Hence, our results are effectively based on a phenomenological extension of [12,13] for the Planckian Kerr case. Furthermore, some recent studies [31] strongly suggest that the detailed formation of black holes will depend on the spacetime quantisation prescription, posing a challenge for our hypothesis, since no quantisation prescription is indeed known for the Kerr black hole. These points certainly deserve a deeper analysis.

A further issue is the implicit assumption that there is no new physics beyond the standard model of particles up to the Planck length, that could open new channels for neutrinos interactions or, for instance, change their magnetic moments and the resulting black hole mass. We have argued that quantum gravity effects are only important at trans-Planckian lengths, where the quantised nature of space-time emerges. We are assuming here that this is also true for the other fundamental interactions. Nevertheless, even in the realm of the standard model, the leading term in the neutrinos magnetic moment may change for energies above the electroweak scale, because of the presence of gauge bosons in the loops involved in its derivation. As the electroweak phase transition is smooth, possible changes can in principle be absorbed by renormalisation, as already pointed out. This is an issue, however, that deserves further investigation.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix

Let us generalise Eq. (15) for the case of non-zero chemical potential. From (7), using  $\bar{\eta} = \bar{\rho}$ , we have

$$\frac{d\bar{\rho}}{dt} \approx \eta_v^2 f_P a^3. \quad (20)$$

Using  $\eta_v a^3 = \eta_{v0} \approx T_{v0}^3$  (since  $\mu \approx 0$  at low energies),  $\rho_{v0} a^4 = \rho_{v0} \approx T_{v0}^4$ , and  $H^2 \approx \rho_v$ , we can derive

$$d\bar{\rho} \approx -T_{v0}^3 f_P \frac{d\eta_v}{\eta_v^{2/3}}. \quad (21)$$

The neutrino number density is given (ignoring again multiplicative factors of the order of unity) by

$$\eta_v(T_v, \mu) \approx \int_0^\infty \frac{p^2 dp}{e^{(p-\mu)/T_v} + 1}. \quad (22)$$

On the other hand, the fraction  $f_P$  can be obtained like in (11), as

$$f_P(T_v, \mu) \approx \eta_v^{-1} \int_1^\infty \frac{p^2 dp}{e^{(p-\mu)/T_v} + 1}. \quad (23)$$

In this way we have

$$\bar{\rho} \approx \left( \frac{m_P}{\ell_P^3} \right) T_{v0}^3 \int_0^{T_{\max}} f_P \eta_v^{-2/3} \left( \frac{\partial \eta_v}{\partial T_v} \right) dT_v, \quad (24)$$

where the factor  $m_P/\ell_P^3$  restores the dimensional units. We integrated (22)–(24) numerically with the help of MATHEMATICA, for a constant  $\mu$ . In Fig. 2 we show the dependence of the reheating temperature on the neutrinos chemical potential. For  $T_{\max} = 10^{16}$  GeV, a comoving density  $\bar{\rho} = 10^{-27}$  kg/m<sup>3</sup> is obtained with  $\mu \approx 0.96$ .

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