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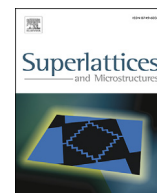
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Localization properties of photonic modes in disordered nonlinear-Kerr/metamaterial heterostructures



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ABSTRACT

The localization properties of electromagnetic waves in one-dimensional disordered nonlinear-Kerr/metamaterial heterostructures are investigated. Structural disorder is introduced via a random fluctuation of layer widths of both nonlinear-Kerr and metamaterial slabs composing the photonic heterostructure. For frequency values in the vicinity of the zero- n gap, multiple electromagnetic modes with different transmission lengths are obtained for a given value of the Kerr defocusing nonlinearity power. Maximum-delocalized photonic states, which are associated with high-transmission electromagnetic modes corresponding to gap-soliton waves, are found to be quite sensitive with respect to the degree of disorder. Moreover, we have found that inclusion of absorption effects leads, as expected, to a decreasing of the transmission length.

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1. Introduction

In the last few decades, the physical properties of one-dimensional (1D) photonic heterostructures have been the subject of considerable interest from both the experimental and theoretical points of view [1–4]. Besides the purely academic interest, 1D photonic crystals have attracted scientific attention due to perspectives in design and construction of optical devices with exotic properties of the refractive index and potential applications in optoelectronics and communication technologies. For example, Dowling and Bowden [5] have theoretically demonstrated the possibility of a less-than-unity refractive index in 1D multilayered heterostructures for allowed frequency values in the vicinity of a bandgap. Kerr and Faraday effects in 1D magnetophotonic crystals were experimentally and theoretically studied by Inoue et al. [6] whereas the transmission properties of 1D magnetic photonic crystals were also investigated by Linden et al. [7]. The optical properties of 1D Si/SiO₂ photonic crystal were studied by Patrini and co-workers [8] and, more recently, the experimental realization of an all-dielectric zero- n metamaterial has been performed in a similar semiconductor/oxide multilayered system [9].

Advances in material science have made possible the fabrication of optical materials with enhanced nonlinearity [10,11], yielding a new thrust on investigations in the optical, localization, and transmission properties of linear-dispersive/nonlinear 1D heterostructures. In that respect, stationary gap solitons have been reported for frequency values in the vicinity of the Bragg [12], zero- n [13], and plasmon-polariton [14] gaps in periodic heterostructures made of nonlinear-Kerr/linear-normal-material and nonlinear-Kerr/linear-metamaterial. Spatial gap solitons rely on the balance between diffraction and

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nonlinearity [15,16]. The combination of these phenomena plus the periodicity of the stack leads to various effects, such as optical multistability as well as the switching from low-transmission states to transparent states for discrete values of the incident power-beam [12–14], resulting in the excitation of nonlinear stationary modes with multisoliton profiles.

In spite of the fact that gap-soliton solutions have been theoretically predicted many years ago, experimental observation has resulted quite difficult. Absorption effects inherent to dispersive materials may cause a soliton extinction in a multi-layered heterostructure [17]. In addition, the presence of disorder on the heterostructure geometry may lead to similar results. Thus, it is not surprising that a considerable amount of work has been devoted to the study of disorder effects in photonic systems [18,19]. For instance, Fernández-Marín et al. [20] experimentally demonstrated anomalous localization of microwaves in 1D random waveguides. Moreover, Chiasera et al. [21] performed an experimental and theoretical study of transmission properties in disordered 1D photonic structures, where the disorder was introduced throughout a random variation of layer thicknesses. They found a decreasing of the transmission coefficient with respect to the corresponding periodic heterostructure. Motivated by such results, in the present work we investigate the localization properties of electromagnetic (EM) waves in 1D Kerr/metamaterial heterostructures. Our objective is to elucidate what are the levels of disorder and absorption which are necessary to prevent soliton formation in the photonic heterostructure. A brief theoretical insight is given in Section 2, whereas results and conclusions are given in Secs. 3 and 4, respectively.

2. Theoretical framework

Let us begin by considering a photonic heterostructure, surrounded by vacuum, composed by N double layers AB which are stacked along the z -growth direction. Layers A and B are supposed to be made of a nonlinear-Kerr material and a dispersive metamaterial, respectively. The electric permittivity of the slabs A is given by $\epsilon_A = \epsilon_A^0 + \alpha |E(z)|^2$, whereas the magnetic permeability is taken as μ_A . Both the electric permittivity and magnetic permeability of the metamaterial layers are given by $\epsilon_B = f_\epsilon + F_\epsilon / (\beta_\epsilon - \nu^2 - i\nu\gamma_\epsilon)$ and $\mu_B = f_\mu + F_\mu / (\beta_\mu - \nu^2 - i\nu\gamma_\mu)$, respectively, where ν is the frequency in GHz. In the above expressions we have taken $\epsilon_A^0 = 2$, $\mu_A = 2$, $f_\epsilon = 1.6$, $F_\epsilon = 40 \text{ GHz}^2$, $\beta_\epsilon = 0.81 \text{ GHz}^2$, $f_\mu = 1.0$, $F_\mu = 25 \text{ GHz}^2$, and $\beta_\mu = 0.814 \text{ GHz}^2$ [13]. Absorption effects in the heterostructure are taken into account via phenomenological damping constants γ_ϵ and γ_μ [22,23] which we have chosen, for simplicity, as equal, i.e., $\gamma_\epsilon = \gamma_\mu = \gamma$. For the transversal-electric (TE) EM modes, the electric-field amplitude $E = E(z)$ of the electromagnetic field propagating along the growth direction of the heterostructure satisfies the differential equation

$$-\frac{d}{dz} \left[\frac{1}{\mu(z)} \frac{d}{dz} E(z) \right] - \left[\frac{\omega^2}{c^2} \epsilon(z) - \frac{q^2}{\mu(z)} \right] E(z) = 0, \quad (1)$$

where $\epsilon = \epsilon(z)$ and $\mu = \mu(z)$ are the stepwise electric permittivity and magnetic permeability of the heterostructure, respectively, $\omega = 2\pi\nu$, $q = \omega/c \sin\theta$, and θ is the incidence angle relative to the vacuum.

Here, structural disorder is introduced by assuming random uncorrelated variables to represent the widths of the heterostructure layers. In this sense we denote as a_k and b_k ($1 \leq k \leq N$) the widths of the k -th slabs A_k and B_k , respectively, and suppose that a_k and b_k are uniformly distributed over the intervals $[a - \Delta_A/2, a + \Delta_A/2]$ and $[b - \Delta_B/2, b + \Delta_B/2]$, respectively. For simplicity we choose $\Delta_A = \Delta_B = \Delta$ in the present calculations. Other distribution functions may be used to characterize the structural disorder of the heterostructure. For instance, the Gaussian distribution is of particular importance for experimentally describing the random fabrication errors in two-dimensional photonic-crystal cavities [24]. However, it has been shown [25,26] that choosing a uniform or normal distribution does not dramatically affect the theoretical description of the optical properties of the photonic system. Actually, such properties are more dependent on the presence (or absence) of disorder correlations [25,26]. In this respect, we have chosen to follow the works of Izrailev and Makarov [27] and Asatryan et al. [28] and used an uniform distribution. The transmission coefficient T_N corresponding to a heterostructure with N double layers AB may be computed from Eq. (1) by combining the analytical solutions of Eq. (1) in the metamaterial layers with the numerical solutions of Eq. (1) in each Kerr-material slab [14]. Investigations of the disorder effects on a certain physical quantity associated with the heterostructure may be obtained by constructing a sufficiently large ensemble of systems, and computing the quantity of interest for each single element of the ensemble. Then, one may average it over the whole ensemble. The number M of elements of the ensemble (i.e., the number of disorder realizations) should be large enough so that the average does not depend on M . By following the above-described procedure we have computed the transmission length associated with the EM modes propagating throughout the heterostructure. The N -dependent transmission length is defined as [28]

$$\xi_N = - \left\langle \frac{\ln(T_N)}{2\lambda_N} \right\rangle^{-1}, \quad (2)$$

where λ_N is the length of each element of the ensemble and the symbol $\langle \dots \rangle$ represents the heterostructure ensemble average. One should note that the transmission length tends toward the Anderson localization length in the limit $N \rightarrow \infty$ [29]. The transmission length, however, accounts for the localization properties of the EM modes in a **finite** heterostructure. In finite-size systems, the transmission length should be large enough with respect to the heterostructure size in order that delocalized states are (theoretically or experimentally) observed. A systematic study of the transmission length in nonlinear/

metamaterial disordered heterostructures allows one to understand not only the EM transport properties of such systems as functions of the incident beam power, but also to elucidate the influence of disorder on the existence and stability of the nonlinear modes excited by the incident power in the system.

3. Results and discussion

One of the nonlinearity effects on the transmission properties of nonlinear-Kerr/metamaterial ordered heterostructures is the transmission-switching phenomenon, consisting of a soliton-mediated transparency switching from a state of no transparency, in the vicinity of a gap in the linear regime, to total transparency in the nonlinear regime [12–14]. The transparency states occur for discrete values of the incident nonlinearity power $-\alpha|E_i|^2$, where E_i is the electric-field amplitude of the electromagnetic field associated with the incident wave. The transmission-switching phenomenon is illustrated in Fig. 1 in the case of normal incidence and for frequency values at the vicinity of the zero- n gap. The heterostructure (without disorder) under consideration is composed of $N = 2000$ double layers AB, with $a = b = 10$ nm. The transmission coefficient as a function of the frequency in the linear regime is depicted in Fig. 1(a). At $\nu = 3.089114$ GHz [cf. vertical dashed line in Fig. 1(a), for which the transmission coefficient is negligible in the linear regime] one notes that, when

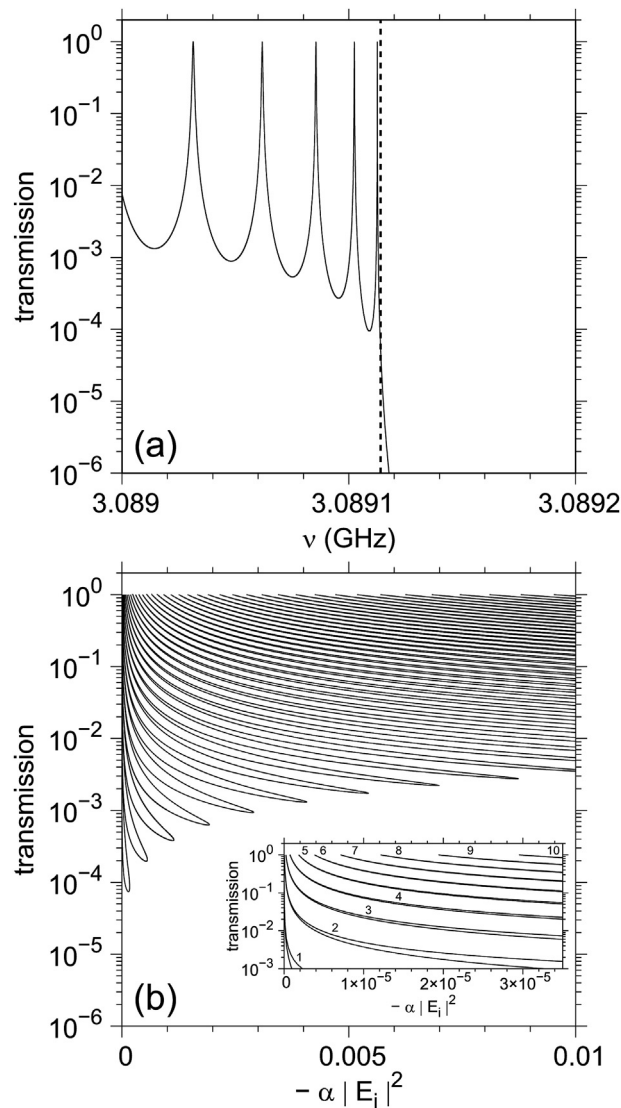


Fig. 1. Transmission coefficient (a) as a function of the frequency in the linear regime for $N = 2000$ double layers and (b) as a function of the defocusing nonlinearity power. Calculations were performed, in the absence of absorption and disorder, for $a = b = 10$ nm, normal incidence, and at the vicinity of the lower edge of the zero- n gap. Results shown in panel (b) were obtained with $\nu = 3.089114$ GHz, for which the transmission coefficient of the ordered system is negligible [see the vertical dashed line in panel (a)]. A magnified view of the first ten peaks of the transmission coefficient is displayed in the inset of panel (b).

nonlinear effects are taken into account in the A Kerr-material slabs, the transmission coefficient as a function of the defocusing nonlinearity power switches from a low-transmission to a transparent state at discrete values of the defocusing nonlinearity power [see Fig. 1(b)]. The multistability of the transmission coefficient as a function of the defocusing nonlinearity power is clearly observed. We have also computed the corresponding transmission length of the ordered system as a function of the defocusing nonlinearity power, which is depicted as solid lines in Fig. 2. Of course, the transmission length diverges for power values corresponding to transparent states.

In the linear regime, it is well known that disorder may cause a broadening of the transmission peaks, a fact related to the loss of the spatial coherence of the EM field inside the heterostructure. As a consequence, the peak structure of the transmission length observed in ordered system may dramatically change in disordered heterostructures. We have observed a similar effect in the transmission length as a function of the defocusing nonlinearity power corresponding to nonlinear disordered heterostructures. In that respect, the transmission length is shown in Fig. 2 as a function of the defocusing nonlinearity power for photonic heterostructures with $N = 2000$ double layers of average widths $a = b = 10$ nm. Calculations were performed for normal incidence at $\nu = 3.089114$ GHz, in the vicinity of the zero- n gap, for $\Delta = 2 \times 10^{-4}$ nm and

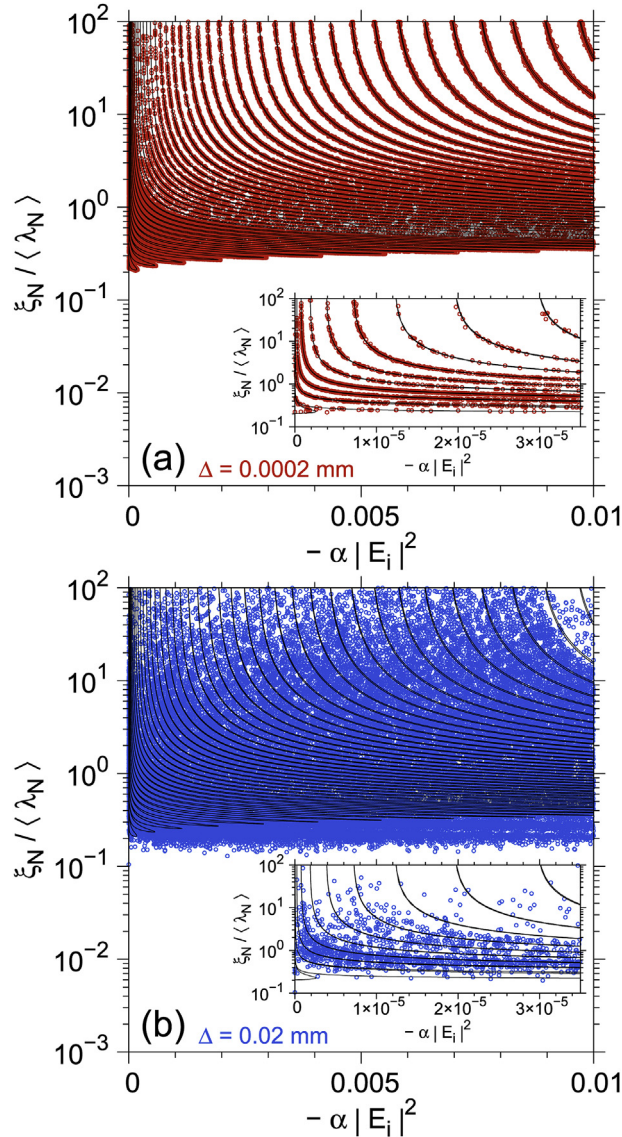


Fig. 2. Transmission length, in units of the average system length $\langle \lambda_N \rangle = N(\mathbf{a} + \mathbf{b})$, as a function of the defocusing nonlinearity power for normal incidence and in the absence of absorption. The heterostructure is supposed to be composed of $N = 2000$ double layers of average widths $a = b = 10$ nm. Calculations (open symbols) depicted in panels (a) and (b) were performed for 200 realizations of disorder, for $\Delta = 2 \times 10^{-4}$ nm and $\Delta = 2 \times 10^{-2}$ nm, respectively, and $\nu = 3.089114$ GHz at the lower edge of the zero- n gap. Solid lines in both panels correspond to the transmission length of the heterostructure without disorder. Enlarged views of the first ten peaks of the transmission length are depicted as insets in both panels.

$\Delta = 2 \times 10^{-2}$ mm [cf. open symbols of Fig. 2(a) and 2(b), respectively], 200 realizations of disorder, and in the absence of absorption. The delocalization of EM modes observed in Fig. 2(a) is due to the survival of the transmission-switching phenomenon for relative low values of the disorder amplitude. In this case, one may note the existence of multiple EM modes with different transmission lengths for a given value of $-\alpha|E_i|^2$, a consequence of the multistability of the transmission coefficient as a function of the defocusing nonlinearity power in disordered heterostructures (results are not shown here). Maximum-delocalized modes correspond to gap-soliton waves associated with transparency states in the heterostructure. In other words, each delocalization peak of the transmission length is associated with a gap-soliton wave corresponding to a high-transmission state. As expected, an increasing of the disorder amplitude leads to a large variation of the localization properties, as one may note from Fig. 2(b). Results of Fig. 2(b) clearly indicate the strong sensitiveness of the transmission length with respect to the disorder amplitude, i.e., an increase of Δ from 2×10^{-4} mm to 2×10^{-2} mm leads to a dramatic change of the peak structure of the transmission length. We would like to mention that these values of Δ correspond to 0.001% and 0.1%, respectively, of the average period $d = a + b$, which we believe could be experimentally realized in the near future. In that respect, we point out that typical tolerances in the manufacturing of two-dimensional photonic-crystal cavities are found to be of the order of 1.5 nm [24].

One may also expect a change of the EM field associated with gap solitons in disordered heterostructures as Δ increases. Bearing this in mind, we display in Fig. 3 the zero- n -gap soliton corresponding to the absolute maximum of the 10^{th}

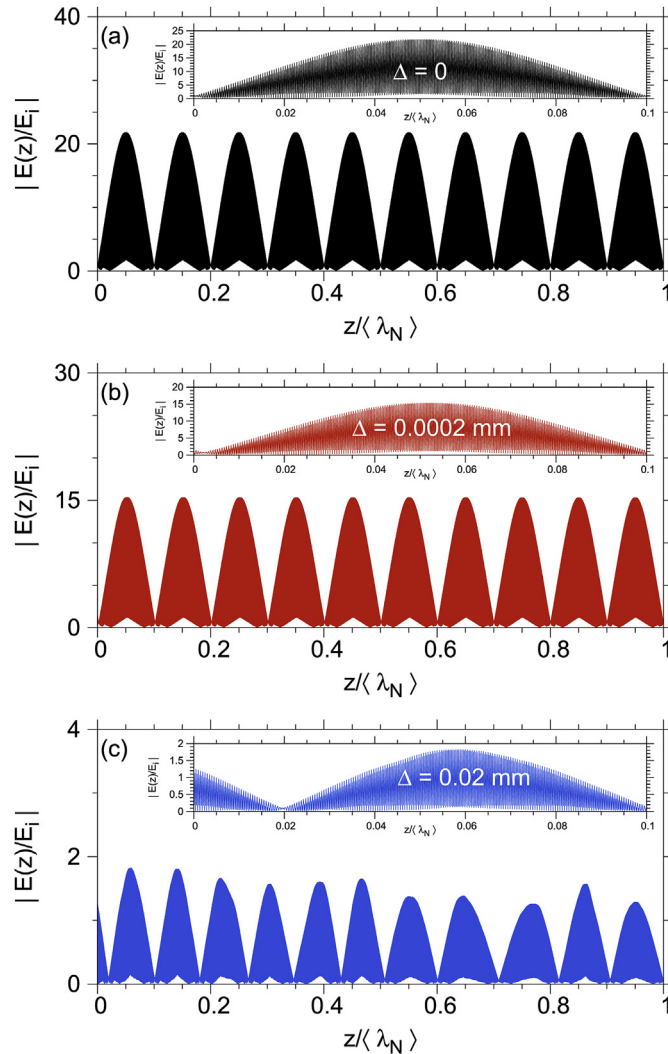


Fig. 3. Zero- n -gap solitons in photonic heterostructures with $N = 2000$ double layers of average widths $a = b = 10$ mm. Calculations were performed for normal incidence, in the absence of absorption, and for $\nu = 3.089114$ GHz at the bottom of the zero- n gap. Numerical calculations in panels (a), (b), and (c) correspond to disorder amplitudes $\Delta = 0$, $\Delta = 2 \times 10^{-4}$ mm, and $\Delta = 2 \times 10^{-2}$ mm, respectively. Results were obtained for a defocusing nonlinearity power of $-\alpha|E_i|^2 = 2.951 \times 10^{-5}$ corresponding to the absolute maximum of the 10^{th} transmission peak of the ordered heterostructure [cf. inset of Fig. 1(b)]. A magnified view of the first soliton peak is displayed in the inset of each panel.

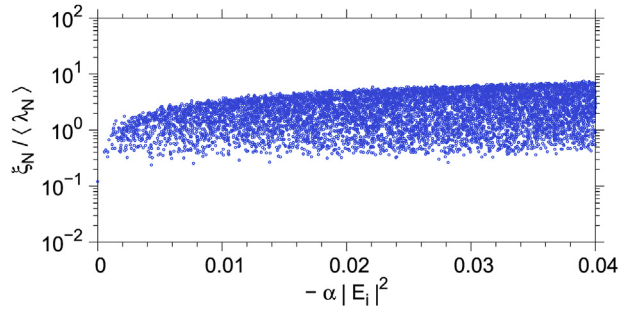


Fig. 4. Transmission length, in units of the average system length $\langle \lambda_N \rangle = N(\mathbf{a} + \mathbf{b})$, as a function of the $-\alpha|E_i|^2$ defocusing nonlinearity power. Calculations were performed for 200 realizations of disorder and $\Delta = 2 \times 10^{-2}$ mm in a heterostructure of $N = 2000$ double layers of average widths $a = b = 10$ mm, for normal incidence, and $\nu = 3.089114$ GHz at the lower edge of the zero- n gap. Absorption effects are taken into account via a $\gamma = 10^{-5}$ GHz phenomenological damping constant.

transmission peak ($-\alpha|E_i|^2 = 0.002951$) for $\Delta = 0$ [see Fig. 1(b)]. Calculations were performed in heterostructures with $N = 2000$ double layers of average widths $a = b = 10$ mm, for normal incidence, in the absence of absorption and for $\nu = 3.089114$ GHz at the lower edge of the zero- n gap. Results depicted in Fig. 3(a), 3(b), and 3(c), correspond to disorder amplitudes $\Delta = 0$, $\Delta = 2 \times 10^{-4}$ mm, and $\Delta = 2 \times 10^{-2}$ mm, respectively. Calculations displayed in Fig. 3 clearly indicate that soliton-induced transparency states do survive for low values of Δ , whereas a large increase of the disorder amplitude may cause the attenuation of the soliton waves inside the heterostructure. In a similar way, absorption effects may lead to the extinction of the soliton waves in the heterostructure and to a broadening of the nonlinearity-power-dependence of the transmission length. We have calculated the transmission length as a function of the defocusing nonlinearity power corresponding to the disordered heterostructure in the same conditions of Fig. 2(b), except that absorption effect are taken into account via a phenomenological $\gamma = 10^{-5}$ GHz damping parameter. Numerical results are displayed in Fig. 4. A comparison between Figs. 4 and 2(b) indicates that absorption effects result in a decrease of the transmission length. Finally, we would like to stress that similar results for the transmission length may be obtained for oblique incidence in both TE and TM configurations, and also in connection with solitons associated with other gaps.

4. Conclusions

To conclude, the localization properties of EM modes in finite Kerr/metamaterial disordered photonic heterostructures have been investigated. Structural disorder is taken into account by considering the slab widths of the heterostructure as uncorrelated random variables, uniformly distributed. For frequency values in the vicinity of the zero- n gap one finds the coexistence of multiple EM modes, with different transmission lengths, for a given value of the $-\alpha|E_i|^2$ defocusing nonlinearity power. Maximum-delocalized states, associated with high-transmission EM modes corresponding to gap-soliton waves, are quite sensitive with respect to the degree of disorder. Furthermore, inclusion of absorption effects leads to a decrease of the transmission length. Both absorption and disordered effects may be thought to constitute the prime impediment before one may succeed in practice to observe these nonlinear modes. Recent breakthroughs in the theoretical understanding and experimental fabrication of gain-enhanced metamaterials may open a new perspective for one to overcome these nuisances [30,31]. In that sense, we do hope the present theoretical results will be of interest in future experimental work on nonlinear-heterostructure systems.

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