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On higher derivative corrections to the $R + R^2$ inflationary model

Ana R. Romero Castellanos,^a Flavia Sobreira,^a Ilya L. Shapiro^{b,c,d}
and Alexei A. Starobinsky^{e,f}

^aInstituto de Física Gleb Wataghin, Universidade Estadual de Campinas,
13083-859, Campinas, SP, Brazil

^bDepartamento de Física, ICE, Universidade Federal de Juiz de Fora,
36036-330, Juiz de Fora, MG, Brazil

^cTomsk State Pedagogical University,
Tomsk, 634041, Russian Federation

^dTomsk State University,
Tomsk, 634050, Russian Federation

^eL. D. Landau Institute for Theoretical Physics RAS,
Moscow, 119334, Russian Federation

^fKazan Federal University,
Kazan 420008, Republic of Tatarstan, Russian Federation

E-mail: arromero@ifi.unicamp.br, sobreira@ifi.unicamp.br, shapiro@fisica.ufjf.br,
alstar@landau.ac.ru

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Abstract. The $R + R^2$ model is successful in describing inflation, as it provides an excellent fit to the full set of available observational data. On the other hand, the same model is the simplest extension of general relativity which does not produce higher derivative ghosts and related instabilities. Long ago, it was proposed to treat all terms which cause higher derivative instabilities as small perturbations that could avoid the presence of ghosts in the spectrum. We put this proposal into practice and consider an explicit example of treating more complicated higher derivative terms as small perturbations over the $R + R^2$ model by introducing the $R\Box R$ term into the action. Within the described scheme, it is possible to obtain an upper bound on the coefficient of this non-scale-free sixth-derivative term by mapping the theory into a one-scalar field potential. It is shown that the result differs from treating this term on equal footing with other terms that requires mapping to a two-scalar field model, and in general leads to different observational consequences.

Keywords: inflation, modified gravity

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1 Introduction

The role of fourth order higher derivative gravitational terms can be seen from different perspectives. In quantum field theory in curved space-time and in the semiclassical approach to gravity, these terms are required to provide a renormalizable theory and to obtain a finite renormalized average value of the energy-momentum tensor of non-gravitational quantum fields [1] (in cosmology this was first done in [2], see [3] for a review and further references). The same situation holds in quantum gravity, where these terms provide renormalizability [4]. An important advantage of higher derivative terms is that their effects are strongly suppressed below the Planck scale, and hence the classical solutions of general relativity can be seen as a very good approximation at the cosmological, astrophysical and laboratory scales. For this reason, theories with high derivative terms do not contradict experimental and observational tests of Einstein’s gravity.

On the other hand, these terms lead to the violation of stability of the classical solutions. At the quantum level, one can find violation of unitarity due to the presence of states with negative energy or negative norm [4]. No comprehensive solution to this problem is known, but the main expectations were always posed on the theories with complex poles, which were supposed to emerge due to loop contributions [5–9]. Indeed, our knowledge of quantum corrections to the gravitational propagator is insufficient to know whether the solution of the ghost problem can be achieved in this way [10]. At the same time, there are versions of super-renormalizable quantum gravity [11], which have complex conjugate poles already at the classical level, and in this case one can prove unitarity in the Lee-Wick sense [12]. These models have sixth or higher order derivatives in the action, and represent a prospective subject of investigation, in particular in cosmology.

At low energies, the terms with higher derivatives can be regarded as small perturbations of the fiducial theory of general relativity. This approach has been suggested as an *ad hoc* universal solution to the ghost problem [13]. This proposal leads to the following dilemma: trying to consider *all* higher derivative terms as objects to be avoided at the fundamental level and treated as perturbations, one has to ‘forbid’ the $R + R^2$ (Starobinsky) model of inflation — the simplified variant of the model introduced in [14] — arguing that it is ‘non-perturbative’ [15] (and it does, in fact, though it is not non-perturbative with respect

to general scalar-tensor gravity). Indeed, this part of the proposal is something difficult to accomplish. First of all, the R^2 -term does not produce ghosts and hence there is no reason to avoid it. At the same time, this inflationary model is the most successful from the observational and phenomenological point of view, so it is not easy to give it up without a real motivation. Finally, the inflation scenario requires the value of the numerical coefficient of the R^2 -term to be quite big, about 5×10^8 [16] (see also the recent papers [17, 18]). This makes this term dominant at curvatures much less than the Planck one, in particular, at curvatures during inflation. As a result, it would be quite natural to replace the R^2 -term into another side of the perturbation scheme of [13] and include it into the basic action along with the Einstein-Hilbert term. The present work is devoted to the practical application of this idea to inflation. Namely, we add a small sixth-derivative term to the standard inflationary $R + R^2$ action and find an upper bound on the coefficient of the new term, treating it as a small perturbation.

Previous attempts to analyze higher derivative corrections to the Einstein-Hilbert actions have been performed, e.g., in [19–23], making a transformation of the gravitational terms to scalar fields, among those one or more ghosts appear, or by considering non-local generalizations of gravity which does not contain ghosts [24, 25] and in which solutions of a local higher derivative theory like the $R + R^2$ one can appear as exact particular solutions of non-local equations.¹ Let us also mention an earlier work [27], where the dS-type solutions were explored in the framework of string-induced fourth-derivative gravity. In the present paper, we follow a different approach and treat the term which may produce ghosts as a small perturbation, such that it becomes harmless. In what follows one can find the perturbative analysis for a special sixth-order term which leads to the constraints on its coefficient derived from observational predictions.

Let us give a comment on the choice of the particular form of the term which will be used below to represent higher derivatives. Since all such terms are supposed to be small perturbations, one can consider them one by one, and in the leading-order approximation the effects of these terms will not depend on each other. As a consequence, we can actually start with an especially simple example by considering the $R\Box R$ -term. As we shall see in what follows, this term is simple to deal with, and gives a clear idea of a way in which one can consider generic higher derivative terms as perturbations. Finally, since the present cosmological constant does not play any noticeable role in the $R + R^2$ inflationary model, we set it to zero.

The paper is organized as follows. In section 2 we review the standard mapping of $R + R^2$ gravity into a metric-scalar model. In section 3 it is shown how an extra $R\Box R$ -term can be introduced as a small perturbation of this model, analyzing the observational consequences of the theory in the slow-roll regime. It is shown that when the $R\Box R$ -term is treated as a small perturbation, the model can still be mapped into a *one-scalar theory*, different from the *two-scalar* mapping within the approach which is a standard procedure in cosmology. The approach with the mapping to the two-scalar model is considered in parallel, for the sake of comparison, and the well-known necessary details of this presentation are postponed to the appendix. Finally, the reader can see that when the $R\Box R$ -term is treated at the same level with other terms, it is mapped into the model with *two* scalars, and the observational consequences are in general different. Finally, in section 4 we draw our conclusions.

¹When this manuscript was prepared for publication, a new paper on this topic [26] appeared which belongs to the former class.

2 The $R + R^2$ model

Among different models of inflation [28–30], the $R + R^2$ model introduced in [14] is one of the most appealing from both theoretical and observational perspectives. It has the least number (one) of free parameters fixed by observations only. The action of this model is closely related to vacuum quantum corrections [2, 31] (see also [32, 33] and [18] for the recent advances in this direction) and, on the other hand, its predictions are consistent with recent bounds including the ones set by the Planck collaboration [34–36].

The model is described by the Einstein-Hilbert action with an extra term proportional to the square of the Ricci scalar R ,

$$S_0 = \frac{M_P^2}{2} \int d^4x \sqrt{-g} (R + \alpha R^2), \quad (2.1)$$

where M_P is the reduced Planck mass, $\alpha = (6M^2)^{-1}$ where M is the low-curvature ($|R| \ll M^2$) value of the rest mass of the scalar degree of freedom (dubbed scalaron in [14]) appearing in $f(R)$ gravity, and we put $\hbar = c = 1$. The theory (2.1) can be easily mapped into a metric-scalar model (see, e.g., [37] and [38] where the procedure is described for the general $f(R)$ extension)

$$S_0^* = \frac{M_P^2}{2} \int d^4x \sqrt{-g} [\phi_0 R - U_0(\phi_0)], \quad (2.2)$$

where the scalar field ϕ_0 is related to the Ricci scalar by the relation

$$\phi_0 = 1 + 2\alpha R, \quad (2.3)$$

and the potential function $U_0(\phi_0)$ is

$$U_0(\phi_0) = \frac{1}{4\alpha} (1 - \phi_0)^2. \quad (2.4)$$

It proves useful to make a conformal transformation, introducing a new scalar field χ_0 ,

$$\bar{g}_{\mu\nu} = g_{\mu\nu} \exp \left\{ \sqrt{\frac{2}{3}} \frac{\chi_0}{M_P} \right\}. \quad (2.5)$$

The action which results from this procedure has a standard kinetic term, and reads

$$S_0^* = \int d^4x \sqrt{-\bar{g}} \left[\frac{M_P^2}{2} \bar{R} - \frac{1}{2} (\bar{\nabla} \chi_0)^2 - V(\chi_0) \right], \quad (2.6)$$

where $(\bar{\nabla} \chi_0)^2 = \bar{g}^{\mu\nu} (\bar{\nabla}_\mu \chi_0) (\bar{\nabla}_\nu \chi_0)$ and $V(\chi_0)$ is a potential given by the expression

$$V(\chi_0) = \frac{M_P^2}{8\alpha} \left(1 - e^{-\frac{2}{\sqrt{6}} \frac{\chi_0}{M_P}} \right)^2, \quad (2.7)$$

which drives the evolution of the scalar field χ_0 (scalaron) and satisfies the slow roll conditions in the large field regime.

3 Treating $R\Box R$ term as a small perturbation to the $R + R^2$ model

Let us now consider the modification of the scheme described above when introducing an extra term $R\Box R$ treated as a perturbation. We shall start from a brief review of the previously known way of dealing with this term, while technical details can be found in the appendix. It is expected that the comparison of the two approaches will make their differences clear.

The new action is

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} [R + \alpha R^2 + \gamma R\Box R], \quad (3.1)$$

where the parameters α and γ have dimensions of $[\text{mass}]^{-2}$ and $[\text{mass}]^{-4}$, respectively. The new term is the simplest one leading to a ghost, as described in [39], and therefore it is interesting to see how it can be treated as a small perturbation, while the ghost problem is avoided.

The action (3.1) can be written in terms of two scalar fields [19, 22] (see appendix A for details):

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} [\bar{R} - 6(\bar{\nabla}\varphi)^2 - \gamma e^{-2\varphi}(\bar{\nabla}\phi_1)^2 - U(\phi_1, \varphi)], \quad (3.2)$$

where the potential is defined as

$$U(\phi_1, \varphi) = e^{-4\varphi} [\phi_1(e^{2\varphi} - 1) - \alpha\phi_1^2]. \quad (3.3)$$

From eq. (A.1), it can be seen that in this case the fields φ and ϕ_1 are related with the Ricci scalar R and $\Box R$ by

$$\begin{aligned} \varphi &= \frac{1}{2} \ln(1 + 2\alpha R + 2\gamma\Box R), \\ \phi_1 &= R, \end{aligned} \quad (3.4)$$

that is in complete agreement with the results presented in [19]. From equations (3.2) and (3.3), it is clear that when $\gamma = 0$ we recover the $R + \alpha R^2$ case [14] in the Einstein frame. In appendix A we present the derivations of the Einstein equation in the weak field approximation up to the second order in the fields that reproduces the results of [19]. Also, the Ansatz $\Box R = r_1 R + r_2$ proposed by [24], is used to verify that the results are consistent with the ones presented in [25]. Indeed, the action (3.2), which follows from the standard approach, has a non-standard kinetic term, of the type that were analyzed in several works [40–43], always in the slow roll approximation.

Nevertheless, this kind of analysis is not completely free of problems, because equations following from the action (3.2) does not satisfy the slow roll conditions generically, as discussed in [44]. In this situation, definition of new slow roll parameters and alternative treatments have been proposed, for instance, in [45–47].

However, the problem is that the “standard” treatment of the situation in cosmology which was done in the references mentioned above is opposite to the one which is usually considered “standard” in dealing with higher derivative theories [13]. Thus, we will close this gap and consider the last term in eq. (3.1) as a perturbation. The main point is that then

we cannot use the standard scheme of mapping to the metric-scalar models (see, e.g., [37]). Instead, we have to follow the treatment of the new term as a perturbation that means that the number of degrees of freedom is not increased in contrast to the action eq. (3.2).

Treating the $R\Box R$ -term as a perturbation, one can suppose that in mapping to a scalar-metric model, the term $R\Box R$ should be substituted by $R(\phi_0)\Box R(\phi_0)$, where ϕ_0 is a scalar field, similar to ϕ_0 in eq. (2.2). The action is perturbed by the inclusion of the γ term,

$$S = S_0^* + S_\gamma, \quad (3.5)$$

where S_γ is defined as

$$\begin{aligned} S_\gamma &= \frac{M_P^2 \gamma}{2} \int d^4x \sqrt{-g} R \Box R \Big|_{R=\frac{(\phi_0-1)}{2\alpha}} \\ &= \frac{M_P^2 \gamma}{8\alpha^2} \int d^4x \sqrt{-g} \phi_0 \Box \phi_0. \end{aligned} \quad (3.6)$$

Let us note that the relation between R and ϕ_0 in this formula is exactly the same as the obtained for the unperturbed $R + R^2$ model given by equation (2.3) without any changes. This procedure means that we disregard all possible terms of higher orders in γ .

In order to obtain the scalar mapping under this approximation, we use the relation between the $\sigma = e^{2\varphi}$ and ϕ_1 fields given by the equation of motion (A.7), taken up to linear order in the $\frac{\gamma}{2\alpha}\Box$ operator. This leads to

$$\phi_1 = \frac{1}{2\alpha}(e^{2\varphi} - 1) - \frac{\gamma}{2\alpha^2}\Box e^{2\varphi}. \quad (3.7)$$

When eq. (3.7) is substituted back into the action, this gives us

$$\begin{aligned} S &= \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[\bar{R} - 6(\bar{\nabla}\varphi)^2 (1 + k e^{2\varphi}) - U(\varphi) \right], \\ \text{where } k &= \frac{\gamma}{6\alpha^2} \quad \text{and} \quad U(\varphi) = \frac{e^{-4\varphi}}{4\alpha} (e^{2\varphi} - 1)^2 \end{aligned} \quad (3.8)$$

which has exactly the same form as the one in eq. (A.14) in the appendix. Nevertheless, in this case the kinetic term has a non-canonical form. Thus, to find the mass of the field in the Minkowski limit, it has to be transformed into the standard form. It is important to emphasize that all the analysis made in this work are concerned with the case $|k| \ll 1$, in order to allow the treatment of the $\Box R$ term as a small perturbation to the $R + R^2$ theory. It is easy to see that when the Ansatz (A.10) is used alongside with the previous approximation, we recover the results of [24, 25]. In order to check this, we write the action (3.8) in the equivalent way,

$$\begin{aligned} S &= \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[\bar{R} - 6(\bar{\nabla}\varphi)^2 + 3k(\sigma - 1)\Box\varphi - U(\varphi) \right], \\ \text{where } \sigma &= e^{2\varphi}, \end{aligned} \quad (3.9)$$

as defined above. Now, we can use eq. (3.7) to find the relation

$$\left(1 - \frac{\gamma}{\alpha}\Box\right)^{-1} \left[r_1 + \Box\sigma - \frac{\gamma}{\alpha}\Box(\Box\sigma) \right] = r_1\sigma. \quad (3.10)$$

When expanded up to the linear order in the operator $\frac{\gamma}{\alpha}\square$ we arrive at the relation $\square\sigma = r_1(\sigma - 1)$.

With these considerations, the action (3.9) becomes

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} [\bar{R} - 6(\bar{\nabla}\varphi)^2 - \mathcal{U}(\varphi)], \quad (3.11)$$

where the potential is defined as

$$\mathcal{U}(\varphi) = \frac{\alpha - r_1\gamma}{4\alpha^2} e^{-4\varphi} (1 - e^{2\varphi})^2. \quad (3.12)$$

Comparing the last expression with the potential corresponding to the nonperturbed Starobinsky model in the Einstein frame given by eq. (2.7), one can see that the values of the spectral index n_s and the tensor-to-scalar ratio r will not be modified that is consistent with the results for these quantities in [25] which are based on the use of the specific Ansatz (A.10).

Let us stress that although the term $\gamma R\square R$ studied here is a particular case of the $F(\square)$ analyzed in previous works, in our case the simplifying Ansatz (A.10) is not an additional general requirement but only a special case. Thus, our study in this paper is not a particular case of that in [24, 25], but represents a qualitatively new approach to the $R\square R$ term in the action.

The bilinear part of the action (3.9) can be cast into the form

$$S = \int d^4x \sqrt{-\bar{g}} \left\{ \frac{M_P^2}{2} \bar{R} - \frac{1}{2} (\bar{\nabla}\chi)^2 - \frac{1}{2} M_\chi^2 \chi^2 \right\}, \quad (3.13)$$

where fields χ and φ are related by the relation

$$\varphi = \frac{\chi}{\sqrt{6(1 + \gamma/6\alpha^2)} M_P}. \quad (3.14)$$

Let $k \equiv \frac{\gamma}{6\alpha^2}$. If $|k| \ll 1$, the mass of the field χ is

$$M_\chi^2 \approx M^2 - \frac{\gamma}{36\alpha^3}, \quad (3.15)$$

where M^2 (defined below eq. (2.1)) is the scalaron mass in the Starobinsky model. From eq. (3.15) one can see that the mass of the scalar χ can be greater or smaller than M depending on the sign of the parameter γ . This behavior is preserved by small values of $|k| \ll 1$ in the general case of (3.8), as it will be seen from the estimated values obtained using the Planck results [36].

Returning to the general expression for the action (3.8), the field transformation that turns the kinetic term in a canonical form should satisfy

$$\left(\frac{d\chi}{d\varphi} \right)^2 = 6M_P^2 (1 + k e^{2\varphi}), \quad (3.16)$$

and then the action becomes

$$S = \int d^4x \sqrt{-\bar{g}} \left\{ \frac{M_P^2}{2} \bar{R} - \frac{1}{2} (\bar{\nabla}\chi)^2 - V(\varphi(\chi)) \right\}. \quad (3.17)$$

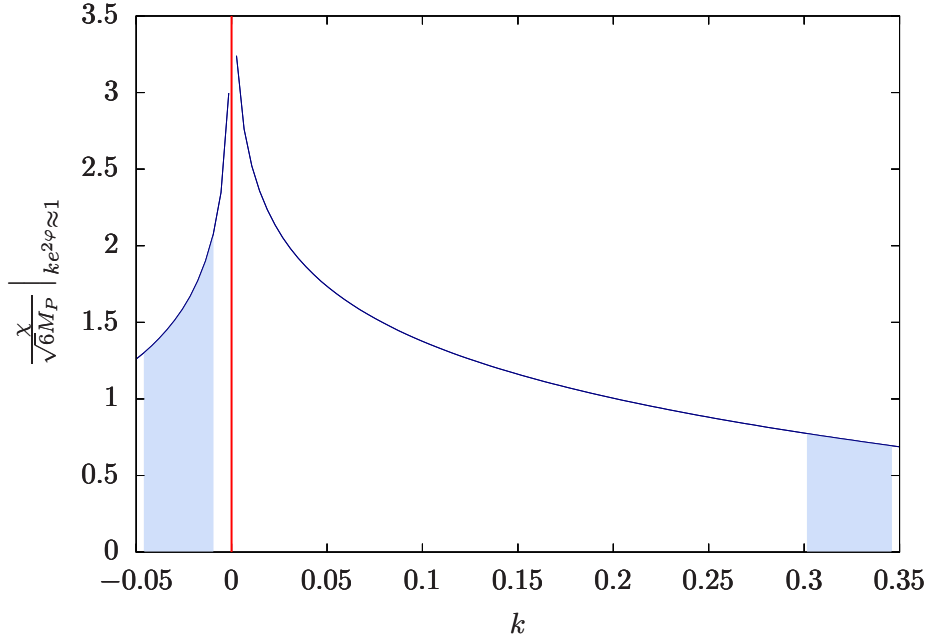


Figure 1. The energy scale of the new scalaron χ as a function of the perturbative parameter k . The shaded regions are excluded by the constraints imposed by Planck data [36], as will be shown later.

In the Einstein frame, the potential is given by

$$V(\chi) = V(\varphi(\chi)) = \frac{M_P^2}{8\alpha} \left(1 - e^{-2\varphi(\chi)}\right)^2, \quad (3.18)$$

where the dependence of the intermediate field φ on the scalar field χ in eq. (3.18) (which may be called new scalaron) can be obtained by solving the transcendental equation that follows from (3.16),

$$\frac{\chi}{\sqrt{6}M_P} = \ln \left[\frac{1 - \sqrt{1 + ke^{2\varphi}}}{e^{\varphi}(1 - \sqrt{1 + k})} \right] + \sqrt{1 + ke^{2\varphi}} - \sqrt{1 + k}. \quad (3.19)$$

In the last expression for the particular case $ke^{2\varphi} \approx 1$, and taking into account that $|k| \ll 1$, but $k \neq 0$ we have

$$\frac{\chi}{\sqrt{6}M_P} \Big|_{ke^{2\varphi} \approx 1} \approx -\frac{1}{2} \ln |k| - \frac{k}{4} + \frac{1}{4} \text{Sign}(k), \quad (3.20)$$

which shows that a large field inflation can be developed when $|k| \ll 1$, as can be seen from figure 1, where we have plotted eq. (3.20) which shows the behavior of the new scalaron χ in the particular case $ke^{2\varphi} \approx 1$, where the shaded regions are excluded by the Planck data, as will be discussed below.

Let us note that the condition $ke^{2\varphi} \approx 1$ has been chosen since it corresponds to the maximal value of the parameter k which is compatible with our approximation $|k| \ll 1$. For smaller values of $|k|$ the effect of the $R \Box R$ term will be less significant.

Let us note that the general expression (3.19) can be used as a basis of metric-scalar cosmological model even for the values of k which do not satisfy the condition $|k| \ll 1$.

However in this case there is no direct link with the main perturbative in $R \Box R$ approach for the $R + R^2 + R \Box R$ theory, which we aim to develop in this work.

In figure 2, the plot of the potential $V(\chi)$ is shown for different values of the parameter k , with $k = 0$ corresponding to the $R + R^2$ model, and the other values to the extra $R \Box R$ term with different coefficients. One can observe that the presence of the $R \Box R$ term changes the shape of the potential including its flatness. For $k > 0$, the slow-roll inflationary regime ends for slightly larger values of the scalar field, implying that for larger values of k , inflation happens at higher energy scales than for the standard $R + R^2$ model.

For positive k , all expanding spatially-flat FLRW universes evolve to the dust-like one filled by massive scalarons at rest at late times, like in the case of the Starobinsky model. As we will see later, at earlier times, they can develop inflation in the slow roll regime. For the case of negative values of k (remember $|k| \ll 1$), there is a maximum in the potential for a critical value of the χ field, given by the expression

$$\frac{\chi_{\max}}{\sqrt{6}M_P} = \ln \left(\frac{\sqrt{|k|}}{1 - \sqrt{1 - |k|}} \right) - \sqrt{1 - |k|}, \quad (3.21)$$

which comes from the constraint of a real χ field in eq. (3.19). In this case, slow roll inflation can take place only for values of k which are close to zero, as will be seen from the behavior of the slow roll parameters ϵ and η . In fact, this is not too relevant, since all our analysis is valid only for small values of the parameter $|k|$.

The nonzero k or, equivalently, the γ term, modifies the value of χ in which the last 60 e -folds of inflation begin, leading to a modification of observable parameters such as the tilt of the primordial power spectrum of scalar (adiabatic) metric perturbations n_s and the scalar-to-tensor ratio r , as we will see in below. Furthermore, the $R \Box R$ -type perturbation modifies the symmetry of the scalar potential near its minimum that can affect oscillations of the field χ (new scalaron) after inflation and gravitational creation of particles and antiparticles by these oscillations through parametric resonance [14, 48–50]. The next important question is whether a non-zero γ modifies the conditions of a slow roll inflation.

3.1 Slow-roll conditions

As far as the model with non-zero γ -term is mapped into a single scalar field action, the analysis of the slow roll conditions can be performed in a standard way.

In order to let inflation last for a sufficient amount of time, the time derivative of the Hubble parameter H has to be sufficiently small. As a result, the slow roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \epsilon - \frac{\dot{\epsilon}}{2\epsilon H} = -\frac{\ddot{\chi}}{H\dot{\chi}}, \quad (3.22)$$

have to be much smaller than unity by modulus, leading to a negligible contribution of the kinetic energy of the field during inflation. Using the Friedmann equations, one can express the slow roll parameters in terms of the potential as

$$\epsilon = \frac{M_P^2}{2} \left[\frac{V'(\chi)}{V(\chi)} \right]^2 \quad \text{and} \quad \eta = M_P^2 \frac{V''(\chi)}{V(\chi)}. \quad (3.23)$$

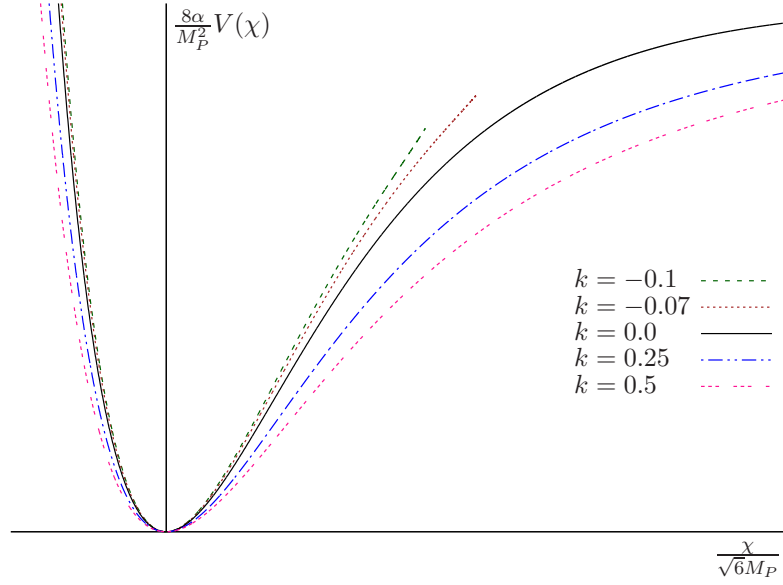


Figure 2. $R \square R$ correction to the Starobinsky model of inflation for different values of the parameter k . The black continuous line corresponds to the original $R + \alpha R^2$ model, and the colored dashed lines to nonzero values of k .

Finally, using eq. (3.19), the two parameters can be written in terms of the field φ ,

$$\epsilon = \frac{4}{3} \frac{1}{(1 + ke^{2\varphi})(1 - e^{2\varphi})^2}, \quad (3.24)$$

$$\eta = -\frac{4}{3} \left[\frac{e^{-2\varphi}(1 - 2e^{-2\varphi}) + \frac{k}{2}(3 - 5e^{-2\varphi})}{(1 + ke^{2\varphi})^2(1 - e^{-2\varphi})^2} \right]. \quad (3.25)$$

In the limit $k = 0$ the slow roll parameters of the $R + \alpha R^2$ model are recovered. For small values of $k > 0$, one can see that the slow roll conditions are satisfied for large enough fields, while for $k < 0$ there is a maximum value of the χ field where the slow roll regime is valid: when $|k| \ll 1$. For the values $k < 0$ we have slow roll for a wide range of the field, as shown in figure 3.

The number of inflationary e -folds N in the Einstein frame is given by

$$N(\chi) = \frac{1}{M_P^2} \int_{\chi_e}^{\chi} \frac{V(\chi)}{V'(\chi)} d\chi, \quad (3.26)$$

where χ_e corresponds to the end of inflation. In terms of the field φ we get

$$N(\varphi) = \frac{3}{4} \left[-2\varphi + (1 - k)e^{2\varphi} + \frac{k}{2}e^{4\varphi} \right] \Big|_{\varphi_e}^{\varphi}. \quad (3.27)$$

The standard results for the $R + \alpha R^2$ model are recovered for $k = 0$. Assuming that $\chi_e \ll \chi$ (that is equivalent to $\varphi_e \ll \varphi$ in the case of large field inflation), one can neglect the linear terms in φ in eq. (3.27), which boils down to

$$N(\varphi) \approx \frac{3}{4} \left[(1 - k)e^{2\varphi} + \frac{k}{2}e^{4\varphi} \right]. \quad (3.28)$$

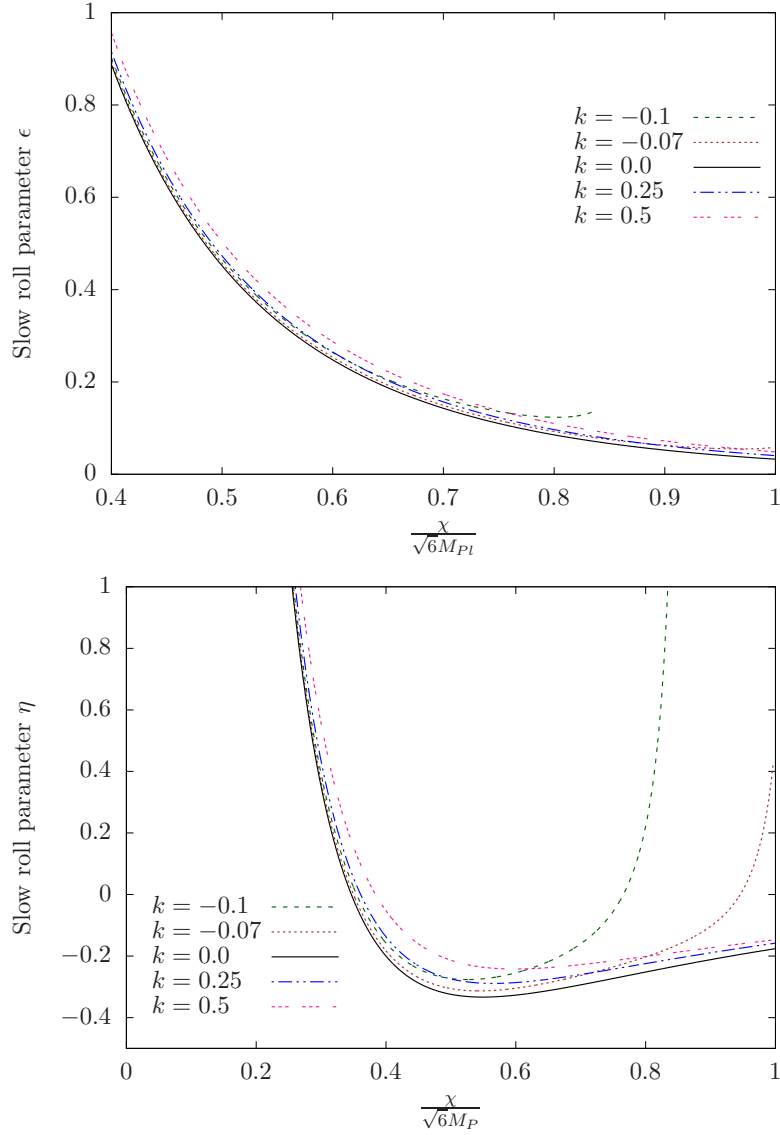


Figure 3. The slow roll parameters ϵ (up) and η (down) for the model with $R \square R$ term as functions of the inflaton field χ .

One can use this relation to derive the value φ_N of the field φ , corresponding to the instant when the universe expanded by N e -folds,

$$\varphi_N \approx \frac{1}{2} \ln \left[1 - \frac{1}{k} \pm \sqrt{\left(1 - \frac{1}{k}\right)^2 + \frac{8N}{3k}} \right], \quad (3.29)$$

where the positive sign has to be taken in order to recover the results for the $R + R^2$ model. Taking into account that $|k| \ll 1$ we can simplify this expression as

$$\varphi_N \approx \frac{1}{2} \ln \left[-\frac{1}{k} + \sqrt{\frac{1}{k^2} + \frac{8N}{3k}} \right]. \quad (3.30)$$

When the slow roll conditions are satisfied, the amplitude of scalar (curvature) and tensor perturbations can be written in terms of the potential $V(\chi)$ and its derivatives at the moment when their physical wavelength $\lambda = p/a(t)$, $p = \text{const}$ crosses the Hubble radius $H^{-1}(t)$ during inflation. Here $a(t)$ is the scale factor of an isotropic universe and $H(t) = \dot{a}(t)/a(t)$. The same matching condition helps to express N as a function of the present physical scale $\lambda = a(t_0)/p$ where t_0 is the present moment. Then one gets the standard expressions for the spectral index $n_s(p)$ of the power spectrum of primordial curvature perturbations and the tensor-to-scalar ratio $r(p)$ in the leading order of the slow-roll approximation:

$$n_s - 1 = -6\epsilon + 2\eta = M_P^2 \left(2 \frac{V''}{V} - 3 \left(\frac{V'}{V} \right)^2 \right), \quad (3.31)$$

$$r = 16\epsilon = -8M_P^2 \left(\frac{V'}{V} \right)^2. \quad (3.32)$$

Because of the conformal transformation between the Jordan and Einstein frames, the same value of a perturbation as a function of N and, finally, p corresponds to somewhat different physical scales in the Jordan and the Einstein frames. Since the standards of length and time intervals are defined in Jordan frame which can be considered as the physical one from the measurement point of view, the number of e-folds in the Jordan frame N_J is more directly related to observations. However, difference between N and N_J is small, of the order of the next correction to the slow-roll approximation (less than a few percent for the model in question), so we may neglect it in the leading order.

Using equations (3.24) and (3.25), we can obtain the following analytical expressions for n_s and r as functions of k and the number of e-folds N :

$$\begin{aligned} n_s - 1 &= \frac{9k [18k^3 + 6k^2(Sq_{k,N} + 12N - 29) + 8N(9 - 2Sq_{k,N}) + 40Sq_{k,N} - 138]}{(Sq_{k,N} - 3)^2(Sq_{k,N} + 3k)^2} \\ &\quad + \frac{9k^2 [16N(Sq_{k,N} + 4N - 18) - 52(Sq_{k,N} + 261)] - 54Sq_{k,N} + 18}{(Sq_{k,N} - 3)^2(Sq_{k,N} + 3k)^2}, \\ r &= \frac{576k^2}{(Sq_{k,N} - 3)^2(Sq_{k,N} + 3k)^2}, \end{aligned} \quad (3.33)$$

where we have defined $Sq_{k,N} = \sqrt{24kN + 9(k-1)^2}$.

From equations (3.33), we can realize that when $k = 0$, we recover the predictions of the $R + R^2$ model [16, 51],

$$n_s - 1 \approx -\frac{2}{N} \quad r \approx \frac{12}{N^2}, \quad (3.34)$$

and up to the order kN there is no shift in $n_s - 1$ and r , and their corrections are of order k/N or k/N^2 , respectively.

The comparison of this inflationary model with the observational constraints set by the Planck collaboration [35, 36] is illustrated in figure 4. The last Planck data constrain these quantities as

$$n_s = 0.9649 \pm 0.0042, \quad r < 0.10. \quad (3.35)$$

In figure 4 we show the Planck constraints on the values of n_s and r , and the prediction for this quantities in the $R + \alpha R^2 + \gamma R \square R$ model. This figure shows the 68% (dark blue and dark yellow) and 95% (light blue and light yellow) CL regions for the measurements of r and

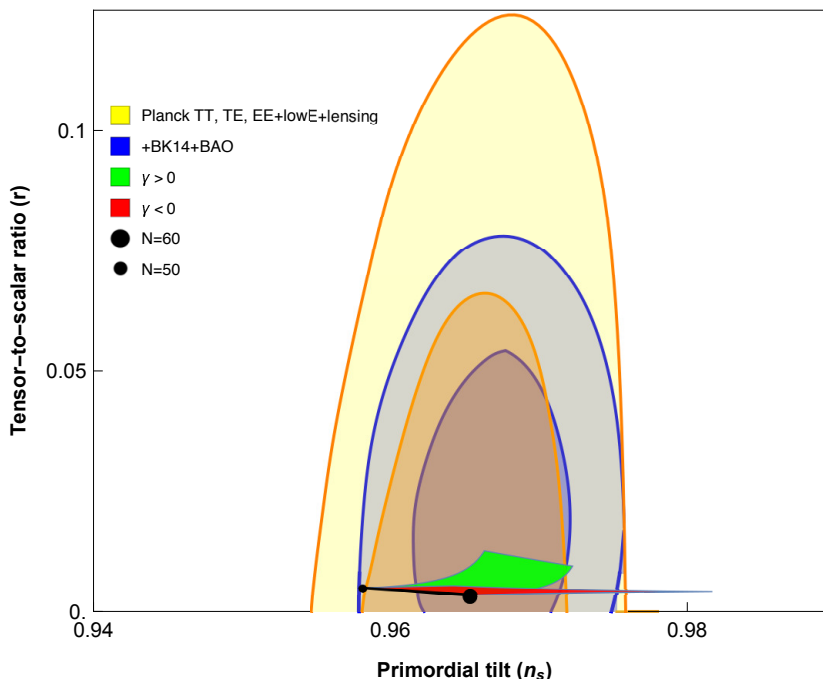


Figure 4. Observational constraints set by the Planck collaboration [35, 36] on the scalar-to-tensor ratio r and scalar spectral index n_s , and the prediction for these quantities in the $R + R^2 + R^3$ model.

n_s , taking the combined data as stated in the plot key, and the variation of the $R + \alpha R^2$ model (black line) due to the inclusion of the γ term regarded as a small correction for $k > 0$ and $k < 0$ in green and red, respectively.

Using a numerical routine, from figure 4 we have found the maximum positive value of k in order to keep the predictions of the R^3 model inside the 68% CL region, concluding that, for $N = 50$ and $N = 55$ e-folds, any value of k satisfying the perturbative condition $|k| \ll 1$ keeps the predictions of the R^3 model inside the 68% CL region. For the case $N = 60$ the maximum value of k that satisfies this condition is $k_{\text{max}} \approx 0.30$.

On the other hand, for negative values of k , Planck results constraints the minimum value of k to -0.0060 , -0.0052 and -0.0045 , when $N = 50$, 55 and 60 , respectively.

Furthermore, figure 5 shows the Planck constraints [35, 52] for the relation between the slow roll parameters (3.23), at the 68% (dark blue) and 95% CL (light blue). Also in this figure one can see the prediction of our present model (light green).

Finally, one can check how the upper bound for the energy scale M may change in the case of the $R + R^2 + R^3$ model. Using a procedure similar to the one presented in [17], we use the expression (3.18) for the inflationary potential and eq. (3.24) for the first slow roll parameter, written in terms of the number of e -folds N , from which we get the curvature perturbations

$$\Delta_{\mathcal{R}}^2 = \frac{1}{M_P^2} \sqrt{\frac{8V}{3\epsilon}}, \quad (3.36)$$

and make a comparison with the last Planck results [36]. For the case of 60 e -folds and using the value of k which assures that the correction due to the R^3 term lies inside the 68% CL

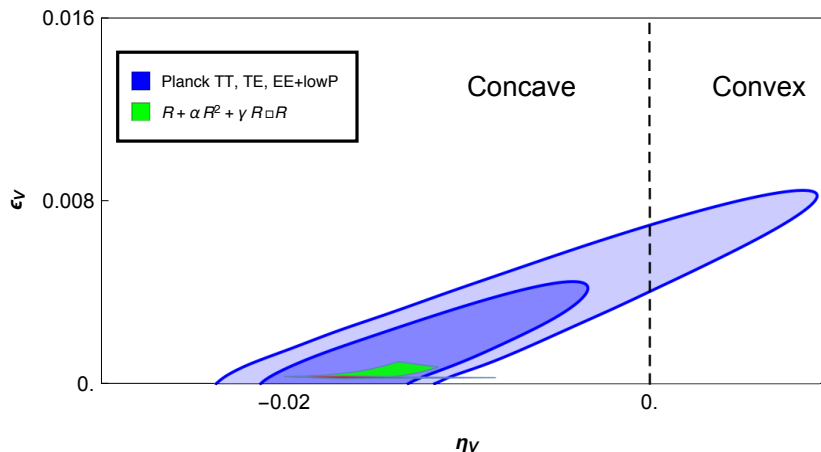


Figure 5. Observational constraints on the slow roll parameters set by Planck collaboration [52], and the prediction for these quantities in the $R + \alpha R^2 + \gamma R \square R$.

regions of both figures 4 and 5 (hence $k \approx 0.30$), the maximum value of M would be

$$M_{\max} = (2.21 \pm 0.01) \times 10^{-5} M_P, \quad (3.37)$$

which is consistent with the result of [16].

4 Conclusions

We consider the extension of the $R + R^2$ inflationary model by adding a small perturbation of the form $R \square R$. Treating terms with higher than four derivatives as small perturbations has been suggested as a general approach to deal with higher derivative terms in quantum gravity, and following this approach we included, for the first time, the R^2 term into the main, non-perturbative part of the action. We have shown that there is no kN relative correction to the main terms in $n_s - 1$ and r when the $R \square R$ term is treated as a small perturbation to the Starobinsky model. First of all, this means that the presence of this term does not increase the amount of degrees of freedom, and one has to perform the mapping of the modified gravity theory to the scalar-tensor model with only one effective scalar field. This approach opens the way for a simple and explicit analysis of observational constraints on extra terms. The case which we considered here provides an especially simple mapping procedure, but there is a good chance that the same result can be achieved for other higher derivative extensions of general relativity.

A Action in terms of scalar fields

Let us briefly review the standard treatment of the model under discussion, which implies using two scalar fields. For this end we write the action (3.1) as [22] (see also [37])

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} [F(\phi_1, \phi_2) + F_1(R - \phi_1) + F_2(\square R - \phi_2)], \quad (\text{A.1})$$

where $F(\phi_1, \phi_2) = \phi_1 + \alpha\phi_1^2 + \gamma\phi_1\phi_2$, $F_1 = \frac{\partial F}{\partial \phi_1}$, and $F_2 = \frac{\partial F}{\partial \phi_2}$, in such a way that the action takes the form

$$\begin{aligned} S &= \frac{M_P^2}{2} \int d^4x \sqrt{-g} [\phi_1 + \alpha\phi_1^2 + \gamma\phi_1\phi_2 + (1 + 2\alpha\phi_1 + \gamma\phi_2)(R - \phi_1) + \gamma\phi_1(\Box R - \phi_2)] \\ &= \frac{M_P^2}{2} \int d^4x \sqrt{-g} [(1 + 2\alpha\phi_1 + \gamma\phi_2)R - \alpha\phi_1^2 + \gamma\phi_1\Box R - \gamma\phi_1\phi_2]. \end{aligned} \quad (\text{A.2})$$

In order to eliminate the term with $\Box R$, we integrate by parts [53],

$$\int d^4x \sqrt{-g} \phi_1 \Box R = - \int d^4x \sqrt{-g} \nabla^\mu \phi_1 \nabla_\mu R = \int d^4x \sqrt{-g} R \Box \phi_1, \quad (\text{A.3})$$

and arrive at the following expression for the action

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} [(1 + 2\alpha\phi_1 + \gamma\phi_2 + \gamma\Box\phi_1)R - \alpha\phi_1^2 - \gamma\phi_1\phi_2]. \quad (\text{A.4})$$

Defining $\sigma = 1 + 2\alpha\phi_1 + \gamma\phi_2 + \gamma\Box\phi_1$, the action can be written in terms of two scalar fields in the form

$$\begin{aligned} S &= \frac{M_P^2}{2} \int d^4x \sqrt{-g} [\sigma R + \gamma\phi_1\Box\phi_1 - U(\phi_1, \sigma)], \\ \text{where } U(\phi_1, \sigma) &= \phi_1(\sigma - 1) - \alpha\phi_1^2. \end{aligned} \quad (\text{A.5})$$

From eq. (A.5) one can see that setting $\gamma = 0$, we recover the $R + \alpha R^2$ case [14].

In order to analyze the theory with two scalar degrees of freedom, we have to perform the conformal transformation of the metric, $\bar{g}_{\mu\nu} = e^{2\varphi} g_{\mu\nu}$ [54, 55],

$$\begin{aligned} S &= \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} e^{-4\varphi} \{ \sigma e^{2\varphi} [\bar{R} - 6(\bar{\nabla}\varphi)^2 - 6\bar{\Box}\varphi] \\ &\quad + \gamma\phi_1 e^{2\varphi} (\bar{\Box}\phi_1 - 2\bar{\nabla}^\mu \varphi \bar{\nabla}_\mu \phi_1) - U(\phi_1, \sigma) \}. \end{aligned} \quad (\text{A.6})$$

In this case, taking $\sigma = e^{2\varphi}$, it is straightforward to obtain the action in the Einstein frame (3.2).

One can show that the fields ϕ_1 and φ satisfy the equations of motion

$$\begin{aligned} \bar{\Box}\phi_1 - 2\bar{\nabla}^\mu \varphi \bar{\nabla}_\mu \phi_1 - \frac{e^{-2\varphi}}{2\gamma} [e^{2\varphi} - 1 - 2\alpha\phi_1] &= 0, \\ \bar{\Box}\phi_1 + \frac{\alpha}{\gamma}\phi_1 &= -\frac{1}{2\gamma}(1 - e^{2\varphi}), \end{aligned} \quad (\text{A.7})$$

which are consistent with the relation between the original fields $\Box\phi_1 = \phi_2$ and reproduces the results of [19].

A.1 Weak field approximation

The Einstein tensor can be found from equation (3.2), taking the variation with respect to $g^{\mu\nu}$,

$$\begin{aligned} \bar{G}_{\mu\nu} &= \gamma e^{-2\varphi} \left(\bar{\nabla}^\mu \phi_1 \bar{\nabla}_\nu \phi_1 - \frac{1}{2} \bar{g}_{\mu\nu} \bar{\nabla}^\lambda \phi_1 \bar{\nabla}_\lambda \phi_1 \right) + 6 \left(\bar{\nabla}_\mu \varphi \bar{\nabla}_\nu \varphi - \frac{1}{2} \bar{g}_{\mu\nu} \bar{\nabla}^\lambda \varphi \bar{\nabla}_\lambda \varphi \right) \\ &\quad + \frac{1}{2} \bar{g}_{\mu\nu} e^{-4\varphi} [\alpha\phi_1^2 + (1 - e^{2\varphi})\phi_1]. \end{aligned} \quad (\text{A.8})$$

The last expression is consistent with the Einstein tensor obtained for the first time in [19], where the weak field approximation was worked out, showing that the action in this case is given by

$$S_{wf} \approx \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} \left[\bar{R} - 6\bar{\nabla}^\lambda \varphi \bar{\nabla}_\lambda \varphi - \gamma \bar{\nabla}^\lambda \phi_1 \bar{\nabla}_\lambda \phi_1 - 2\phi_1 \varphi + \alpha \phi_1^2 \right]. \quad (\text{A.9})$$

The mixed term involving ϕ_1 and φ can be removed by performing a rotation of these fields, leading to the identification of scalar fields that can be tachyonic or physical, depending on the α and γ parameters [20]. In order to have a stable Minkowski space it is necessary that $\gamma < 0$, within this approximation. Let us note that this condition does not apply within our approach to the problem.

A.2 Simplifying Ansatz $\square R = r_1 R + r_2$

In this section we consider the simplifying Ansatz $\square R = r_1 R + r_2$ proposed in [24], and later on used in [25] to analyze non-local modifications of gravity with general form factors depending on the D’Alambertian operator \square applied to the Riemann, Ricci tensors and to the Ricci scalar.

Our main goal is to show that the application of this simplifying Ansatz to the action (3.2), under certain conditions for the parameters α and γ , reproduces the results obtained in [24, 25] for this particular case.

As far as we are not considering the cosmological constant term in the action, the r_2 contribution vanishes, and the ansatz becomes

$$\square \phi_1 = r_1 \phi_1. \quad (\text{A.10})$$

The action given by (3.2), can be written as

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} \left[\bar{R} - 6(\bar{\nabla} \varphi)^2 + \gamma \phi_1 e^{-2\varphi} (\square \phi_1 - 2\bar{\nabla}^\mu \varphi \bar{\nabla}_\mu \phi_1) - U(\phi_1, \varphi) \right], \quad (\text{A.11})$$

or equivalently, using eq. (A.10), as

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} \left[\bar{R} - 6(\bar{\nabla} \varphi)^2 + \gamma r_1 \phi_1^2 e^{-4\varphi} - U(\phi_1, \varphi) \right]. \quad (\text{A.12})$$

Finally, using the relation between the fields $\sigma = e^{2\varphi}$ and ϕ_1 , we arrive at the one-scalar representation

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}} \left[\bar{R} - 6(\bar{\nabla} \varphi)^2 - U(\varphi) \right], \quad (\text{A.13})$$

with the potential

$$V(\varphi) = \frac{U(\varphi)}{2} = \frac{1}{8(\alpha + \gamma r_1)} (1 - e^{-2\varphi})^2. \quad (\text{A.14})$$

From (2.7) and (A.14), it is clear that assuming that the scalar field φ evolves within the slow roll approximation, the scalar spectral index n_s and the tensor-to-scalar ratio r , which depend on the potential, are exactly the same as obtained in R^2 inflation, as was previously shown by [24] for the model $R + R^n$, and after them, in the case of a non-local framework. There is no change in the tensor-to-scalar ratio r , because the Weyl term C^2 is excluded and the non-local operator $F_C(\square)$ is absent [25] in our analysis. It is important

to remark that, as was shown in refs. [24, 25], any solution of the $R + R^2$ theory satisfying the ansatz (A.10), is also a solution of the $R + R^2 + R\Box R$ theory, but this is not the whole solution to the theory, as there could be solutions to $R + R^2 + R\Box R$ which do not satisfy the simplifying ansatz (A.10). On the other hand, if we assume that inflation is just as in the $R + R^2$ model, we can take the $R\Box R$ term as a small perturbation, in the same spirit in which ref. [13] analyzes higher order terms as corrections to the Einstein-Hilbert action, leading to the determination of new solutions of the corrected theory. In our case, these solutions are independent of the simplifying ansatz (A.10), but give the same results when this assumption is taken as a particular case.

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